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Rough representations of rough topological groups

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Abstract

In this paper, the concept of rough representation of a rough topological group on a Banach space is explored. Mainly, the continuity and the irreducibility of rough representations are studied.

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1. Introduction

The rough set theory was introduced by Pawlak in 1982 [14]. The original definition of rough set was slightly modified by Bryniarski in [9]. The rough sets theory is related to theories which deal with analysis of intelligent systems under imperfect knowledge and incomplete information such as fuzzy sets theory, bayesian inference, etc.

The theory of rough sets has been deepened in many directions. For instance, Al-shami et al. in [1, 2] and Mustafa et al. in [11] constructed the supra-topology and the infra-topology spaces by using N_k-neighborhood systems induced from any binary relation; therefore, new rough set models emerged along

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with their lower and upper approximation spaces with medical applications. It appears that neighborhood systems conduct to the improvement of the value of accuracy measure.

The theory of rough sets has many applications in several areas, such as business and finance, chemistry, computer engineering and electrical engineering, including data compression, control, digital image processing, digital signal processing, parallel and distributed computer systems, power systems, sensor fusion, fractal engineering, decision analysis and systems, economics, environmental studies, digital image processing, computer science, medicine, molecular biology, musicology, neurology, robotics, social science; see for instance [3, 4, 5, 13, 15] and references therein.

In [8], Biswas and Nanda introduced the concept of rough group. In [7], Bagirmaz et al. introduced the concept of topological rough group in an approximation space. Altassan et al. in [6] deepened the study of topological rough groups by defining rough subgroup, rough homomorphism and other related topics and studied their major properties.
The main purpose of this paper is to study the representations of rough topological groups on a Banach space. Roughly speeking, a representation of a group is a map that sends an element of the group to a linear transformation of some vector space such that the group properties are preserved. Thus, a representation of a rough group enables to transform any problem to be solved on the rough group to a problem on the vector space that can be solved by tools from linear algebra. Since linear algebra is a well-mastered theory with valid applications, a representation theory of rough group can bring a new perspective and an effective tools to the theory of rough sets.

The organization of the paper is as follows: Section 2 collects basic definitions about rough sets, rough groups and a topological rough groups. In Section 3, we prove our main results on rough representations of rough topological groups.

2. Rough topological groups

This section is devoted to the collection of definitions and results that we may need. Our main reference here is the article [7].

Definition 2.1. Let U be a non-empty set (called the universe). Let R be an equivalence relation on U. The pair (U, R) is called an approximation space.

Let (U, R) be an approximation space. For $x \in U$, the equivalence class of x is denoted by $[x]$. For $X \subset U$, set

$$\underline{X} = \{x \in U \,:\, [x] \subset X\} \text{ and } \overline{X} = \{x \in U \,:\, [x] \cap X \neq \varnothing\}.$$

The sets $\overline{X}$ and $\underline{X}$ are called the *upper approximation* and *lower approximation* of X in (U, R) respectively. We have $\underline{X} \subset X \subset \overline{X}$.

Assume that U is endowed with a binary operation $U \times U \longrightarrow U$. The product of two elements x and y is denoted by xy.

Definition 2.2 ([8])**.** Let (U, R) be an approximation space with a binary operation. A subset G of U is called a rough group if the following properties hold :

(1) $\forall x, y \in G$, $xy \in \overline{G}$,
(2) $\forall x, y, z \in \overline{G}$, $(xy)z = x(yz)$,
(3) $\exists e \in \overline{G}$, $\forall x \in G$, $ex = xe = x$,
 (e is called a rough identity element of G.)
(4) $\forall x \in G, \exists y \in G, xy = yx = e$.
 (y is called the rough inverse of x and is denoted by x^{-1}).

To define a topological rough group [7], one may assume that there is a topology on $\overline{G}$. Thus G is endowed with the induced topology.

Definition 2.3. A topological rough group is a rough group G together with a topology on $\overline{G}$ such that

(1) the map $(x, y) \longmapsto xy$ is continuous from $G \times G$ into $\overline{G}$,
(2) the map $x \longmapsto x^{-1}$ is continuous from G into G.

Example 2.4. Consider the approximation space (Q_8, R) where $Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$ is the quaternion group. Let $*$ be the quaternion multiplication.
Let $Q_8/R = \{\{\pm 1\}, \{\pm i\}, \{\pm j, \pm k\}\}$. Let $G = \{\pm i, -1\}$, then $\overline{G} = \{\pm i, \pm 1\}$ and $\underline{G} = \{\pm i\}$. From Definition 2.2, since the conditions

(1) $\forall x, y \in G, x * y \in \overline{G}$,
(2) Association property holds in G,
(3) $1 * (\pm i) = (\pm i) * 1 = (\pm i)$ and $1 * (-1) = (-1) * 1 = -1$ then 1 is the rough identity element of G,
(4) $(-i)^{-1} = i \in G$ and $(-1)^{-1} = -1 \in G$

hold, then G is rough group.
Let $T = \{\varnothing, \overline{G}, \{-1\}, \{-i\}, \{i\}, \{-1, -i\}, \{-1, i\}, \{-i, i\}, \{-1, -i, i\}\}$ be a topology on $\overline{G}$, then $T_G = \{\varnothing, G, \{-1\}, \{-i\}, \{i\}, \{-1, -i\}, \{-1, i\}, \{-i, i\}\}$ is the relative topology on G.
Since the maps $G \times G \to \overline{G}$, $(x, y) \mapsto x * y$ and $G \to G$, $x \mapsto x^{-1}$ are continuous, then G is a topological rough group according to Definition 2.3.

3. Banach space representations of rough topological groups

In this section, we construct a representation theory of a locally compact topological rough group on a Banach space. We follow the approach in [16] for locally compact groups.
Let G be a rough group and let E be a complex separable Banach space. Denote by $\mathcal{L}(E)$ the space of invertible bounded linear operators on E.

Definition 3.1. A rough representation of G on E is a homomorphism $\sigma : x \longmapsto \sigma(x)$ from $\overline{G}$ into $\mathcal{L}(E)$, that is,

$$\forall x, y \in G,\ \sigma(xy) = \sigma(x)\sigma(y). \tag{3.1}$$

Example 3.2. An example of rough group from [12] and an example of a representation of a finite group from [10] inspired us to provide the following example of rough representation.

Consider U as the symmetric group $S_3 = \{e, (12), (13), (23), (123), (132)\}$ where e is the identity. The binary operation on S_3 is the composition law. Consider the subgroup $H = \{e, (12)\}$ of S_3 and endow S_3 with the equivalence relation $\mathcal{R}$ defined by

$$x\mathcal{R}y \text{ if } x^{-1}y \in H.$$

The left cosets are $eH = \{e, (12)\}$, $(1, 2)H = \{e, (12)\}$, $(13)H = \{(13), (123)\}$, $(23)H = \{(23), (132)\}$, $(123)H = \{(123), (13)\}$ and $(132)H = \{(132), (23)\}$.

Consider $G = \{e, (12), (123), (132)\}$ a subset of S_3. Then

$$\underline{G} = \{e, (12)\} \text{ and } \overline{G} = S_3.$$

One may check that the conditions of Definition 2.2 are satisfied. Therefore G is a rough group.

Let $\{e_i : i = 1, 2, 3\}$ be the canonical basis of $\mathbb{R}^3$. Denote by $\mathcal{M}_3(\mathbb{R})$ the space of 3×3 matrices with real entries. The space $\mathcal{M}_3(\mathbb{R})$ is identified with the space of linear (necessary bounded) operators on $E = \mathbb{R}^3$. Let us consider the map

$$\begin{aligned} \sigma : \overline{G} = S_3 &\longrightarrow \mathcal{M}_3(\mathbb{R}) \\ x &\longmapsto \sigma(x) = (L_x)_{ij} \end{aligned}$$

where $(L_x)_{ij}$ is the matrix associated with the linear map L_x such that $L_x e_i = e_{x(i)}$. The explicit computation shows that $\sigma(e) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\sigma(12) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $\sigma(13) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $\sigma(23) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $\sigma(123) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ and $\sigma(132) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

Finally, it is straightforward to verify that $\sigma(xy) = \sigma(x)\sigma(y)$ for $x, y \in S_3$. Therefore, the map σ is a rough representation of G.

Definition 3.3. A locally compact rough group is a topological rough group G such that $\overline{G}$ is a locally compact space, that is, every point in $\overline{G}$ has a compact neighborhood.

Definition 3.4. Let G be a locally compact topological rough group. A rough representation σ of G on E is said to be continuous if the map $(x, a) \longmapsto \sigma(x)a$ is continuous from $\overline{G} \times E$ into E.

Through the rest of this article, $B(a, r)$ stands for the open ball of centre a and of radius r.

Theorem 3.5. *If a rough representation σ of G on E is continuous then for every $a \in E$, the map $x \longmapsto \sigma(x)a$ is continuous from G into E.*

Proof. Suppose that a rough representation σ of G on E is continuous. Since G is endowed with the induced topology, then a neighborhood of x in G is a set of the form $G \cap U$ where U is a neighborhood of x in $\overline{G}$. Fix $x_0 \in G$, given $\varepsilon > 0$ and $a \in E$, and let the open ball $B(\sigma(x_0)a, \varepsilon)$ be a neighborhood of $\sigma(x_0)a$ in E. The map $\Phi : (x, a) \mapsto \sigma(x)a$ from $\overline{G} \times E \to E$ is continuous, therefore there exists a neighborhood U of x_0 in $\overline{G}$ such that $\Phi(U \times B(a, r)) \subset B(\sigma(x_0)a, \varepsilon)$. Especially, $\Phi(U \times \{a\})) \subset B(\sigma(x_0)a, \varepsilon)$. Thus $\forall\, x \in U$, $\|\sigma(x)a - \sigma(x_0)a\| < \varepsilon$. The set $G \cap U$ is a neighborhood of x_0 in G such that $\|\sigma(x)a - \sigma(x_0)a\| < \varepsilon$ whenever x lies in $G \cap U$; that is, if $x \in G \cap U$, then $\sigma(x)a$ belongs to the set $B(\sigma(x_0)a, \varepsilon)$ i.e $\sigma(G \cap U)a \subset B(\sigma(x_0)a, \varepsilon)$. Thus the map: $x \mapsto \sigma(x)a$ is continuous from G into E. □

Theorem 3.6. *If a rough representation σ of G on E is continuous then for every compact subset Ω of G, the set of operators $\{\sigma(x) : x \in \Omega\}$ is equicontinuous.*

Proof. Assume that σ is a continuous rough representation of G on E. Set $H = \{\sigma(x) : x \in \Omega\}$ where Ω is a compact subset of G. Let $a \in E$ and $\varepsilon > 0$. Let $x \in \Omega$. Since the representation σ is continuous, then the map $(y, b) \mapsto \sigma(y)b$ is continuous at (x, a). Hence there exists an open neighborhood of (x, a) of the form $(U_x \cap \Omega) \times B(a, \eta)$ where U_x is an open neighborhood of x in $\overline{G}$ and η is a positive real such that $\forall (y, b) \in (U_x \cap \Omega) \times B(a, \eta)$, $\|\sigma(y)b - \sigma(x)a\| < \varepsilon/2$. On the other hand, according to Theorem 3.5, as the representation σ is continuous, the map $s \mapsto \sigma(s)a$ is continuous at x. Thus there exists an open neighborhood $(V_x \cap \Omega)$ of x in G, where V_x is a neighborhood of x in $\overline{G}$ such that if $y \in V_x$, then $\|\sigma(y)b - \sigma(x)a\| < \varepsilon/2$. Set $O_x = U_x \cap V_x \cap \Omega$. We have $\Omega = \bigcup\limits_{x \in \Omega} O_x$. As Ω is compact, then there exists a finite list of elements $x_1, ..., x_n$ of Ω such that $\Omega = \bigcup\limits_{i=1}^{n} O_{x_i}$. For $z \in \Omega$, there exists $i \in \{1, ..., n\}$ such that $z \in O_{x_i}$. Then for $b \in B(a, \eta)$ we have $\|\sigma(z)b - \sigma(z)a\| \leq \|\sigma(z)b - \sigma(x_i)a\| + \|\sigma(x_i)a - \sigma(z)a\| < \varepsilon/2 + \varepsilon/2 = \varepsilon$. Thus H is equicontinuous at a. Since a is an arbitrary element of E, then H is equicontinuous. □

Theorem 3.7. *Let σ be a rough representation of G on E. If for every $a \in E$, the map $x \longmapsto \sigma(x)a$ is continuous from G into E and if for every compact subset Ω of G the set $\{\sigma(x) : x \in \Omega\}$ is equicontinuous, then σ is continuous.*

Proof. We have to prove that the map $\Psi : (x, a) \mapsto \sigma(x)a$ is continuous from $\overline{G} \times E$ into E. Let $\varepsilon > 0$ and let $(x, a) \in \overline{G} \times E$, then we have $[x] \cap G \neq \varnothing$, thus there exists $t \in [x] \cap G$. By Theorem 3.5, there exists a neighborhood $U_t = (tV_e) \cap G$ of t (where V_e is a compact neighborhood of e) such that,

$$\forall y \in U_t, \|\sigma(y)a - \sigma(x)a\| < \frac{\varepsilon}{2(\|\sigma(xt^{-1})\| + 1)}. \tag{3.2}$$

On the other hand, set $U_x = xV_e$. Then U_x is a compact neighborhood of x in G. By hypothesis, the set $\{\sigma(y) : y \in U_x\}$ is equicontinuous. Thus there exists $\eta > 0$ such that $\forall\, b \in B(a, \eta)$ and forall $y \in U_x$, we have,

$$\|\sigma(y)b - \sigma(y)a\| < \frac{\varepsilon}{2}. \tag{3.3}$$

Put $V_x = \{y \in U_x : tx^{-1}y \in U_t\}$. Then V_x is a neighborhood of x. The set $V_{(x,a)} = V_x \times B(a, \eta)$ is a neighborhood of (x, a). Then $\forall (y, b) \in V_{(x,a)}$, we have,

$$\begin{aligned}
\|\Psi(y,b) - \Psi(x,a)\| = \|\sigma(y)b - \sigma(x)a\| &\leq \|\sigma(y)b - \sigma(y)a\| + \|\sigma(y)a - \sigma(x)a\| \\
&\leq \|\sigma(y)b - \sigma(y)a\| + \|\sigma(xt^{-1}tx^{-1}y)a - \sigma(xt^{-1}t)a\| \\
&\leq \|\sigma(y)b - \sigma(y)a\| + \|\sigma(xt^{-1})\sigma(tx^{-1}y)a - \sigma(xt^{-1})\sigma(t)a\| \\
&\leq \|\sigma(y)b - \sigma(y)a\| + \|\sigma(xt^{-1})(\sigma(tx^{-1}y)a - \sigma(t)a)\| \\
&\leq \|\sigma(y)b - \sigma(y)a\| + \|\sigma(xt^{-1})\|\|\sigma(tx^{-1}y)a - \sigma(t)a\| \\
&\leq \frac{\varepsilon}{2} + \|\sigma(xt^{-1})\|\|\sigma(tx^{-1}y)a - \sigma(t)a\|.
\end{aligned}$$

Furthermore, since for $y \in V_x$, $tx^{-1}y \in U_t$ according to (3.2), then

$$\|\sigma(tx^{-1}y)a - \sigma(t)a\| < \frac{\varepsilon}{2(\|\sigma(xt^{-1})\| + 1)}.$$

Hence

$$\begin{aligned}
\|\Psi(y,b) - \Psi(x,a)\| &\leq \frac{\varepsilon}{2} + \|\sigma(xt^{-1})\|\|\sigma(tx^{-1}y)a - \sigma(t)a\| \\
&\leq \frac{\varepsilon}{2} + \frac{\varepsilon\|\sigma(xt^{-1})\|}{2(\|\sigma(xt^{-1})\| + 1)} \\
&< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.
\end{aligned}$$

Thus $\Psi : (x, a) \mapsto \sigma(x)a$ is continuous from $\overline{G} \times E$ into E. Hence, σ is continuous. ☐

Theorem 3.8. *Let σ be a homomorphism from $\overline{G}$ into $\mathcal{L}(E)$ such that*

(1) *the map $x \longmapsto \sigma(x)a$ from G into E is continuous for every $a \in E$, and*

(2) *there is a neighborhood $\mathcal{N}$ of e in $\overline{G}$ such that $\forall x \in \mathcal{N}$, $a \in E$, $\|\sigma(x)a\| \leq \|a\|$.*

Then σ is a continuous rough representation of G.

Proof. To prove that σ is a continuous rough representation of G, it suffices to prove that the map $\Psi : (x, a) \mapsto \sigma(x)a$ is continuous at $(e, a) \in \overline{G} \times E$. By (2) there exists a neighborhood $\mathcal{N}$ of e in $\overline{G}$ such that $\forall x \in \mathcal{N}, \forall b \in E, \|\sigma(x)b\| \leq \|b\|$. Since $e \in \overline{G}$ we have $[e] \cap G \neq \varnothing$. Thus there exists $t \in [e] \cap G$. According to (1), the map $x \longmapsto \sigma(x)a$ is continuous at t. Hence there exists a neighborhood

$V_t = (t\mathcal{N}) \cap G$ of t in G such that,

$$\forall x \in V_t,\ \|\sigma(x)a - \sigma(t)a\| \leq \frac{\varepsilon}{2(\|\sigma(t^{-1})\| + 1)}$$

Set $V_e = \{x \in \mathcal{N} : tx \in V_t\}$. Then V_e is a neighborhood of e in $\overline{G}$. Set $W_{(e,a)} = V_e \times B(a, \frac{\varepsilon}{2})$. Then $W_{(e,a)}$ is a neighborhood of (e, a) in $\overline{G} \times E$. Then $\forall (x, b) \in W_{(e,a)}$, we have

$$\begin{aligned}
\|\Psi(x,b) - \Psi(e,a)\| &= \|\sigma(x)b - \sigma(e)a\| \\
&\leq \|\sigma(x)b - a\| \\
&\leq \|\sigma(x)b - \sigma(x)a\| + \|\sigma(x)a - a\| \\
&\leq \|\sigma(x)(b-a)\| + \|\sigma(t^{-1}tx)a - \sigma(t^{-1}t)a\| \\
&\leq \|\sigma(x)(b-a)\| + \|\sigma(t^{-1}tx)a - \sigma(t^{-1}t)a\| \\
&\leq \|b-a\| + \|\sigma(t^{-1})(\sigma(tx)a - \sigma(t)a)\| \\
&\leq \|b-a\| + \|\sigma(t^{-1})\|\|\sigma(tx)a - \sigma(t)a\| \\
&< \frac{\varepsilon}{2} + \|\sigma(t^{-1})\|\|\sigma(tx)a - \sigma(t)a\|
\end{aligned}$$

As $x \in V_e$ then $tx \in V_t$ and we have

$$\|\sigma(x)a - \sigma(t)a\| \leq \frac{\varepsilon}{2(\|\sigma(t^{-1})\| + 1)}$$

$$\begin{aligned}
\text{So } \|\Psi(x,b) - \Psi(e,a)\| &< \frac{\varepsilon}{2} + \|\sigma(t^{-1})\| \frac{\varepsilon}{2(\|\sigma(t^{-1})\| + 1)} \\
&< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon
\end{aligned}$$

Hence the map $\Psi : (x, a) \mapsto \sigma(x)a$ is continuous at $(e, a) \in \overline{G} \times E$. $\square$

Definition 3.9. Let σ_i, $i = 1, 2$ two rough representations of G on E_i, $i = 1, 2$ respectively. They are said to be rough equivalent if there exists a homeomorphism $S : E_1 \longrightarrow E_2$ such that

$$\forall x \in \overline{G},\ S\sigma_1(x) = \sigma_2(x)S.$$

S is called an intertwining operator.

Definition 3.10. The rough representation σ of G in E is said to be rough irreducible when E and $\{0\}$ are the only two closed subspaces of E that are invariant by all the operators $\sigma(x)$ with $x \in \overline{G}$.

Definition 3.11. A rough representation σ is said to be rough unitary when $E = H$ is a Hilbert space and $\langle \sigma(x)u, \sigma(x)v\rangle = \langle u, v\rangle$, $\forall x \in \overline{G}, \forall u, v \in E$.

Theorem 3.12. *Let σ be a rough unitary rough representation of G in the Hilbert space H. Let H_1 be a subspace of H. If H_1 is invariant by σ then the orthogonal $H_1^{\perp}$ of H_1 in H is also invariant by σ.*

Proof. Assume that H_1 is invariant. Let $u \in H_1^{\perp}, v \in H_1, \ x \in \overline{G}$. we have $\langle \sigma(x)u, v\rangle = \langle u, \sigma(x)^* v\rangle = \langle u, \sigma(x^{-1})v\rangle = 0$ where $\sigma(x^{-1})v \in H_1$ by assumption that H_1 is invariant. This implies $\sigma(x)u \in H_1^{\perp}$. □

Theorem 3.13. *Let σ and π be two rough representations of G on E and F respectively and let $S : E \longrightarrow F$ be an* intertwining operator. *Then the kernel* $\ker(S)$ *of S is a subspace of E invariant by σ and the range $ran(S)$ of S is a subspace of E invariant by π.*

Proof. It is clear that $\ker(S)$ is a subspace of E. If $a \in \ker(S)$, then for all $x \in \overline{G}$, $S(\sigma(x)a) = (S \circ \sigma(x))a = (\pi(x) \circ S)(a) = \pi(x)(S(a)) = \pi(x)(0) = 0$. Hence $\sigma(x)a \in \ker S$.
It is clear that $ran(S)$ is subspace of F. If $b \in ran(S)$, then there exists $a \in E$ such that $S(a) = b$. For all $x \in \overline{G}$, $\pi(x)b = \pi(x)(S(a)) = (\pi(x) \circ S)(a) = S \circ \sigma(x)(a) = S(\sigma(x)a)$. Since $\sigma(x)a \in E$, then $\pi(x)b \in ran(S)$.

□

Theorem 3.14. *Let $S : E \longrightarrow F$ be an* intertwining operator *of two rough irreducible rough representations (σ, E) and (π, F) of a rough group G. If (σ, E) and (π, F) are not rough equivalent, then $S = 0$.*

Proof. If (σ, E) and (π, F) are not rough equivalent, then S is not invertible. Then we have two possibilities:

i) If S is not injective, then $\ker(S) \neq \{0\}$. According to Theorem 3.13, $\ker(S)$ is a subspace of E invariant by σ. But σ is irreducible. Therefore $\ker(S) = E$. Hence $S = 0$.

ii) If S is not surjective, then its range $ran(S)$ is a proper subset of F. By Theorem 3.13, $ran(S)$ is a subspace of F which is invariant by π. But π is irreducible. Therefore $ran(S) = \{0\}$. Hence $S = 0$.

□

4. Conclusion

Representations of rough topological groups have been studied. Mainly, some of their algebraic and topogical properties have been scrutinized. As a perspective, we plan to consider rough sets modeled from different types of neighborhood systems (references [3, 5] will be useful). We will study their realizations as objects of linear algebra.

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