Original Research Article

N-way modeling for wavelet filter determination in Multivariate Image Analysis

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Abstract

When trying to analyze spatial relationships in image analysis, wavelets appear as one of the state-of-the-art tools. However, image analysis is a problem dependent issue and different applications might require different wavelets in order to gather the main sources of variation in the acquired images with respect to the specific task to be performed. This paper provides a methodology based on N-way modeling for properly selecting the best wavelet choice to use, or at least to provide a range of possible wavelets choices (in terms of families, filters and decomposition levels), for each image and problem at hand. The methodology has been applied on two different data sets with exploratory and monitoring objectives.

Keywords: Wavelets, Tucker3, Multivariate image analysis.

1 INTRODUCTION

Wavelet-based image analysis is still one of the state-of-the-art techniques for texture analysis [1-5], both in terms of image characterization as well as image segmentation.

However, image wavelet decomposition is problem dependent, because different images are better characterized by different types of wavelet families. Moreover, each wavelet family consists of different filters, which makes the selection a critical issue.

Different works have addressed this problem: wavelet family and filter choice. In [6] different properties of the decomposition filter were analyzed in terms of texture description capability, proposing different general criteria for the selection of the best wavelet family. Daubechies filters, Haar filter and biorthogonal filters were analyzed. Other works have focused on image compression [7], by comparing the goodness of the reconstructed image in comparison with the original one, which also gives an idea about the ability of the wavelet transform for characterizing the image and problem at hand.

Svensson et al. [8] applied a Design of Experiments (DoE) in order to determine the best wavelet for the texture characterization of pharmaceutical tablets. They used Daubechies 2D-Discrete Wavelet Transform (DWT) in comparison with Dual Tree-Complex Wavelet Transform (DT-CWT) for texture characterization. More recently, the same authors [9] again compared different types of wavelets for evaluating texture properties in images. Dual Tree-DWT was used in comparison with Stationary Wavelet Transform from Daubechies 4, and with continuous wavelet in the form of Mexican hat, but still the final conclusions are problem dependent.

In many research papers, just one wavelet family is used, especially when wavelet transform is only considered as a denoising tool and not as an analysis tool to resolve different patterns in the image. In other occasions [3] trial and error is the selection criterion, no justification is provided [8, 9], or even no comment on the wavelet family applied is provided [10].

The decomposition level (i.e. scale) has also to be addressed. Wavelets can be applied on the images recursively starting with the first decomposition level up to the maximum possible level depending on the image size. At each level more coarse aspects (smoothed version) of the image are captured in the approximations, while other finer aspects are separately filtered in the details. Thus, depending on the problem at hand, one may need to consider different or some specific decomposition levels for properly characterizing some phenomenon.

In this context, a methodology is needed for properly selecting the best wavelet choice to use, or at least to provide a range of possible wavelets choices (in terms of families, filters and decomposition levels), for each image and problem at hand. Adaptive wavelets [11] may represent a solution to this issue, since they allow developing wavelets specifically for a particular set of data and data analysis task. Nevertheless, this approach needs to optimize other settings besides the meta-parameters of the multivariate model, *e.g.* the number of components to use, etc.; so it suffers the drawback of requiring a proper double validation scheme, as well as being computationally costly.

This paper presents an alternative approach, based on the selection of several wavelets sensible to be applied on some type of images, and integrate them all in an N-way data structure. The complexity of the data sets, coming from RGB images with multitude of pixels, wavelet families with different filter lengths, decomposition levels and directions, needs a careful consideration of the array assembling. In this sense, analyzing these datasets by N-way models such as Tucker3 [12-13] allows evaluating the similarity or dissimilarity of the given set of wavelets investigated.

The paper is organized as follows. Section 2 introduces the materials and methods used. First, the different wavelet families used in the paper are presented and the decomposition scheme applied explained. Afterwards, the data structure built from the image decomposition is presented, and the N-way model applied introduced. Section 3 illustrates the results from an exploratory case (orange images), and a monitoring case (artificial stone plate images). Finally, section 4 reports some conclusive remarks.

2 MATERIALS AND METHODS

2. 1. Image data sets

Two types of images are used to illustrate the proposed methodology. The first type is an RGB image (size 100x100) from oranges affected by insects that act as a major pest of citrus fruits (Fig. 1). Resolution of the images was 0.25 mm/pixel.

The second type of images (RGB) relates to two artificial stone plates from a manufacturing process: one produced under Normal Operating Conditions (Fig. 2a, size 64x64) and another one with some defects (Fig. 2b, size 128x128). Resolution of the images was 0.32 mm/pixel.

[INSERT FIGURES 1 and 2 ABOUT HERE]

2. 2. Wavelets families and filters

Wavelets can be defined as functions that have sufficient compact support in both time (localization) and frequency [14], which decompose a signal (or image in this case) through high-pass and low-pass filters. They transform the images into a new representation by splitting their features at different resolution scales, yielding the so-called approximation and detail images, while maintaining the textural structure of the analyzed images (decomposed as the sum of the detail coefficients at each scale and at each direction). This way, they constitute a multiscale representation of the image that can be used to analyze the texture present in an image at different scales.

When dealing with digital images, the discrete wavelet transform (DWT) is usually applied. In principle, DWT is implemented through the fast Mallat algorithm [15], which requires a dyadic structure (*i.e.* image size to be some power of two in both directions). In practice the algorithm works also if this is not the case, although some border distortion may arise in the convolution step. Different extension methods can be applied [16] in order to minimize this problem. Another aspect of this decomposition is that it is not shift invariant, i.e. a shifted version, also to a small extent, of the original image (what is of concern is a slightly shift of the same pattern/features in different images that we want to recover) can bring to very different wavelet coefficients, hence in decomposition images. In this case some artifacts can be introduced in the reconstructed single decomposition image, as in our application, with respect to the original, also resulting in low resolution.
One way to overcome this drawback is to use the stationary wavelet transform, SWT [17-19], also known as undecimated wavelet transform, UWT [20]. In this case decimation (down-sampling) of coefficients is not applied; instead the wavelet filter

coefficients are up-sampled when passing to the next decomposition level. Thus, each block of coefficients at every level maintains the same size as the original image and captured features may be localized directly in the wavelet transform (WT) coefficients space without reconstruction, hence obtaining congruent images that may be better analyzed, without interfering-neighboring-pixels effects.

Out of all the wavelet families and filters proposed in the literature in this paper the following wavelet families and filters were tested:

- Daubechies, DbN (asymmetric, 2N filter length): N 1 to 5;
- Symlets, SmN (near symmetric, 2N filter length): N 2 to 5;
- Coiflets, CfN (near symmetric, 6N filter length): N 1 to 5.

These wavelets are selected because they are widely used in literature and cover different characteristics, such as degree of symmetry or regularity and number of vanishing moments [21-24]. These characteristics are linked to the analysis capability of the given wavelet filter, *e.g.* degree of compression, smoothness of reconstructed signals, or capacity to capture signal variation.

2. 3. Data structure

Once an image is decomposed by some wavelet family, from level one to the maximum level permitted for some filter length and given image size, we end up with a quite complex structure related to all the approximation and details blocks obtained in the decomposition.

For a given color band (R, G or B) of an image analyzed by a given wavelet filter, *e.g.* Db2, we have the following 3-way data structure:

- The first mode is related to the unfolded image, hence being formed by the pixels of each of the reconstructed images.
- The second mode is related to the **A** approximation image, and **H**, **V** or **D** details images. For DWT scheme **A**, **H**, **V** and **D** stands for the images reconstructed by

using only the **CA**, **CH**, **CV** and **CD** coefficients at a given decomposition level, respectively. For SWT scheme reconstruction is not necessary.

- The third mode is related to each decomposition level (scale) and wavelet filter.

For an RGB image, the second mode has the approximation (**A**) and details (**H**, **V** and **D**) images repeated 3 times, *i.e.* as many times as color bands we have.

Finally, if we want to compare several wavelet filters belonging to different families and having different filter length, it is necessary to build up a common 3-way array, just concatenating in the third mode the different decomposition levels (scales) for each wavelet filter (a member of a wavelet family) one after the other; then repeating this procedure for the different wavelet families. The final 3-way data structure arranged is shown in Fig. 3.

[INSERT FIGURE 3 ABOUT HERE]

2. 4. Pre-processing

Preprocessing has a potential relevance in the analysis results [25]. When dealing with 3-way data, there are multiple choices regarding the modes and the preprocessing to apply. A thorough discussion about preprocessing can be found in [26] and [27].

The most applied preprocessing techniques are centering and scaling. Centering the data structure can be easily thought as translating our variable space to deviations from its gravity center or removing a common offset from the data. This is commonly performed subtracting the column averages across one of the modes. Regarding scaling three-way data sets, there are different alternatives to perform (assuming observations, pixels of the image in this case, to be placed in the first mode, as in Fig. 3):

- Slab scaling (SSc) within the second mode, *i.e.* color bands and directions. This means to give all color bands and decomposition blocks (each block is capturing a different pattern in the image, approximation being a smoothed version of the raw image and details capturing pattern changes in the three directions) the same opportunity to contribute to the model.

- Slab scaling within the third mode, *i.e.* wavelet families, filter lengths and their corresponding levels (scales). Thus, we give the same relevance to the different wavelets filters and levels, maintaining the original variance structure in the second mode.
- Double slab scaling (DSSc) both within the second and third mode, which lets all variables in both second and third modes to have the same global variance, and at the same time maintaining the original internal variance structure ratio.
- Column scaling (CSc). This typical two way scaling option gives all variables in the second and the third mode the same weight (calculated on the **X**(*I*, *JK*) columns), without maintaining the original variance structure within each mode. The risk of this scaling is to give high weights to variables with low signal-to-noise ratio.

In summary, since preprocessing is problem dependent, we have considered the different options. Because in this paper scaling only within the third mode is not an option (we need all color bands to a priori have the same weight), scaling within the second mode, scaling within the second and third modes, and column scaling, have been considered for DWT and SWT decompositions schemes.

2. 5. N-way modelling

Since the data structures analyzed here (see Fig. 3) do not show any trilinearity (components in all modes may present interactions between them), Tucker3 models have been used for analyzing them.

For any *D*, *E* and *F* components in the first, second and third modes of a 3-way data array, the mathematical model for the Tucker3, also known as N-way-PCA [12-13] is:

$$x_{ijk} = \sum_{d=1}^{D}\sum_{e=1}^{E}\sum_{f=1}^{F} a_{id} b_{je} c_{kf} g_{def} + r_{ijk} \qquad (2)$$

The model parameters are estimated by minimizing the residuals sum of squares $\sum r^2_{ijk}$. Expressed in matrix notation, with $\underline{\mathbf{X}}$ and $\underline{\mathbf{G}}$ in matricized form $\mathbf{X}(I,JK)$, $\mathbf{G}(D,EF)$, and using the Kronecker product ⊗ the model is:

$$\mathbf{X} = \mathbf{AG}(\mathbf{C} \otimes \mathbf{B})^{\mathrm{T}} + \mathbf{R} \tag{3}$$

Where matrices **A**, **B** and **C** gather the loadings of the first (a_{id}), second (b_{je}) and third (c_{kf}) modes, respectively; and **R** holds for the residuals (r_{ijk}). **G** matrix, commonly known as the core matrix with elements g_{def}, indicates the amount of variance explained by the different combinations of the components of the modes. It can be rotated to maximum variance [28] or to maximum diagonality [12, 28-29] for a better interpretation of the results. Since the number of components does not necessarily need to be the same for the different modes, the dimensions of the loading matrices can be individually accommodated in each mode. This enables the model to maximally explain the variance in the data, usually imposing as the only restriction the orthogonality between the components in each mode.

From the N-way model fitted to the data structure, a set of loadings is obtained. Careful inspection of the third mode loadings allows assessing which of the wavelet families and filter lengths carry similar and/or unique information and at which scaling levels. Salient color bands and wavelet coefficients from approximation (**A**) or details (**H**, **V** and **D**) images for describing the image pattern can be depicted from the second mode loadings. Finally, considering the first mode loadings and refolding the pixels located in them the scores images are obtained. This way it is possible to analyze which type of information is being provided by those wavelets and color bands most influencing the model. This will be illustrated in Section 3.

2. 6. Multivariate Image Analysis

Once the best wavelet filter has been selected, as well as the scales to use, the final stage is to apply the image analysis technique, in order to deal with the problem at hand. Since we have already decomposed the image using the WT's, after the analysis with the Tucker3 model we only have to extract out, from the corresponding three-way array, the frontal slabs of interest corresponding to the chosen settings: wavelet family and filter length, as well as decomposition levels or scales selected. Remember that each frontal slab corresponds to one scale of one specific wavelet family and filter length: i.e. the first frontal slab corresponds to Daubechies family, filter length 1, first decomposition level; the second frontal slab corresponds to Daubechies family, filter length 1, second decomposition level, and so on (see Fig. 3).

This way of performing can be included under the Multi-Resolutional MIA framework [30], since we select, for some wavelet and filter length, the scales that better characterize some feature or NOC image; which can also be extended for selecting those scales (and directions) that maximize some predictive ability.

By stacking the resulting matrices for the decomposition levels selected one beside the other, we end up with an **X** matrix that can be analyzed by Principal Component Analysis, PCA [31] as $\mathbf{X} = \mathbf{TP}^T + \mathbf{E}$, where **T** holds the new latent variables, orthogonal and linear combinations of the original ones; **P** gathers the loadings that explain the inner relations between variables; and **E** contains the residuals of the PCA model.

From these results, several analyses can be carried out [1]. For the exploratory analysis case, we have used the score images, spatial representation of the score values obtained by each pixel, according to its original $n_1 \times n_2$ pixels image spatial location, in order to check whether the phenomenon of interest is gathered in the model.

For the monitoring case study, T^2 and Residuals Sum of Squares (RSS) images have been used. T^2 and RSS images are, in analogous way to score images, $n_1 \times n_2$ spatial representations of the T^2 and RSS pixels values, but only for those pixels having T^2 or RSS values above some pre-specified threshold. This way, those pixels with abnormal behaviors are shown up, revealing the defects of the image.

Thus, the proposed methodology will be validated by checking if the different selected wavelet filter and scales for the WT decompositions are able to:

i) gather the phenomenon of interest in the exploratory study;
ii) detect the defects in the monitoring case.

2. 7. Methodology scheme

The proposed methodology, explained through two different applications in Section 3, is summarized in a three-step approach, as follows.

STEP 1:

- Decompose the image using all the wavelet families and related filters from one to the maximum level reachable.
- Build the three-way data structure as described in section 2.2.
- Fit 3-way models, after preprocessing. Considering that in this context N-way modeling serves as exploratory tool a rather standard procedure can be adopted, such as the use of Tucker3 model, assuming a [2 2 2] model dimensionality. This is very rough but generally works for exploration [12, 32-33]. Anyhow, the following general procedure can also be suggested: assess by preliminary testing a range of components for each mode, inspecting the scree-like plot and selecting few models using parsimony as main criterion; rotate the core array to maximum variance and consider only the first two/three core elements for further analysis of loadings. Nevertheless, for simplicity and comprehension reasons we decided to illustrate the methodology with a Tucker3 [2 2 2] model. Results with more complex models (not shown) did not provide better results.

STEP 2

- Inspect third mode loadings plot for the Tucker3 components (Factors). Each point corresponds to a specific wavelet filter and decomposition level. Look for the highest loadings (in absolute value) in each component or for those ones further from the center. Any of the families and filters pointed out would provide the image features gathered by the Tucker3 model. Features that are shown in the scores images of the corresponding components.

STEP 3

- By using the wavelet family, filters and decomposition levels selected from STEP 2, build MIA models on them, using all the approximation and details images and color bands. This is accomplished by selecting the corresponding slabs from the array shown in Fig 3. Afterwards, unfold the data by stacking each slab one beside the other. This way, we obtain a matrix of dimensions

($n_1 \times n_2$, $J\times$levels of the selected wavelets family and filter length) that can be directly analyzed by PCA. Finally, check whether they gather or not the information of interest for the problem at hand.

3 RESULTS

When performing any image analysis, the wavelet selection may be dependent on the goals pursued. In this paper two potential applications are illustrated: exploratory analysis and process monitoring.

- In an exploratory analysis, the focus is in process understanding or object characterization. Therefore, we will try to find those wavelets that best extract out the different sources of variations (phenomena) in the images.
- In process monitoring, the goal is to detect any abnormal behavior (mainly defects) from images obtained under Normal Operating Conditions, NOC, i.e. when the process is working properly [34].

In both cases, results from Double slab scaling (DSSc) both within the second and third mode [35] are shown, which lets all variables in both second and third modes to have the same global variance, and at the same time maintaining the original internal variance structure ratio.

Different data sets and/or applications may require different preprocessing options in order to obtain optimal results. In these two cases, other preprocessing options (results not shown), such as Slab Scaling or typical 2-way autoscaling [27] did not provide better results.

3.1 Exploratory analysis

This first approach deals with trying to find the best wavelet family that better helps in characterizing some pits that appear in the oranges when one disease is present (See Fig. 1). So, it is necessary to perform an exploratory analysis in order to search for those (latent) variables that describe these defects in detail.

Steps 1 and 2: Image wavelet decomposition, Tucker3 modeling and analysis

For computational restrictions associated to the (huge) data structure derived from the image, a 100×100 pixels area representative of the events in the original image was selected (Fig. 1).

As indicated in Section 2.7, the proposed methodology is applied. The [2 2 2] Tucker3 model explained a 7.82% of the data. Higher Tucker3 models did not provide insight on information of interest. The main core elements are (1,1,1), explained variance ratio of 56.83%, and (2,1,2), with 42.47%. Fig. 4 shows the loadings plots indicating the color bands (R, G and B) and decomposition directions (a, h, v and d) (Mode 2); as well as the wavelet families (Db, Sm and Cf), with their filter lengths (1 to 5) and scales (1 to maxlevel) most influencing the model (Mode 3). In this case, Daubechies (Db), filter lengths 1 to 3, decomposition levels 1 to 3; Symlets (Sm), filter lengths 2 and 3, decomposition levels 1 to 3; and Coiflets (Cf), filter lenght 1, levels 1 and 2.

[INSERT FIGURE 4 ABOUT HERE]

By refolding the loadings of the first mode into the original image size, the Tucker3 model score images are obtained (Fig. 5). It is possible to see how they mainly gather textural information linked to the high and medium frequencies (fine and medium roughness) pointed out by the loadings plot of the third mode from Fig. 4. These features are related to the characteristics of the phenomenon we are dealing with in this example, so it makes sense to use them for next Step 3.

[INSERT FIGURE 5 ABOUT HERE]

Step 3: Multivariate image analysis

This analysis provides different options for Multivariate Image Analysis, regarding the wavelet families and filters that can be used. In this case, from the SWT results, some wavelet families were selected (most prominent in the third mode from previous Tucker

3 model), and the consistency of the proposals was checked. In the following, the results for Symlets 2 (Sm2) wavelet filter, for decomposition levels 1 to 3, are shown.

When inspecting the MIA score images (Fig. 6), it is possible to see that PC's 4, 11 and 13 are the ones more related to the phenomenon of interest (pits in the oranges, Fig 1). From the MIA loadings (Fig. 7), the most important color bands and wavelet decomposition images are green horizontal detail images for levels 2 and 3 (PC 4, inverted score values for illustration purposes), diagonal and vertical details for green and blue color bands for level 3 (PC 11), and diagonal and vertical details for green color bands for level 3 (PC 13). These directions are supported in the score images by the type of pits detected.

[INSERT FIGURE 6 ABOUT HERE]

[INSERT FIGURE 7 ABOUT HERE]

In order to improve the final image picture, it is possible to form a T^2 image based on the PC's of interest (Fig. 8). Comparison with the original image supports the goodness of the proposed methodology for finding proper wavelet families for some specific purpose (in this case, pits detection).

[INSERT FIGURE 8 ABOUT HERE]

It must be stressed that, by considering any other wavelet filter different from the ones proposed by the methodology, the results (see the Supplementary material) were not satisfactory.

3.2 Monitoring

The second studied approach, illustrated on artificial stone plates images (see Fig 2), deals with process monitoring. The goal is finding out whenever a process behaves in some abnormal way. These abnormalities show up as defects in images as spots, scratches, color changes, etc. that can be detected applying MIA with monitoring purposes, using T^2 and RSS images [31] as image sensors.

Steps 1 and 2: Image wavelet decomposition, Tucker3 modeling and analysis

Again, for computational restrictions associated to the data structure, a 64×64 pixels area representative of the original image was selected (Fig. 2a). For illustration purposes, the commented results are those obtained by the stationary wavelets (SWT). The [2 2 2] Tucker3 model built explained a 9.93% of the data. The main core elements are (1, 1, 1) with explained variance ratio of 57.24%, and (2, 2, 2) with 41.62% ratio. Fig. 9 shows the corresponding loadings plots.

[INSERT FIGURE 9 ABOUT HERE]

From the main core element (57.24% of the core variance), the first component of the second mode, dominated by the diagonal details images is mainly related to the first component of the third mode, mostly influenced by Daubechies, filter lengths 2 and 3, levels 3 and 4; Symlets, filter lengths 2 and 3, levels 3 and 4; and Coiflet 1 and 2, levels 3 and 4 (Fig. 9).

Regarding the second factor of the second mode, which interacts with factor 2 of mode 3, accounting for 41.62% of the core element variance, respectively, it is related to the horizontal details. In this case, this second component is associated to Daubechies, filter length 1, levels 1 to 3; Daubechies and Symlets filter length 2, levels 2 to 4; Daubechies and Symlet, filter length 3, level 4; Coiflets filter length 1, levels 2 to 4; and Coiflet filter length 2, levels 3 and 4.

By refolding the loadings of the first mode into the original image size, the score images derived from the Tucker3 model are obtained (Fig. 10). They are mainly gathering textural information linked to high and medium frequencies, which are related to the nature of the defects supposed to appear in the images.

[INSERT FIGURE 10 ABOUT HERE]

Step 3: Multivariate image analysis

Using e.g. Db1, levels from 1 to 3, by projecting the test image (Fig 2b) we obtain the score images from the MIA model (4 PCs) shown in Fig. 11. It is possible to see how they gather the defect properly. Afterwards, the T^2 and RSS images (Fig. 12) unravel those pixels with extreme values associated to defects of different nature. This procedure is known as the *Fit to a pattern model* approach [1, 34].

[INSERT FIGURES 11 AND 12 ABOUT HERE]

In this case, the proposed methodology indicated that some other pre-processing options would be advisable (see the Supplementary material), although the corresponding results did not seem as good as the ones shown here.

4 CONCLUSIONS

A methodology for coping with the problems that arise when trying to apply wavelets in MIA for each problem at hand: selecting the best type of wavelet family, filter and decomposition scheme; as well as the best decomposition levels, has been presented.

The three way analysis applied has shown very efficient when trying to reveal these critical aspects, moreover confirming the closeness between Daubechies and Symlet; and that Coiflet is not so ambivalent in general. Moreover, the segregation of the loadings in separate modes facilitates the decision about the scaling level to reach for each problem at hand. In general, it has been possible to get a limited set of suitable wavelets filters/decomposition levels to choice. Moreover, considering that SWT has always yielded a suitable solution and it does not suffer shift distortions, nor it needs a reconstruction step, we may suggest this is a better alternative with respect to DWT scheme. Regarding the type of pre-processing, DSSc also has provided good results in these two cases.

Results have shown that wavelet selection is problem dependent, not only in terms of the type of image analyzed, but also depending on the final approach under analysis: exploratory or monitoring. Therefore, this methodology represents an alternative for deciding between a set of possible wavelet families, within the whole bunch of associated decomposition possibilities.

5 ACKNOWLEDGEMENTS

This research work was partially supported by the Spanish Ministry of Economy and Competitiveness under the project DPI2011- 28112-C04-02.

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