Optimal Fleet Replacement.
A Case Study on a Spanish Urban Transport Fleet

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Abstract
Optimize the average annual cost of a bus fleet has become an increasing concerning in transport companies management around the world. Nowadays, there are many tools available to assist managerials decisions and, one of the most used, is the cost analysis of the life cycle of an asset, known as ''Life Cycle Cost''. Characterized by performing deterministcs analysis of the situation, allows the administration evaluate the process of fleet replacement however is limited by not contemplating certain intrinsic variations related to vehicles and for disregarding variables related to contingencies of fleet use. The main purpose of this study is to develop a combined model of support to asset management based in the association of the Life Cycle Cost tool and the math model of Monte Carlo Simulation, by performing a stochastic analysis considering both, age and average annual mileage, for optimum vehicle replacement. The utilized method was applied in a spanish urban transport fleet and the results indicates that the use of the stochastic model was more effective than the use of the deterministic model.

Keywords: Fleet Replacement, Optimization, Operation and Maintenance Costs, Transport, Life Cycle Cost Analysis, Monte Carlo Simulation.

INTRODUCTION

"Replacement decision" is a classical operation research topic in the industrial engineering. The replacement theory can indicate the optimal life of equipment. "Optimal life" can be defined as the period between the time that the equipment enters into service and the time that it should be replaced for economic reasons. Generally, the operating cost of an equipment or asset rises as its condition deteriorates over time. When the cost reaches a certain level, the long-run costs associated with investing in a new equipment becomes less than that of keeping the old equipment (Di and Hauke, 2000). At this point, replacement is carried out. Thus, a basic replacement analysis usually examines both the trend in operating and maintenance costs (O&M) and the net cost of replacement, which is defined as the difference between the cost of the new equipment and the salvage value of the old. In some cases, replacement analysis also considers the resale value of equipment at various stages of its service life.

For fleet replacement, two kinds of models are usually suggested in the literature: economic engineering (EE) and operational research (OR) models (Pinar and Hartman, 2004; Khasnabis et al. 2002). EE models are restricted to economic and financial aspects, with technological, management, and strategic variables considered as exogenous. This limitations force management to avoid formal investment analysis and to use unstructured subjective analysis (Collan and Liu, 2003). Traditional OR models focus on a single objective to be maximised/minimised by modelling multiple variables. The adoption of a single criterion limits practical applications where cost, efficiency and service level should be assessed simultaneously. Methods such as multi-attribute utility theory (MAUT), analytical hierarchy process (AHP) and quality function deployments (QFD) have been also proposed to solve these problems (Erol et al., 2003; Ensslin et al., 2001).

These methodologies are complemented by a management tool known as Life Cycle Cost Analysis (LCC), which is determined by summing up all of the potential costs associated with equipment over its lifetime, and it should be restricted to costs that can be controlled. In this method, the operation, maintenance, and purchase costs of the asset, as well as its resale value, are estimated. All values are then organised as cash flows. Once this is accomplished, all cash inflows and outflows during the life of the asset are converted to present values based on interest rates and inflation, applicable for the considered period. These rates allow a comparison between the costs of present and future costs. In other words, the financial consequences of something done today can be compared with the cost that will be incurred tomorrow. Besides, the LCC analysis helps decision makers to justify equipment replacement based on the total costs over the equipment's lifespan. It allows maintenance manager to specify the optimal replacement time at the moment of the equipment's purchase. Cost function models can be used for different categories to allow an easy estimation of the total cost. Such models can be generally classified as detailed models, analogous models and parametric models. A detailed model uses approximation of material quantities and prices, labour time and rates to estimate the direct costs of equipment. Analogous models identify similar equipment and adjust costs to account for differences between it and the target equipment. Cost estimation with a parametric model is based on predicting the equipment's total cost by using regression analysis based on technical information and historical cost.

A viable alternative to conduct a LCC study on vehicles is to use economic engineering criteria in conjunction with optimisation models (Laver et al., 2007; Clark et al., 2009 and Kim et al., 2009). Depending on the type of vehicles in a fleet, optimization models can be divided into two categories: homogeneous and heterogeneous (Feng et al., 2012; Bourdat et al., 2012). In homogeneous models, the main objective is to find the best time for the replacement of a set of identical vehicles (same type and age) that must be replaced together; this is also known as the “no cluster splitting rule”. These models are usually developed using a dynamic programming approach. Heterogeneous models are more appropriate when different types of vehicles need to be optimised simultaneously or if there are budget constraints. These models can solve more practical problems and the input variables are generally deterministic. Most heterogeneous models are mathematical models, fairly representative of the respective dynamic systems studied; they are intended to maximise or minimise the performance index (PI) and to find the “optimal solution” for the problem, i.e., the solution resulting in the best possible system performance, according to the performance criterion previously defined (Fan et al., 2012).

Emblemsvag (2003) and Dhilon (2010) studies indicates that the LCC life cycle cost analysis tool is increasingly used to support taken decisions which directly and indirectly are related with equipments and/or engineering systems in the management of the company’s assets. The key issue is not the technique but the economics, which presents as main purpose, maximize the return on assets investments obtaining a better performance and a lower operation and maintenance cost. In this context, there are many jobs that use this tool, as shown at Zambujal and Duque (2011), Feng and Figliozzi (2012) and Mishra et al. (2013) studies.

However, because *LCC* tool is a deterministic approach, it presents some limitations when analysis vehicles fleet scenarios, being necessary your combination with a math model more robust that allows a stochastic analysis. This combined model allows broader and more reliable analysis of variables that interfere directly and indirectly in the fleet performance and consequently from the company.

According to Rubinstein (2016), there are several math models with simulation for solving stochastics problems available in the literature highlighting the Monte Carlo Simulation (MCS) based on the generation of random numbers and probability of occurrence of associated values to the studied phenomenon under analysis. Among the advantages of the MCS, highlight that the input data can present any distribution type being able to analyse scenarios in an agile way (changing only input data) and due to non-existence of a single algorithm for MCS, one can adjust the simulation procedure in the timeliest manner to the situation. Associated to this premise, McLeish (2011) understand MCS as a universal stochastic numerical method for the solution of math problems proposing your utilization for management risks and MCS has been successfully apply in projects management, allowing better cost analysis.

The present study demonstrates the combined model of fleet management based on the *LCC* Life Cycle Cost Analysis tool and on the Monte Carlo Simulation math model. The bigger and better the data availability and reliability, the greater will be the model success. The collected data belongs to a Spanish urban transport fleet. The main objective is to develop a robust method that allows the fleet managers maximize the fleet profitability, optimizing the life cycle value of each vehicle.

METHODOLOGY

Model Formulation

Being $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{y})$ a function that represents the total maintenance and operational cost for a single bus converted to actual value since the beginning from its operational life until it reaches simultaneously the total mileage "x" and the age "y". So, the description of the problem consists in finding the best fit from mileage and age $(\boldsymbol{x}^{*},\ \boldsymbol{y}^{*})$ that converge them to the optimal fleet replacement point with life cycle cost (LCC) analysis applying a Monte Carlo Simulation method inserting random variables in the total maintenance an operational cost model.

For the LLC analysis, the first step is to define the entire cash flow in function of $G(\boldsymbol{x}, \boldsymbol{y})$ as shown in the equation (1):

$$G(x, y) = f(x, y) + \ Vc - VR \qquad (1)$$

Being:

Vc: Purchase Cost

VR: Resale value

With the cash flow defined, it is necessary to find mileage and age $(x^{*},\ y^{*})$ for the optimal fleet replacement, which means to find the lower values of the average annual cost per mile (AAC) defined in the equation below:

$$H\ (x, y) = \frac{G(x, y)}{x\ *\ y} \qquad (\mathbf{2})$$

So, the problem can be written as:

$$(P1) \begin{cases} Find(x, y) \in R^2\ , such\ as \\ (x, y) = \arg min\ [H(x, y)] \end{cases} \qquad (3)$$

Being:

$x \in [x_{min}\ , y_{min}]$

$y \in [y_{min}\ , y_{max}]$

$x_{min}\ , x_{max,}\ , y_{min}, y_{max} \in R$

The problem studied in this work considers variations in the cost function that have no predictability or a well-defined pattern. Thus, random modeling will be used for estimation. The problem is defined as $(x, y)X(\Omega, F.P)$; in which " Ω" is the sample space of events,

"F" the algebra of events and "P" a probability measure. From this the following hypotheses are necessary:

$H1$) the total **c**ost function is differentiable;

$H2$) the random variables that define the cost function are limited and statistically independent (Avila et. al., 2015)

As a result, the problem $P1$ is reformulated as:

$$(P2)\begin{cases} Find(x^*, y^*) \in [R^2, P], such\ as \\ (x^*, y^*) = \arg min\,[H(x, y, w)]; \\ \quad (x, y, w) \in R^2\ X(\Omega, F.P) \end{cases} \qquad (4)$$

The problem defined in Eq. (4), will be solved through Monte Carlo simulation-based methods. The methods of this class are developed in three stages:

I. Generate, according to the probability functions of each parameter, N - samples of random variables that model the uncertainty on the parameters that define the function total cost;

II. For the sample of the parameters, solve the following problem:

$$(P3)\begin{cases} Find(x_i^*, y_i^*) \in [R^2, P], such\ as \\ (x_i^*, y_i^*) = \arg min\,[H(x, y, w)]; \\ \quad (x, y, w) \in R^2\ X(\Omega, F.P) \end{cases} \qquad (5)$$

III. With all the results, analyze the result with the graphical distribution from the function $H_i^*\ (x_i^*, y_i^*)$ and the variables x_i^* and y_i^*.

Database analysis

The methodology was demonstrated through of a case study using real data of a Spanish urban bus transport fleet, consisting of 80 vehicles. For this work, a sample of 34 vehicles were selected and named as “Type A” vehicles. The buses belonging to the sample have the same technical characteristics, mechanical configurations, fuel and were exposed to similar operating conditions such as average speed, stops per mileage, passenger loading, climate conditions and the largest mileage during lifespan. The analysis period considered was of 10 years (2005-2014), and all the costs were converted and updated using the Spanish economic indicators. The results obtained were extrapolated to the entire lifespan of the vehicles.

The following restraints were applied and considered:

1) Averaged fuel consumption was considered constant along the vehicle's lifespan.

2) Annual averaged mileage was constant, and determined through among all of sample selection buses. For this work was considered 61.597 km/year.

3) To determine the operational costs: fuel, insurances and taxes costs were summed up.

4) Resale Value ($\boldsymbol{VR}$) was calculated by a linear model used by the company and based on own experience, which is obtained by formula 6:

$$VR = R + \left[\frac{Vc - R}{N} * 0{,}7778 * Rv\right] \quad (6)$$

Where:

$0{,}7778 =$ Factor dependent on service conditions.

$Rv =$ Remaining vehicle's life.

(R) Residual Value: In accounting, residual value is another name for salvage value, the remaining value of an asset after it has been fully depreciated. This cost is calculated using a number of factors, generally a vehicle market value for the term and mileage required is the start point for the calculation, followed by seasonality, monthly adjustment, lifecycle and disposal performance. It describes the future value of a good in terms of absolute value in monetary terms and it is sometimes abbreviated into a percentage of the initial price when the item was new. In other words, the residual value could be defined as an estimated amount that an entity can obtain when disposing of an asset after its useful life has ended. When doing this the estimated costs of disposing of the asset should be deducted. The formula to calculate the residual value for this case study was established in 10% of the purchase cost.

(Vc) Purchase Cost: the investment cost considered to acquisition a new vehicle, similar as type A. To simplify, the investment was considered paid full at the purchase moment. Value: € 240.000,00.

(N) Estimated Lifespan: The estimated age indicated by the company and adopted for this study was of 14 years, which is similar to other ones used by other Spanish companies. Notice that this parameter is above the average value in other countries such as United States, France and Italy, where the vehicles' lifespan considered is 12 years. Probably, the very important economic crisis suffered by Spain on this period can be the explanation for this increase on the estimated lifespan.

Operational and maintenance total cost model

To generate the model that predicts the operational and maintenance total cost in function of the age and mileage simultaneously $f(x, y)$, a regression analysis with a minimum distance algorithm was performed using ®Minitab software to find the best fit for a quadratic equation as shown below:

$$f(x, y) = a^* x^2 + b^* y^2 + c^* x^* y + d^* x + e^* y + f \qquad (7)$$

Where:

$\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}, \boldsymbol{e}, \boldsymbol{f} \in \boldsymbol{R}$

In order to add to the Monte Carlo simulation the variation that a deterministic approach cannot give, the constants from the quadratic function will be considered as uniform random variables with a "p" variation of the data field:

$$X_i \in [(1-p) * x_i ; (1+p) * x_i] \quad (8)$$

Where:

X_i : Set of possible results for a random variable x_i .

where: $x_i = a$; $x_i = b$; $x_i = c$; $x_i = d$; $x_i = e$; $x_i = f$

and $p \in [0; 1]$.

RESULTS

For a better understanding, firstly, the optimum replacement moment was determined using the conventional Life Cycle Cost Analysis (LCC) methodology. After that, an analysis using the LCC and Monte Carlo Simulation method combined was performed and finally, results can be compared.

Conventional LCC Method

The conventional life cycle cost analysis is based on engineering economics to identify a point of an a given assets life where the cumulative cost of operating (O), maintenance (M) and ownership costs reaches its minimum value. According to Fan and Jin (2011), the most widely accepted approach is called the "cost minimization method". Gransberg and O" Connor (2015) describes it as "the most appropriate analysis method" and proposes that it "yields an optimum replacement timing cycle and a corresponding **E**quivalent **A**nnual **C**ost (EAC). In order to establish their useful life, particularly for buses, it is of key importance the understanding of the concepts explained in the following figure 1.

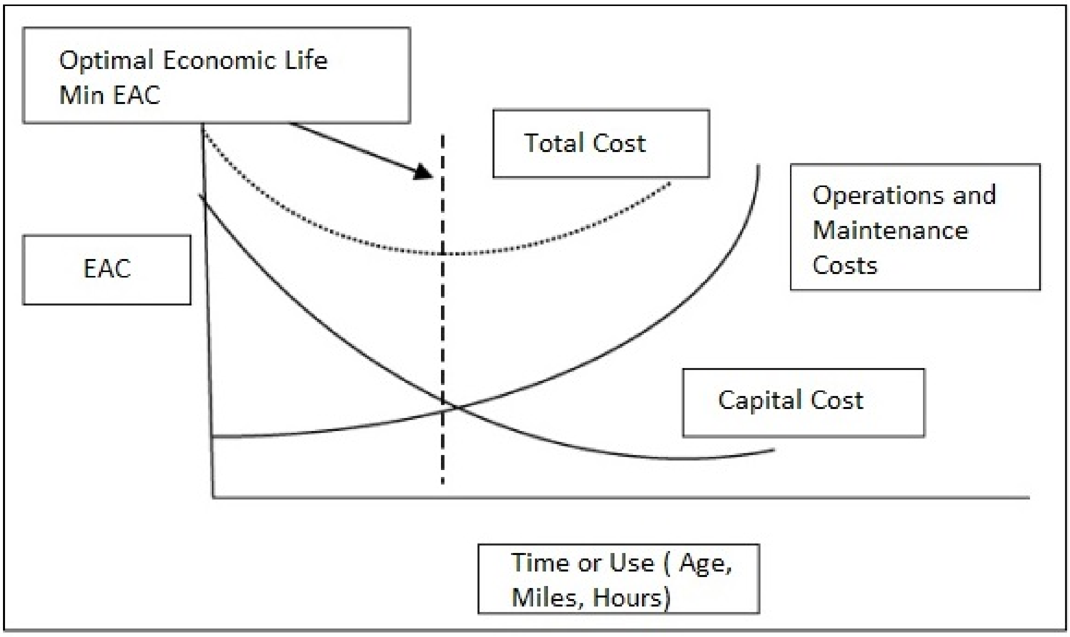


Figure 1 - EAC

It shows that as its capital value decreases while its operation and maintenance costs increase. The theoretical optimum service life is the point where cumulative costs are at the minimum and defines the economic life. From a financial standpoint, the cost object of minimum life cycle is the ideal age of retirement and/or replacement this, shown in years of life or mileage travelled. This analysis has to be subject to the same working conditions, in order to observe a similar trend. According to the asset type, design specifications and the service to be performed, it's clear that total vehicle mileage is, in most cases, a better indication of asset ware than the vehicle's age.

For this case study, some aspect should be taken into account:

1. The maintenance costs were selecting and adjustments based on Extrapolation Mathematics Technique, to obtain a set of observations and extend this pattern into the future.

2. The **T**otal **A**ccumulated **C**ost (TAC) until a certain year is the result of the total investment cost plus the maintenance costs and the operating costs. All of the costs were accumulated until that year less the resale value of that year.

3. The **A**verage **A**nnual **C**ost (AAC) indicates the cost accrued until the vehicle’s life divided by its life time thereof, so that the minimum average annual cost will determine the optimal time for the vehicle’s renewal, which presents the lowest possible cost for the vehicle operation (formula 9).

$$AAC = \frac{[\sum Investment + \sum Maintenance \quad + \sum Operating - (Resale\ Value)]}{n\ (year)} \quad (\mathbf{9})$$

4. The total costs incurred by the company for the vehicle are represented on table 1.

Year (n)	∑ Purchase Cost	∑ Maintenance Cost	∑ Operation Cost	VR Resale Value	TAC	AAC	AAC/km
1	€ 240.000,00	€ 8.581,30	€ 30.143,09	€ 180.656,22	€ 98.068,16	€ 98.938,36	€ 1,61
2	€ 240.000,00	€ 17.913,63	€ 62.778,49	€ 168.612,43	€ 152.079,69	€ 76.475,59	€ 1,24
3	€ 240.000,00	€ 28.062,72	€ 96.514,27	€ 156.568,64	€ 208.008,36	€ 69.627,09	€ 1,13
4	€ 240.000,00	€ 39.100,07	€ 129.365,81	€ 144.524,84	€ 263.941,04	€ 66.203,87	€ 1,07
5	€ 240.000,00	€ 51.103,41	€ 166.922,78	€ 132.481,05	€ 325.545,15	€ 65.284,25	€ 1,06
6	€ 240.000,00	€ 64.157,28	€ 196.474,96	€ 120.437,25	€ 380.194,99	€ 63.512,15	€ 1,03
7	**€ 240.000,00**	**€ 78.353,63**	**€ 229.795,11**	**€ 108.393,46**	**€ 439.755,29**	**€ 62.947,89**	**€ 1,02**
8	€ 240.000,00	€ 93.792,45	€ 267.635,53	€ 96.349,66	€ 505.078,32	€ 63.245,05	€ 1,03
9	€ 240.000,00	€ 110.582,47	€ 307.604,76	€ 84.305,87	€ 573.881,36	€ 63.862,86	€ 1,04
10	€ 240.000,00	€ 128.841,96	€ 346.550,49	€ 72.262,08	€ 643.130,38	€ 64.401,73	€ 1,05
11	€ 240.000,00	€ 148.699,52	€ 384.039,34	€ 60.218,28	€ 712.520,58	€ 64.855,48	€ 1,05
12	€ 240.000,00	€ 170.295,02	€ 423.066,24	€ 48.174,49	€ 785.186,77	€ 65.506,63	€ 1,06
13	€ 240.000,00	€ 193.780,56	€ 462.093,14	€ 36.130,69	€ 859.743,00	€ 66.203,00	€ 1,07
14	€ 240.000,00	€ 219.321,56	€ 501.120,04	€ 24.086,90	€ 936.354,70	€ 66.946,73	€ 1,09

Table 1

In summary, the results demonstrate the minimum AAC is located between year 7 and 8, with an average annual cost per km of 1,02 €/km and 1,03 €/km (AAC/km)**,** which includes the investment amortization, maintenance and operation, excluding infrastructure and management costs.

Life Cycle Cost and Monte Carlo Simulation

Before applying, the combined methodology previously exposed, in order to prove the real necessity of this type of approach, in figure 2 the real annual mileage data can be observed from a group of five different vehicles in a Spanish transport fleet and the high variability present between different vehicles. These buses have the same assembling characteristics (engine, load capacity and dimensions), and they are of the type A (same model used on the LCC method), identified by the numerical code: 1; 2; 3; 4 and 5. This different mileages weights differently over the maintenance and operating costs, which are not identified by the LCC method at first because it needs a more detailed analysis. As a solution to this discrepancy, it is necessary to adopt the combined LCC and MCS method.

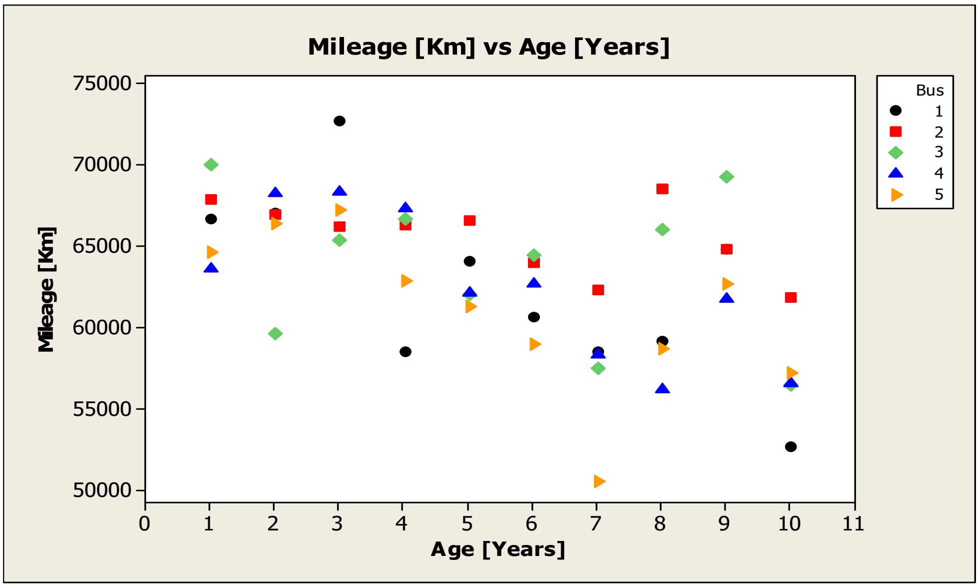


Figure 2

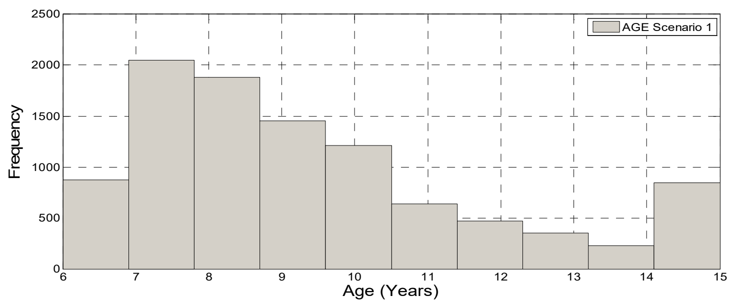
The method was developed considering two different scenarios for analysis. In scenario 1, an annual mileage restriction was considered based on real data of the company between 55.000 km and 80.000 km. In scenario 2, a hypothetical optimum situation was selected, considering in this case an annual mileage ranging from 55.000km to 135.000 km (that last value can be considered the maximum theoretical mileage per year). The idea is to add successive solutions of troublesome scenarios to the solution of the stochastic problem.

This technique gives a reliable mathematical basis for solutions derived from individual scenarios and it can be applied to linear problems in order to improve pure scenario analysis. The data used for the development of the Monte Carlo Simulation were processed using software *Matlab*.

Results Life Cycle Cost and Monte Carlo Simulation Combined

1) Age Indicated

The ages indicated for replacement fleet were obtained and are graphed shows as histogram (figure 3).

Scenario 1	

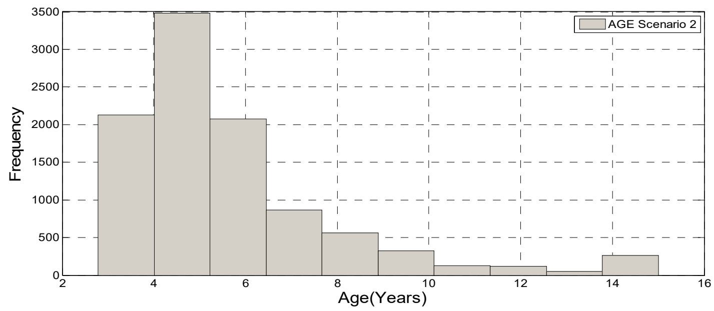
Scenario 2	

Figure 3

The analysis of the histograms for both scenarios show us which is the best option for replacement decision taking into account restrictions defined before. For Scenario 1, 7th year is the highest value but 8th year is quite close and it can be observed that values obtained are more uniformly distributed. For Scenario 2, 5th year is clearly the most convenient option.

2) Mileage Indicated

The mileages indicated for replacement fleet were obtained and are graphed shows as histogram (figure 4).

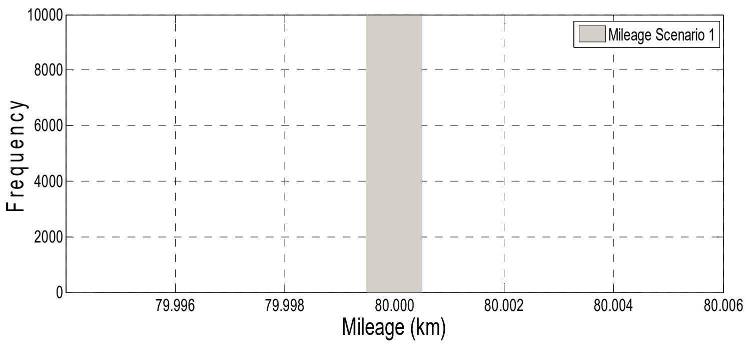
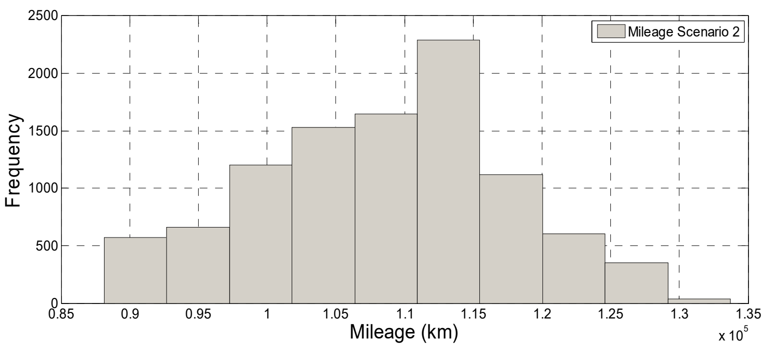
Scenario 1	
Scenario 2	

Figure 4

The histogram's analysis of Scenario 1, which presents the best mileage to be used by the vehicle, shows that the values in km are inclined to the established value of 80.000km (simulation's restraining condition), which proves that the restraining conditions do not represent the minimal cost real points of an unrestrained scenario. This fact is proved in Scenario 2, which presents an obtained superior value between 110.000 and 115.000 km.

3) General Cost Analysis

This analysis shows which parameter(s) would be the best for the fleet's vehicle use during its lifespan, optimizing its costs. For this analysis were correlated the parameters: Annual Mileage, Age of the vehicle and the Annual Average Cost (AAC). Figure 5 shows one sample out of ten thousand possibilities obtained by scenario 2.

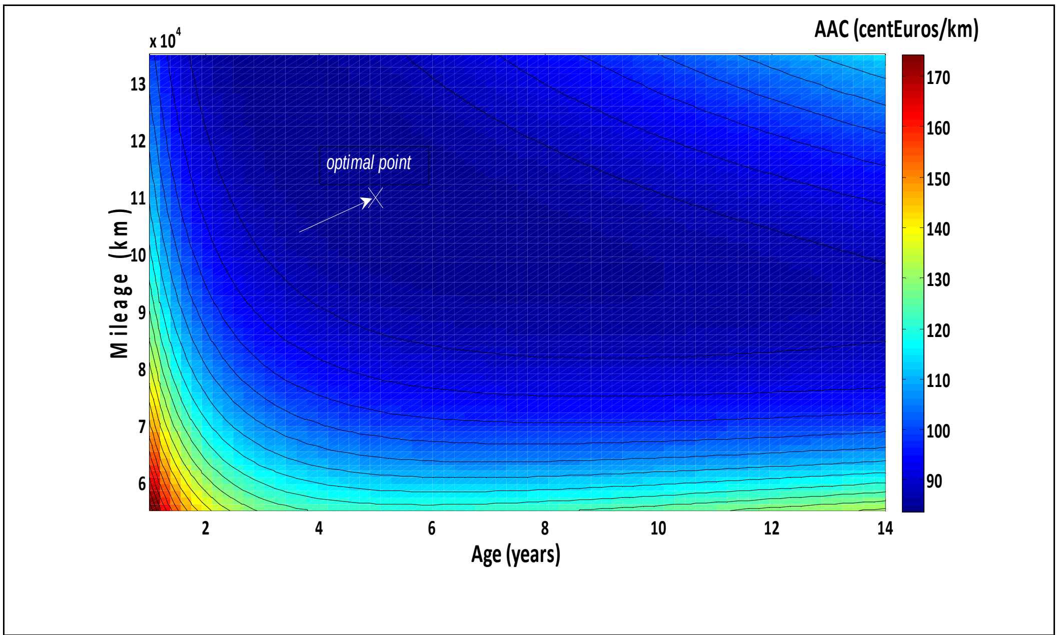


Figure 5

Furthermore the fact to be possible to indicate an optimal point (figure 5), this analysis demonstrates another combinations over its lifespan, and it might effectively support fleet managers in the developing of their strategies and activities, especially when fleet managers need to access the trade-off between costs and benefits related to different parameters.

CONCLUSION

This paper demonstrated and applied an economic model for the determination of the adequate replacement moment of bus fleets in function of Random Variables. Such model has two development parts. On one hand, a deterministic or conventional methodology Life Cycle Cost Analysis, on the other hand a probabilistic model, which uses the LCC with Monte Carlo Simulation. In order to prove this methodology, a real study case of an urban

Spanish fleet was realised, in which fixed parameters and random variables were correlated simultaneously.

The combination between these two models allows not just the evaluation of the proper replacement moment, but also a general cost analysis over the fleet' s useful life, which indicates the age, run mileage and the unit cost per mileage in an organised and simultaneously way over the vehicle's useful life.

Besides, this research permits the management and the evaluation of the vehicle's life cycles, since it is a tool that assesses the replacement decision of the vehicles' for a similar one.

Nevertheless, a final replacement decision should take into account not just economic criterions, but also a variety of factors different from those previously studied, such as fleet's size, real mileage, number of workers and passengers, service quality, government transport policies, environment, annual budget, among others.

Finally, another possible function of this model, which will be studied on future researches, could be to improve the management with an ideal replacement strategy, regarding how much and which vehicles should be replaced.

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