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TECHNICAL STUDY FOR THE RESTORATION OF MURAL PAINTINGS THROUGH THE TRANSFER OF A PHOTOGRAPHIC IMAGE TO THE IRREGULAR SURFACE OF THE VAULT OF THE SANTOS JUANES CHURCH IN VALENCIA (SPAIN)

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Abstract:	This study, which is part of the project for the restoration of the pictorial set that Antonio Palomino painted on the vault of the central nave of the Church of the Santos Juanes in the city of Valencia (Spain), describes the complete geometric study for the digital restitution of these paintings, after their partial destruction in a fire caused during the Spanish Civil War.
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IN VALENCIA (SPAIN)**

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Abstract

This study, which is part of the project for the restoration of the pictorial set that Antonio Palomino painted on the vault of the central nave of the Church of the Santos Juanes in the city of Valencia (Spain), describes the complete geometric study for the digital restitution of these paintings, after their partial destruction in a fire caused during the Spanish Civil War.

Two analogic black and white photographs taken before the fire (1920), will be the key to the rehabilitation of these valuable paintings. 3D modeling with laser scanning systems of the vault provides the geometry with precision and a high level of detail of the surface on which the paintings were carried out, in addition to the structural elements that make up the architecture of the temple.

The solution proposed approach a new methodology for the digital reconstruction of the rectified image and its subsequent georeferencing, as well as the entire environment to determine the correct location of the artistic work. Today, digital (or virtual) restoration is considered an accepted technique for the restoration of old photographs, but it is also an excellent opportunity to develop possible solutions in restoration of paintings and frescoes.

This paper describes the initial process of transferring the paintings of this Spanish artist, from a photographic image of the original frescoes (2D) to the irregular surface and volume (3D) of the current cover of this Valencian Royal Parish, through a restoration of the mural set.

Keywords

Digital restoration, Cultural Heritage, 3D model, laser scanner

1. INTRODUCTION

Antonio Palomino (Córdoba, 1655-1726), a chamber painter of Carlos II, has his most ambitious piece, made between 1699 and 1701, displayed in the vault of the Royal Parish of Santos Juanes in Valencia. It is a fresco mural painting, which shows an extensive pictorial program that occupies an area of approximately 750 square meters that is intended to be recovered.

This great pictorial composition, a work partially destroyed in July 1936 by fire, is complex and takes place on a continuous barrel vault which is 31 meters long, 15.5 meters wide. The surface of this vault is not uniform, but warped and with an appreciable irregularity.

In addition to the fire, a restoration team carried out an intervention that fragmented these paintings between 1958 and 1963. The Gudiol team tore off some of the paint that adhered to a plywood support and after touch-ups and repaints, they replaced the piece on 90 panels.

In this proposed intervention, the whole complex will be acted upon, considering that around 70% are original pictorial remains. This action will allow the recovery of Antonio Palomino mural painting and contribute to the preservation of the Spanish historical artistic heritage [1], its own cultural heritage of the past in a period of transition between monarchical dynasties. Living cultural heritage is inherited by the current societies and should be preserved in the most optimal conditions for the next generations [2].

The solution proposed in this research is carried out from the digital treatment of two photographs of the previous state of the paintings. These black and white photographs provide the only almost complete overhead view of the vault before the fire.

1 Digital restoration technology opens up a new field for wall painting and allows
2 simulations before a painting is restored [3]. To provide geometric or dimensional
3 validity, these images must be rectified to approximate a geometric representation free of
4 perspective and scale errors. Laser Scanner (TLS) and photogrammetry techniques are
5 now commonly used in heritage recording because of their potential to generate 3D point
6 clouds efficiently and reliably. [4-7].
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14 Subsequently, it is necessary to transfer a single rectified image onto the vault through a
15 projection that preserves its metric properties. Traditional projection systems are used for
16 regular surfaces or geometric bodies, but they are not useful when the surface to which to
17 transfer the image is irregular, as it is in this case. In this project, new methodologies have
18 been developed that allow a process of rectification, straightening and scaling of the
19 photographic image to be carried out, where linear and angular deformations are
20 minimized as much as possible, trying not to alter the aesthetic properties of the paintings.
21 This process allows the digital reconstruction of the missing fragments, as well as the
22 composition and placement of the pictorial composition in its original place.
23

24 **2. MATERIALS AND METHODS**

25 ***2.1. The church and the paintings***

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The `Santos Juanes Real Parish` is a church has been listed as a National Historic-Artistic Monument since 1947. The temple, located outside the medieval city, was one of the thirteen parishes established by King Don Jaime after the conquest of Valencia in 1238. The structural base of the building is of Gothic origin, but later renovated (1693-1702) with a Baroque aesthetic on the inside and a Mannerist on the outside. In this reform, a partitioned barrel vault was built as a support for the pictorial decoration; it was not assigned any overload, just enough stiffness to support its own weight. This vault hides the Gothic structure with its form arches and transept, which are preserved intact. The

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main vault is completed with twelve lateral lunettes or secondary vaults, which in this case, in addition to providing light to the interior of the temple, increase its pictorial surface.

The pictorial composition was carried out by Antonio Palomino, influenced by Luca Giordano and by the great compositions that were made in Italy. In the vault and the apse of the central nave of the temple, the artist developed a well-articulated and executed iconographic program. For the interpretation of the paintings, Palomino left in his treatise *‘Museo Pictórico y Escala Óptica’* a detailed description of them.

2.2. Photographs

The two available photographs were taken before the 1936 fire by Juan Alcón and belong to the Luis Roig D’Alos archive. The main photograph offers an almost complete image of the paintings from an overhead view of the vault, using a nadir angle shot (Figure 1a). Figure 1b shows the image of the second photograph and includes the area of the vault closest to the apse, which has been used to complete this common area with the main photograph.

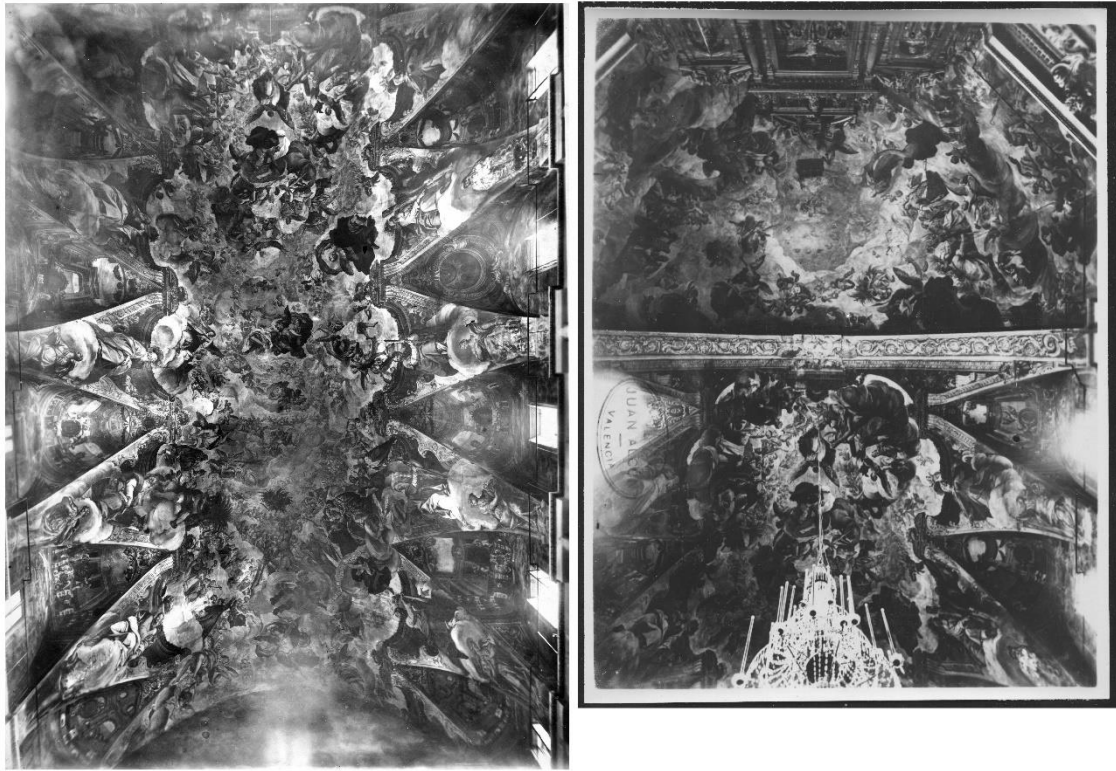


Figure 1. Original photographs of the painting before the 1936 fire (J. Alcón) (a)

Painting in the vault of central nave (b) Painting on the vault of presbytery

To obtain these two nadir angle photographs, the camera had to be placed near the ground in a vertical direction, achieving a central perspective, since the lines of flight tend towards the centre of the scene. In both cases, the geometry of the camera is unknown, but being an older camera that used a glass support with a photosensitive emulsion, the type of lens used minimized image distortion. Therefore, it has been found that the radial distortion is negligible and does not introduce any error in the process.

Once the photographs have been digitized, a coordinate system is defined with the origin at the centre of the image. To relate the image to the paintings in their original position, it is necessary to determine the image coordinates (x_i, y_i) , measured in pixels, of a sufficient number of homologous points. These identifiable points in reality have allowed the calculation of the camera's capture point (from where the photographs were taken), as well as the rectification, straightening and scaling of the pictorial images.

2.3. Generating the 3D model

The geometry of the pictorial composition is a determining element because any alteration affects the aesthetic and dimensional properties of the iconography. The first phase consisted in defining as accurately as possible the area where the paintings had been located. The vault, which a priori, is a geometric element in which half a cylinder could be adapted, has turned out to be a surface with irregular curvature, with holes and protuberances, and on which the complex surface formed by the lunettes extends.

There are several ways to reconstruct objects digitally, however, when the geometry of the object is complex and the size of its surface is large, 3D surveying using laser scanning techniques is the technology that best allows a digital model of the object to be obtained with a high level of detail (Figure 2a). Three-dimensional modelling of cultural heritage with metrological documentation aims, is an expanding application area [8, 10]. Using the 3D laser scanning technique, several models of the church (exterior, interior and vaults) has been recorded in millimetre detail [11, 12].

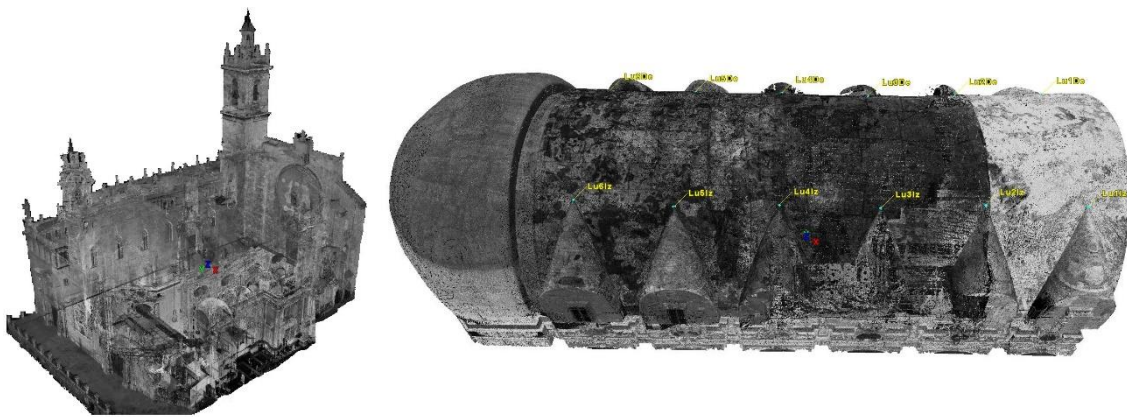


Figure 2. 3D points cloud (a) Church (b) Vault

To obtain the 3D model of the surface of the vault a Leica RTC360 laser scanner was used. Figure 2b shows the 3D geometry of the area, which is a half cylinder with a radius of 7,75 m, 31 metres in length and a total area of 750 m².

To define the coordinate system of the model, the direction of the central aisle of the church nave is adopted as the x-axis. In this reference system the model coordinates (X , Y , Z) of the homologous and identifiable points in the image are determined. The selected points are spatially distributed by the vertices of the lunettes and the corners of the railings.

3. METHODOLOGY

The fundamental process to be carried out is based on a novel method in which photogrammetric techniques are applied, complemented by laser scanner systems to establish the geometric relationships that allow the images to be rectified and moved to their original location. A graphic version of the workflow is provided in Figure 3.

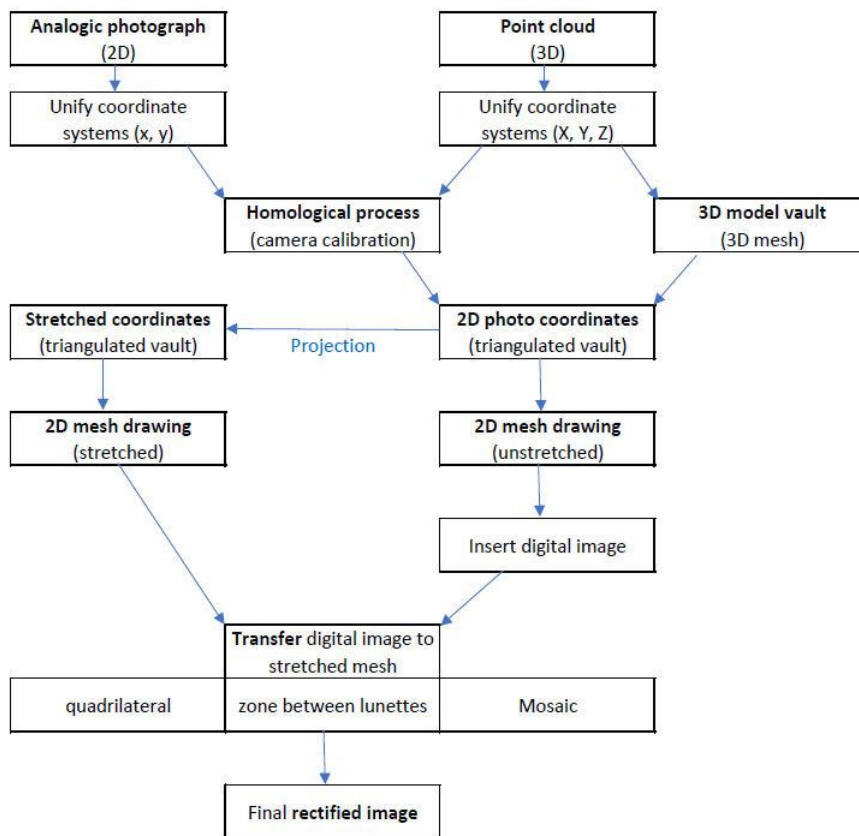


Figure 3. Workflow

Through different processes the images have been calibrated, establishing the relationship between the image and spatial coordinates, as well as the cutting of each

quadrilateral and its projection on the rectified surface of the vault. In another phase, the 3D model of the vault has been generated, simplifying the point cloud to flat quadrilaterals and stretching said surface with minimal deformation. And finally, the union of the cut images (tiles) is carried out as a mosaic to generate the rectified image.

3.1. Homological process

Since a photograph is a 2D document, it is necessary to project the image onto the 3D model of the irregular surface of the vault. To do this, it is necessary to establish a relationship of each pixel of the image (x_i, y_i) with its corresponding spatial position in reality (X, Y, Z) . The equations [1] that govern that the image of a straight line in space is a straight line in the image, are:

$$x_i = \frac{a*X+b*Y+c*Z+d}{a_2*X+b_2*Y+c_2*Z+d_2}; \quad y_i = \frac{a_1*X+b_1*Y+c_1*Z+d_1}{a_2*X+b_2*Y+c_2*Z+d_2} \quad [1]$$

where

a, b, c, d: projective parameters including the Euler matrix angles and the scale factor between object space and image space;

(X, Y, Z) : coordinates of a point in the 3D model;

(x_i, y_i) : coordinates of the same point in the photograph.

As the photographs are practically vertical, the coefficient $c_2 = 1$ is taken, adopting the vertical axis of the model as the z axis of the coordinate system. The approach of the homological method would be the following [2]:

$$a_2 * X * x_i + b_2 * Y * x_i + c_2 * Z * x_i + d_2 * x_i - a * X - b * Y - c * Z - d = 0$$

$$a_2 * X * y_i + b_2 * Y * y_i + c_2 * Z * y_i + d_2 * y_i - a_1 * X - b_1 * Y - c_1 * Z - d_1 = 0$$

[2]

This linearized mathematical model can be expressed as a system of equations in matrix form of the type $[A]*[x]=[k]$, which allows the determination of the coefficients of the

1 equation, and from them, the shooting position, its direction and the focal length of the
2 camera, whose solution is determined by [3]:
3

$$4 \quad [x] = ([A]^t * [P] * [A])^{-1} * ([A]^t * [P]) * [k] \quad [3]$$

6 where

7 [A]: matrix of the coefficients

8 [x]: matrix of unknowns containing the coordinates of the camera

9 [K]: matrix of independent terms

10 [P]: weight matrix

11 It is now possible to establish the correspondence of all the pixels of the images with their
12 corresponding points on the surface of the dome.
13

14 **3.2. Development of irregular surfaces in 2D**

15 The next step is to fit a Delaunay triangulation to the point cloud of the vault obtained
16 with the laser scanner, obtaining a continuous surface of triangles that fit the real surface
17 with great precision.
18

19 To project this network of triangles onto a plane, a cylindrical development like those
20 used in cartographic projections could be used, but this type of projection only preserves
21 one of the three metric properties (angles, distances or areas). Furthermore, as it is an
22 irregular surface, the vault is not a perfect cylinder, so it is necessary to develop an
23 algorithm that builds flat triangles with minimal deformation, in order to maintain the
24 aesthetics and shape of the original paintings.
25

26 The solution used in this study consists of a combination of projections to simultaneously
27 preserve the distances and angles of the triangles formed in the point cloud mesh [13].
28

29 For the algorithm to conserve the distances between points in space (3D), whose distance
30 D_{ij} is equation [4]; the distance d_{ij} between those two projected points (2D) must be like
31 equation [5]:
32

$$D_{ij} = \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2} \quad [4]$$

$$d_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} + r_{ij} \quad [5]$$

Similarity, for the algorithm to conserve the angle between two vectors in space Ω [6], defined by three points (i, j, k) , the projected angle ω must be given by its scalar product [7]:

Ω

$$= \text{arc cos} \frac{(X_j - X_i) * (X_k - X_i) + (Y_j - Y_i) * (Y_k - Y_i) + (Z_j - Z_i) * (Z_k - Z_i)}{\sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2} + \sqrt{(X_k - X_i)^2 + (Y_k - Y_i)^2 + (Z_k - Z_i)^2}} \quad [6]$$

ω

$$= \text{arc cos} \frac{(x_j - x_i) * (x_k - x_i) + (y_j - y_i) * (y_k - y_i) + (z_j - z_i) * (z_k - z_i)}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2} + \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2 + (z_k - z_i)^2}} + r_i \quad [7]$$

where

$[X_i, Y_i, Z_i]$: coordinates of a point in space 3D

$[x_i, y_i, 0]$: coordinates of a point on the projected plane 2D

$[r_{ij}]$: the remainder of the distance in the development

$[r_i]$: the remainder of the development angle

The developed system is based on the resolution of a free network with two position constraints and one rotation constraint, so that, by assigning weights to the angle and distance equations, the solution minimizes the sum of the squares of the residuals. In other words, the defined weights act as elastic modules that act on the angles and distances of the mesh. It is therefore a question of obtaining the three-dimensional surface of the stretched-compressed vault in a network of 2D triangles with minimal deformations.

3.3. Rectified and scaled historical photography

A photograph from the point of view of projective geometry is a central projection of the image on the plane of the negative. The original photographs of the vault that contain the image of the paintings before the fire are in analogic format, and have previously been converted to a digital format.

The next step is to apply a two-dimensional projective transformation [8] that incorporates the image of the photographs in the coordinate system defined by the geometry of the developed vault.

$$X_i = \frac{ax+by+c}{a_2x+ b_2y+1} ; Y_i = \frac{a_1x+ b_1y+c_1}{a_2x+ b_2y+1} \quad [8]$$

In this projective rectification, the image is moved, rotated and deformed, defining the mathematical transformation between the coordinates of a minimum of four identifiable points, both in the photographic image and in the vault model. For this digital rectification, the photograph has been divided into quadrilaterals (trapezoids), so that each area is defined by the photo coordinates and model coordinates of its four vertices. Subsequently, these rectified quadrilaterals are assembled into a single image.

This phase is completed by resampling pixels of the original image to create the rectified image, since the projected position will not match the original. In this case, the nearest neighbour method has been used to assign the gray value (ND) of the nearest pixel in the original image. The equation applied to each position in space provides the coordinates that each pixel of the calculated image must have in order to be projected in its correct position in space.

4. RESULTS AND DISCUSSION

4.1. Calibration of the photographs

To solve the system of equations [3] in section 3.1., 20 common points have been selected in space and in the image (Figure 4b), defining a system of 40 equations and 11

unknowns, in the case of photograph 1. As it is a practically vertical image, then coefficient $c_2 = 1$, the shooting direction is established as the z axis, the keystone of the vault as the x axis, and the y axis perpendicular to the other two. Table 1 shows the value obtained for the coefficients in photograph 1:

Coefficient	Value	Coefficient	Value	Coefficient	Value
a	181,4255067	a_1	-5724,23365	a_2	0,02134364
b	-5607,84664	b_1	-201,704881	b_2	-0,01183005
c	-94,6362768	c_1	160,226527	c_2	1
d	-36204,4472	d_1	9697,78589	d_2	0,76502309

Table 1. Values resulting from the coefficients (first frame)

Derived from the calculation, the spatial position of the camera ($X_c = 1.894$ m; $Y_c = -6.380$ m; $Z_c = -0.881$ m), the direction of the shot ($\Delta X = -0.428$; $\Delta Y = -0.090$; $\Delta Z = 0.899$) and the main distance ($F_x = 5.610$; $F_y = 5.728$) were determined.

In the case of photograph 2, 20 common points in space and in the image have been selected (Figure 4c), defining a system of 40 equations and 11 unknowns. Table 2 shows the value obtained for the coefficients:

Coefficient	Value	Coefficient	Value	Coefficient	Value
a	117,9054083	a_1	-5895,36798	a_2	0,00221848
b	-5932,03125	b_1	-24,121732	b_2	-0,02050882
c	-518,329987	c_1	230,02115	c_2	1
d	-35222,3524	d_1	-90412,7465	d_2	1,28302141

Table 2. Values resulting from the coefficients (second frame)

Derived from the calculation, the spatial position of the camera ($X_c = -15.365$ m; $Y_c = -6.123$ m; $Z_c = -1.374$ m), the direction of the shot ($\Delta X = -0.428$; $\Delta Y = -0.090$; $\Delta Z = 0.899$), and the main distance ($F_x = 5.954$; $F_y = 5.898$) were determined.

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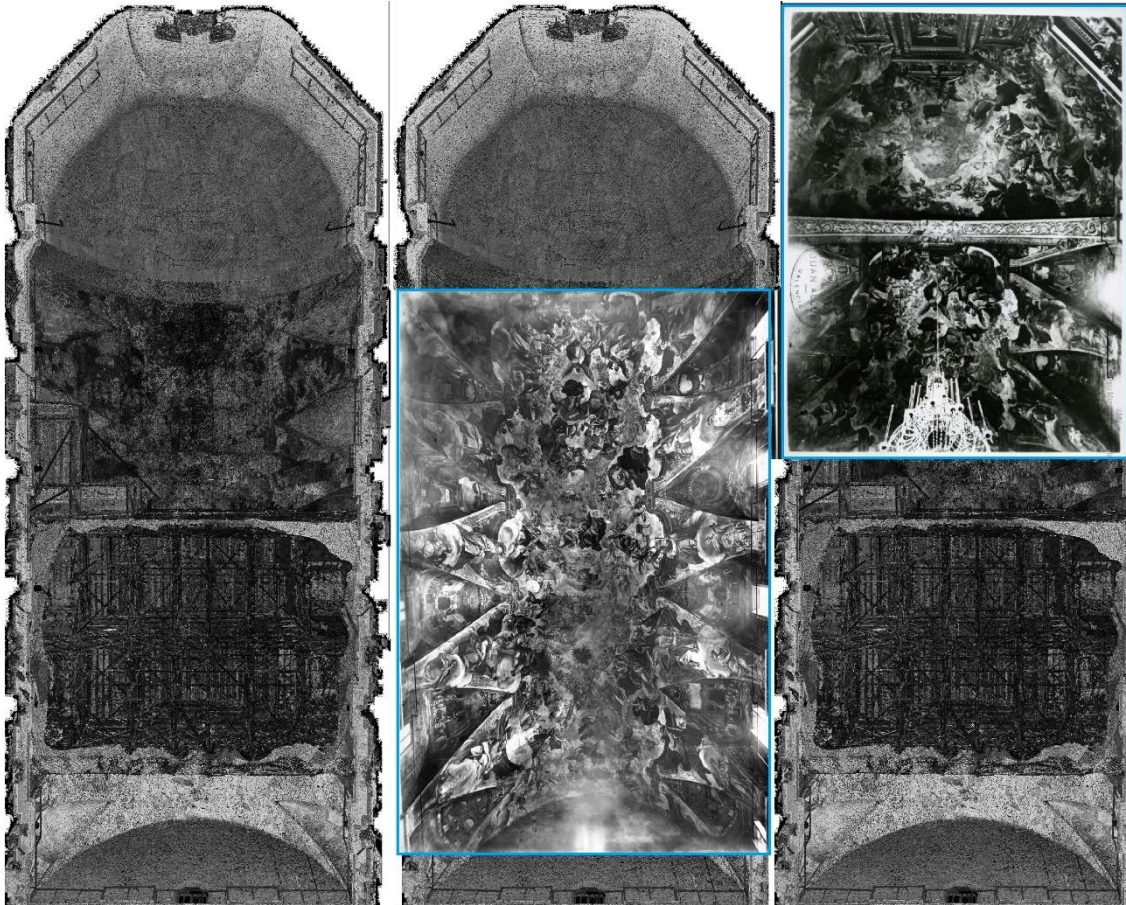


Figure 4. Selection of points. (a) 3D model. (b) First image. (c) Second image

Figure 5 shows an elevation drawing of the church and the point cloud of the vault, the location of the cameras and the direction of each one of them.

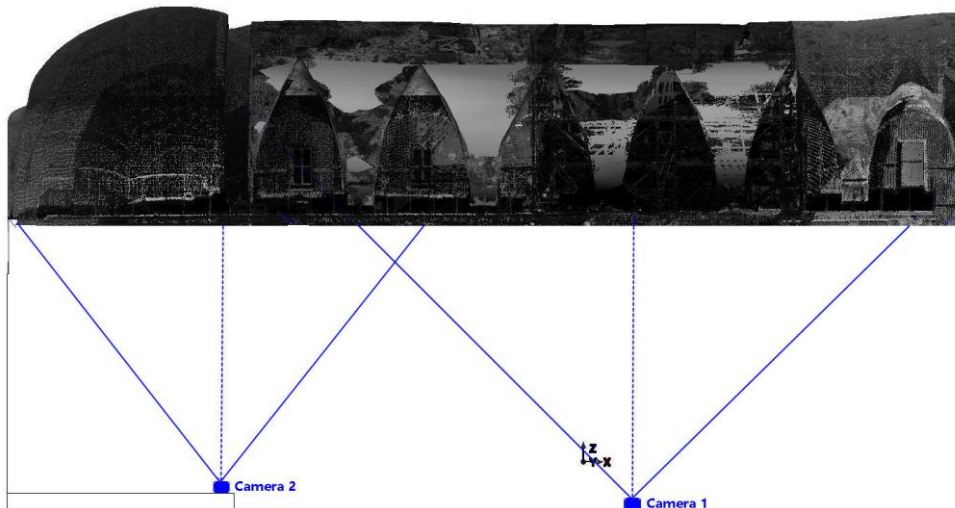


Figure 5. Shooting position of cameras

4.2. Vault development in 2D

The developed algorithm works as a perimechoic perspective cartographic projection, making linear alterations minimal [13]. This development positions the image of the photograph (2D) in the barrel vault (quasi-regular geometry) and in its oculi and lunettes (irregular geometry) modelled three-dimensionally (3D).

Previously, a triangulation of the vault has been carried out, forcing the formation of elongated triangles that better adapt to its geometry (Figure 6a). In this mesh defined analytically by points, lines and triangles, the system of equations (section 3.2) that solves this projection has been proposed. From the coordinates (x, y, z) of a series of singular points (Figure 6b) that define the vault and the area between the lunettes, the system has been solved by least squares, which has allowed obtaining a new flat triangulated surface (Figure 6c), where the differences between the real and projected measurements are minimized to simultaneously preserve angles and distances.

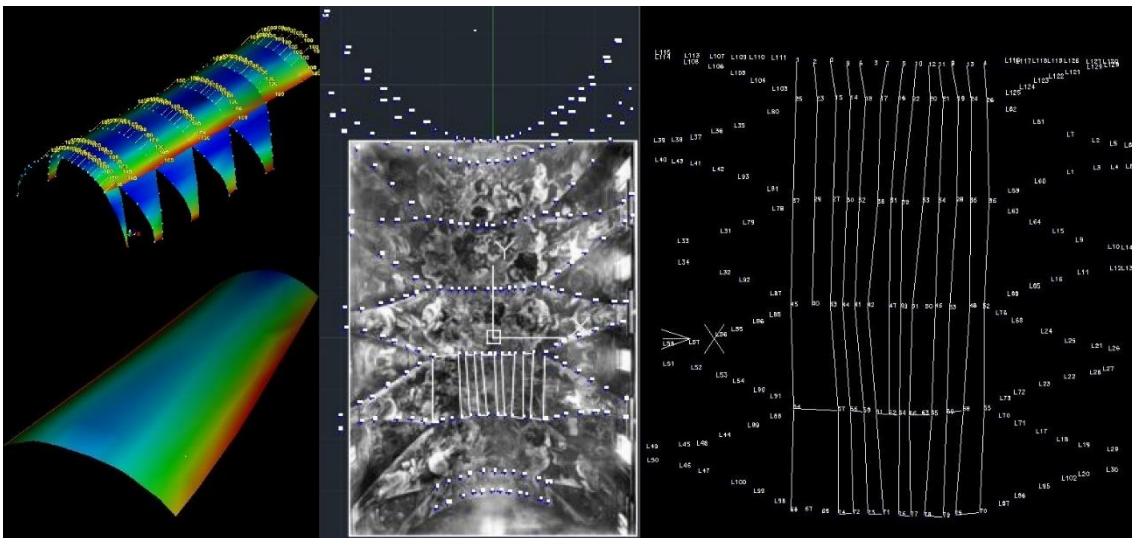


Figure 6. Vault development. (a) 3D model. (b) Singular points in 3D. (c) Singular points in 2D

4.3. Rectified and scaled historical photographs

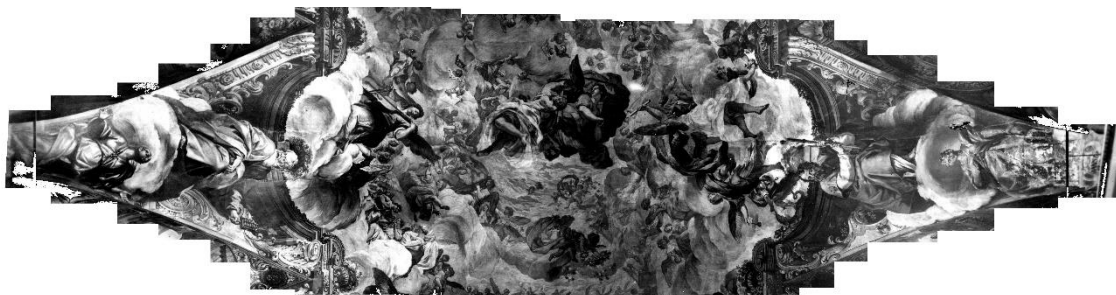
Once the architecture of the vault has been determined, the photographic image is transferred to the roof surface by rectifying, straightening and scaling the two

1 photographs, using the projective transformation. Since it is not possible to perform a
2 constant scaling of the complete image, a series of unitary areas have been designed for
3 quadrilateral rectification of the image. Each quadrilateral is defined by the coordinates
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5 quadrilateral rectification of the image. Each quadrilateral is defined by the coordinates
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7 (x, y) of its 4 vertices, thus proposing a system of 8 equations that solve the projective
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9 transformation [8].
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31 *Figure 7. Rectified digital images (areas 34-19, 34-20 y 34-21)*

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33 The entire vault has been divided into 7 zones, delimited between the vertices of the
34 lunettes, and each zone has been divided into 24 quadrilaterals. Figure 7 shows 3
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36 consecutive images of area 34 (areas 19, 20 and 21), where areas of overlap can be
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38 observed.
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54 *Figure 8. Zone 34 mosaic*

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56 The result has been a differential rectification of the photographs, stretching and
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58 compressing the surface of each trapezoid to adjust each one of the images, minimizing
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the residuals between the real measurements and the projected measurements. Figure 8 shows the mosaic formed by the union two by two, of all the quadrilaterals of zone 34 of photograph 1.

The next step was to form the mosaic that includes all the zones to obtain the complete rectified image of photograph 1 (figure 9a). Following the same steps as used for photograph 1, the mosaic is generated to obtain the complete rectified image of photograph 2 (figure 9b).

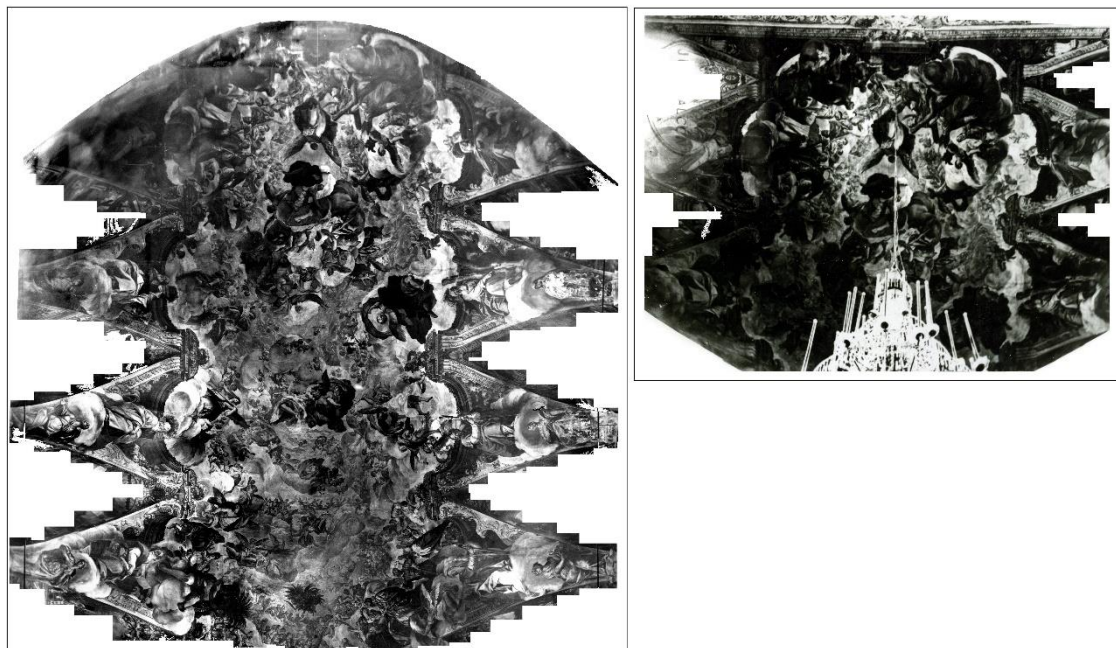


Figure 9. Rectified image (a) Full mosaic of photograph 1. (b) Full mosaic of photograph 2

The image in the upper zone of the photographs, corresponding to the zone in the vault of the nave closest to the apse of the church (zones 5-5 and 6-A), is more complete and with better definition and sharpness in photograph 2 than in 1. In this last step of the process, the montage of both photographs has been carried out, to complement part of photograph 1 and obtain a complete rectified image of the painting to be restored (Figure 10).

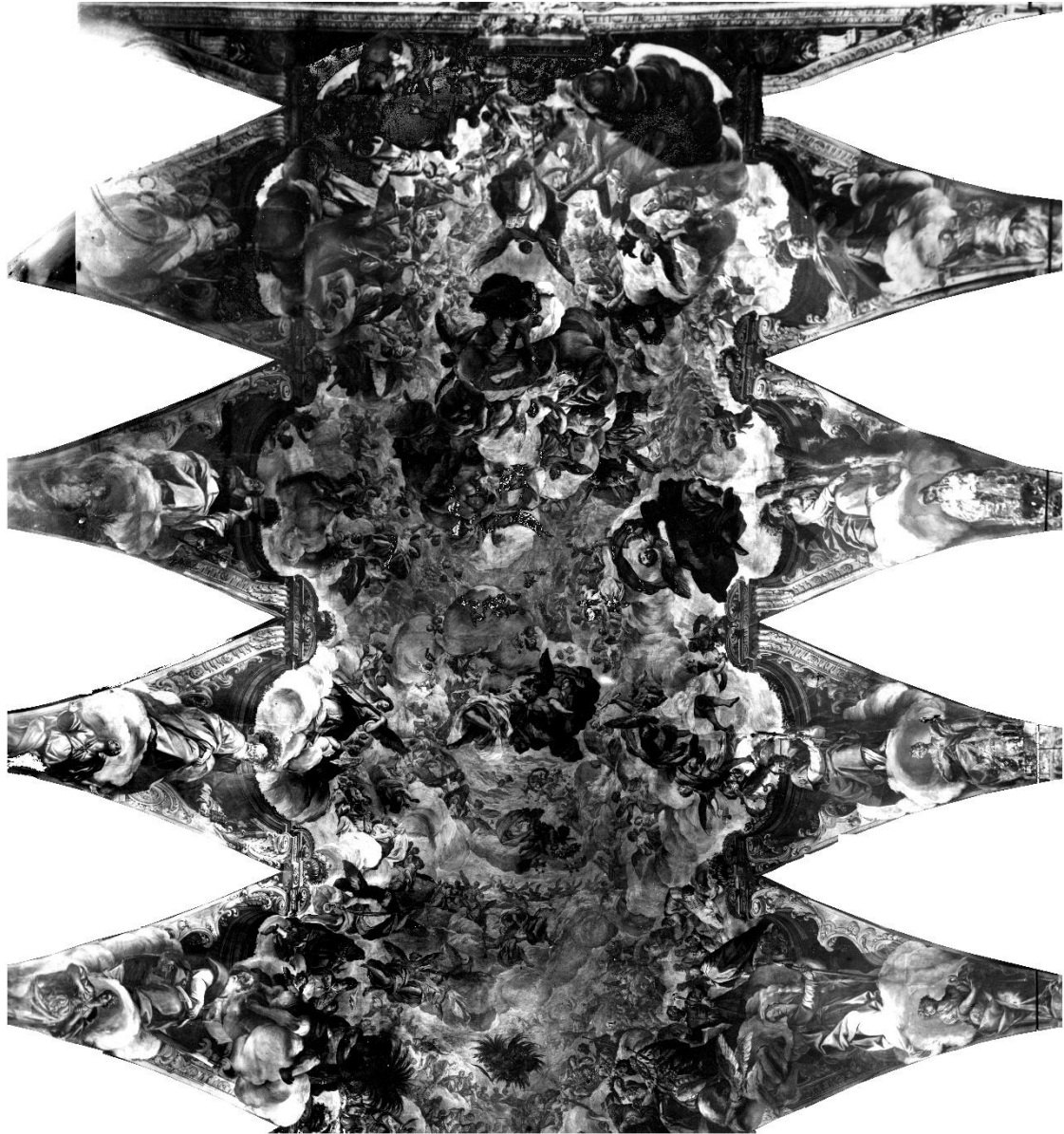


Figure 10. Rectified image of the Antonio Palomino paintings on the vault of the church of Los Santos Juanes

5. CONCLUSIONS AND FUTURE WORK

Once the digital treatment of the photographs of the original frescoes has been carried out, the method constitutes a novel approach for the restoration of images that develop on non-irregular surfaces. It is based on using a projection that, minimizing metric deformations, reconstructs the original shapes of the images with the greatest possible fidelity. The rectified and georeferenced photograph obtained is precise enough to allow

1 the image to be assembled on a real scale (1/1) with the original painting located in the
2 vault.
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4 The application of this technique that allows extracting said information, contributes to
5 the restitution of damages and losses of the original paintings. The proposed solution is
6 being applied to the restoration of the pictorial group that Antonio Palomino painted on
7 the vault of the central nave. In addition to replacing the loss of images, it will allow in a
8 future phase, to place each one of the disassembled panels for cleaning and reconstruction
9 in its correct position.
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19 This digital reconstruction process has been the basis for the transfer of images from the
20 original photographs to the irregular surface of the vault. This method can be exported to
21 other image transfers, since it is capable of correcting the distortions and deformations
22 that a geometric or perspective projection produces.
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