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Discrete Fuzzy System Orbits as a Portfolio Selection Method

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Abstract The purpose of this work is to approach the portfolio selection problem from a particular System Theory framework. The System will be formed by the set of public companies in the portfolio and a set of fuzzy relations on those companies. Starting with an equally split portfolio represented by a fuzzy set B, the orbit of B is computed for a particular period obtaining a portfolio to invest in the next period. We present an example finding nine portfolios to invest in nine months and we compare them with some optimal portfolios in the efficient frontier given by the Modern Portfolio Theory and with some random generated portfolios. We find a better performance in returns for the portfolio based on the systemic method.

Keywords Abstract System · Discrete Orbits · Fuzzy Finance

1 Introduction

A widely used approach in recent work on financial problems is the use of the fuzzy set theory, where fuzzy terms are used to model the uncertain environments (Tiryaki & Ahlatcioglu, 2009; Agliardi & Agliardi, 2011). A discrete Abstract System Theory in a fuzzy environment, based on binary fuzzy relations between objects forming the system and using fuzzy subsets of the set of those objects, can be a useful framework to deal with the portfolio selection problem under a systemic approach. The purpose of this work is to propose an example of that systemic approach. Firstly, for each day we build a fuzzy relation on the set M of public companies based on the closing values of that day. Then, starting with an equally split portfolio represented by a fuzzy set B, we compute the orbit of B for a particular period and we obtain a portfolio to invest in the next period. The portfolio obtained in that way has been built

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by making changes to the initial set B in a way that companies performing better obtain more weight, whereas companies performing worse lose weight. The strategy is to use the trends in the market known as momentum investing presented by Narasimhan Jegadeesh and Sheridan Titman (1993). Finally we test our example comparing the returns of the systemic portfolios with random portfolios and the ones obtained by applying Modern Portfolio Theory (Markowitz, 1952).

2 Previous Concepts

Although there are several references to the concept of system throughout history, there is a consensus identifying Von Bertalanffy as the father of the System Theory (Von Bertalanffy, 1968). Since Von Bertalanffyś work, several definitions of system have been made. Ma and Lin (1987) define a System as a pair (M,R) being M a set of objects and R a set of relations on those objects. Later Lloret, Villacampa and Us (1998) limited the Ma and Lin definition of System to a pair (M,R) where R is a set of binary relations on M and based on that definition, Esteve and Lloret (2006) adapted the concept of orbit for an abstract discrete System. On the other hand, fuzzy mathematics was born with the theory of fuzzy sets Zadeh (1962) and since this work many mathematical concepts have been generalized using a fuzzy framework where crisp set operations are generalized through triangular norms (Klement, Mesiar & Pap, 2013) so fuzzy set operations can be made. We will use the fuzzification of the concept of orbit (Perez-Gonzaga, Lloret-Climent and Nescolarde-Selva, 2015) to obtain a portfolio with a systemic method. The system dynamics are based on the concept of structure of relations defined bellow:

Definition 1 *Let S=(M,R) be an abstract system and given a fuzzy set $F \subseteq M$ and a fuzzy relation $r \in R$. The structure of the relations associated to r and applied to the fuzzy set F is defined as:*
$\varphi_r(F) = \{(y, \mu_\varphi(y)) : y \in M\}$ *where* $\mu_\varphi(y) = C_{x \in suppp(F)} \{T(\mu_r(x, y), \mu_F(x))\}$
Being C a T-conorm and T a T-norm.

Note 1 $C_{x \in suppp(F)} \{T(\mu_r(x, y), \mu_F(x))\}$ *represents the application of the T-conorm C with as many arguments as the cardinal of supp(F). Figure 1 shows an example of structure of relations.*

Definition 2 *Let S=(M,R) be an abstract system and given a fuzzy set $F \subseteq M$ and a fuzzy relation $r \in R$. We call the orbit of A:*
$Orb_r(A) = \bigcup_{i=0}^{\infty} O_i(A)$
Where $O_0(A) = A$ *and* $O_{n+1}(A) = \varphi_r(O_n(A))$ *being* φ_r *the structure of fuzzy relations for r.*

Note 2 *Each O_i is a fuzzy set, like its own orbit, formed by the fuzzy union (with the corresponding T-conorm) of these O_i.*

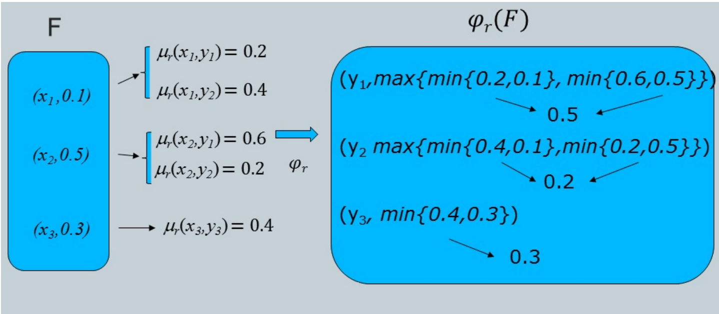

Fig. 1 Example of structure of relations for maximum T-conorm and minimum T.norm

3 Portfolio selection method

Considering a fuzzy system S = (M, R) such as M is a set of public companies, a fuzzy set $F \in M$ is the set $F = \{(x_i, \mu_F(x_i)), x_i \in M\}$ where $\mu_F(x_i) \in [0, 1]$ is the grade of membership of x_i in F and it will represent the fraction of the budget invested in the company i. Finally, $R = \{r_k : k \in K\}$ is a set of fuzzy relations built by an algorithm based on the stocks closing value data at day k, being card(K) the number of days for the decision period. So let $(x_i, x_j) \in MxM$ for instance, $\mu_{r_k}(x_i, x_j)$ may represents the portion of money invested in i to be transferred to j.

Discrete system dynamics are based on the concept of structure of relations defined above. Starting with the fuzzy set $B = [\frac{1}{card(M))}, ..., \frac{1}{card(M))}]$ which represents the initial situation where the budget is equally split between the public companies belonging to M. Let $O_0 = B$ and $O_{n+1} = \varphi_{r_n}(O_n)$ being φ_{r_n} the structure of fuzzy relations for r_n. Applying iteratively the structure of fuzzy relations φ_{r_n} as we will show in the example below, each fuzzy set O_i is interpreted as the new portfolio distribution according to the closing price at day i. (Fig 2)

By computing the union $Orb_r(A) = \bigcup_{i=0}^{\infty} O_i(A)^*$ using a T-conorm, we obtain a fuzzy set forming the orbit of the system. That orbit will be the chosen portfolio to buy right after the decision period. That portfolio would depend on how the fuzzy relation r is built, on the T-norm and the T-conorm used by the structure of relations and on the T-conorm C used to compute the fuzzy union. By choosing an appropriate r and a suitable T-norm and T-conorm the system can be adapted to different situations. In the example below we obtain a portfolio setting a particular r and particular triangular norms and we compare them with some optimal portfolios in the efficient frontier given by the Modern Portfolio Theory and with some random generated portfolios.
*To compute that infinite union we consider $O_n = \emptyset \ \forall n \geq card(K)$

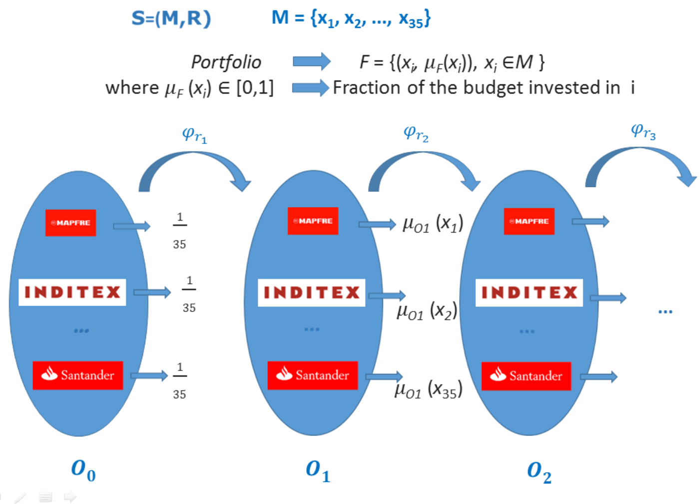

Fig. 2 Example of the portfolio selection model

4 Example

In this example we consider a portfolio formed by a combination of stocks belonging to the Spanish IBEX35 index. We will have therefore a fuzzy system S = (M, R) where $M = \{x_1, x_2, ..., x_{35}\}$ will correspond to the stocks for the 35 companies making up the index. A fuzzy set $F \in M$ is the set $F = \{(x_i, \mu_F(x_i)), x_i \in M\}$ where $\mu_F(x_i) \in [0, 1]$ is the grade of membership of x_i in F and will represent the fraction of the budget invested in the company i. On the other hand, R will be the set of fuzzy relations defined below.

For this example the product T-norm and the bounded sum T-conorm will be used. We have collected data from the period T between Jul 2nd 2015 and May 13th 2016, for each day k we define a fuzzy relation $r(k) \in R$ depending on the increment of the stocks closing value at day k:

Definition 3 *We define the fuzzy relation $r(k) \in R$ for $k \in \{1, ..., card(T)\}$ as:*

$$r_{ij}(k) = \begin{cases} \frac{Ret_j - Ret_i}{\sum_{n=1}^{35} r_{in}(k)} & if Ret_j - Ret_i \geq 0 \\ 0 & Otherwise \end{cases}$$

So let $(x_i, x_j) \in M \times M$: $\mu_{r_k}(x_i, x_j) = r_{ij}(k)$ represents the portion of money invested in i to be transferred to j. Note that the relation r thus defined is a symmetric relationship.

Following the concept of orbits presented before (Perez-Gonzaga, Lloret-Climent and Nescolarde-Selva, 2015) we will consider a series of fuzzy sets $\{O_i\}$. We begin with a fuzzy set $B = [\frac{1}{card(M))}, ..., \frac{1}{card(M))}]$ which represents the initial situation where the budget is equally split between the 35 public

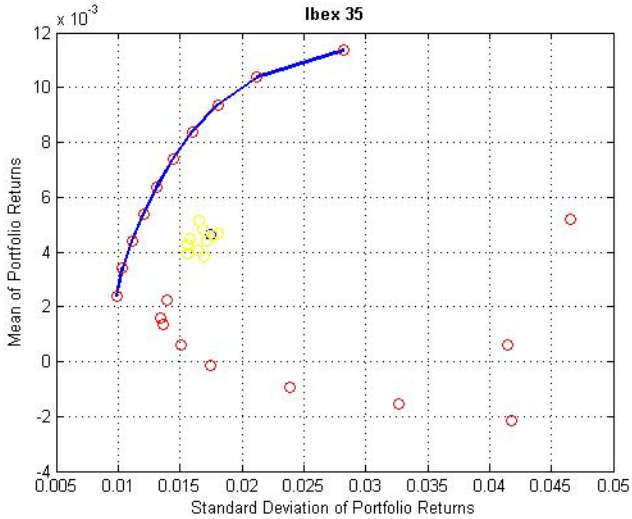

Fig. 3 Comparison Between MPT, Systemic and Random Portfolios

companies. Let $O_0 = B$ and $O_{n+1} = \varphi_r(O_n)$ being φ_r the structure of fuzzy relations for r. Applying to B the structure of fuzzy relations φ_r iteratively (using the product T-norm and the bounded sum T-conorm) a sequence O_i of fuzzy sets is obtained. Each fuzzy set O_i is interpreted as the new portfolio distribution according to the closing price at day i. The union of those O_i for a period will give us the particular portfolio to invest in the next period. The total days of data has been divided in 10 periods of 30 days each. In the first period we will obtain the portfolio to invest in the next period, whereas in the next nine periods we are investing the portfolio obtained in the previous period and studying to obtain the next portfolio to invest in the next period.

Finally we analyse the mean and standard deviation of portfolio returns for a portfolio given by this Systemic method and compare it with random portfolios and the efficient frontier given by the MPT. Figure 3 shows a representation of the efficient frontier for the period 6 (blue line), together with the systemic portfolio (blue circle), the portfolios given by MPT (red circles) and some random portfolios (yellow circles) all three obtained with data from the previous period. Table 1 summaries return means for the nine periods.

5 Discussion

Although the main goal of this work was to present a theoretical proposal, the example presented above supports the hypothesis of getting a better return performance by choosing a portfolio under a systemic method rather than a random method or the classic Modern Portfolio Theory. Further research has to be done in order to confirm this result. Since Modern Portfolio Theory was

Period	MPT	Systemic	Random
1	Only Study	Only Study	Only Study
2	0,0026	0,0039	0,0042
3	0,00088	0,0019	0,0014
4	-8,95E-07	2,54E-07	-1,44E-07
5	0,0019	0,0016	0,0012
6	6,88E-04	0,0046	0,0044
7	0,006	0,0033	0,0033
8	-0,0065	-0,0079	-0,008
9	-2,85E-04	0,0017	0,0019
10	0,0022	0,0011	8,32E-04
Return Means	0,000831346	0,00113336	0,001025734

Table 1 Comparison between means of the return

born to optimize returns while minimizing the risk, it is also necessary to study the variance behaviour of the systemic selection. One of the main problems to be solved is to determinate a right choice for the fuzzy relations and for the triangular norms.

6 Conclusion

We have chosen some portfolios by using a systemic approach. In that system money is moving around the stocks of 35 public companies forming the IBEX35 index. The orbit of that particular abstract fuzzy system, starting from an equally split situation, will lead to the systemic portfolio. Each portfolio has been built by using a particular fuzzy relation based on stocks closing price information for a particular decision period. Finally, we compare those portfolios obtained by the systemic approach with some optimal portfolios in the efficient frontier given by the Modern Portfolio Theory and with a set of random portfolios for different periods.

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Conflict of Interest The authors declare that they have no conflict of interest.

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