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Submission Template for IET Research Journal Papers

Equivalent circuit and calculation of unbalanced power in three-wire three-phase linear networks

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Abstract: For analysis of three-wire three-phase linear systems, the transformations 'wye-delta' and 'delta-wye' from Kennelly's theorem are used. These transformations can be applied to balanced systems but not to unbalanced systems. Depending on the type of connection that is used, zero-sequence voltages and currents appear in generators and loads and are not transferred over the network. The zero-sequence voltage in a delta-connected load and the zero-sequence current that is obtained using Kennelly's theorem in a star-connected load, or vice versa, cause different imbalance effects. In this work, the equivalent circuit for any point of the system is developed. The impedances of the equivalent circuit in any node are calculated using line-to-line voltages and line currents. This equivalent circuit incorporates all energetic phenomena, including the imbalance. For its calculation, the phasor unbalance power is used.

1. Introduction

In order to analyse an electrical power system correctly, it is necessary to know all the phenomena involved in the energy balance. In an unbalanced linear system, in addition to reactive and active power, there are other inefficient powers caused by imbalances that increase the apparent power of the system [1]. The quality of the system is significantly degraded by these phenomena. The value of the apparent power will depend on the asymmetry of the voltages in the nodes of the distribution network and the currents that circulate in the electric lines [2-5]. The configuration of the type of distribution and transmission network significantly affects the propagation of the imbalance in the system nodes. In a radial network, there will be higher levels of imbalance than in a mesh network [6].

In an electrical system, the imbalances are mainly caused by the connected loads. However, they can also be caused by the asymmetry of the voltages [7]. In most countries, there are standards that restrict the values of asymmetric voltages. An unequal distribution of loads causes an imbalance in the voltage and the circulating currents of the nodes [6].

The effects of imbalances on the electrical equipment in a system are multiple [8-11]: decrease of power and energy capacity in equipment, increase in the power losses of the lines and in the windings of machines and transformers, additional heating in electrical equipment, mechanical vibrations, unexpected failure of the protection, and propagation of imbalances from one node to another.

In an electrical system, measuring the unbalanced power allows us to correct and eliminate these inefficiencies and to estimate more reliably the apparent power of the system. The IEEE 1459-2010 standard [12] formulates the unbalanced power S_U according to (1). This power occurs at the fundamental frequency, and its unit is VA. S_e is the effective apparent power that is deduced from an approximate equivalent circuit having the same losses as the actual circuit, and S_+ is the positive-sequence apparent power [13-15]:

$$S_U = \sqrt{S_e^2 - S_+^2} \tag{1}$$

Unified power measurement (UPM) formulates the unbalanced power D_U from (2). S_1 is the modulus of the total apparent power defined by Buchholz [16-18]. It is expressed by the sequence components from (3) [19]. In a four-wire linear unbalanced three-phase system, the values S_e and S_1 are approximate but not equal, and therefore the unbalanced powers S_U and D_U are also unequal. However, the values are the same in three-wire linear systems:

$$D_U = \sqrt{S_1^2 - S_+^2} \tag{2}$$

$$S_1 = 3\sqrt{(V_+^2 + V_-^2 + V_0^2)\,(I_+^2 + I_-^2 + I_0^2)} \tag{3}$$

In 2016, from the expressions for instantaneous power, the authors of [20] formulated the total phasor of the unbalanced power from (4) and the total phasor of the apparent power from (5). These expressions are applicable to linear systems with unbalanced loads and asymmetric voltages:

$$\overrightarrow{D_u} = \sqrt{1 + \delta_-^2 + \delta_0^2}\;(A\,u_a + B\,u_b + C\,u_c + D\,u_d) + \sqrt{\delta_-^2 + \delta_0^2}\;(P_+u_x + Q_+u_y) \tag{4}$$

$$\overrightarrow{S_1} = \sqrt{1 + \delta_-^2 + \delta_0^2}\,(A\,u_a + B\,u_b + C\,u_c + D\,u_d + P_+u_x + Q_+u_y) \tag{5}$$

The three-wire three-phase systems have particular characteristics that must be taken into account in order to correctly estimate the energetic phenomena in an unbalanced system. These features involve many issues for the calculation of the unbalanced power:

- The first problem is the variety of connection types of the network's elements. The generators and loads can be star-connected or delta-connected. In unbalanced systems, depending on the type of connection that is used, there are zero-sequence voltages or currents. These zero-sequence values are not transferred over the network [21], and therefore the unbalanced power and total apparent power that are calculated at any point in the system will have different values, depending on whether the calculations are performed from the source or from the load.
- The second problem is due to the electrical parameters measured at any point in the system. These parameters are the line-to-line voltages and the line currents. Moreover, in a network, it is impossible to know the nature of the connection system of the generators and loads.
- Finally, it is difficult to obtain an equivalent circuit that takes into account all energetic phenomena. Kennelly's transformations of 'star-delta' and 'delta-star' are only valid for the calculation of active and reactive power flows, but not for the calculation of powers caused by the phenomenon of imbalance. The zero-sequence voltage in a delta-connected load and the zero-sequence current that are obtained using Kennelly´s theorem in a star-connected load, or vice versa, cause different imbalance effects.

In this paper, the equivalent circuit at any point in the three-wire three-phase linear system is proposed from the line-to-line voltages and line currents. For that, it is not necessary to know the nature of the connection types of the generators and loads in a system; in this circuit, all energetic phenomena are represented, including those that are caused by the imbalances. Accordingly, in Section 2, considering the different forms of connection systems in generators and loads, it is shown that the classical energy balance is not valid for determining the unbalanced power. In Section 3, the equivalent circuit is developed. This circuit is calculated from the line-to-line voltages and line currents measured in a node. In Section 4, the values A, B, C, and D of the phasor total unbalanced power in a three-wire three-phase linear system is analysed. In Section 5, in order to facilitate an understanding of the concepts developed in this work and its application, a practical case study of a three-bus system with unbalanced loads and voltages is studied. The connection systems of the loads are different.

2. Energy balance of unbalanced three-phase systems

In this section, we will demonstrate that the total apparent power produced by the generator is not the same as that consumed by the load in circuits with different configurations, under certain considerations. Finally, and for the balance to be identical, an equivalent circuit will be obtained that will replace the load.

2.1. Case A: Three-phase electrical system with generator and star-connected load

Fig.1 shows a three-phase electrical system with generator and star-connected load. The value of the total apparent power is given by equation (3) of the UPM theory. That is, from the standpoint of the generator:

$$S_{1G} = 3\sqrt{\left(V_{ga+}^2 + V_{ga-}^2 + V_{ga0}^2\right)\left(I_{a+}^2 + I_{a-}^2 + I_{a0}^2\right)}$$

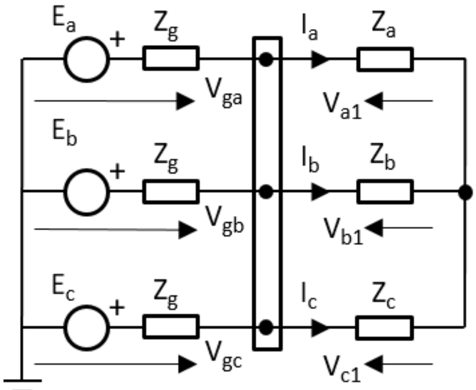

Fig. 1. *Three-phase electrical system with generator and star-connected load.*

Suppose $E_a = E_b = E_c$ are balanced, and that the impedance Z_g is the same for every phase. We then have that the total apparent power of the above-mentioned generator S_{1G} is given by

$$S_{1G} = \sqrt{9\left(V_{ga+}^2 + V_{ga-}^2\right)\left(I_{a+}^2 + I_{a-}^2\right)} \qquad (6)$$

This is because the generator, with the above-mentioned conditions, will not have zero-sequence components, neither of voltage nor of current, under normal conditions of operation.

On the other hand, the load will consume a different total apparent power S_{1L}, provided that the sum of V_{a1}, V_{b1} and V_{c1} voltages is not null. This is because a zero-sequence voltage is present on the load. Therefore:

$$S_{1L} = \sqrt{9\cdot\left(V_{a1+}^2 + V_{a1-}^2 + V_{a10}^2\right)\left(I_{a+}^2 + I_{a-}^2\right)} \qquad (7)$$

Moreover, taking into account that $V_{ga+} = V_{a1+}$ and $V_{ga-} = V_{a1-}$:

$$S_{1L} = \sqrt{S_{1G}^2 + 9\left(V_{a10}^2\right)\left(I_{a+}^2 + I_{a-}^2\right)} = \sqrt{S_{1G}^2 + D_U^2} \quad (8)$$

The last addend of the final sum D_U corresponds to the unbalanced power that the zero-sequence voltage provides, or the voltage difference between the generator neutral and the neutral of the load.

Therefore, to correct the energy balance, we will have to cancel the zero-sequence voltage and obtain the equivalent impedances. To do this, we include the zero-sequence voltage in the star load, as shown in Fig. 2:

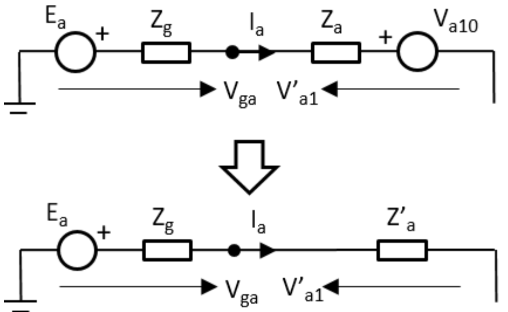

Fig. 2. *Initial and equivalent circuit.*

Here:

$$Z'_a = Z_a + \frac{V_{a10}}{I_a} = Z_a\, \frac{V'_{a1}}{V'_{a1} - V_{a10}} = Z_a\, \frac{V_{ga}}{V_{ga} - V_{a10}} \quad (9)$$

With this change in each of the branches, the total apparent power produced by the generator coincides with the total apparent power consumed by the load.

2.2. Case B: Three-phase electrical system with star-connected generator and delta-connected load

Fig.3 shows a three-phase electrical system with star-connected generator and delta-connected load. Making the same assumption as in the previous section, the total apparent power produced by the generator is given by equation (6). On the other hand, the load will have a different total apparent power, provided that the sum of the currents of the triangle. I_{ab1}, I_{bc1} and I_{ca1} is not null. This is because a zero-sequence current is present inside the triangle. Therefore:

$$S_{1L} = \sqrt{9\, (V_{ab1+}^2 + V_{ab1-}^2)\, (I_{ab1+}^2 + I_{ab1-}^2 + I_{ab10}^2)}$$

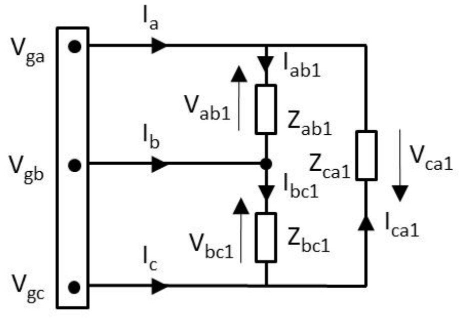

Fig. 3. *Three-phase electrical system with star-connected generator and delta-connected load.*

Moreover, taking into account that

$$V_{ga+} = \frac{V_{ab1+}}{\sqrt{3}} \qquad V_{ga-} = \frac{V_{ab1-}}{\sqrt{3}}$$

$$I_{a+} = \sqrt{3}\, I_{ab1+} \qquad I_{a-} = \sqrt{3}\, I_{ab1-}$$

we obtain

$$S_{1L} = \sqrt{S_{1G}^2 + 9\, I_{ab10}^2\, (V_{ab1+}^2 + V_{ab1-}^2)} = \sqrt{S_{1G}^2 + D_U^2} \quad (10)$$

As before, the last addend of the final sum corresponds to the unbalanced power D_U. In this case, the zero-sequence current generated in the unbalanced delta-connected loads produced it.

With this result, and to correct the energy balance, we will have to cancel the zero-sequence current and obtain the equivalent impedances. Accordingly, a voltage source, the product of the zero-sequence current and the impedance of this branch, will be included in the branches as:

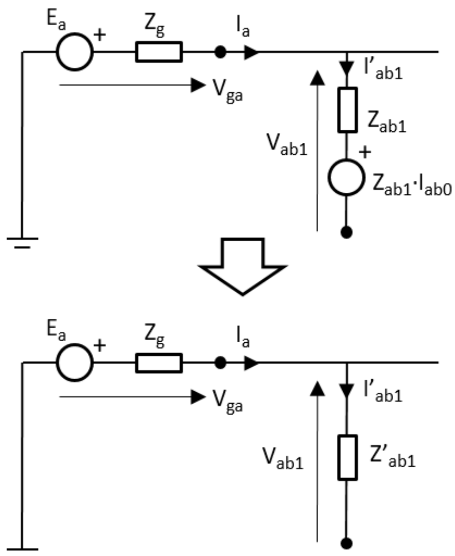

Fig. 4. *Initial and equivalent circuit.*

Here:

$$Z'_{ab1} = Z_{ab1} + \frac{Z_{ab1}\, I_{ab10}}{I'_{ab1}} = Z_{ab1}\, \frac{V_{ab1}}{V_{ab1} - Z_{ab1}\, I_{ab10}} \quad (11)$$

With this change in each of the branches, the total apparent power produced by the generator coincides with the total apparent power consumed by the load.

2.3. Case C: Three-phase electrical system with delta-connected generator and star-connected load

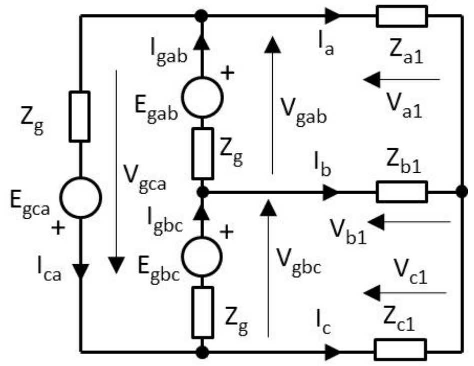

Fig. 5. *Three-phase electrical system with delta-connected generator and star-connected load.*

Fig.5 shows a three-phase electrical system with delta-connected generator and star-connected load. Making the same assumption as in the previous section, the total apparent power produced by the generator is given by the equation (12).

$$S_{1G} = \sqrt{9\left(V_{gab+}^2 + V_{gab-}^2\right)\left(I_{gab-}^2 + I_{gab-}^2\right)} \quad (12)$$

On the other hand, the load will consume a different total apparent power S_{1L}, provided that the sum of the voltages V_{a1}, V_{b1} and V_{c1} is not null. This is because a zero-sequence voltage is present on the load. Therefore:

$$S_{1L} = \sqrt{9\,(V_{a1+}^2 + V_{a1-}^2 + V_{a10}^2)\,(I_{a+}^2 + I_{a-}^2)}$$

Moreover, taking into account that

$$V_{a1+} = \frac{V_{gab+}}{\sqrt{3}} \qquad V_{a1-} = \frac{V_{gab-}}{\sqrt{3}}$$

$$I_{a+} = \sqrt{3}\, I_{gab+} \qquad I_{a-} = \sqrt{3}\, I_{gab-}$$

we obtain

$$S_{1L} = \sqrt{S_{1G}^2 + 9\,(V_{a10}^2)\,(I_{a+}^2 + I_{a-}^2)} = \sqrt{S_{1G}^2 + D_U^2} \quad (13)$$

This equation is the same as in the previous section, Eq. (8). Because the energy balance is correct, we will have to cancel the zero-sequence voltage and obtain the equivalent impedances, which yields equation (9).

2.4. Case D: Delta-connected generator and delta-connected load

Fig.6 shows a three-phase electrical system with delta-connected generator and delta-connected load. Making the same assumption as in the previous section, the total apparent power produced by the generator is given by equation (12).

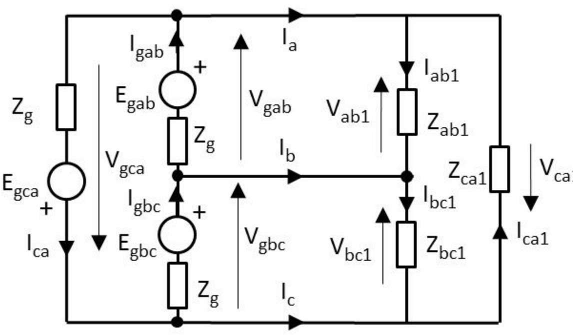

Fig. 6. *Three-phase electrical system with delta-connected generator and delta-connected load.*

On the other hand, the load will have a different total apparent power provided the sum of the currents of the triangle I_{ab1}, I_{bc1} and I_{ca1} is not null. This is because a zero-sequence current is present inside the triangle. Therefore:

$$S_{1L} = \sqrt{9\,(V_{ab1+}^2 + V_{ab1-}^2)\,(I_{ab1+}^2 + I_{ab1-}^2 + I_{ab10}^2)}$$

Moreover, taking into account that

$$V_{gab+} = V_{ab1+} \qquad V_{gab-} = V_{ab1-}$$
$$I_{gab1+} = I_{ab1+} \qquad I_{gab1-} = I_{ab1-}$$

we obtain

$$S_{1L} = \sqrt{S_{1G}^2 + 9\, I_{ab10}^2\,(V_{ab1+}^2 + V_{ab1-}^2)} = \sqrt{S_{1G}^2 + D_U^2} \quad (14)$$

To correct the energy balance, we will have to cancel the zero-sequence current and obtain the equivalent impedances, which yields equation (11).

3. New equivalent circuits of a three-wire three-phase sinusoidal power system

According to Kennelly's theorem, we can transform a star-connected three-phase load to another equivalent delta-connected load, and vice versa (see Fig.7). From the point of view of the energy balance, these changes are only valid when the load is balanced; otherwise they are not valid. First, we are going to see the star-to-delta transformation.

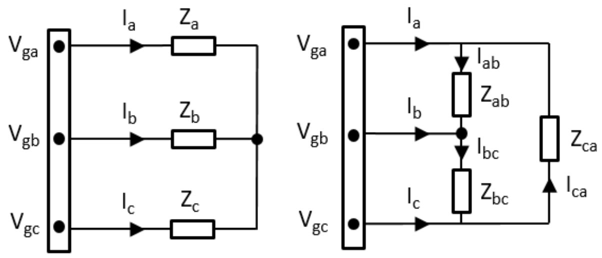

Fig. 7. *Original circuit with star-connected load and its delta-connected equivalent.*

According to Kennelly:

$$Z_{ab} = Z_a + Z_b + \frac{Z_a\, Z_b}{Z_c}$$
$$Z_{bc} = Z_b + Z_c + \frac{Z_b\, Z_c}{Z_a} \quad (15)$$
$$Z_{ca} = Z_c + Z_a + \frac{Z_c\, Z_a}{Z_b}$$

Obviously, from the standpoint of the energy balance, the transformation will be correct when the sum of the currents I_{ab}, I_{bc} and I_{ca} is null; otherwise, it will not be equivalent, as shown in the previous sections.

To achieve equivalence, we can utilise equation (9). However, we can also use the expressions obtained from the line-to-line voltages and phase currents, because they are easily measurable parameters.

If we call the positive-sequence current of Kennelly's equivalent delta I_{ab+} and the positive-sequence current of the delta energetically equivalent with the generator I'_{ab+}, we then have:

$$I'_{ab+} = I_{ab+} \qquad I'_{ab-} = I_{ab-} \qquad I'_{ab0} = 0 \quad (16)$$

Developing the system, we get

$$
\begin{aligned}
I'_{ab} &= I_{ab+} + I_{ab-} \\
I'_{bc} &= a^2\, I_{ab+} + a\, I_{ab-} \\
I'_{ca} &= a\, I_{ab+} + a^2\, I_{ab-}
\end{aligned} \tag{17}
$$

On the other hand, and according to Fig. 8, we have

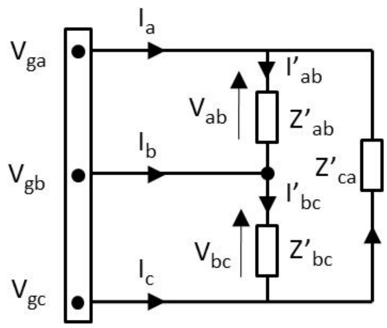

Fig. 8. *Equivalent circuit with unbalanced delta-connected load and without zero-sequence current.*

$$
\begin{aligned}
I_a &= I'_{ab} - I'_{ca} = (I_{ab+} + I_{ab-}) - (a\, I_{ab+} + a^2\, I_{ab-}) \\
I_b &= I'_{bc} - I'_{ab} = (I_{ab+} + I_{ab-}) - (a^2\, I_{ab+} + a\, I_{ab-})
\end{aligned}
$$

$$
\begin{aligned}
I_a &= \sqrt{3}\, I_{ab+}\, e^{-30j} + \sqrt{3}\, I_{ab-}\, e^{30j} \\
I_b &= \sqrt{3}\, I_{ab+}\, e^{-150j} + \sqrt{3}\, I_{ab-}\, e^{150j}
\end{aligned} \tag{18}
$$

Solving the previous system (18), we obtain

$$
I_{ab+} = \frac{a\, I_b}{3} - \frac{a^2\, I_a}{3} \qquad I_{ab-} = \frac{a^2\, I_b}{3} - \frac{a\, I_a}{3}
$$

Substituting in (17), we have

$$
I'_{ab} = \frac{I_a - I_b}{3} \qquad I'_{bc} = \frac{I_b - I_c}{3} \qquad I'_{ca} = \frac{I_c - I_a}{3}
$$

From the currents circulating in the delta energetically equivalent with the generator, and knowing the line-to-line voltages V_{ab}, $V_{bc}$and V_{ca}, impedances that form this triangle are the following:

$$
\begin{aligned}
Z'_{ab} &= \frac{V_{ab}}{I'_{ab}} = \frac{3\, V_{ab}}{I_a - I_b} \\
Z'_{bc} &= \frac{V_{bc}}{I'_{bc}} = \frac{3\, V_{bc}}{I_b - I_c} \\
Z'_{ca} &= \frac{V_{ca}}{I'_{ca}} = \frac{3\, V_{ca}}{I_c - I_a}
\end{aligned} \tag{19}
$$

Now we consider the reverse transformation, namely, from delta to star. According to Kennelly's theorem see Fig.9, where:

$$
\begin{aligned}
Z_a &= \frac{Z_{ab}\, Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \\
Z_b &= \frac{Z_{ab}\, Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}} \\
Z_c &= \frac{Z_{bc}\, Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}
\end{aligned} \tag{20}
$$

Obviously, from the standpoint of the energy balance, the transformation will be correct when the sum of the voltages V_a , V_b and V_c is null; otherwise, it will not be equivalent, as shown in the previous sections.

As in the previous case, to achieve equivalence, we can employ equation (11). However, we can also use the expressions obtained from the line-to-line voltages and phase currents, because they are easily measurable parameters.

If we call the positive-sequence voltage of the equivalent Kennelly's star V_{a+} and the positive-sequence voltage of the star energetically equivalent with the generator V'_{a+}, we have

$$
V'_{a+} = V_{a+} \qquad V'_{a-} = V_{a-} \qquad V'_{a0} = 0 \tag{21}
$$

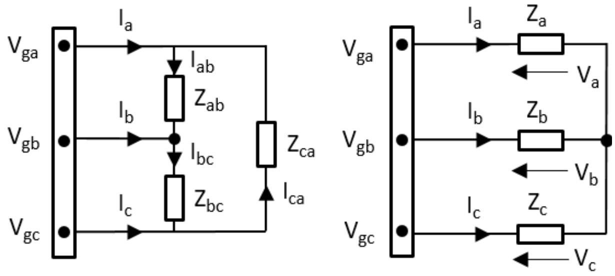

Fig. 9. *Original circuit with delta-connected load and its star-connected equivalent.*

Developing the system, we obtain

$$
\begin{aligned}
V'_a &= V_{a+} + V_{a-} \\
V'_b &= a^2\, V_{a+} + a\, V_{a-} \\
V'_c &= a\, V_{a+} + a^2\, V_{a-}
\end{aligned} \tag{22}
$$

On the other hand, according to Fig. 10, we have

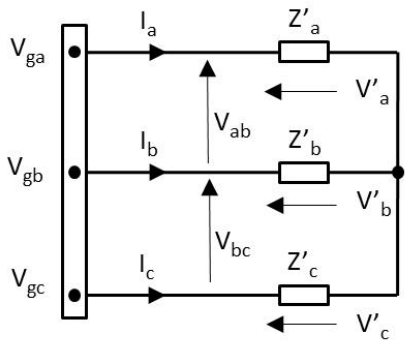

Fig. 10. *Equivalent circuit with unbalanced star-connected load and without zero-sequence voltage.*

$$
\begin{aligned}
V_{ab} &= V'_a - V'_b = (V_{a+} + V_{a-}) - (a^2\, V_{a+} + a\, V_{a-}) \\
V_{bc} &= V'_b - V'_c = (a^2\, V_{a+} + a\, V_{a-}) - (a\, V_{a+} + a^2\, V_{a-})
\end{aligned}
$$

$$
\begin{aligned}
V_{ab} &= \sqrt{3}\, V_{a+}\, e^{-30j} + \sqrt{3}\, V_{a-}\, e^{30j} \\
V_{bc} &= \sqrt{3}\, V_{a+}\, e^{-90j} + \sqrt{3}\, V_{a-}\, e^{90j}
\end{aligned} \tag{23}
$$

Solving the previous system (23), we obtain

$$
V_{a+} = \frac{V_{bc}\, e^{60j} + V_{ab}}{3} \qquad V_{a-} = \frac{V_{bc} \cdot e^{-60j} + V_{ab}}{3}
$$

Substituting in (22), we have

$$V'_a = \frac{V_{ab} - V_{ca}}{3} \quad V'_b = \frac{V_{bc} - V_{ab}}{3} \quad V'_c = \frac{V_{ca} - V_{bc}}{3}$$

Once we have the voltage applied to the star-connected impedances energetically equivalent with the generator, and knowing the phase currents I_a, I_b and I_c, the impedances that form such a star are as follows:

$$\begin{aligned} Z'_a &= \frac{V'_a}{I_a} = \frac{V_{ab} - V_{ca}}{3\, I_a} \\ Z'_b &= \frac{V'_b}{I_b} = \frac{V_{bc} - V_{ab}}{3\, I_b} \\ Z'_c &= \frac{V'_c}{I_c} = \frac{V_{ca} - V_{bc}}{3\, I_c} \end{aligned} \qquad (24)$$

Thus, if the phase currents and line-to-line voltages of a transmission line are known, we can obtain the equivalent circuit in delta or star configuration by using Eqs. (19) and (24), always from the standpoint of the total apparent power delivered by the generator.

This allows us to obtain the total apparent power that is consumed, without knowing the connection or the nature of downstream loads, at any point in an unbalanced network. It should be noted that the proposed equivalent circuits allow the active and reactive powers consumed individually in each of the phases to be obtained. On the contrary, in the equivalent circuits derived from the transformations of Kennelly, only the sum of these two powers is known, but not the phase-by–phase values.

4. Unbalance total phasor and apparent power phasor

According to [20], the unbalanced total phasor and apparent power phasor are formulated from Eqs. (4-5). The unbalanced factors of the voltages are given by $\delta_- = V_-/V_+$ and $\delta_0 = V_0/V_+$. The values A, B, C, and D are calculated from Eqs. (25-28):

$$A = -\sqrt{2} \sum_{Z=a,b,c} V_{z+}\, I_z \cos\theta_Z^{Z+} \cos 2\alpha_{Z+} \qquad (25)$$

$$B = \sqrt{2} \sum_{Z=a,b,c} V_{z+}\, I_z \cos\theta_Z^{Z+} \sin 2\alpha_{Z+} \qquad (26)$$

$$C = -\sqrt{2} \sum_{Z=a,b,c} V_{z+}\, I_z \sin\theta_Z^{Z+} \sin 2\alpha_{Z+} \qquad (27)$$

$$D = -\sqrt{2} \sum_{Z=a,b,c} V_{z+}\, I_z \sin\theta_Z^{Z+} \cos 2\alpha_{Z+} \qquad (28)$$

In a three-wire three-phase electrical system, the zero-sequence line current is null. Decomposing each of the terms A, B, C, and D under these conditions, we observe that the following equations are satisfied:

$$C = A = -\sqrt{2} \sum_{Z=a,b,c} V_{z+}\, I_z \cos\theta_Z^{Z+} \cos 2\alpha_{Z+} \qquad (29)$$

$$D = B = \sqrt{2} \sum_{Z=a,b,c} V_{z+}\, I_z \cos\theta_Z^{Z+} \sin 2\alpha_{Z+} \qquad (30)$$

5. Practical case study

In this section, a practical case study is developed to check all the concepts discussed in the previous sections.

Fig. 11 shows a three-wire three-bus electrical system with two unbalanced three-phase linear loads. Load 1 is star-connected, and Load 2 is delta-connected. The loads are modelled at a constant impedance. The line-to-line voltages are unbalanced and sinusoidal in bus 1 (Slack node), where

$$\begin{aligned} V_{ab} &= 394{,}00 \cdot e^{j30{,}21} \\ V_{bc} &= 380{,}92 \cdot e^{-j91{,}10} \\ V_{ca} &= 379{,}95 \cdot e^{j151{,}28} \end{aligned}$$

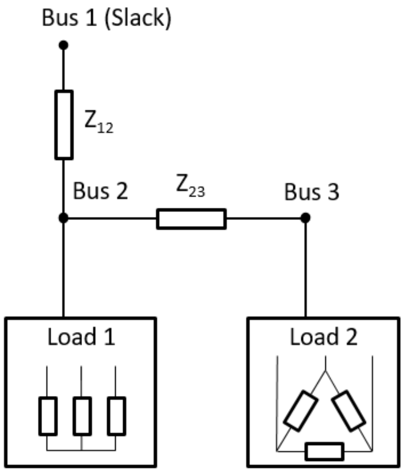

Fig. 11. *Three-wire three-bus electrical system with two unbalanced three-phase linear loads.*

The line impedances are displayed in Table 1. The impedances of Load 1 and Load 2 are displayed in Tables 2 and 3, respectively.

Table 1 Line impedances

	R (Ω)	X (Ω)
Line 1-2	0,06	0,02
Line 2-3	0,04	0,01

Table 2 Load 1 impedances

	R (Ω)	X (Ω)
Z_a	17	3
Z_b	7	1
Z_c	10	2

Table 3 Load 2 impedances

	R (Ω)	X (Ω)
Z_{ab}	24	3
Z_{bc}	8	1
Z_{ca}	3	2

After solving the system, the following results are obtained (Tables 4-5):

5.1. Equivalent circuits in any of the nodes

The impedances of the equivalent circuit at any of the nodes can be represented as star-connected or delta-connected configurations. Both representations are valid. We are going to use a star-connected circuit. Fig. 12 shows the equivalent circuits 'star-connected' in any of the nodes. In this circuit, the line-to-line voltages and line currents are known (Tables 4 and 5). The values of the equivalent impedances are displayed in Table 6 are calculated from Eq. (24).

Considering the values in Tables 4 and 5, the following results (Tables (6–9)):

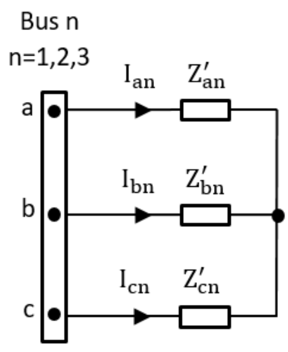

Fig. 12. *Equivalent circuit 'star-connected' in any of the nodes.*

Table 4 Line-to-line voltage

	V_{ab} (V)		V_{bc} (V)		V_{ca} (V)	
	Modulus	Angle	Modulus	Angle	Modulus	Angle
Node 2	387,16	30,46	367,77	- 91,85	364,60	151,98
Node 3	384,18	30,78	360,81	- 92,29	355,67	152,56

Table 5 Line currents

	I_a (A)		I_b (A)		I_c (A)	
	Modulus	Angle	Modulus	Angle	Modulus	Angle
Node 1-2	107,45	- 45,72	81,19	- 116,75	154,30	104,44
Node 2-3	101,33	- 52,15	55,05	- 113,40	136,62	107,16
Load 1	13,20	- 13,52	26,42	- 123,74	18,98	84,44

Table 6 Line equivalent impedances

	Z'_a		Z'_b		Z'_c	
	R (Ω)	X (Ω)	R (Ω)	X (Ω)	R (Ω)	X (Ω)
Node 1	1,424	1,530	2,766	- 0,205	1,354	0,378
Node 2	1,364	1,510	2,706	- 0,225	1,294	0,358
Node 3	1,218	1,743	3,924	- 0,574	1,439	0,325
Load 1	16,240	- 3,257	8,336	0,325	8,894	6,337
Load 2	1,218	1,743	3,924	- 0,574	1,439	0,325

Table 7 Line-to-neutral voltage

	V_a (V)		V_b (V)		V_c (V)	
	Modulus	Angle	Modulus	Angle	Modulus	Angle
Node 1	224,62	1,33	225,17	- 120,99	216,97	120,04
Node 2	218,68	2,18	220,44	- 121,50	207,23	119,91
Node 3	215,51	2,90	218,34	- 121,73	201,60	119,88

Table 8 Positive-, negative-, and zero-sequence line-to-neutral voltage

	V_+ (V)		V_- (V)		V_0 (V)	
	Modulus	Angle	Modulus	Angle	Modulus	Angle
Node 1	222,23	0,13	5,26	63,61	0	0
Node 2	215,37	0,20	8,21	67,44	0	0
Node 3	211,70	0,35	10,24	69,75	0	0

Table 9 Voltage unbalance factors

	δ-	δ_0
Node 1	0,02368	0
Node 2	0,03814	0
Node 3	0,04839	0

Table 10 Positive-sequence active and reactive powers

	P_+ (W)	Q_+ (VAr)
Node 1	67657,22	25423,00
Node 2	65541,63	24717,81
Node 3	52802,35	22349,91
Load 1	11761,15	2123,37
Load 2	52802,35	22349,91

5.2. Unbalanced power and apparent power in the loads, lines and nodes

The values A, B, C, and D displayed in Table 11 are calculated from Eqs. (25–30), and the following results are obtained:

Table 11 Values A, B, C and D of the nodes and loads

	A and C	B and D
Node 1	12477,16	18422,15
Node 2	12071,04	17868,66
Node 3	9385,82	20414,17
Load 1	2466,60	2874,60
Load 2	9385,82	20414,17

The unbalanced power phasors are calculated from Eq. (4), and the following results are obtained (see equation at the bottom of the page).

For lines 1–2 and 2–3, we apply the following:

$$\overrightarrow{D_{u(Line\ 1-2)}} = \overrightarrow{D_{u(Node\ 1)}} - \overrightarrow{D_{u(node\ 2)}}$$

$$\overrightarrow{D_{u(Line\ 2-3)}} = \overrightarrow{D_{u(Node\ 2)}} - \overrightarrow{D_{u(Load\ 1)}} - \overrightarrow{D_{u(Node\ 3)}}$$

The unbalanced power phasors are calculated from Eq. (5), and the following results are obtained (see equation at the bottom of the page).

For lines 1–2 and 2–3, we apply the following:

$$\overrightarrow{S_{1(Line\ 1-2)}} = \overrightarrow{S_{1(Node\ 1)}} - \overrightarrow{S_{1(node\ 2)}}$$

$$\overrightarrow{S_{1(Line\ 2-3)}} = \overrightarrow{S_{1(Node\ 2)}} - \overrightarrow{S_{1(Load\ 1)}} - \overrightarrow{S_{1(Node\ 3)}}$$

The moduli of the unbalanced power phasor and apparent power phasor are displayed in Table 12. These values are the same as those obtained by Buchholz, UPM, and IEEE Std. 1459-2010.

$$\overrightarrow{D_{u(Node\ 1)}} = 12480{,}7\,u_a + 18427{,}3\,u_b + 12480{,}7\,u_c + 18427{,}3\,u_d + 1601{,}9\,u_x + 601{,}9\,u_y\ VA$$

$$\overrightarrow{D_{u(Node\ 2)}} = 12079{,}8\,u_a + 17881{,}6\,u_b + 12079{,}8\,u_c + 17881{,}6\,u_d + 2499{,}7\,u_x + 942{,}7\,u_y\ VA$$

$$\overrightarrow{D_{u(Node\ 3)}} = 9396{,}8\,u_a + 20438{,}1\,u_b + 9396{,}8\,u_c + 20438{,}1\,u_d + 2555\,u_x + 1081{,}5\,u_y\ VA$$

$$\overrightarrow{D_{u(Load\ 1)}} = 2468{,}4\,u_a + 2876{,}7\,u_b + 2468{,}4\,u_c + 2876{,}7\,u_d + 448{,}6\,u_x + 81\,u_y\ VA$$

$$\overrightarrow{D_{u(Load\ 2)}} = 9396{,}8\,u_a + 20438{,}1\,u_b + 9396{,}8\,u_c + 20438{,}1\,u_d + 2555\,u_x + 1081{,}5\,u_y\ VA$$

$$\overrightarrow{D_{u(Line\ 1-2)}} = 400{,}8\,u_a - 545{,}7\,u_b + 400{,}8\,u_c - 545{,}7\,u_d - 897{,}8\,u_x - 340{,}8\,u_y\ VA$$

$$\overrightarrow{D_{u(Line\ 2-3)}} = 214{,}6\,u_a - 320{,}3\,u_b + 214{,}6\,u_c - 320{,}3\,u_d - 503{,}9\,u_x - 219{,}7\,u_y\ VA$$

$$\overrightarrow{S_{1(Node\ 1)}} = 12480{,}7\,u_a + 18427{,}3\,u_b + 12480{,}7\,u_c + 18427{,}3\,u_d + 67676{,}2\,u_x + 25430{,}1\,u_y\ VA$$

$$\overrightarrow{S_{1(Node\ 2)}} = 12079{,}8\,u_a + 17881{,}6\,u_b + 12079{,}8\,u_c + 17881{,}6\,u_d + 65589{,}3\,u_x + 24735{,}8\,u_y\ VA$$

$$\overrightarrow{S_{1(Node\ 3)}} = 9396{,}8\,u_a + 20438{,}1\,u_b + 9396{,}8\,u_c + 20438{,}1\,u_d + 52864{,}1\,u_x + 22376{,}1\,u_y\ VA$$

$$\overrightarrow{S_{1(Load\ 1)}} = 2468{,}4\,u_a + 2876{,}7\,u_b + 2468{,}4\,u_c + 2876{,}7\,u_d + 11769{,}7\,u_x + 2124{,}9\,u_y\ VA$$

$$\overrightarrow{S_{1(Load\ 2)}} = 9396{,}8\,u_a + 20438{,}1\,u_b + 9396{,}8\,u_c + 20438{,}1\,u_d + 52864{,}1\,u_x + 22376{,}1\,u_y\ VA$$

$$\overrightarrow{S_{1(Line\ 1-2)}} = 400{,}8\,u_a - 545{,}7\,u_b + 400{,}8\,u_c - 545{,}7\,u_d + 2086{,}9\,u_x + 694{,}4\,u_y\ VA$$

$$\overrightarrow{S_{1(Line\ 2-3)}} = 214{,}6\,u_a - 320{,}3\,u_b + 214{,}6\,u_c - 320{,}3\,u_d + 955{,}4\,u_x + 234{,}8\,u_y\ VA$$

Table 12 Moduli of D_u and S_1

	D_u (VA)	S_1 (VA)
Node 1	67657,22	25423,00
Node 2	65541,63	24717,81
Node 3	52802,35	22349,91
Load 1	11761,15	2123,37
Load 2	52802,35	22349,91

6. Conclusion

This article has analysed the behaviour of unbalanced three-wire three-phase linear systems from the standpoint of the total apparent power generated. It has been shown that the total apparent power produced by the generator does not coincide with the power consumed by the load. Expressions have been developed to transform unbalanced loads with zero-sequence components to unbalanced loads without zero-sequence components, to correct the energy balance. This occurs because the apparent power imbalance generated by the zero-sequence components is not transmitted over the network, and therefore it only affects the load that produces it.

It has been shown that the equivalent circuits derived from Kennelly transformations are not valid from the standpoint of the power imbalance. New equations have been developed to obtain a circuit star-connected or delta-connected in any node of the network, equivalent to the connected unbalanced downstream loads of this network. It should be noted that such equivalent circuits are independent of the type of connection of the downstream loads. These equations are obtained from the node voltages and phase currents. These parameters can be measured easily. This simplifies the analysis of electrical networks as well as individual determination of the active and reactive powers in each of the phases. These powers cannot be calculated from the equivalent circuits deduced from Kennelly's transformations.

In three-wire three-phase systems, the conclusion is that it is sufficient to calculate the values A and B used in the imbalance power phasor, because A = C and B = D. This equality simplifies the calculations in this type of system compared to four-wire three-phase systems.

Finally, to validate the applicability of the proposed equations and to clarify the underlying principles, a practical three-wire system case study has been developed.

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