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Textile formwork for concrete shells

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Abstract

Fabric formwork is a new application for textile membranes that provides numerous advantages and new opportunities for architecture and engineering compared to well known traditional formworks. The installation of fabric formwork requires less manual labor and has reduced material, storage, and transportation costs. But the most significant advantage of fabric moulds is the form freedom and structural performance they offer to shell design. This paper presents the state of the art of the preceeded research on fabric formed shells. Subsequently it includes a numerical form finding method based on the force density for an anticlastic geometry. An initial saddle will be 'formfinded' to obtain shell shape that under self weight only experiences axial stresses. Finally a numerical finite element analysis of the initial and 'formfinded' saddle will be done. The different results are summarized and discussed in terms of implications for future research

Keywords: Fabric Formwork, Concrete shell, Anticlastic, Funicular equilibrium shapes, form finding, force density, Finite element analysis.

1. Introduction

Worldwide textile membranes are used as building applications, such as mobile tents and shelters. The last decades the knowledge and technical materials developed enormously, nowadays textile can be applied for permanent membrane constructions and even for new implementations. One of these new implementations is the use of fabric as flexible formwork for architectural concrete elements.

Traditional formworks, such as rigid wooden or steel panel formwork, are rather stiff and straight-lined. Consequently, these frameworks are the limiting factor for realizing more organic or curved concrete shapes. Concrete is a very plastic material, as fresh and humid concrete can be poured into any formwork shape. Compared to well-known traditional formworks, textile formwork constructions provide numerous advantages and new opportunities for architecture and engineering in. It is not only less expensive than conventional panelized formwork; the installation requires less manual labor and it contains reduced material, storage, and transportation costs. However the most significant advantage is the form freedom and structural performance that is related to the use of textile membranes.[1]

Limited exploration is done on this new construction method using textile formwork. The method was proposed by Prof. West at the CAST laboratory of the University of Manitoba (Canada). Besides his this pioneer research, other scholars must be mentioned as well. In Japan Unno developed different methods to optimize the practical use of cast-in-place fabric-formed concrete walls. And in Belgium the WTCB project recently explored this new formwork technique investigating several case studies. Both, the West and WTCB project will be discussed in detail in this paper.

Textile architecture consists of a vocabulary of double-curved shapes. Textile membranes with tensioned surfaces, typically take anticlastic geometrical forms. This implies that concrete shell structures casted by tensioned textile membranes will have anticlastical shapes. Since concrete resists only to compression stresses and not to tension stresses, these double-curved shells need to approximate three dimensional compression geometries. The structural efficiency of these compression shapes will therefore result in a structural system that requires a “minimal” amount of non-renewable material and reduced reinforcement. To achieve the optimal compression geometries, a particular form finding procedure needs to be done. This procedure is a modeling process, in which an equilibrium shape is calculated based on the “force densities” method.

Thin shell concrete anti-clastical shells rarely appear. In 1935 Torroja designed and constructed a double-curved surface for the roof of the Zarzuela Hippodrome in Madrid. The top of this cantilever roof has a hyperboloid shape. Another shell builder is Candella. In the 1950s he designed and built innovative double curved thin shell concrete roof structures in Mexico. The hyperbolic paraboloid is a pattern he often used for designing his roof constructions. Some of his most important realizations are the roof of the restaurant Los Manantiales in Xochimilco (1958), the Chapel Lomas de Cuernavaca (1958) and the Bacardi Rum Factory (1960).

2. Textile formwork for concrete shells: State of the Art

2.1. Mark West: Thin barrel vaults [2]

Prof. Mark West is the director of The Centre for Architectural Structures and Technology (CAST), the first laboratory that is dedicated to the development and testing of fabric formwork and fabric-formed concrete structures. This laboratory is located at the University of Manitoba’s Faculty of Architecture in Winnipeg (MB) Canada. It provides a unique facility for interdisciplinary research on beams, trusses, columns, and thin shell vaults involving architects, engineers, and industry. In this paragraph the building procedure of fabric formed thin-shell concrete barrel vaults will be briefly discussed (see Figure 1).

Since concrete only has resistance to compression, these barrel vaults must have a defined shape that will lead to compression stresses in the vault under his self weight. Hence the vaults must be shaped to follow very efficient natural compression geometries. These funicular compression vaults are constructed by spraying a uniform thickness of Glass fiber Reinforced Concrete (GFRC) onto sheets of fabric hung on a steel frame. Consequently the fabric deflects under the load of the wet concrete into the tension resistant shape dictated by its load and support conditions. Because tension and compression are each others geometric inversions, the resulting funicular tension shape can be flipped over to provide a thin-shell funicular compression vault. This vault is used as a mold for precast production.

Figure 1 CAST's funicular fabric formed compression vaults

2.2. WTCB: Double curved shell [3]

Another study of fabric formed thin shells was realized by the Belgian WTCB in cooperation with Centexbel and the department Architectural Engineering of the Vrije Universiteit Brussel. It concerns an investigation to the material requirements of textile membranes, the assembly of these fabrics, and the formwork building. In comparison to West's research, during the WTCB exploration, the starting point remained the modeling of formwork. In order to minimize the deformations when applying the concrete they proposed a pattern for the assembly of the fabric shape and minimum requirements regarding formwork pretension. The fabric for the formwork was composed out of 3 pieces, which is a minimum to guarantee the shape (see Figure 2). Next, these numerical modeling models were evaluated through physical experiments. The fabric was fixed with integrated keder and rope to the primary construction, two boundary circular arches (Figure 2). For the concreting up to a shell shotcrete was used with an overall thickness of 5 cm.

**Figure 2 Left: Fabric formwork assembled for the double-curved shell.
Right: Resulting concrete shell**

2.3. Conclusion

Although research on fabric formed shells is rather new in architectural engineering, some pioneer studies were executed. The first to focus on this in an experimentally point of view was Prof. Mark West and his C.A.S.T. laboratory, followed by the research by W.T.C.B. in Belgium, that paid more attention to the numerical modeling of the textile membrane construction used as formwork material.

In the author's research the focus is not only on the modeling of the fabric formwork, but also on the structural optimal concrete shape. The fabric formed concrete shell elements will be preceded by a form finding procedure. This methodology will be discussed in the next paragraph.

3. Form finding an anticlastic shell

When wanting to design and building very thin wide spanning surface structures of concrete, the shape has to be found and adopted following the forces acting in them, therefore the form is derived from 'forms of equilibrium'. These forms of equilibrium can be found following different strategies. Concrete is limited in resistance to compression forces, while cables only resist tensile forces. Due to these opposite capacities and the fact that tensile and compression are geometrically inverse, hanging equilibrium forms can approximate three dimensional compression geometries by turning them upside down including their boundary conditions.

A well known example of such a figure of equilibrium is the shape of a freely hanging chain suspended between two anchoring points. A catenary is typically formed. In each pin-joint of successive chain links we have equilibrium between the tension forces and the force of gravity acting simultaneously on the chain links. Imaging such a chain line fixed in its form and reversed by 180° around the connection line of its anchoring points, we get an upright catenary-like arch. The former tension forces are converted to compression forces again being in equilibrium with the forces of gravity. No bending occurs.

The same reasoning can be applied for surfaces derived with a chain network. Here the same reverse procedure can be performed with the 'hanging shells'. Thus, the geometrical compression shapes are created by exploring funicular shapes in suspension that are inverted afterwards. This can be done either by physical experiments (physical hanging shells like Isler (1955, Switzerland) did), or by numerical simulations, such as the force-density method.

3.1. Force-density

This analytical form finding tool is applicable for each network of linear elements hinged to each other. Therefore hanging shells will be approached as bidirectional cable nets. The force-density tool starts the calculation from a charged cable network of which it needs the coordinates of fixed points, the topology of the network and the force-densities of all links. The force-density of a link is defined by the ratio of the internal normal force and the length of this link. If all three parameters are known, the unknown surface of equilibrium and the internal forces can be calculated.

The force-density method is based on the fact that in an equilibrium form of a double curved cable net, the sum of forces in each single point is equal to zero. This is the case for one single point (like for example in Figure 3) if the following non-linear equations for x, y and z are valid [4].

$$\begin{aligned} x: &\ \frac{(x_a - x_i)}{l_a}.N_a + \frac{(x_b - x_i)}{l_b}.N_b + \frac{(x_c - x_i)}{l_c}.N_c + \frac{(x_d - x_i)}{l_d}.N_d = p_x \\ y: &\ \frac{(y_a - y_i)}{l_a}.N_a + \frac{(y_b - y_i)}{l_b}.N_b + \frac{(y_c - y_i)}{l_c}.N_c + \frac{(y_d - y_i)}{l_d}.N_d = p_y \\ z: &\ \frac{(z_a - z_i)}{l_a}.N_a + \frac{(z_b - z_i)}{l_b}.N_b + \frac{(z_c - z_i)}{l_c}.N_c + \frac{(z_d - z_i)}{l_d}.N_d = p_z \end{aligned} \tag{1}$$

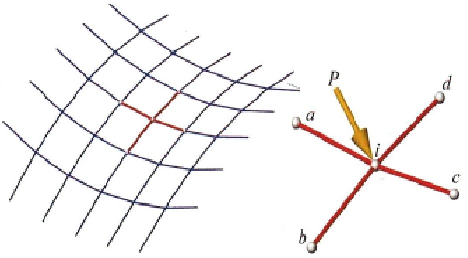

Figure 3 One node with neighbours in a network

The distances in three dimensions are calculated with the formula of Pythagoras.

$$l_a = \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2 + (z_a - z_i)^2} \tag{2}$$

And the normal force in the link is defined by Hooke's law.

$$\sigma = E.\varepsilon \quad => \quad \frac{N}{A} = E.\frac{\Delta l}{l_0} \quad => \quad N_a = EA.\frac{l_a - l_{oa}}{l_{oa}} \tag{3}$$

This calculation of the equilibrium form is an iterative process that results in a shape in which there is an equilibrium between the tension forces and the external force, acting simultaneously on the chain links.

3.2. Formfinding a saddle

In this paragraph a form-finding procedure with the force density tool will be applied on a saddle. A saddle is a double ruled surface shaped (left image of Figure 4). It was Candela that introduced this shape for his concrete roof constructions. One of the advanced examples is the Chapel Lomas de Cuernavaca in Mexico, built in 1951 (right image of Figure 4).

**Figure 4 Left: One hypar generating form for Chapel Lomas de Cuernavaca
Right: Chapel Lomas de Cuernavaca (Mexico, 1951) [5]**

3.2.1. Initial saddle

The initial saddle that will be 'formfinded', is derived from a surface of revolution. A hyperbolic line segment is rotated around a straight line. Consequently, a saddle with crosswise circular arches (y-direction) is created (see Figure 5). In plan view the geometrical size is 10 m by 10 m.

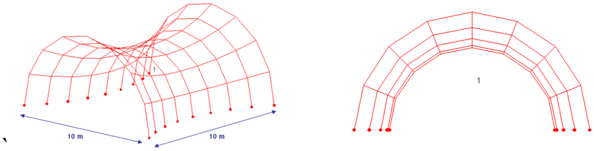

**Figure 5 Left: View of initial saddle
Right: Section view of initial saddle**

This initial shape will be 'formfinded' under his dead weight with the force-density tool.

3.2.2. Load conditions

The shell as proposed by the model will have an arbitrarily chosen thickness of 5 cm. This results in an area load equal to:

$$p = 2400\,kg/m^3 .\, 0{,}05\,m .\, 9{,}81\,N/kg = 1{,}2\,kN/m^2 \tag{5}$$

Under this area load the form finding procedure will take place. This is done by the software program Easy.

3.2.3. First equilibrium shape

In Figure 6 the first equilibrium shape resulted from a form finding calculation the shape of the arches in cross direction evolved from a circular form to a catenary. This phenomenon can be clearly viewed on the central image of Figure 6, where the red color corresponds to the initial shape and the pink color to the 'formfinded' shape in equilibrium. The lower crosswise links have inclined inwards. Therefore the middle links at the top sag deeper. The right image of Figure 6 shows the vertical (z-direction) deflections between the two shapes. Positive deflections in z-direction are indicated with black hatched circles, negative deflections with red hatched circles. The maximal deflection between the two shapes in z-direction comes to 0,47 m and is positioned at the top of the exterior cross-arches.

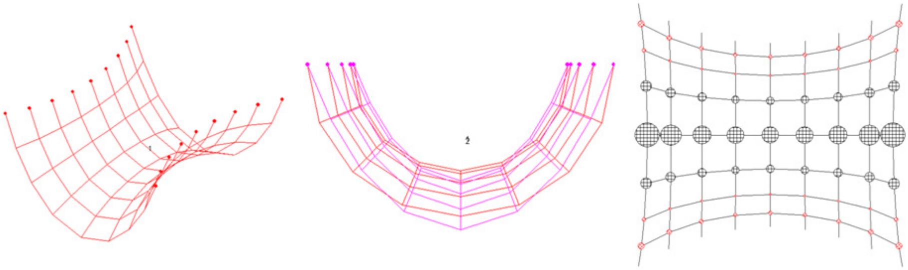

Figure 6 Left: Geometry of first equilibrium shape
Central: Section view of the initial saddle and the first equilibrium shape
Right: Deformations between the initial saddle and the first equilibrium shape

Figure 7 shows the internal forces of the links in the equilibrium shape. All forces in the links in cross direction (y-direction) are positive (tension). These will develop in compression forces by inverting the shape. But not all internal forces in the longitudinal links are positive. The links indicated with a red circle in Figure 7 are compression links in the hanging model. These lead to links in tension in the inverted shape. Hence, the first 'formfinded' shape will not lead to a full compression geometry.

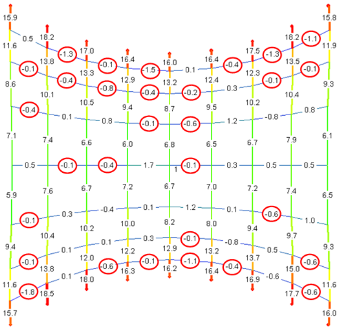

Figure 7 Internal forces in the links of the first equilibrium shape.

To optimize this shape to an inverted full compression geometry (all the forces in the longitudinal links in tension), the value of the force-density of the longitudinal links can be fixed to an arbitrary value. In a first test, the force-density value is set equal to 1,0 kN/m. A new equilibrium shape is converged after different iterations calculations. Figure 8 shows the precedent equilibrium shape in red and the new equilibrium form in purple. The increase of the force-density in the longitudinal links results in an inwards incline. A quite large variation exists between the two different equilibrium shapes (1,2 m between the tops of the exterior cross arches). Therefore a decrease of the values of the force-density in the longitudinal links is accomplished. The force-densities are vary from 1,0 kN/m to 0,2 kN/m with an intermediate value of 0,2 kN/m. The right image on Figure 8 shows the new equilibrium shapes according to the different force-densities. The lower the force density, the less the crosswise arches will incline inwards. The force-density equal to 0,2 kN/m is the lowest value, other wise the saddle will not converge into an equilibrium shape.

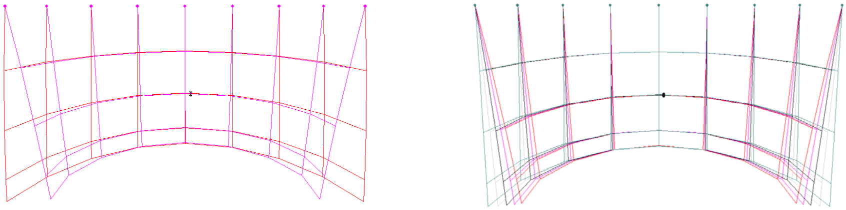

Figure 8 Left: First equilibrium form and one with a force density equal to 1,0 kN/m in longitudinal links Right: Equilibrium shapes with decreasing force-densities in the longitudinal links

The equilibrium shape corresponding with the 0,2 kN/m force-densities in the longitudinal links converges most to the initial shape. The crosswise arches are still a bit inclined inwards (3,4°). The maximum deflections between this new equilibrium shape and the first one amounts to 0,30 m and is located again at the top of the exterior cross-arches.

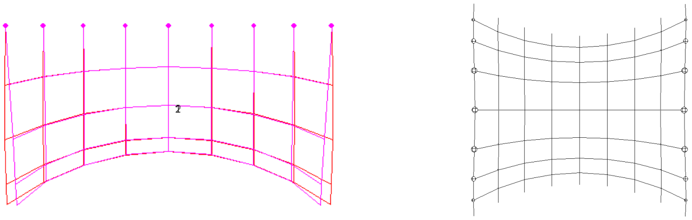

Figure 9 Left: First equilibrium form and one with a force density equal to 0,2 kN/m in longitudinal links Right: Deformations between these two equilibrium shapes

The internal force distribution is showed in Figure 10. An important determination is the fact that all links are in tension in this hanging network. A theoretical anticlastic compression geometry is created. The internal force ratio between the links in the cross-direction (y) and longitudinal direction (x) is practically equal to 10. This implies that the load transfer will mainly happen in cross direction in the inverted geometry.

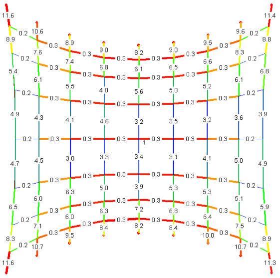

Figure 10 Internal forces in the links of the final equilibrium shape (Easy)

3.3. Conclusion

The purpose to create anticlastic compression geometries is because of the fact that tensioned textile membranes (used as formwork) form anticlastic surfaces. To create these anticlastic geometries, a form finding procedure need to be accomplished. In this case the force-density tool is used for form finding a ‘saddle’, considered as a network of hinged linear elements, to equilibrium. A hang model in equilibrium with all elements in tension is achieved. This was only possible by bringing the force-densities of the longitudinal links into coincidence. They were set equal to 0,2 kN/m. It resulted in the ‘formfinded’ saddle shown in Figure 10. This equilibrium shape will be inverted and considered as a concrete shell structure in the next paragraph. A structural analysis of this shell will be done in the finite element program Ansys. The occurring stresses due to the dead weight in the initial saddle and the ‘formfinded’ one will be compared. Furthermore the ‘simplification’ of the definition of a non linear concrete shell by a network of linear hinged links will be discussed.

4. Finite element analysis

The initial saddle shape and the shape that resulted from the final formfinding computation will be structurally analyzed in this section. Initially they will be materialized. The material parameters of concrete (Young modulus, Poisson, density) are assigned to the three dimensional models with a uniform thickness of 5 cm. Figure 11 shows both solid models in the finite element software program Ansys.

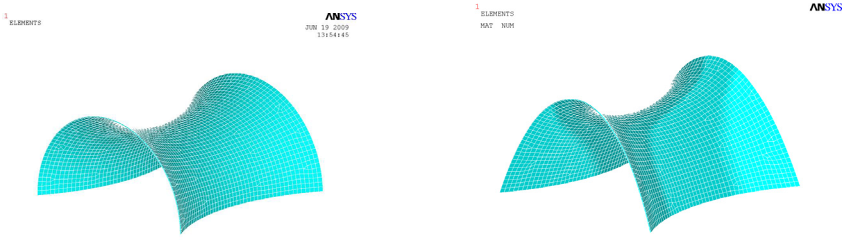

Figure 11 Left: Initial saddle (Ansys)
Right: ‘formfinded’ saddle (Ansys)

4.1. Finite element analysis of initial shell

Due to the dead weight the normal stresses in the y-direction add up to a maximum value of 0,30 MPa (tension) and a minimum value of -0,22 MPa (compression). The normal stresses in the x-direction (longitudinal) reach a maximum tension stress of 0,37 MPa and a maximum compression stress of -0,36 MPa. A clear clarification is the fact that the load transfer in both directions (x- and y-direction) are of the same order of magnitude. Thus, there is no clear preference path for the load transfer. Another conclusion concerns the fact that there are tension areas in both stress directions. These areas are colored in red in the images at the bottom of Figure 12. The tension areas are more significant for the x-stresses than for the y-stresses. In both directions they reach a maximum in the middle of the saddle.

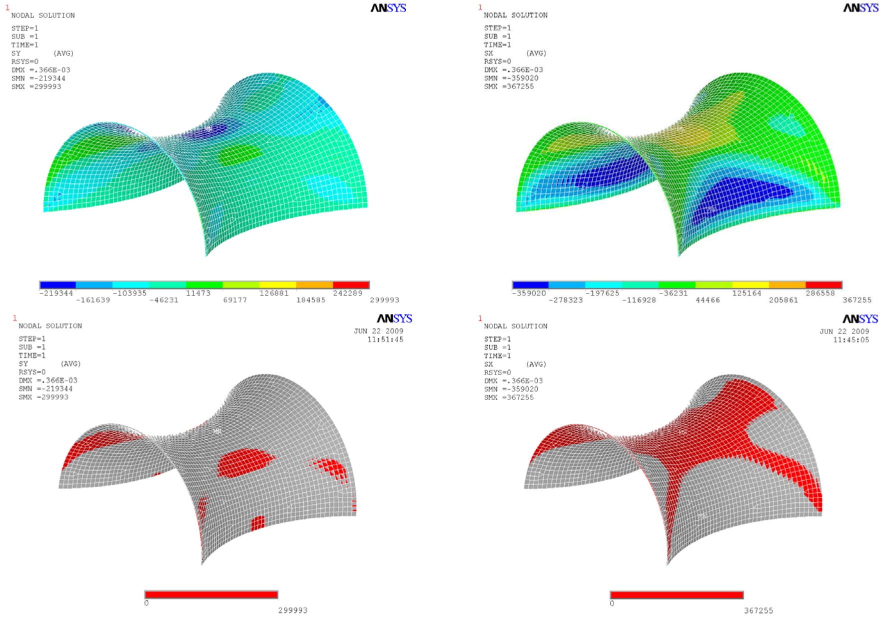

Figure 12 Left & right up: Stresses in y- and x-direction of initial saddle under deadweight
Left & Right under: Localization of tension stresses in y- and x-direction of initial saddle under deadweight

4.2. Finite element analysis of ‘‘formfinded’ saddle’ shell

The same structural analysis will be accomplished for the final ‘formfinded’ saddle. The structural behavior of this shell under dead weight is different in both directions. The stresses in y-directions are all negative (with a minimum value of -0,91 MPa). This means that load transfer occurs only by compression in this direction. In the opposite direction tension stresses are still present. They are located at the corner points and the flanks of the saddle shape. They are indicated on the right image of Figure 13 by the yellow and orange spots. Hence a complete compression shell is not achieved. However in y-direction the load transfer happens only by compression stresses. Furthermore on average the stresses in this direction are also globally dominating in comparison with the other direction.

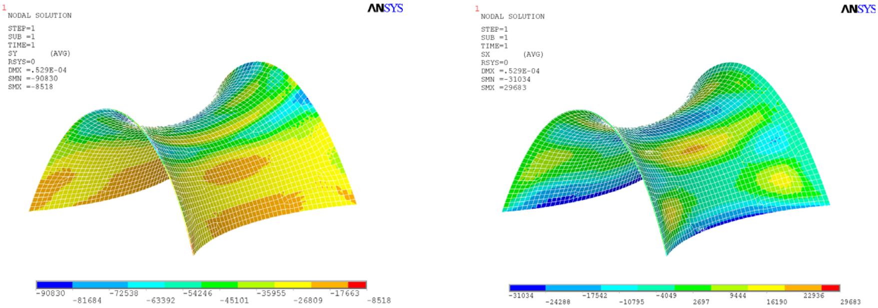

Figure 13 Left: Stresses in y-direction of 'formfinded' saddle under deadweight
Right: Stresses in x-direction of 'formfinded' saddle under deadweight

4.3. Conclusion

Due to dead weight of the concrete shell the operating stresses in y-direction, are all compression stresses in the 'formfinded' saddle. This, contrary to the initial saddle, where tensile stresses will occur in the load transfer. The maximum value of the appearing compression forces is practically four times greater in the 'formfinded' 'saddle' than in the initial one. For the stresses in x-directions owing to the self weight, both saddles show zones where tensile stresses occur. Nevertheless these areas are limited in the 'formfinded' saddle. Finally the operating maximum tensile stresses are also smaller in this saddle.

5. Equilibrium shape versus finite element analysis

In this final paragraph a comparison will be made between the results of the inverted cable-net in Easy and the solid model in Ansys. For this purpose the stresses will be compared in two spots of the anticlastic shape. One spot is located in the middle of the saddle, the other in the middle of one of the exterior boundary arches.

Table 1 compares the internal stresses in the middle of the saddle. The left part of the table shows the stresses, derived from the internal forces distribution in Easy (see Figure 10). The right part gives the corresponding stresses from Ansys. The stresses derived from Easy are calculated by dividing the internal forces by the thickness and the width of the elements. Table 1 summarizes the results of the same order of magnitude for the stresses in the y-direction. In the opposite direction a larger difference is remarked.

	EASY				ANSYS
	Force (kN)	Thickness (m)	Width (m)	Stress (MPa)	Stress (MPa)
x-direction	-0,3	0,05	1,33	-0,005	-0,019
y-direction	-3,4	0,05	1,30	-0,052	-0,061

Table 1 Comparison between the stresses derived from the form finding procedure in Easy and the finite element analysis in Ansys in the middle of the 'formfinded' saddle

Table 2 compares the internal stresses at the top of the external boundary arch of the anticlastic shape. On average the stresses in both directions are from the same order of magnitude in both models. There is a good identity between the Easy and the Ansys model.

	EASY				ANSYS
	Force (kN)	Thickness (m)	Width (m)	Stress (MPa)	Stress (MPa)
x-direction	-0,2	0,05	1,33	-0,0030	-0,0049
y-direction	-4,9	0,05	1,30	-0,075	-0,041

Table 2 Comparison between the stresses derived from the form finding procedure in Easy and the finite element analysis in Ansys at the top of the boundary arch in the 'formfinded' saddle

Moreover, there is a good match between the stresses derived from Easy and those from Ansys for the two arbitrarily chosen points.

6. Conclusions

The use of textile formwork can be seen as a new methodology to solve the feasibility problems while building complex concrete shells. Textile formwork can widen the range of concrete shell shapes. As a tensioned textile membrane takes an anticlastic shape, the fabric formed shell will also take this form. These anticlastic shapes though need to approximate three dimensional compression geometries. Therefore, a preceding form finding procedure of an anticlastic shell (based on force-density method) needs to be executed. Attempting to create a complete compression surface of a saddle shape with this method, it is needed to change the parameter of the force-density to achieve a cable net in full compression. A finite element analysis of the same geometry results in a full compression transfer load in cross direction. In the opposite direction areas with bending moments still occur. Nevertheless, the final shape is structurally more optimized compared to the initial saddle. However, further research to optimize the final shell geometry needs to be done.

7. Acknowledgements

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