Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

Mapping two-way continuous elastic grid on an imposed surface: Application to grid shells

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Abstract

This paper presents a method to generate a grid shell on an imposed shape and imposed boundary conditions. The proposed method is done by mapping a two-way continuous elastic grid on an imposed surface using an explicit dynamic finite element method. An initially flat two-way grid, with free boundary conditions, is set up over a formwork like surface that has the desired shape. Then, the grid is dropped on the fixed surface. Moreover, different strategies of formfinding are presented and a comparison with other methods is done. Finally, many examples are shown to illustrate the proposed method.

Keywords: Grid Shells, formfinding, finite elements, explicit dynamic.

1. Introduction

Architects, through the development of computer-aided design, propose forms of increasingly complex, these structures are usually referred as 'Blob architecture'. Grid shell structures could be an acceptable answer to these complex forms. Whatever the form proposed for a grid shell the technology required remains the same. Few grid shells were constructed in the world; most of them are made of wood. The grid shell Bundesgartenschau in Mannheim was the first timber grid shell designed and built by Frei Otto in the 70's. Recently, two grid shell prototypes made of glass fibre reinforced polymers were built at the Ecole des Ponts ParisTech (France) (Douthe [6], Douthe et al. [7-8]). In this context, a numerical algorithm using the dynamic relaxation method was developed. The implementation was done using the free scientific software Scilab™ that is developed by INRIA and the Ecole des Ponts ParisTech.

The authors classify grid shells in two families. The first family is that of grid shells with imposed boundary conditions. Their shape is a result of the calculation and is difficult to control. The grid shells of Frei Otto designed by the inversion form method (Otto [9]) are in this category, and those designed according to the dynamic relaxation method. The grid shells of the second family have their form imposed as well as their boundary conditions. Frei Otto proposed a method called "compass method" (Otto [9]) that allows this kind of grid shells. This last is based only on geometry principles. Another method for such designs is proposed in this paper. This method, through an explicit finite element code, gives the possibility to generate a grid to almost any form. An application of this method on an hemisphere is shown and a comparison with the compass method is done. In addition, examples are shown to illustrate the proposed technique.

2. Grid Shells: A structural innovation

Grid shells are often defined as structures that have the shape and rigidity of a double curvature shell but they consist of a grid and not a continuous surface. The particularity of these structures is to be able to cross large span with a low quantity of materials. They can be made of steel, aluminium, wood or even cardboard. In general, metallic structures are composed of rectilinear elements that define facets on the surface of the grid. The complexity of the obtained geometry requires the development of a great number of pieces of complex and expensive assemblies. Therefore, an original process of construction was developed. This process consists of long continuous beams that are assembled on floor and articulated between them. This gives the grid a lack of shear stiffness and allows large deformations. The grid is then deformed elastically until the desired shape is reached. It is then rigidified using a third direction of bars. In the world, a dozen grid shells were constructed using the described method, among them the building Bundesgartenschau Mannheim and Downland Museum in the United Kingdom.

In the existing grid shells, wood was chosen because of its low density and its equivalent limit strain (about 2%). Moreover, Glass Fibre Reinforced Polymers (GFRP) have Young modulus much higher (of 20 GPa to 40 GPa) than wood (around 10 GPa), so that, for a given geometry of grid Shell, the buckling load of a grid Shell in composite materials will be higher than for a timber grid Shell (Douthe [6], Douthe et al. [7-8]). Furthermore, as the efforts in the bars are exclusively axial efforts, fibres are required only in the main direction of the bar. Such unidirectional profiles can be obtained using the industrial process of pultrusion, a very economic method of production. This also allows the manufacture of tubes of great length. Consequently, composite materials seem to be appropriate candidate for grid shell structures.

3. Grid Shells and formfinding

The authors classify the grid shells in two families according to their formfinding strategies. The first family is that of grid shells with imposed boundary conditions whose shape is a result of the calculations. Whereas, the second family is that of grid shells having imposed form and boundary conditions.

3.1. Grid Shells with imposed boundary conditions, form as a calculation result

The funiculars forms with imposed boundary conditions were studied by Frei Otto and his team at the Institut für leichte Flächentragwerke [4]. Their work was based on the assumption that the flexural stiffness of the grid elements is negligible. This assumption allows them working only on suspended nets and establishing a typology of the grid shells founded on imposed boundary conditions. However, the structures obtained by this method are ideal structures without bending.

The inversion form method can be summarized in the following steps: A net with square meshes is constructed form chains of ten metallic links which figure the grid shell bars and circular rings which represent the assemblies. In the next stage, a set of boundary conditions is chosen and the net is suspended; the lengths of the bank cables are then modified until all the chains are tightened and the whole curvature is visually satisfactory (figure 1). Then, the geometry of the net is raised by photogrammetry and a first approximation of the coordinates of each ring is calculated. Finally, the positions just been evaluated, are recomputed with the force density method so that the equilibrium of the structure under actual weight is verified in each point.

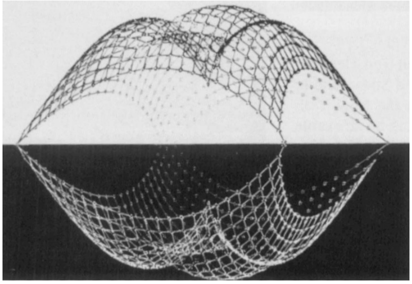

Figure 1: Suspended model of IL and, by inversion, the corresponding Grid Shell

An alternative method was proposed in the Navier institute at the Ecole des Ponts, ParisTech (Douthe [6], Douthe et al. [7-8]). This method was developed using the dynamic relaxation method by taking into account the bending stiffness of the elements.

Figure 2 shows different steps for generating a dome like grid shell. First a flat two way direction grid is chosen. Then it is pushed upwards at every connecting node. The boundary conditions remain in the initial plane of the grid. Finally, a third direction beam is set in place to rigidify the structure. This method shows that it is difficult to control the final shape of the grid shell.

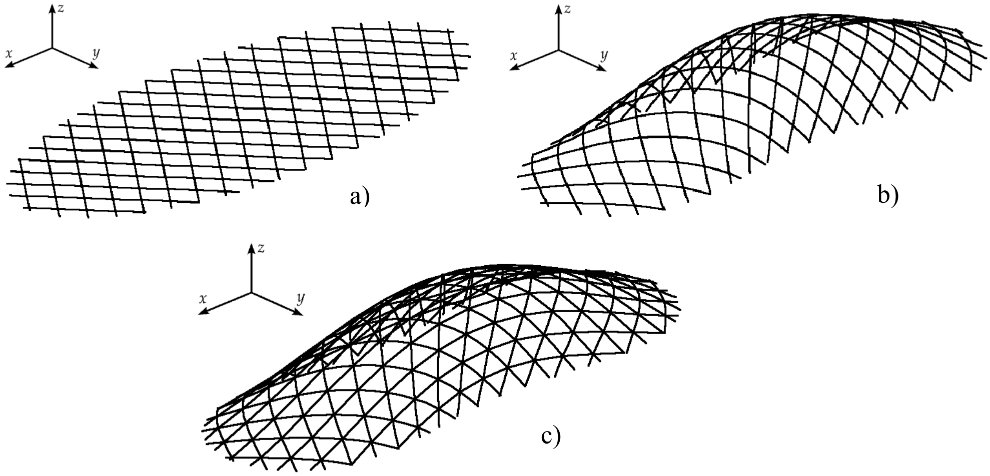

Figure 2: a) plane geometry, b) final geometry, c) geometry after triangulation

The dynamic relaxation method is a numerical tool that uses a dynamic calculation to find the static equilibrium state of a mechanical system. The concept dynamic relaxation appeared for the first time in an article by Day [5]. Micheal Barnes was the first who uses the dynamic relaxation for the formfinding and the analysis of tension structures (Barnes [1]). According to Barnes, the dynamic relaxation method consists of tracing step by step for a small increment of time the dynamic behaviour of the structure studied, from an initial perturbed time to a static equilibrium state. The later is reached by adding a kinetic damping (Barnes [2]), where kinetic damping (Cundall [4]) is an artificial damping. In this procedure, the oscillations of the structure are free until a maximum of kinetic energy is reached. Then, the structure is stopped, and all velocities are put back artificially to zero. After that, the structure is free to oscillate again until the next maximum kinetic energy. And so on, until the kinetic energy of all modes of vibration has been dissipated and that the equilibrium is reached.

Figure 3 shows two prototypes of composite materials grid shells that were built on the site of the Ecole des Ponts ParisTech. These grid shells are made of pultruded glass fibre reinforced tubes having a diameter of 42mm and a thickness of 3.5mm. The roofs are made of polyester canvas coated PVC, with the help of Ferrari SA, Esmery Caron and Abaca consulting engineers. The accurate geometric measurements were carried out by surveyors from the National School of Geographical Sciences ENSG, Paris.

Figure 3: First prototypes of composite materials Grid Shells

3.2. Grid Shells with imposed boundary conditions and form

The objective of the development of surfaces method, known as "the compass method", is therefore the definition of a methodology of construction of a network of parallelograms on any surface. This method was described in IL10 Gitterschalen of Frei Otto [9]. Figure 4 shows different steps of the method on a plane surface. The steps of this method are summarized as follow. First, a square is chosen as the form for the edge. The task is to construct a grid using only a compass. Two arbitrary, curved axes that intersect are laid down. Then, a mesh width is selected and serves as the compass radius. The spacing of the grid is marked along each axis, starting from the point of intersection of the axes. Two neighbouring knots are defined. The fourth knot is the intersection of the two circles drawn around each of the neighbouring points with radii equal to the mesh width. Then, gradually, new points are determined in the same way. Finally, the net knots are connected rectilinearly.

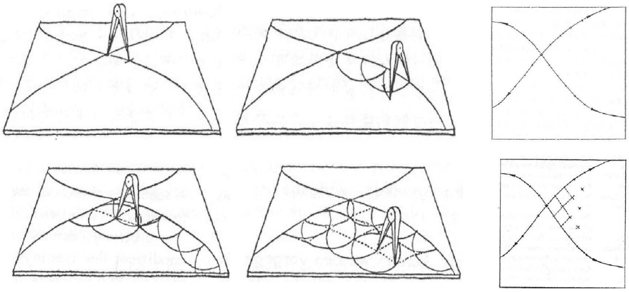

Figure 4: Construction of the directors and the grid using the compass method (Otto [9])

For other forms, the principle of the construction of the grid is exactly the same one as previously explained. This method is only based on geometrical parameters. However, the second method uses the dynamic explicit finite element method as described in detail in the section 4.

4. Dynamic explicit finite elements simulation

So far, the simulations carried out at the Navier institute allow determining the form of a grid shell from the geometry of the flat grid and the boundary conditions using a home made software 'AlgoRD'. Thus, the final form is the result of calculation (Case 3.1). However, a designer usually knows the final form he is looking for and therefore a designer will prefer the (Case 3.2). Since the compass method does not take into account the mechanics of material, the proposed method described in this section, allows the designer to determine a grid shell with imposed form and imposed boundary conditions using the dynamic explicit finite elements analysis (Abaqus™). This method is inspired from the formfinding of composite fabric (Boisse et al. [3], etc). It consists of mapping a two-way continuous elastic grid on an imposed surface using an explicit dynamic finite element method.

Figure 5 shows the steps for finite element simulation for a hemisphere. First, a two-way grid, initially flat, with free boundary conditions, is set up over a fixed surface having a defined form. Then, a system of force that is applied vertically on the grid enables it to take the form of the desired surface. During the calculation, the friction between the grid and the surface is permitted. A convergence study helps to define the system of force in each case. Calculation is done for tubes having the form and the characteristics of those pultruded used in the construction of the prototype.

Several calculations were done by varying the angle between the two bars of the flat grid. It is noticed that for various initial angles (30, 45, 60 and 90 degrees), the calculation converges to a final angle of 90 degrees in the final result. This is a very important point, since the method seems to allow an optimization of the bars directions as it minimize the energy of the system.

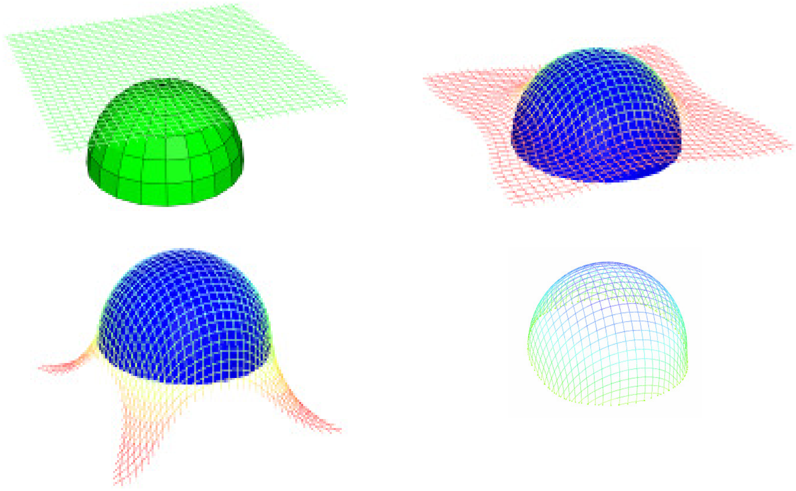

Figure 5: Steps for mapping a half of a sphere using finite elements

Figure 6 shows the various stages for mapping half of a sphere using the compass method described in section 3.2. A comparison was made between the two methods for a half of a sphere with a radius of 10m. Both methods give similar final geometries.

Table 1 summarizes the difference between the two methods. This variation is calculated on the final geometry of the grid where x, y and z are the coordinates of the points in space. The average value is calculated by eliminating the aberrant values.

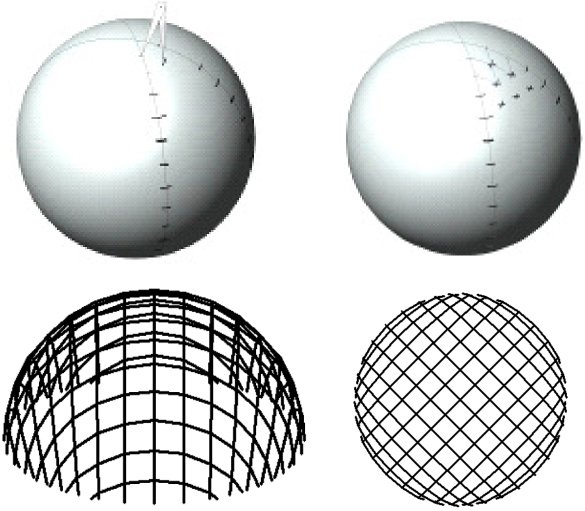

Figure 6: Mapping half a sphere using the compass method

	dx (in m)	dy (in m)	dz (in m)	d (in m)	dx/x (%)	dy/y (%)	dz/z (%)
max	0.015	0.014	0.008	0.040	2.998	0.608	4.226
min	-0.032	-0.012	-0.038	0. 001	0.001	0.000	0.003
moy	-0.008	0.000	-0.010	0.018	0.488	0.099	0.397

Table 1: Variation between the two methods for a half of a sphere of 10m radius

It should be noted that the compass method is purely geometric; it does not include the mechanical behaviour. In the numerical method, the impactor imposes the geometry on the grid where mechanical properties are those of the real grid shell, but the efforts applied are those needed for the algorithm to converge.

However, it is obvious that after removing the impactor and the imposed forces and after fixing the boundary conditions, the final shape will evolve. This step can easily be simulated by dynamic explicit finite element method.

In addition to the spherical shape, other forms of surface are considered. Figure 7 shows different steps for mapping an ellipsoidal surface. On the other hand, figure 8 illustrates steps for mapping an arbitrary surface using finite elements.

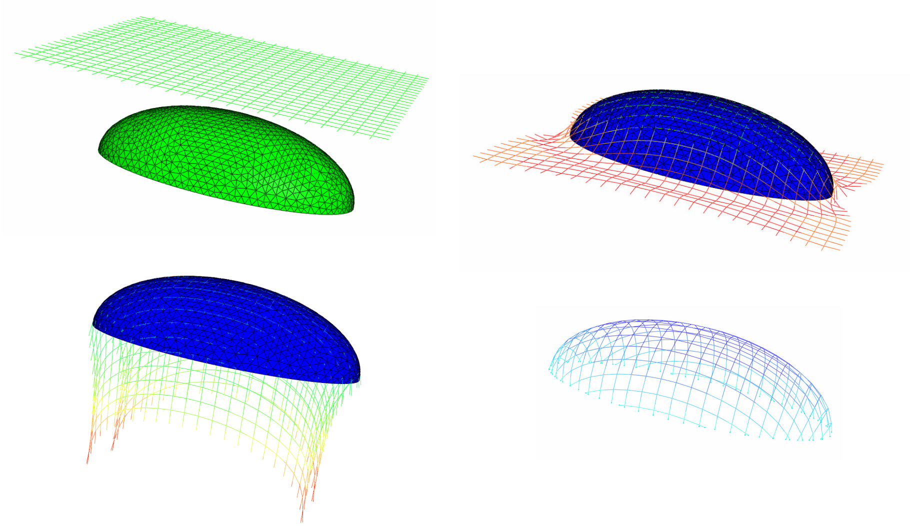

Figure 7: Steps for mapping a 3D ellipse using finite elements

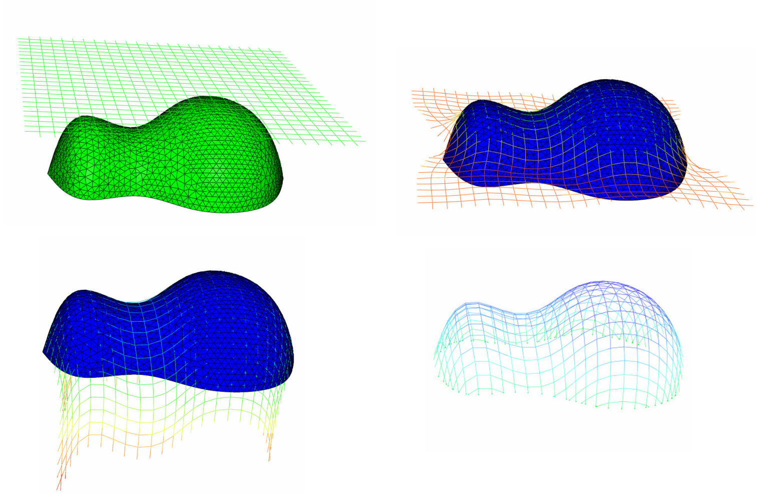

Figure 8: Steps for mapping an arbitrary surface using finite elements

5. Conclusion

A method using the dynamic explicit finite element for the grid shells formfinding is presented in this paper. Two strategies for the formfinding of grid shells are detailed. The first strategy is to have imposed boundary conditions with a final form resulting from the calculation. The second strategy is to have imposed forms and boundary conditionss. In the first strategy, it is difficult to control the final shape of the structure. The methods used for this strategy are the hanging net method and an alternative method developed at the Navier Institute using the dynamic relaxation algorithm. The later consists of pushing elastically upwards a two-way flat grid having constrained boundary conditions. The second strategy involved the development of a tool using a dynamic explicit finite element code.

This method consists of mapping a bidirectional grid initially flat over a defined surface shape using explicit finite elements. The method can simulate the geometry which should be obtained and quantify deviations from the desired shape by the architect. It also allows taking into account the rigidity in the bars. To perform a structural analysis, boundary conditions have to be fixed, a third direction of bars for the bracing of the grid shell has to be added and external loads also need to be added. Optimization work on the bars direction for a given form to minimize the initial stress in the grid shells is under study.

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