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Computational morphogenesis in architecture: the cost optimization of free form grid-shells

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Abstract

In this paper an optimization problem related to the tessellation of a free form grid shell is presented. This kind of structures is generally composed by a supporting grid that defines the geometry of a large number of cladding elements always different one from another. From the constructive point of view it means that every single piece needs to be designed and produced "ad hoc", then marked and positioned with the aid of an assembling table. In order to reduce the heterogeneity of grid-shells elements, several optimization strategies referring both to evolutionary and gradient-based techniques, have been tested and compared. In view of future development, a multi-objective procedure that involves static analysis combined with the discussed geometrical optimization is finally proposed. All the free form geometries are defined and handled by means of a commercial NURBS based software. On the contrary, the development of all the presented optimization procedures has been possible thanks to the implemented VB based programming language of the same NURBS based software. Due to the smoothness of the solution domain of this specific problem, gradient based procedures seem to be the most efficient in the rapidity of convergence to the optimal solution.

Keywords: computational morphogenesis, form finding, multi-objective optimization, cost optimization, grid-shells, genetic algorithms, force density method.

1. Introduction

Grid shells belong to a constructive typology initially studied and developed by engineers from the constructive point of view (Schlaich and Shober [1]) in order to improve mainly the efficiency of this kind of structures. With the development of computer technologies, many designers, with the aid of parametric surfaces, gradually replaced regular and quasi-regular shapes with more complex geometries, raising a set of new problems related to the constructive rationality of free-form structures, that we can handle by means of morphogenesis and optimization techniques.

In huge free-form glass roofing, such as in the long covering designed by Fuksas and Schlaich for the trade fair in Milan, structural elements might be chosen from a catalogue and the risk to deal with a puzzle of numbered pieces on the building site could be avoided. Moreover, a limited typology of cladding elements may not be a decisive factor in the case of glass slabs, easily 'mass customized', but quite important, for instance, in the case of solar panels that are themselves a composition of different elements.

Starting from these considerations, what the present study aims to do, in relation with free form grid-shells, is to explore the potentiality of acting "a priori" on morphogenesis instead of "a posteriori" on manufacturing to achieve the same cost benefits by avoiding the wide heterogeneity of elements. Four optimization procedures of grid shell structures tessellation have been developed and compared. Firstly a discussion about the limits of an analytical approach is presented, then the generative problem has been changed in a shape improvement problem and faced with the aid of traditional optimization methods and evolutionary techniques. The aim is not to present a detailed comparison among different strategies but to focus on the most significant problems which have been faced and to underline the advantages and disadvantages of each procedure in a sort of "step by step" solution process.

Algorithms have been carried out by personalizing the NURBS modelling software Rhinoceros™ through its implemented VB programming language.

2. Analytic approach: Sphere Packing

This first developed algorithm, named "sphere packing", is based on a recursive generation of spheres over a given surface in order to build a triangular mesh with the desired characteristics.

An "a priori" database of radii measures defines the number of possible sphere typologies involved in the algorithm. Each sphere has to be tangent to all the others around it and the center of each sphere has to be as near the primitive surface as possible. The generation procedure starts from a chosen point on the surface and develops in a radial way (Fig. 1). The final mesh is generated by the connection of all the tangent spheres' centers and the number of frame typologies derives from the combinations of radii measures.

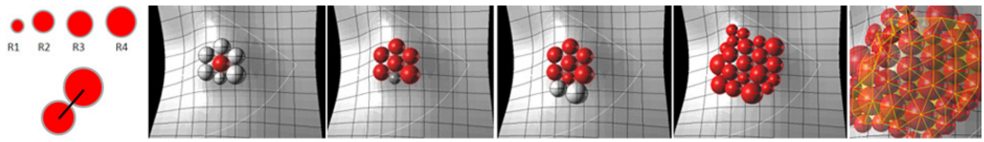

Fig. 1. A sequence of the "sphere packing" algorithm.

The results of this first approach (see ch.5) show problems mainly connected to computational speed. In particular, an increasing number of database measures leads to many more possible combinations of spheres. Moreover the necessity to take care of many particular cases, in order to avoid crashes, does not fit very well with a smart algorithm's structure. For all these reasons the procedure has not been developed anymore.

3. From generative to improving procedure

Looking at the ineffectiveness of the first developed algorithm the idea was no longer to create a new mesh from scratch but to start from a given mesh and adapt mesh frames lengths to a set of chosen measures (database).

The change from a "generation" process to an "improvement" process increased significantly computational speed. In fact commercial modeling software offers internal procedures to mesh generic shape surfaces and the resulting meshes are always an optimal approximation of the NURBS. Another advantage is the possibility to decide "a priori" constraints for mesh vertices. It is important to remember the mesh is the representation of the aimed structure; if we suppose the structure is, for example, a covering, it is obviously important to take care of the position of columns during the optimization process.

Three new different algorithms have been developed, one "ad hoc" whereas the other two taken from literature and suitably adapted to this problem. The "ad hoc" developed algorithm, named Progressive Move Rotate and Fix (PMRF), it is a simple translation of the sphere packing concept to the case of a given starting mesh: the algorithm develops from a chosen mesh knot in a radial way changing at each step a frame length with the nearest one taken from the database. The first procedure taken from literature is a Genetic Algorithm (GA), a meta-heuristic optimization method based on the concept of human evolution. Since it is a consolidated method only references about this technique (Mitchell [2]) and its usage (Pugnale and Sassone [3]) are provided here. The last procedure tested on the problem is a particular implementation of the gradient-based technique called Force Density Method. In ch.4 a brief description is reserved to this method considered the most suitable for the previously set goal.

All the presented algorithms have been tested on benchmarks explained in ch.5.

2.1. Objective function

The improving process of the starting mesh can be analyzed as a comparison between the frames lengths at each step of the optimization process and a set of referential measures, chosen "a priori" as a database for the final tessellation of the initial shape.

The fitness function that allows to monitor the effectiveness of the developed algorithms is:

$$f = \sum_{i=1}^{n}(l_i - l_{dat}^{*}) \rightarrow 0 \tag{1}$$

$where:$

$n = number\ of\ frames\ ;$

$l_i = length\ of\ the\ frame\ i;$

$l_{dat}^{*} = the\ nearest\ database\ measure\ to\ l_i$

The convergence of the fitness function f to zero is the optimal searched solution.

2.1. Optimum database

A particularly effective step, in order to improve previously shown procedures, has been the development of an auxiliary algorithm, the function of which is to optimize the database by choosing a set of "smart" measures.

Starting from the mesh given by the software it is possible to know exactly the measure of each frame and is also possible to define a mean value from all these measures. In the same way, it is possible to divide the range of measures in smaller intervals and find out the mean value of each one. Consequently it is immediate to understand that the fitness calculation result is improved by assuming these mean values as database measures and moreover this is true if intervals are designed to contain as many frame measures as possible.

To perform this process a standard "divide et impera" algorithm has been implemented. It has to be noticed that avoiding a direct choice of database measures does not mean a loss of control on the final result as the lengths of starting mesh frames are managed by the designer.

4. (Virtual) Force Density Method

The Force Density Method (FDM), since Linkwitz and Schek's first development [4] in 1970's, has been well known as a powerful tool for analytical form-finding and static analysis of self-stressed structures like tensile membranes and cable networks (Southwell [5]).

Actually force density is always associated to a real stress state of the structure under a field of applied forces that, combined with other boundary conditions (constraints, etc.), allows the shape to evolve and improve. However, looking at the problem from the mathematical point of view, it can be realized that the method works the same without an external force field and by replacing real cable tension with a virtual one.

For the defined purpose, stress state for each mesh frame has been defined by geometrical vectors representing the difference between the length of the frame and the nearest measure of the database (Fig.2 and equations n.2). This way the algorithm acts the same as a traditional FDM but for the form finding process that is guided only by geometrical rules.

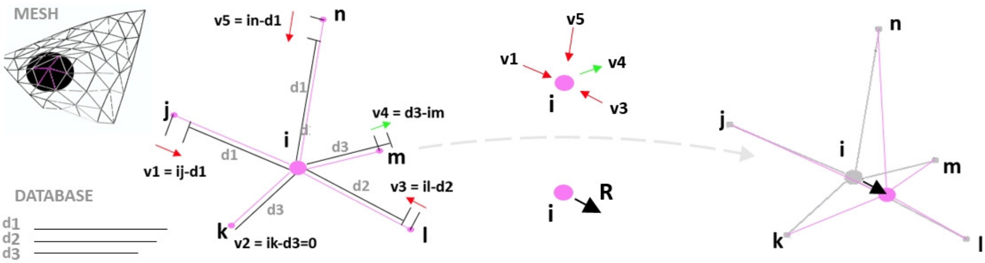

Figure 2: VFDM concept.

$$\begin{aligned}
\frac{v1_x}{d1}+\frac{v2_x}{d3}+\frac{v3_x}{d2}+\frac{v4_x}{d3}+\frac{v5_x}{d1}&=R_x\\
\frac{v1_y}{d1}+\frac{v2_y}{d3}+\frac{v3_y}{d2}+\frac{v4_y}{d3}+\frac{v5_y}{d1}&=R_y\\
\frac{v1_z}{d1}+\frac{v2_z}{d3}+\frac{v3_z}{d2}+\frac{v4_z}{d3}+\frac{v5_z}{d1}&=R_z
\end{aligned} \qquad (2)$$

5. Applications and Results

A first comparison among the four developed algorithms has been made over three simple benchmarks representing three surfaces with different Gaussian curvature. This test (fig.3) highlights the VFDM as the most effective algorithm both for the computational speed and for the number of frames adapted to the database (red colour in fig.3).

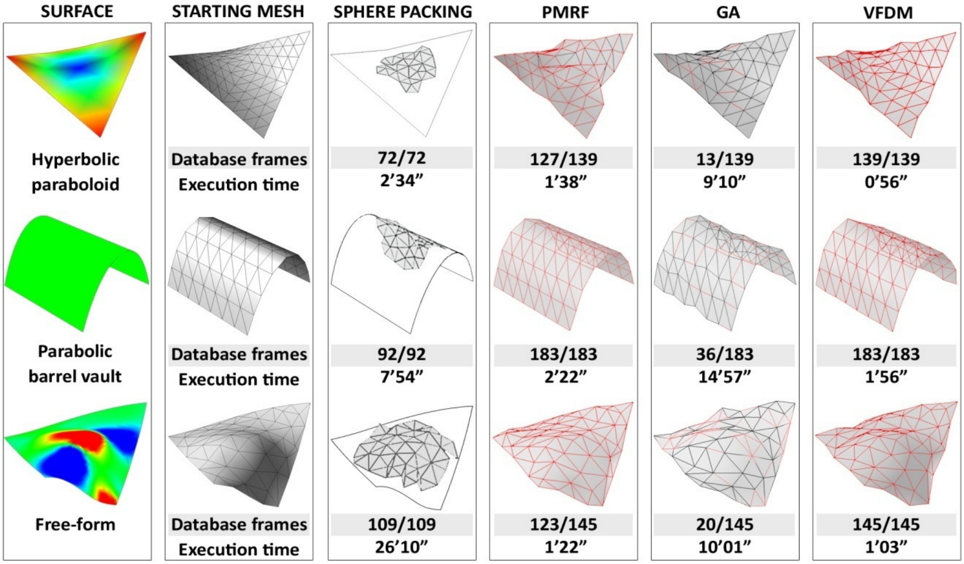

Figure 3: Comparison among developed algorithms on benchmark surfaces with different Gaussian curvature.

To better analyze the behavior of the VFDM algorithm, another two applications have been developed. The first one (fig.4) shows the consequences of using different databases in the same optimization process: an increasing number of database measures allows a time saving in terms of computation and also a better approximation of the original surface and consequently smoother shapes. Anyway the algorithm seems to work quite well, adapting all the frames lengths to database measures, even if the database is 'small'.

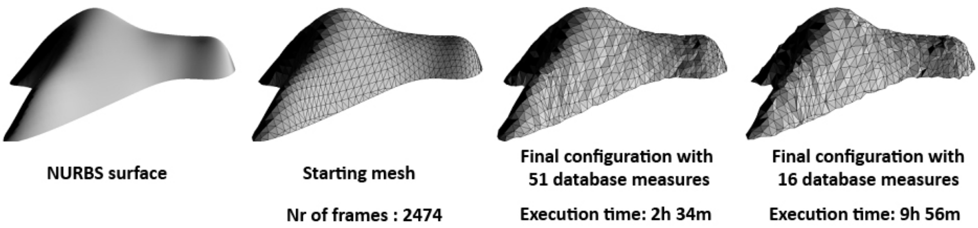

Figure 4: VFDM application 1 - shape smoothness evaluation.

The second application shows consequences in algorithm efficiency of a significantly high number of constrained joints. When the original shape to approximate has a very irregular geometry or there are characteristic lines the maintenance of which is of primary importance, the possibility to fix some joints or to link their movement to curves or surfaces during the optimization process is requested. On the other hand, too rigid boundary conditions could make a total convergence of the algorithm impossible as in the real case study shown in fig.5.

Mesh frames	2743
Database measures	36
Database frames	2326
% of database frames	85
Execution Time (min)	47
Nr of constrained joints	392
% of constrained joints	15

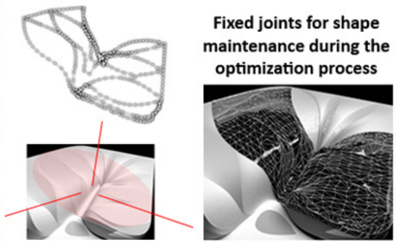

Figure 5: VFDM application 2 - shape maintenance evaluation.

VFDM algorithm seems to be an effective optimization procedure to face the discussed geometrical problem. It has to be noticed, in particular, that the greater is the number of elements composing the structure to optimize, the better is the solution found. In fact a significant increasing number of elements usually does not require a similar increase in database measures to achieve a smooth approximation of the initial shape. Consequently the number of database measures becomes a smaller percentage of the total number of frames.

6. Multi-objective optimization

The possibility of a combination between the presented geometrical optimization process and a static enhancement of structure starting from the research of Pugnale and Sassone [3] has been tested. The procedure, written in VB, sees the interaction of a commercial NURBS modeler as Rhinoceros™ with a FEM software as Ansys™ through a Memetic Algorithm (MA) (Elbeltagi *et al.* [6]) that implements inside the previously shown VFDM algorithm.

The MA implements the evolution of a NURBS surface acting on the vertical movement of 16 control points into a square basis parallelepiped volume. All the NURBS surfaces are then changed into a correspondent mesh, automatically generated by the software, and geometrically optimized before the static performance evaluation. The shell static performance evaluation is based on the strain energy of the structure under a uniform force field (Sasaki [7]). The first results obtained from the study of a simple benchmark are shown below.

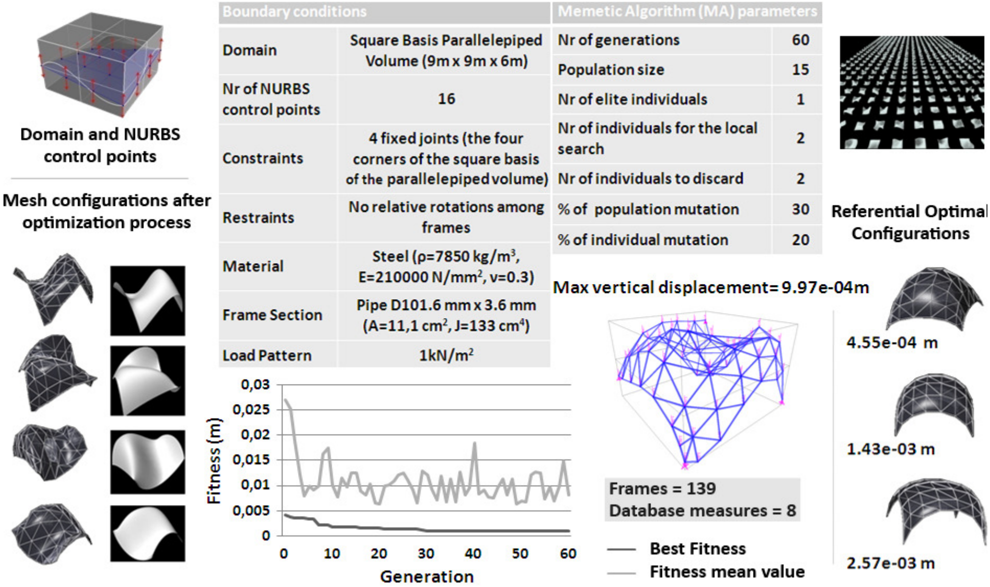

Figure 7: Multi-objective optimization as a combination of geometrical and static performance improvement.

The static behavior of the resulting grid-shell is comparable to other traditionally effective configurations and the free-form structure (139 elements) is made only by 8 frame typologies.

7. Conclusions

Both the field of engineering and architecture need new methods of investigation and analysis and new approaches which can interface the output of a new design. Acting on morphogenesis by using innovative instruments seems an interesting and effective method of planning improvement which can perform economic advantages and guarantee suitable performances.

The results which have been discussed in this paper show the effectiveness of a generative approach linked to the internal logic of the form. This approach allows the designer to decide the design priority aspect and at the same time to have full control of the involved parameters.

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