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Applications of a "Virtual" Force Density Method

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Abstract

The Force density Method [FDM], since Linkwitz and Schek's first development [8] in 1970's, has been well known as a powerful tool for analytical form-finding and static analysis of self-stressed structures like tensile membranes and cable networks [12]. Actually force density is always associated to a real stress state of the structure under a field of applied forces that, combined with other boundary conditions (constraints, etc.), allows the shape to evolve and improve). The approach proposed in this paper enables the technique to become more versatile so that it is possible to extend its application to different kinds of problems. In particular, changing the connection matrix and introducing virtual forces, morphogenesis can be related to general spatial frame structures looking at geometrical requirements as form and function irrespective of the applied stress field. As a demonstration of the capabilities of this tool, three different applications on current issues in free-form architecture are tested by means of a commercial NURBS based software and a VB based algorithm. One of these applications concerns the preliminary design of the roof structure of Ponte Parodi project, by UNStudio Amsterdam, a multifunctional building to be located on the waterfront of Genoa, Italy.

Keywords: computational morphogenesis, form finding, optimization, force density method.

1. Introduction

The development of CAD-CAM applications and its large diffusion in architecture is changing the traditional relation between design and construction, from two different points of view [9]. First, architectural construction rely more and more on the industrial production of components and elements, which are merely assembled in the building site. Secondly, the largest group of problems related to manufacturing and assembling need to be managed almost completely at the design time. Manufacturing and industrial production of elements become a part of the design process.

One effect of such technological evolution in architectural design is the growth of free-form projects, in which the construction of irregular shapes, characterised by a large use of non standard elements is made possible by a strong industrialisation and prefabrication of components. In fact, the design of geometrically complexes structures involves not only complexes analyses and checks but even the solution of constructional problems. In grid-shells and large thin roofs, when the shape of structural members and of elements as roofing, cladding and glass panels must adapt to irregular surfaces and volumes, the design must solve a number of manufacturing problems in order to make the construction efficient and to contain the overall cost. This goal can be achieved through the development and application of optimization procedures, based on robust and flexible computational tools, for the search of solutions that best fit the design requirements. Optimization strategies become central in these kind of projects and frequently they condition the final result.

In this paper we have considered two well known constructional problems, related to the geometry of grid shells, as optimization problems by means of a custom version of the classical Force Density method we have especially developed. We will refer to it as Virtual Force Density Method (VFDM). The optimal solution is approached iteratively starting from configurations that can be sub-optimal or random defined or deriving from the early stages of conceptual design.

In the first case we explored the possibility to improve grid-shells shapes in order to standardize element typologies. In huge free-form glass roofing, such as in the long covering designed by Fuksas and Schlaich for the trade fair in Milan (Figure 1, c), structural elements might be chosen from a catalogue and the risk to deal with a puzzle of numbered pieces on the building site could be avoided. Moreover, a limited typology of cladding elements may not be a decisive factor in the case of glass slabs, easily ‘mass customized’, but quite important, for instance, in the case of solar panels that are themselves a composition of different elements.

In the second case the aim is to make an effective approximation of the original shape by a composition of Planar Quadrilateral (PQ) glass elements. Usually, cladding glass elements are planar because they are directly cut from planar glass plates industrially produced (Figure 1, b). When a plane rigid panel is used to cover a net face with more than three sides, it is then necessary to make sure that all the corner points lay on the same plane. Only if the cladding material is soft and can be freely curved, as in the case of inflated ETFE pillows this requirement loses its importance. We focus on this geometrical problem for quadrilateral grids since almost all NURBS modeling softwares can effectively generate quadrilateral meshes starting from a general free-form surface.

Figure 1, a, b, c: a) Great Court of the British Museum, London, designed by Foster and Partners; b) The Traid Fair in Milan designed by Massimiliano Fuksas.

We present here three applications in which the resolution of these problems were an integral part of the design process. The first one is simple benchmark shape developed through a NURBS modeller. The second one concerns the roof structure design of Ponte Parodi, by UNStudio Amsterdam, a multifunctional building to be located on the waterfront of Genoa, Italy (Figure 8). The last application is on the free form grid shell roof for an historical gallery in the north of Italy (Figure 6**¡Error! No se encuentra el origen de la referencia.**).

2. Force Density Method [FDM]

The force density method (FDM) was initially presented by Linkwitz and Schek 1971 [8] as a form finding technique for cable nets. The properties of this method were subsequently studied and the method could be implemented in an efficient way by applying special sparse matrix techniques for solving the resulting equations. It proved to be a powerful tool for setting up and solving the equations of equilibrium for pre-stressed networks and structural membranes, without requiring any initial coordinates of the structures [3].
Furthermore FDM is an effective method for finding viable configurations, which avoids the problems related to nonlinear analysis, which was first proposed in the context of cable nets. The starting point for the traditional FDM is a pin-joint network consisting of cable or bar elements, in which some of the points are fixed and the others are free. The free points will have to find a position in the equilibrium configuration and the constrained points are the input data for the initial equilibrium configuration problem. The essential ideas are as follows. Pin-jointed network structures assume the state of equilibrium when internal forces s and external forces **p** are balanced.

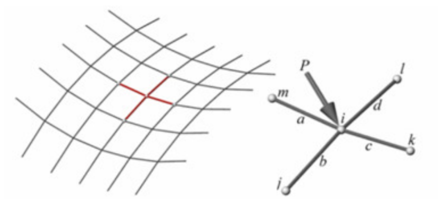

Figure 2 – Part of a cable network, picture from Gründig *et al*. [5].

In the case of the node in Figure 2, the equations read:

$$
\begin{aligned}
\frac{s_a}{a}(x_m - x_i) + \frac{s_b}{b}(x_j - x_i) + \frac{s_c}{c}(x_k - x_i) + \frac{s_d}{d}(x_l - x_i) &= p_x \\
\frac{s_a}{a}(y_m - y_i) + \frac{s_b}{b}(y_j - y_i) + \frac{s_c}{c}(y_k - y_i) + \frac{s_d}{d}(y_l - y_i) &= p_y \\
\frac{s_a}{a}(z_m - z_i) + \frac{s_b}{b}(z_j - z_i) + \frac{s_c}{c}(z_k - z_i) + \frac{s_d}{d}(z_l - z_i) &= p_z
\end{aligned}
\quad (1)
$$

In these equations, the lengths **a**, **b**, **c** and **d** are non-linear functions of the coordinates. This initial information can be brought together in force density parameters $\mathbf{s_a/a=q_a}$, for every link. The resulting linear equations read,

$$
\begin{aligned}
q_a(x_m - x_i) + q_b(x_j - x_i) + q_c(x_k - x_i) + q_d(x_l - x_i) &= p_x \\
q_a(y_m - y_i) + q_b(y_j - y_i) + q_c(y_k - y_i) + q_d(y_l - y_i) &= p_y \\
q_a(z_m - z_i) + q_b(z_j - z_i) + q_c(z_k - z_i) + q_d(z_l - z_i) &= p_z
\end{aligned}
\quad (2)
$$

The system of equations assembled is extremely sparse and can be efficiently solved for the coordinates of the structure using the Conjugate Gradient method as described in [4].
A general scheme of the whole procedure for a traditional FDM is as follows:

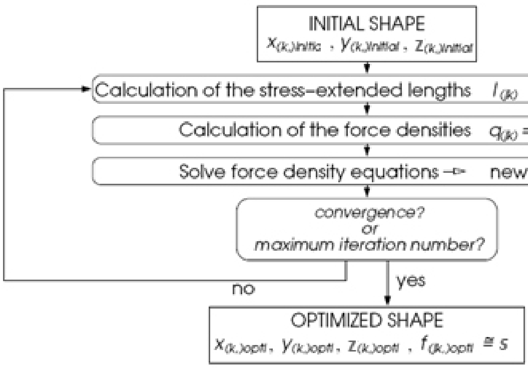

Figure 3 – FDM scheme.

It should be noticed that it is not always feasible to determine a shape that leads to the imposed forces in each element. Nevertheless the iterative process can determine a shape as close as possible to the requirements; this will be the optimized geometry. This is why two classical criteria are used to stop the iteration procedure. On the one hand we have a convergence test and on the other hand a maximum iteration step.

3. Development of a 'Virtual' Force Density Method [VFDM]

The FDM, as previously shown and as the name itself implies, is a well known form-finding technique that has been widely studied and used, above all, with reference to the static behavior of a structure. In other words, in almost all the range of applications of this method, vector generations depends on a force field and/or a stress state defined and applied to the structure itself. In fact structure typologies associated to this method are mainly pre-stressed ones, like cable nets, tensile membranes, etc. even if, for instance, the capability to find minimal surfaces made the method interesting also for grid shells and concrete shells, etc. However, looking at the field of civil engineering, as far as we know, all the applications of this method, , used for optimization purposes deal with static performance.

Now, starting from the classical FDM procedure, we will redefine it in order to maintain the philosophy of the method but to make it suitable for a wider range of purposes such as, for instance, the geometrical optimization of architectural patterns.

Tracing the scheme of the method it can be realized that, from the mathematical point of view, the only elements necessary for the optimization algorithm are:

1- A set of *n* points (nodes) $\mathbf{p} \in \mathbb{R}^n_3$ where $\mathbf{p}_k$ is a 1x3 array representing point k coordinates for k = 0,1…, *n*.
2- A connectivity matrix **M**
3- Boundary conditions **C**
4- A vector's generation rule *r*
5- An objective function *f*

Points 1 and 2, from the classical point of view, give together the geometry definition of the network. Anyway it has to be underlined that the shape of M in the algorithm can vary in function of the set goal (this concept will be resumed in subsection 3.2.).

Point 3 is not strictly necessary because, from the mathematical point of view, the initial position of nodes could represent itself a sufficient boundary condition. However defining constraints or restraints for the nodes coordinates can't practically be avoided when we deal with real projects.

Point 4 is the most important for our purpose and will be explained in the next subsection.

Point 5 represents the controller of the algorithm. The fitness evaluation at each step is the way we can stop iterations. It is possible and, sometimes, convenient to use the objective function itself as a vectors generation rule; in this case points 3 and 4 would be merged into one and $r = f$ (this will be our case).

3.1. The vector generation rule

Vector generation in the traditional FDM is due to the interaction between a force field applied over the structure and the stress state of the structure itself. So, in this case, vectors which are generated in each node are essentially the result of a vectorial sum of forces. From the mathematical point of view the concept can be resumed with a function like:

$$f = f(\mathbf{p}, \mathbf{M}, \mathbf{t}, \mathbf{f}) \tag{3}$$

Where:

- **p** and **M** have been already defined,
- **t** is the 1xe array describing the stress state of the net and t_i is the tension value associated to the i element (connection), for i = 0, 1.. , e
- **f** (optional) is a 3xn array of forces with $\mathbf{f}_k$ the force vector applied on the net node k, for k = 0, 1.. , n .

Assuming **t** and **f** as parameters confined inside f makes it possible to replace or simply avoid them.

For instance, with reference to our problems, where there is no presence of force fields but only geometry is involved, the function for vector generation will read:

$$f = f(\mathbf{p}, \mathbf{M}) \tag{4}$$

In this case vector generations on the nodes of the net will be dependent only on the geometry of the mesh at each step and that is why we will refer to this method as a Virtual Force Density Method. In section 4, two different examples of objective functions, used also as vector generation rules, are presented.

3.2. The connectivity matrix

FDM is essentially based only on one kind of connectivity matrix (**M)** that is an nxe matrix where n is the number of points (nodes) and e is the number of connections (frames) depending on the shape of mesh faces. It is important to point out that, in this case, a connection is always intended as a link between two nodes. Consequently the calculation of the resultant vector in each point is dependent only by the position of the other points which are connected to this through a frame.

This has been obviously the more suitable choice since the aim was the static optimization of structures where stress is strictly connected to frames/cables but, on the other side, the method could be applied for several others purposes by changing the 'classical' connection matrix with a more effective one. Figure 4 shows, for example, a comparison between a classical connection and an alternative connection we have used to apply the method to the planarity problem (section 4.2.).

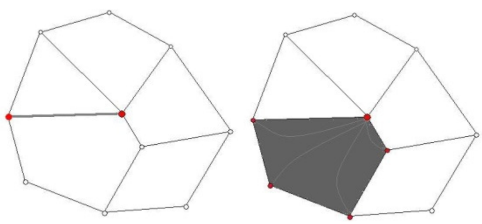

Figure 4 a, b: a) traditional connection between two vertices of the mesh - the connector is a frame; b) a possible alternative connection among more than two vertices - the connector is a face.

4. Applications and results

As we have already explained, the development of the VFDM has been a consequence of the will to find a solution to geometrical problems related to complex shape in architecture. In particular we firstly faced the problem of heterogeneity of elements composing free form grid-shells. The name of grid shell commonly describes a structure with the shape and strength of a double-curvature shell, but made of a grid instead of a solid surface. These structures can cross a large span with very little material. They can be made of any kind of material - steel, aluminum, wood or even cardboard tubes, etc. Generally, metallic structures are made of short straight elements defining a facet geometry. The complexity of geometry ,in this case, very often requires the development of clever and expensive assembly and a large number of non-standard elements. The first application of VFDM was just in order to limit frame typologies and, consequently cladding panels typologies of this kind of structures and in the subsection 4.1. an example of this application is presented.

After testing the effectiveness of the method we had the idea to apply it to another tessellation problem previously studied with another optimization algorithm (Sassone and Pugnale [6]). In this case the aim was to make an effective approximation of the original shape by a composition of quadrilateral planar elements.

Two different applications, related in a different way to the problem of planarity for quadrilateral mesh are presented in subsections 4.2.

4.1. Frames Standardization

4.1.1. Objective function

The improving process of the starting mesh can be analyzed as a comparison between the frames lengths at each step of the optimization process and a set of referential measures, chosen “a priori” as a database for the final tessellation of the initial shape.

The fitness function that allows to monitor the effectiveness of the developed algorithms is:

$$f = \sum_{i=1}^{n} (l_i - l^*_{dat}) \rightarrow 0 \qquad (4)$$

where:

n = *number of frames ;*

l_i = *length of the frame i;*

l^*_{dat} = *the nearest database measure to* l_i

Database is the set of measures, decided ‘a priori’ by the designer, as the only allowable for structure frames.

The convergence of the fitness function f to zero is the optimal searched solution.

4.1.2. Hyperbolic paraboloid

A simple hyperbolic paraboloid (Figure 5) shape has been choosen to test the VFDM over the problem of frames heterogeneity.

No constraints have been applied to vertices and only 8 frame lengths have been allowed (database) for the standardization process.

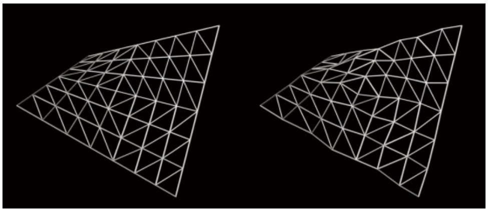

Figure 5

The roughness of the resulting mesh is obviously a consequence of total frames/database ratio but it depends also of the choice of database measures. The better database measures cover the range of frame lengths, the smoother the final shape. However the algorithm seemed to be very effective as it led all the frame lengths to database measures.

4.2. Planarity of quadrilateral meshes

4.2.1. Objective function

A quadrilateral mesh, generated starting from a free-form surface, is in general skewed, and the four vertices lay on different planes. Considering a group of four adjacent points, the 'skeweness' is what is referred to as the 'planarity error'. The simplest way to measure such error is to pick three points out of the four and to measure the distance between the fourth and the plane defined by the first three. Given points ***P1***, ***P2*** and ***P3***, ***a***, ***b***, and ***c*** coefficients of the corresponding plane are the solution of the following linear system:

$$\begin{cases} a x_{P1} + b y_{P1} + c z_{P1} + 1 = 0 \\ a x_{P2} + b y_{P2} + c z_{P2} + 1 = 0 \\ a x_{P3} + b y_{P3} + c z_{P3} + 1 = 0 \end{cases} \tag{1}$$

and the distance ***d*** between the plane and the fourth point ***P4*** is given by:

$$d = \frac{\left| a x_{P4} + b y_{P4} + c z_{P4} + 1 \right|}{\sqrt{a^2 + b^2 + c^2}} \tag{2}$$

A measure of the planarity error based only on one face could be misleading, because each point in general belongs to four different faces, except points along boundaries. Hence the measure should take into account the planarity of all the four adjacent faces, that have one common point. A simple way to do that is to sum the error values of the adjacent four faces: it can be shown that this sum is a good approximation of the local gradient of the planarity error, if only the coordinates of one point are considered as variables.

When a suitable error measure for each face or for each point of the polyhedral configuration is defined, an overall measure of the configuration error is to be evaluated.

Both the whole local errors vector and its Euclidean norm can be used in the optimization process, depending on the computational procedure. Consequently the objective function reads:

$$f = \sum_{i=0}^{n} \sqrt{\sum_{j=0}^{m} d_j^2} \rightarrow 0 \qquad (3)$$

where:

$n = number\ of\ vertices;$

$m = number\ of\ faces\ touching\ the\ vertex\ i;$

$d_j = planarity\ error\ between\ vertex\ i\ and\ the\ plane\ defined\ by\ face\ j\ points;$

The convergence of the fitness function f to zero is the optimal searched solution.

4.2.2. A new roof for the Tergesteo Gallery

This real case study consists in the glass canopy of a cross shaped commercial space, inside a historical building in Trieste (Italy) [2]. Erected in 1842 with a traditional glass covering for the inner gallery (Figure 6,a) it was renovated after the second post-war with a new roof realized in reinforced concrete and glass blocks vaults. Due to the excessive loading on existing walls, generated by this heavy structure, at present it is necessary another renovation process that can be seen as an opportunity to design a lighter and transparent roof, of higher quality also from the architectural point of view.

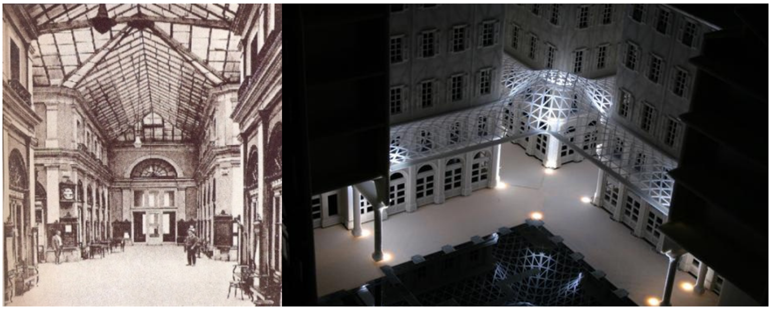

Figure 6, a, b: a)Picture of the first Tergesteo Gallery, 1842; b) New roof of the Fifties.

The general shape of the project for the new glass grid shell is depicted in Figure 6,b. The quadrilateral structural mesh is drawn following the symmetry axes of the gallery. Only the dark portion of the roof has been optimised. The architect has defined the initial shape at the conceptual design stage with the aim to emphasise the central space and to reach the maximum transparency effect. The optimization process is then required to improve the grid geometry without radically altering the architectural shape.

At the end of the iterative process, the obtained optimized shapes are very similar to the initial ones, but their fitness is considerably higher. That means that significant improvements of the shape, from the point of view of the planarity requirement, can be reached with slight modifications of the initial shape, preserving the original concept (Figure 7).

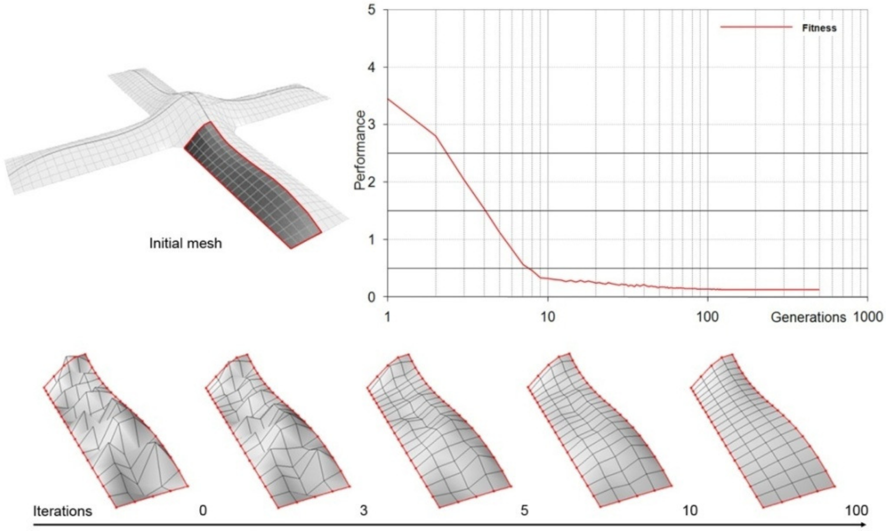

Figure 7: Results of the VFDM application to the Tergesteo gallery project.

4.2.3. The Ponte Parodi project

Ponte Parodi is a project by UNStudio Amsterdam for a commercial building in Genova, Italy, which takes its name from the shore platform where the building will be located (Figure 8). The most complex element of the project, from a geometrical point of view, is represented by the roof which has the function of a “green” public plaza.

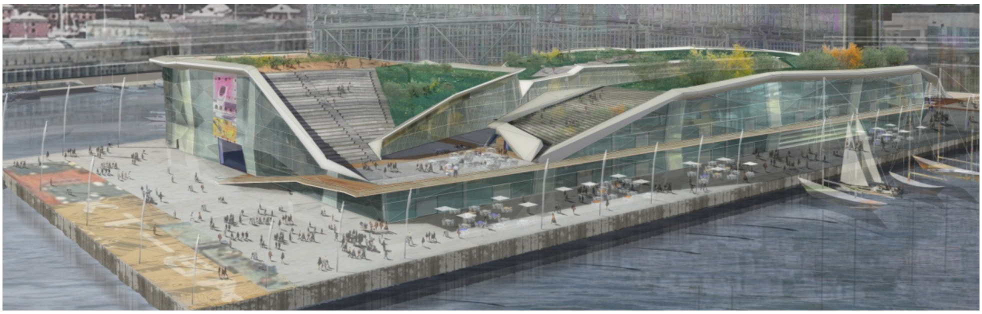

Figure 8: Ponte Parodi – render by UNStudio

The unconventional shape of this covering and the involved dead load (1700kg/mq) imposed a particular study for the supporting structure.

In order to take care of several analyses, such as seismic behavior of the whole structure, suitable approximation of the architectural design and costs optimization, the proposed solution was a composition of planar steel grids which allowed a rigid behavior of the whole structure and a standardization of steel elements. The solution was performed with the aid of the VFDM algorithm and the optimization process took care of several parameters as the position of the columns and the minimum and maximum height allowable for the structure all along the building. Consequently, the algorithm allowed only a vertical movement of planes corners between two values of z coordinate where constraints were represented by columns. Other boundary conditions were necessary to ensure the respect of characteristic lines of the architectural design. The result of the generation process is showed in Figure 9.

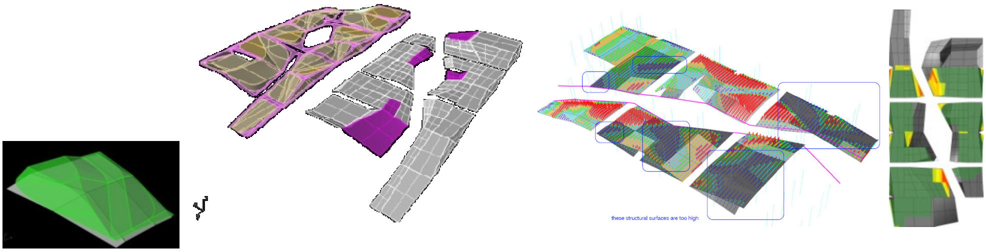

Figure 9: On the left a scheme of the structural plane composition. Purple surfaces were not led to a planar configuration because of the presence of too rigid boundary conditions. On the right the control of the approximation result.

A non-perfect approximation of architectural landscape was allowed considering the possibility to model shapes in a second time with the ground. However the deviation of the solution from the referential envelop was controlled through a point deviation study (Figure 9 on the right) and a similar study was necessary to respect internal heights (Figure 9 on the right) which represent together the solution domain. As a consequence of grid planarity it was largely possible to standardize elements and joints (maintaining angles of 90° among elements).

5. Conclusions

In free form structures, constructive and technological aspects can heavily affect constructive costs if they are not taken into account in early design phases. We can generate free form shapes following defined geometric rules or, more in general, an analytic approach that guarantees results respecting defined requirements. On the contrary, we can define totally free shapes and act at the end by means of an optimization procedure.

This procedure is to be considered a form of optimization, since the complexity of problems which we have to face in a project implies that we can't focus our attention on a single element but we have to aim at the general improvement rather than the attainment of the best solution.

The VFDM development satisfies the need to create both an instrument of optimization and a quick and versatile design instrument, directly developed inside the modelling commercial software, thus avoiding any problem of information conversion and interface.

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