Air-gap force distribution and vibration pattern of Induction motors under dynamic eccentricity

PEDRO JOVER RODRÍGUEZ*, ANOUAR BELAHCEN*, ANTERO ARKKIO*, ANTTI LAHIO**, JOSÉ A. ANTONINO-DAVIU***

**Laboratory of Electromechanics,*

Department of Electrical Engineering

Helsinki University of Technology

P.O. Box 3000, 02015 HUT, Finland

Fax: +358 9 451 2991

E-mail: vicent@cc.hut.fi

***VTT Technical Research Finland*

****Universidad Politécnica de Valencia*

Department of Electrical Engineering

P.O. Box 22012, 46071

Fax: +34 96 3877599

E-mail: joanda@die.upv.es

Abstract: A method for determining the signatures of dynamic eccentricity in the airgap force distribution and vibration pattern of induction machine is presented. The radial electromagnetic force distribution along the airgap, which is the main source of vibration, is calculated and developed into a double Fourier series in space and time. Finite element simulations of faulty and healthy machines are performed. They show that the electromagnetic force distribution is a sensible parameter to the changes in the machine condition. The computations show the existence of low frequency and low order force distributions, which can be used as identifiable signatures of the motor condition by measuring the corresponding low order vibration components. These findings are supported by vibration measurements and modal testing. The low frequency components offer an alternative way to the monitoring of slot passing frequencies, bringing new components that allow to discriminate between dynamic eccentricity and rotor mechanical unbalance. The method also revealed a non linear relationship between loading, stress waves and vibration during dynamic eccentricity.

Keywords: dynamic eccentricity, vibration, stress, FEM, Fourier analysis, induction motor

INTRODUCTION

Condition monitoring of electrical machines is becoming increasingly essential for both industrial and academic sectors. It plays a very important role for the safe operation of industrial plants and enables to avoid heavy production losses, whereas the choice of adequate monitoring methods is a challenging task for the academic world.

The most used indicators for monitoring electrical machines are currents, temperatures, voltages, chemical debris and vibrations. In many cases, the overall vibration level of the machine is sufficient to diagnose mechanical failures [1], [2]. In contrast, the effect of electrical faults on the vibrations is still under investigation. Airgap eccentricity is one of the main faulty conditions of induction machines. It causes excessive stressing of the machine, increasing bearing wear and producing harmful vibrations and noise. In the worst case, it could produce rotor-stator rub, with consequential damage to the stator core and winding. Thus, the online monitoring of rotor eccentricity is highly desirable to prevent serious operational problems.

Pöyhönen et al. [3] showed that the electromagnetic force is the most sensitive indicator of airgap eccentricity. The only drawback of this indicator is its low accessibility. Nevertheless, since vibrations are the consequences of the forces on the machine structure, identifiable signatures should be found in the vibration pattern. Finley et al. [4] compiled a resume table with a comprehensive list of electrically and mechanically induced components in the vibration pattern. Their analysis is based on analytical formulas. The conclusion from this paper is that with solid knowledge of motor fundamental it is possible to ascertain the root cause of a vibration problem.

Cameron et al. [5] developed a monitoring strategy based on monitoring high frequency vibration components (slot passing frequencies). They presented a theoretical analysis based on rotating wave approach whereby the magnetic flux waves in the airgap are taken as the product of permeance and magnetomotive force (mmf). This monitoring strategy has the drawbacks that detailed motor information is needed and the monitored frequencies my be close to the resonance frequencies of the machine.

Dorrell and Smith [6] described an analytical model to study induction motor with a static eccentricity based on airgap permeance approach including the stator and rotor mmf. The model examine the interaction between the harmonics that produce unbalance magnetic pull (UMP). It is verified by experimental investigation carried out in [7]. They obtained good agreement at low slip. They concluded that the effects of the higher order winding harmonics and rotor skew can influence on the magnitude of the UMP.

Dorell et al. [8] analysed the airgap flux and vibration signals as a function of the airgap eccentricity. This paper put forward a theoretical analysis of the interaction between harmonic field components due to eccentricity. It is illustrated how eccentricity faults can be identified from vibration analysis using condition monitoring techniques. However, the paper does not make clear the dependence of the vibration with the machine loading, an important fact to take into account in a monitoring system, and the possible modal pattern of the stress waves are not calculated.

Based on analytical methods, Verma and Balan [9] presented an analysis of the radial force distribution in squirrel cage induction motors; they were concerned with the noise problem. The analytical approaches have drawbacks; they do not take into account the slotting and saturation effects.

Vandevelde and Melkebeek [10] developed a method for numerical analysis of vibration based on magnetic equivalent circuits and structural finite element models. From the combined electromechanical analysis the vibration and noise are predicted. This investigation overcomes the drawbacks from the analytical models and develops the calculation of the radial forces in the (frequency, spatial order)-domain but in this work, the effects of the low order forces due to eccentricity were neglected.

Based on finite element method (FEM), Belahcen et al. [11] presented a similar analysis for a mid-size synchronous generator. The agreement between the simulations and measurements of noise and vibrations was rather good.

In this work, the method used in [11] is applied to predict the excited vibration frequencies due to dynamic eccentricity in the stator of an induction machine fed from a sinusoidal voltage source. The method takes into account the possible mode shapes of the stress waves distribution, as well as the machine slotting and saturation. The simulations and measurements show that this fault has recognisable signatures in the stress waves, and the magnitudes of lower frequency stress waves during a fault event increase greatly, producing forced vibrations [12].

METHODS

Analytical, numerical and experimental methods are used in this work. The experimental methods are used for the measurements of the vibrations of the test machine under healthy and eccentric conditions as well as to obtain the frequency response functions (FRF) of the stator and the mechanical system. The numerical methods are used to solve the magnetic problem in the cross-section area of the test machine, whereas the analytical methods are used to analyse the stress waves and the spectra of the measured vibrations and to compute magnetic force distribution.

Theoretical analysis

In an ideal concentric rotor and stator the radial forces are cancelled out and the resultant net force acting between the two cylindrical bodies is zero. If any abnormality exists, UMP occurs. During dynamic eccentricity events the position of the minimum airgap length rotates with the rotor position making the UMP direction coincide with the position of the minimum airgap, see Fig. 1. During this phenomena significant forces are produced that try to pull the rotor even further from the concentric position.

Figure 1

The equivalent airgap is usually constant with angular position φ, but if dynamic eccentricity is present (see Fig. 1), the airgap length can be expressed as

$$g(\varphi,t) = g[1 - d_e \cos(\omega_r t - \varphi)] \qquad (1)$$

where d_e is the degree of dynamic eccentricity, ω_r is the rotational speed, φ is the angular position from some base line and g is the average airgap. If low value of dynamic eccentricity is assumed, the airgap permeance can be represented as [8]

$$\Lambda(\varphi,t) = \frac{1}{g}(1 + d_e \cos(\omega_r t - \varphi)) \qquad (2)$$

Assuming that the flux crosses the airgap normally even during eccentricity event, the airgap field can be derived as

$$b(\varphi,t) = \Lambda(\varphi,t)\int \mu_0 j_s(\varphi,t)d\varphi \qquad (3)$$

where $j_s(\varphi,t)$ is the linear current density of the stator given by

$$j_s(\varphi,t) = J\ \sin(\omega t - p\varphi) \qquad (4)$$

where p is the fundamental pole-pair number of the motor and ω is the fundamental frequency. By substituting (2) and (4) in (3), equation

$$b_s(\varphi,t) = B_S{}^p \cos(\omega t - p\varphi) + $$
$$+ B_S{}^{p-1} \cos((\omega - \omega_r)t - (p-1)\varphi) + $$
$$+ B_S{}^{p+1} \cos((\omega + \omega_r)t - (p+1)\varphi) \quad (5)$$

is obtained, where

$$B_S{}^p = \frac{\mu_0 J_s}{pg}; B_S{}^{p\pm1} = \frac{\mu_0 J_s}{2pg} d_e$$

At any point, the radial stress on the stator inner surface is approximated by

$$\sigma = \frac{B^2}{2\mu_0} \quad (6)$$

Substituting (5) in (6), some algebraic manipulation gives

$$\sigma(\varphi,t) = \frac{1}{2\mu_0}\Big[(B_s{}^p)^2 + (B_s{}^p)^2 \cos(2\omega t - 2p\varphi) + $$
$$+ B_s{}^p [\ B_s{}^{p-1} \cos(\omega_r t - \varphi) + B_s{}^{p-1} \cos((2\omega + \omega_r)t - \varphi)] + $$
$$+ (B_s{}^{p-1})^2 + (B_s{}^{p-1})^2 \cos(2(\omega - \omega_r)t - 2(p-1)\varphi) + $$
$$+ B_s{}^{p-1} B_s{}^{p+1} \cos(2\omega_r t - 2\varphi) + B_s{}^{p+1} \cos((2\omega - \omega_r)t - \varphi) + $$
$$+ (B_s{}^{p+1})^2 \cos(2(\omega + \omega_r)t - 2(p+1)\varphi) \ \Big] \quad (7)$$

A dynamic eccentricity causes a rotating radial force (UMP) on the rotor due to the interaction of space harmonic field components with pole pair numbers differing by one and rotate in the same direction [6]. From equation (7), it can be concluded that the dynamic eccentricity may produce other non-twice supply frequency vibrations in addition to the rotational frequency (f_r), these are $2f_r$, $2f_s \pm f_r$, $2(f_s \pm f_r)$, where f_s is the supply frequency. These stress components are rotating during dynamic eccentricity with several different modal patterns. It is important to note that equation (7) is obtained under many approximations and the slotting and saturation effects are not taken into account. More accurate results are obtained by means of time stepping FEM simulations.

Magnetic field and magnetic forces

The magnetic field on the cross-section of the test machine is computed by using an in-house 2-D FE program. This software package was designed for the transient magnetic field analysis of electrical machines coupled with the circuit equations of the machine windings. This method allows the simulation of electrical machines fed from measured voltages used in experiments. The software uses the time-stepping method, which takes into account the motion of the rotor and the induced voltage due to this motion. Some of the 3-D effects like flux fringing and end windings are also modelled with analytical and electric circuit approaches. A full description of the software is given in [13]. A detailed description about the fault implementation can be found in [3]. The method models properly the effects of the circulating equalising currents, the slotting and saturation. From the solution of the magnetic field, the force distribution in the air-gap of the machine is calculated by using the Maxwell stress tensor averaged over the cross-sectional air-gap area [11].

Using polar co-ordinates in two dimensions on a cylindrical air-gap surface, the stress is reduce to a radial stress component

$$\sigma_r = \frac{1}{2\mu_0}(B_r^{\ 2} - B_\varphi^{\ 2}) \tag{8}$$

and the circumferential stress component or shear,

$$\sigma_\varphi = \frac{1}{\mu_0} B_r B_\varphi \tag{9}$$

where B_r and B_φ are respectively the radial and circumferential components of the magnetic flux density vector in the airgap.

The radial component of the flux density vector B_r is much higher than its circumferential counterpart B_φ. This fact results in a very high value of the radial stress compared to the shear. The latter one is responsible for the torque production, whereas the first one is mainly responsible for the vibrations of the stator of the machine. The radial component of the stress tensor can be written as double Fourier series in terms of the position φ and time t (space and time harmonic decomposition) [11]

$$\sigma_r = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \lambda_{mn} \begin{Bmatrix} a_{mn}\cos(m\varphi)\cos(n\omega t) + \\ b_{mn}\cos(m\varphi)\sin(n\omega t) + \\ c_{mn}\sin(m\varphi)\cos(n\omega t) + \\ d_{mn}\sin(m\varphi)\sin(n\omega t) \end{Bmatrix} \tag{10}$$

where the coefficients

$$a_{mn} = \frac{\omega}{\pi^2}\int_0^{2\pi}\int_0^{\frac{2\pi}{\omega}} \sigma_r \cos(m\varphi)\cos(n\omega t)\,d\varphi\,dt$$

$$b_{mn} = \frac{\omega}{\pi^2}\int_0^{2\pi}\int_0^{\frac{2\pi}{\omega}} \sigma_r \cos(m\varphi)\sin(n\omega t)\,d\varphi\,dt$$

$$c_{mn} = \frac{\omega}{\pi^2}\int_0^{2\pi}\int_0^{\frac{2\pi}{\omega}} \sigma_r \sin(m\varphi)\cos(n\omega t)\,d\varphi\,dt$$

$$d_{mn} = \frac{\omega}{\pi^2}\int_0^{2\pi}\int_0^{\frac{2\pi}{\omega}} \sigma_r \sin(m\varphi)\sin(n\omega t)\,d\varphi\,dt \tag{11}$$

are integrated from time-step to time-step and λ_{mn} is given by

$$\lambda_{mn} = \begin{cases} \frac{1}{4} & \text{for } m=n=0 \\ \frac{1}{2} & \text{for } m=0, n>0 \text{ and } m>0, n=0 \\ 1 & \text{for } m>0, n>0 \end{cases} \quad (12)$$

The integers “n” and “m” are respectively the order of time and space harmonics of the stress waves.

Equation (10) can also be written as

$$\sigma_{rr} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\{ \frac{1}{2}\lambda_{mn} \begin{Bmatrix} (a_{mn}+d_{mn})^2 + \\ (b_{mn}-c_{mn})^2 \end{Bmatrix}^{\frac{1}{2}} * \cos(-m\varphi + n\omega t + \gamma_+) + \frac{1}{2}\lambda_{mn} \begin{Bmatrix} (a_{mn}-d_{mn})^2 + \\ (b_{mn}+c_{mn})^2 \end{Bmatrix}^{\frac{1}{2}} * \cos(m\varphi + n\omega t) + \gamma_- \} \quad (13)$$

so that, it can be interpreted as stress waves of different wave lengths rotating in either the same direction as the rotor (forward) or opposite direction (backward) with different speeds or frequencies. The spatial order of these waves “m” defines the mode shape of the vibrations that could be caused by the stress wave, whereas the time order “n” determines the corresponding vibration frequency.

Numerical calculation

The rated parameters of the studied machine are shown in Appendix I. The 2D-magnetic problem was solved on the cross-section area of the machine. Both, a healthy and faulty motor with dynamic eccentricity under different load conditions were simulated. The supply is a sinusoidal voltage at a frequency of either 50 or 100 Hz, with voltage of 200 or 400 V, respectively.

The simulations are carried out with a fixed time step of 25 μs and the total number of 40 000 steps. So that, the frequency spectra of the stress waves are calculated for every spatial mode with a frequency resolution of 1 Hz, for both the healthy and faulty conditions. In order to compare the healthy and faulty condition, the normalised stress difference is defined for every spatial mode shape

$$Norm.\,stress = \frac{Stress\ faulty - Stress\ healthy}{Maximum\ stress\ per\ mode} \quad (14)$$

This normalised value allows the comparison between the healthy and faulty condition for each spatial mode shape. It shows the new frequency components of the stress for a given spatial mode shape when the motor is operating under a fault. These stresses are calculated at three load conditions when the motor is either healthy or faulty with 2, 5, 6, 10, 16, 20, 25, 33, 37, 50 and 60 % dynamic eccentricity. We considered only the first seven modes because these low order modes are the most important ones from the vibration point of view [5], [9].

Measuring system

A measuring setup was arranged to obtain data from a working motor. A block diagram of this setup is shown in Fig. 2. The motor used in the measurements was the same as the one used in the FEM simulations. The motor is supported from the front drive-end to avoid the effects of feet fixation on the mode shapes.

Figure 2

The tests were carried out with the motor in healthy condition and with a 37 % dynamic eccentricity. The dynamic eccentricity was obtained by fitting non-concentric support parts between the shaft and bearing, see Fig. 3.

Figure 3

The induction machine was fed from a sinusoidal supply of either 200 V at 50 Hz or 400 V at 100 Hz. A Bruel & Kjaer vibration sensor (accelerometer) was installed on the stator frame of the machine. The signal of the accelerometer was amplified through a charge amplifier Bruel & Kjaer 2651.

The number of samples was 40 000 and the sampling frequency 40 kHz, so that the frequency resolution is 1 Hz. A transient recorder Kontron-WW700 recorded the data. The vibration acceleration signals from the vibration sensor were analysed by using Discrete Fourier Transform (DFT) in MATLAB.

Analysis of mechanical response of the stator and mechanical system

Modal tests were performed on the motor stator and the whole mechanical system [14]. In the stator case, forty Endevco 65-100 triaxial isotron icp accelerometer sensors were evenly distributed on the stator frame. A hammer Endevco E2302-5 icp was used to excite the stator and the mechanical system. The data from the accelerometer were collected through a LMS SC310 data acquisition module.

The modal analysis was made by a LMS Test Lab 6A using PolyMax modal analysis techniques. Figure 4 shows a measured FRF of the stator. Examination of all stator FRF spectra showed that the first natural frequency of the stator is at 936 Hz and it is associated with circumferential vibration mode “2”.

Figure 4

For the whole mechanical system, five sensors were distributed on the motor support and motor end shields. Figure 5 shows a FRF of the mechanical system.

Figure 5

ANALYSIS AND RESULTS

The presented FE analysis and decomposition of the electromagnetic stress into rotating waves showed the existence of remarkable changes in the amplitudes of the stress waves during dynamic eccentricity events. Significant changes in stress occur for the spatial mode shapes (m=1, 2, 3, 5 and 6). In contrast, the modes 0 and 4 hardly change.

The changes in the stress waves cause changes in the vibration patterns of the stator. The fact that these vibrations are forced (at the predicted frequencies neither the stator under study nor the mechanical system have resonance frequencies, thus we have forced vibrations), together with the fact that the cylindrical structure of the stator is most sensitive to the vibrations in the lower modes allow us to predict the way in which the stator will vibrate under dynamic eccentricity conditions.

It is not strictly correct to relate vibration magnitudes directly with stress waves, since the stator frame will react differently to stress waves with varying numbers of pole pairs and rotational speeds. However, our results suggest that under dynamic eccentricity clear forced vibrations will exist because the resonance frequencies of the motor are well above the analysed frequency range.

Calculated and measured results

Figure 6 shows the calculated stress spectra for the relevant mode shapes for a motor working with 37 % dynamic eccentricity at half load (1,9 % slip), and motor fed at 100 Hz, 400 V. In this figure, the predicted peaks of the stress waves forecasted from the analytical solution equation (7) and theirs associated modes can be seen.

Figure 6

Figure 7 shows the calculated normalised stress difference for the spatial mode shapes 1, 2, 3, 5 and 6 for the case of the motor working with a 33 % dynamic eccentricity at full load (3 % slip), and fed from a sinusoidal supply of 200 V at 50 Hz.

Figure 7

Figure 8 shows the calculated normalised stress when the motor is fed at 400 V, 100 Hz and is working with 33 % dynamic eccentricity at full load (3 % slip).

Figure 8

Figure 9 shows the variation of the peak values of the relevant mode shapes with loading, from no load (0,1 % slip to full load 3 % slip), for the motor working with 33 % dynamic eccentricity.

Figure 9

Figure 10 shows the stress peaks of the relevant mode shapes as a function of the dynamic eccentricity degree.

Figure 10

Figure 11 shows the spectrum of the measured vibration acceleration in the healthy condition and for the motor working with 37 % dynamic eccentricity, both cases at half-load operation (1,9 % slip) when the motor is fed from a 400 V at 100 Hz sinusoidal supply.

Figure 11

Figure 12 shows the spectrum of the measured vibration acceleration at two load conditions when the motor is working with a 37 % dynamic eccentricity and fed from the same sinusoidal supply (100 Hz, 400 V).

Figure 12

Figure 13 shows the measured acceleration of the motor working with the same degree of dynamic eccentricity and supply conditions at no load.

Figure 13

Discussion

In a concentric rotor cage, the magnetic stress is symmetrically distributed around the stator (ideal case), so the resultant magnetic radial force does not exist. On the other hand, when a dynamic eccentricity exists, there is asymmetry in the stress distribution that produces new vibration components. The main stress wave rotates at the same speed as the rotor. In addition, there are other rotating stress waves as show equation in (7) and also higher space harmonics due to other winding harmonics. These stresses acting on the stator core will cause vibration of the same frequency to be transmitted to the stator core surface. Therefore, the surface vibration signal will contain frequency components characteristic of dynamic eccentricity. This is supported by the fact that neither the natural frequencies of the stator (Fig. 4) nor of the mechanical system (Fig. 5) coincide with the predicted frequencies from the mathematical analysis and simulations and demonstrated by the measurements.

By comparing the measured vibration pattern shown in Fig. 12, top figure (measured 37 % dynamic eccentricity, 1,9 % slip), and the numerical calculated stress waves shown in Fig. 6 (simulated 37 % dynamic eccentricity, 1,9 % slip), a clear relationship is seen. Fig. 12 shows up the same peaks values presented in Fig. 6. In Fig. 6 the greatest calculated stress wave occurs at the rotational speed (46,2 Hz) in mode 1, which corresponds with the highest vibration frequency represented in Fig. 12. In both figures, the relevant frequencies predicted from equation (7) are observed. They are also predicted from the calculated normalised stress showed in Fig. 8.

From Fig. 10 can be seen how a low level of dynamic eccentricity (10 %) produces stress components associated with modes "1" and "5" with high amplitudes. These components may produce steady vibration in the stator frame. This could be considered as a fault in an incipient state, which is worthwhile to be detected. From Fig. 10 it is deduced that the amplitude of the forced vibrations due to dynamic eccentricity increases with the degree of eccentricity. The amplitude of the stress for modes 1, 2, 3, 5, 6 increases linearly with the level of the dynamic eccentricity. The modes 0 and 4 are not presented because these modes are produced by the main flux, which is about the same in the healthy and faulty condition.

By comparing the cases of a healthy motor and motor working with dynamic eccentricity, see Fig. 11, remarkable changes in the low frequency spectrum can be seen. There are new vibration components in the faulty spectrum. The component at the rotational frequency f_r is clearly visible. There exist also the other frequencies predicted from the analytical results f_r, $2f_r$, $2f_s \pm f_r$, $2(f_s \pm f_r)$. They could help to discriminate if the vibration at the rotational frequency is due to dynamic eccentricity or mechanical imbalance. It is worth to remember that a mechanical imbalance will also produce rotational frequency vibration [1], [2]. In a 2-poles pair motor (as our motor), the synchronous rotation speed is half of the supply frequency. Therefore, the component $2(f_s - f_r)$ coincides with $2f_r$.

It is important for condition monitoring strategies to make clear the dependence between vibration and machine loading. During the loading, the currents, which create the mmf increase their amplitudes, however, the stress waves do not change proportionally with the loading as can be seen in Fig. 9. Similarly, Fig. 12 shows how the loading decreases the amplitudes of some vibration components in the spectra and theirs frequencies decrease proportionally with the machine slip.

By comparing Fig. 12 (measured acceleration) and Fig. 9 (calculated stress waves) as function of the loading important facts are observed. In Fig. 12 the main vibration component at the rotation frequency decreases appreciably its amplitude with loading as it is predicted in Fig. 9, from half load (1,9 % slip) to full load (3 %). This is explained by the fact that when the machine is not loaded, the generated asymmetric flux due to the eccentricity, can easily flow through the airgap and the iron over the rotor bars (through a short path), thus producing a net high stress. When the motor is slightly loaded (small current in the bars), the iron over the bars is not fully saturated, while is saturating, the characteristic for the stress is non-linear. When the load increases enough (thus, the bar currents), the iron over the rotor bars gets saturated, changing the flux path (through a longer path, beneath the rotor bars), reducing the magnitude of the stress waves. Moreover, the asymmetric flux induces current in the motor cage, which opposes to the asymmetric flux, damping the magnitude of the stress waves. Furthermore, the asymmetric flux induces circulating current in the parallel branches (the motor studied has two parallel branches) of the stator winding. These currents tend to equalise the flux distribution, reducing the amplitude of the stress waves [6], [7], [15], [16]. From this analysis is deduced that the no-load test is the most informative for the identification of dynamic eccentricity, see Fig. 13 and compare with Fig. 12. In the no-load condition, there are no rotor currents to damp the asymmetric flux, thus the stress waves have the highest amplitudes, as it is observed in Fig. 9.

Figure 9 (calculated stress) and Fig. 12 (measured vibration) show the dependence of stress wave and vibration with loading respectively. Figure 12 shows that the vibration at the rotational frequency decreases appreciably with the loading. This frequency is associated with mode "1" and decreases appreciably from a rated load of 1,9 % slip to full load (slip 3 %). This fact is observed also in Fig. 9 at the corresponding rotor slips. The vibration component $2f_s + f_r$, associated with mode "5", decreases less. The vibration frequencies $2f_r$, $2f_s - f_r$ and $2(f_s + f_r)$ have little change with loading, see Fig. 12, similarly with the stress waves at the same frequencies associated with the modes 2, 3 and 6, respectively, see Fig. 9. These components remain almost flat with the loading.

The vibration component at twice the supply frequency (seen in Fig. 11 and Fig. 12) is not predicted from the numerical calculation, it is a consequence of inherent static eccentricity of the rotor, which produces this characteristic frequency [8], [12]. We measured a 37 % of dynamic eccentricity, which should include certain degree of static eccentricity. In practice, it is quite difficult to obtain isolated pure static or dynamic eccentricities.

The cases studied in the literature, [4] and [8], showed that the main vibrations during dynamic eccentricity are at the rotation frequency. The simulations and measurements carried out in our work are in agreement with these reported results. However, even if a dynamic eccentricity leads to slip current frequency components [8], our results showed that the dynamic eccentricity does not produce sideband frequencies with similar compositions as the ones produced by the broken bars [12]. Therefore, the vibration patterns presented by these two faults are different. This is an important fact because vibration measurements may allow to discriminate between the two above mentioned faults, which is not always possible by analysing the motor current.

The simulations and measurements carried out show that the vibration pattern is a consequence of the stress waves present during the dynamic eccentricity events.

CONCLUSIONS

A method to predict the vibrations of induction machine under dynamic eccentricity is presented. The method consists of numerical simulation of the machine and decomposition of the electromagnetic stress into rotating waves of different wavelengths and rotating frequencies. The method revealed, in addition to the main vibration at the rotational speed, other components that could help to discriminate between mechanical unbalance and dynamic eccentricity. Further, the load dependency of the main vibration components under dynamic eccentricity is studied, rising the conclusion that a no-load test is the most informative for a condition monitoring system. A non-linear dependence between loading, stress waves and vibration is revealed and explained.

The low frequency components can be used as signatures for the detection of eccentricity in induction motors. The changes in the stress waves are to be monitored through accelerometers fixed on the outer casing of the motor. This offers an alternative way to detect eccentricity without using slot passing frequencies, having the advantages of monitoring a cleaner low frequency spectrum and detailed motor information is not needed.

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Table **Appendix 1,** Motor rated parameters

Parameter	
Rated Power	35 kW
Rated frequency	100 Hz
Rated Voltage	400 V
Rated current	64 A
Connection	Star
Number of pole pairs	2
Number of stator slots	48
Number of rotor bars	40

List of legends

Fig. 1 Illustration of dynamic eccentricity. The left hand figure indicates the initial position, the right hand figure indicates the position after half rotor turn.

Fig. 2 Measuring system.

Fig. 3 Artificially created dynamic eccentricity.

Fig. 4 Measured FRF of the stator.

Fig. 5 Measured FRF of the whole mechanical system.

Fig. 6 Calculated tress waves amplitudes for the relevant mode shapes. Motor working at half load (1,9 % slip) and fed at 100 Hz.

Fig. 7 Calculated normalised stress in case of 33% dynamic eccentricity. The motor is working at full load (3 % slip), fed at 50 Hz.

Fig. 8 Calculated normalised stress for the motor working at 100 Hz, full load (3 % slip).

Fig. 9 Calculated stress waves with loading. Motor fed at 100 Hz (33 % dynamic eccentricity).

Fig. 10 Calculated peak stress per mode as a function of the eccentricity. Motor working at full load (3 % slip), fed at 400 V, 100Hz.

Fig. 11 Measured low frequency vibration pattern for the healthy motor (top) and with 37 % dynamic eccentricity (bottom), slip 1,9 % and motor fed at 100 Hz in both cases.

Fig. 12 Measured acceleration vibration pattern at two load conditions, rotor with a 37 % dynamic eccentricity. Upper figure half load (1,9 % slip), bottom figure full load (3 % slip).

Fig. 13 Measured acceleration at no load, rotor with 37 % dynamic eccentricity.