Second-order Effects in Steel-Concrete Composite Columns of the Unbraced Frames

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Abstract: An unbraced frame is regarded as the one without a bracing system to avoid the horizontal displacement of its nodes. These displacements progressively increase due to their interaction with axial load (second-order effects) reducing the column´s resistant capacity and can lead to the global frame instability. At the present work the Eurocodes criteria related to the way to approach its design will be displayed. Different iterative processes that take into account second-order effects will be exposed as well as a non-linear procedure based on an incremental load application. This procedure will be applied over an unbraced frame under gravity and wind loads with reinforced concrete beams and steel-concrete composite columns. Two geometrical models will be used: one of them starts from geometry without deformation and the other one takes into account the initial imperfections. The results comparison regarding first-order linear analysis will show that bending moments at the critical section of the columns will increase by nearly 50% when their initial imperfections are taken into account.
Keywords: Unbraced frames; Second-order analysis; Eurocodes; Steel-concrete composite columns; Imperfections.

1. Introduction

The most widely used structural system in building are the reinforced concrete rigid frames. Their columns are under axial load as the main effort is usually accompanied by one or two bending moments. When axial forces are increased it is frequent to resort to steel-concrete composite columns, which provides a high resistant capacity with relatively small sections. Especially the ones made up of a steel profile encased in concrete, they also provide fire and corrosion protection [1].

Since the middle of the 20th century, they have been the subject of a lot of resources that includes multiple experimental tests [2]. Most of these studies assume these frames as sufficiently braced, developing several methods of simplified design for the isolated columns of the structure.

A minimum number of tests consider the column as a part of a frame without a bracing system [3]. Moreover, technical codes are unclear about the design process to apply: "second-order effects will be considered, when they could significantly affect the frame´s global stability" [4]. It is difficult to predict if the second-order effects are going to significantly affect the frame.

This paper studies the effect of interaction between the axial force and horizontal displacements at the unbraced frames. Different second-order designing methods will be exposed and a non-linear procedure with an increasing load application will be described. This procedure will be applied on several unbraced frames and will be compared with the results regarding the linear analysis.

2. Structural Analysis

Reinforced concrete frames are structures with a high hyperstaticity degree due to the stiffness provided by its nodes. Nowadays it is common structural analysis by means of matricial methods, where deformation of the frame nodes are related to the loads applied through the named "stiffness matrix". The first approach to this matrix method, in an elastic and linear regime, was made by Livesley [5], the essence of his mathematical model is preserved even today.

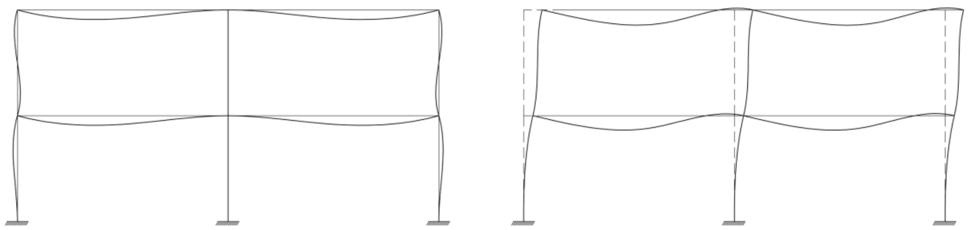


Figure 1 a) Braced Frame, b) Unbraced Frame

Even for braced frames (Figure 1a) this method is acceptable for a global level, for unbraced frames (Figure 1b), the linearity between loads and stresses is questionable. For this interaction between the horizontal displacements and vertical loads, displacements are increased until the equilibrium position is reached. This interaction between vertical loads and the frame horizontal displacements, second-order effects, is known as "*P-Δ*" effect (Figure 22). The columns resistant capacity is reduced while frame global stability is compromised [6].

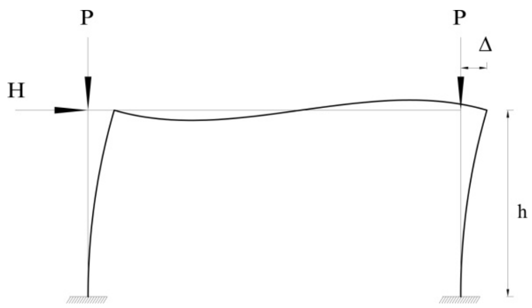


Figure 2 "P-Δ" Effect

The structural analysis of this type of frames is more complex. Jennings [7] was the first one to raise the geometric non-linearity at the metrical methods, having his complex work simplified by Yang and McGuire [8]. Geometric non-linear analysis is nowadays addressed by an incremental approach of loads application [9]. The consideration of the materials non-linear behaviour at the frame analysis process is less common. When an evaluation of its effects is wished, the linear stiffness matrix is replaced by the tangent matrix, among its components a progressive variation of the material stiffness modulus (E·I) is introduced. An interesting comparative study from this one and other design models was compiled in Nethercot´s work [10].

3. Codes Criteria

Some of the Eurocodes sections can generate doubts about the need for taking into account second-order effects when the frame is being analysed.

Section 5.1.4 in Eurocode 2 (EC2) [3] prescribes that second-order effects must be taken into account if they can significantly affect the global frame stability. Section 5.8.2(6) from the same code indicates that second-order effects can be ignored if they are 10% lower than the correspondents for the first-order effects.

Section 5.2 in Eurocode 3 (EC3) [11] advises the consideration of deformed geometry (second-order effects) when it raises the load effects or when it significantly modifies the global frame response.

It is clear that a frame second-order analyses is not necessary when the horizontal nodes displacements are insignificant, that is for frames with an adequate bracing system. However for unbraced frames is complex to know, previously to their assessment, if the “added efforts” produced by nodes horizontal displacements will be lower than a 10% of the first-order ones. Also it is not easy to estimate if second-order effects will modify significantly the global frame response.

Other international standards as the American Concrete Institute ACI-318[12] or the American Institute of Steel Construction AISC [13] establish an increase of a 10% of the solicitations too, in order to require a design method that takes into account second-order effects.

4. P-Δ effect

The requirement of taking into account the *P-Δ* effect for the unbraced frames, together with the complexity of addressing the problem by means of “exact methods”, has led to the development of iterative methods with a simpler application that, nevertheless provide reliable results. Some of these procedures have been adopted as the design method for computer software to carry out their analysis in the second-order theory [14]. Two of the best known of these iterative methods are explained below.

4.1 Equivalent lateral load method

The procedure involves replacing the fame nodes horizontal displacement with a virtual horizontal force V' (Figure 3). The equation of moment’s equilibrium on the bar lets determine its value.

$$V' = \frac{P \cdot \Delta}{h} \tag{1}$$

Where P is the axial force applied over the column, Δ the relative horizontal displacement between the ends of the column in a first-order analyses and h the column length.

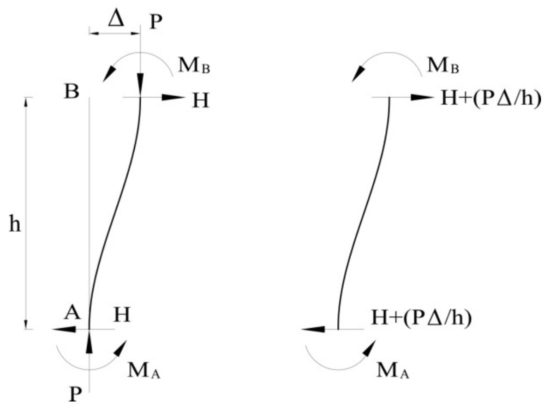


Figure 3 Equivalent lateral load determination (Own resource)

The obtained horizontal forces are added to the initial loads and a frame second-order analyses is carried out. With the new horizontal displacements, the virtual horizontal forces value is corrected and a new analysis is performed. The efforts, obtained in a cycle where the deformations increase is negligible from the previous cycle, are adopted as the final efforts.

4.2 Iterative gravity load method

The process requires a frame first-order analysis where stresses and strains produced only by the horizontal loads are evaluated. A second analysis is performed by applying vertical loads over the frame deflected geometry this will produce new displacements. An iterative process is started at the vertical load application over the deflected geometry at the previous cycle (figure 4). The process ends when the deformation increase is negligible. Once the equilibrium position is reached, the final position can be achieved by the addition of the displacement caused by the horizontal forces plus the ones caused by the application of gravity loads at the n cycles.

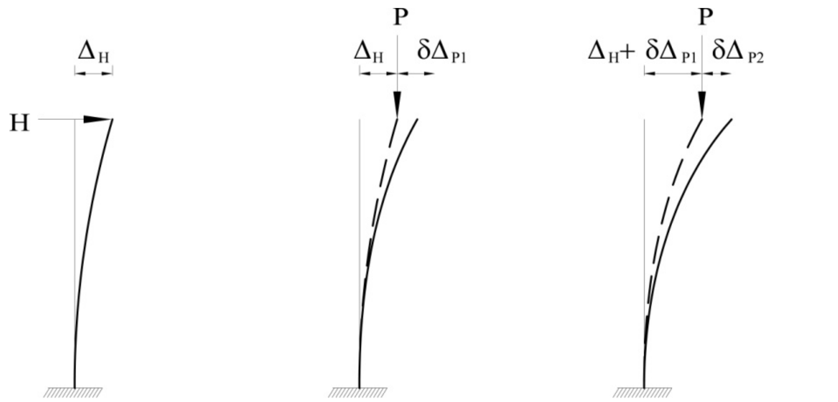
5. Adopted method.

5.1 Structural analysis

The method adopted in this paper [15] begins with the discretization of all the bars in a set of sections (Newmark´s method [16]) and the nodes initial coordinates are obtained; taking into account imperfections. By means of the ANGLE [17] software for structural design, a geometric and mechanical frame non-linear analysis is carried out, with a load incremental treatment until its application is completed or until the collapse in the first bar is produced. This can occur by cross-section failure or by instability, if equilibrium is not reached.

Stiffness matrix at the first load step is determined by the original geometry and with the gross section inertia. For the successive load steps, a tangent stiffness matrix [*Kt*] is used, which takes into account the frame nodes deflected position at the previous step (geometric non-linearity) and the stiffness modulus value *E·I* (mechanical non-linearity) whose determination is exposed in the next section. At the different load steps, the process to get the frame equilibrium is carried out by means of applying Newton-Raphson method [18].

It has to be noted that even mechanical non linearity consideration can be taken into account for any material. It significantly affects concrete structures due to the fact that its stiffness depends on the elasticity modulus, which varies depending on the stress applied, and the cross-section inertia affected by the material cracking.



Horizontal load analysis Gravity load analysis Gravity load analysis

Cycle 1 Cycle 2

Figure 4 Application process of the iterative gravity load method

5.2 Obtaining the section equilibrium

Known the efforts (*N, Mz, My*), locating of the neutral axis position and the deformed plane curvature requires the application of some iterative process, due to the non-linear materials behaviour. At the present paper a procedure based on the bisection method has been applied. It consists adopting an upper and lower limit values for the neutral axis position (z_n-θ) and for the deformed plane curvature (ϕ) in its interval the right value is contained [19].

By means of three loops in chain that iteratively correct the three variables involved, a deformed plane curvature and the distance from the neutral axis to the section gravity centre are located, and the third one determines neutral axis rotation.

The section response for a determined neutral axis position (z_n-θ) and a strain plane curvature (ϕ) have been obtained by the following process.

First all the points coordinates defining the section (vertices on the perimeter, vertices at the structural profile and the gravity centre at reinforcing bars), are changed to a reference system defined by the neutral axis. The section is discretized in a set of reduced thickness fibers (figure 5) and at each fiber the deformation of its middle line is determined (equation 2).

$$\varepsilon_i = \phi \cdot y_i \tag{2}$$

Being:

ε_i Deformation of the fiber gravity centre

ϕ Strain plane curvature

y_i Distance between the fiber gravity centre and the neutral axis

Stress, considered as constant for each fiber, is obtained by the materials constitutive equations application. When a fiber involves several materials, it is subdivided into segments in order to determine the area and the stress belonging to each one of them.

The section response is obtained by numerical integration (equation 3):

$$\begin{aligned} N &= \int \sigma_c \cdot dA_c + \int \sigma_y \cdot dA_y + \sum \sigma_s \cdot A_s \\ M &= \int \sigma_c \cdot y \cdot dA_c + \int \sigma_y \cdot y \cdot dA_y + \sum \sigma_s \cdot y \cdot A_s \end{aligned} \tag{3}$$

Being:

σ_c, σ_y, σ_s Stress applied at each concrete, structural and reinforcing steel segment respectively

A_c, A_y, A_s Area of each concrete, structural and reinforcing steel segment respectively

y Distance from the segment gravity centre to Z axis

Obtaining the equilibrium, the section stiffness modulus, implied at the before mentioned structural analysis process, is deduced by the expression $E \cdot I = M/\phi$.

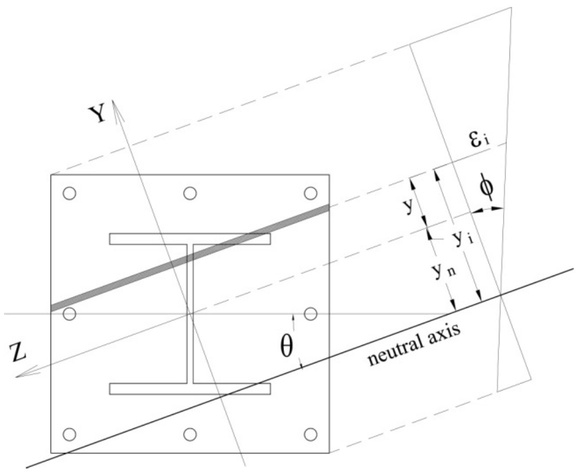
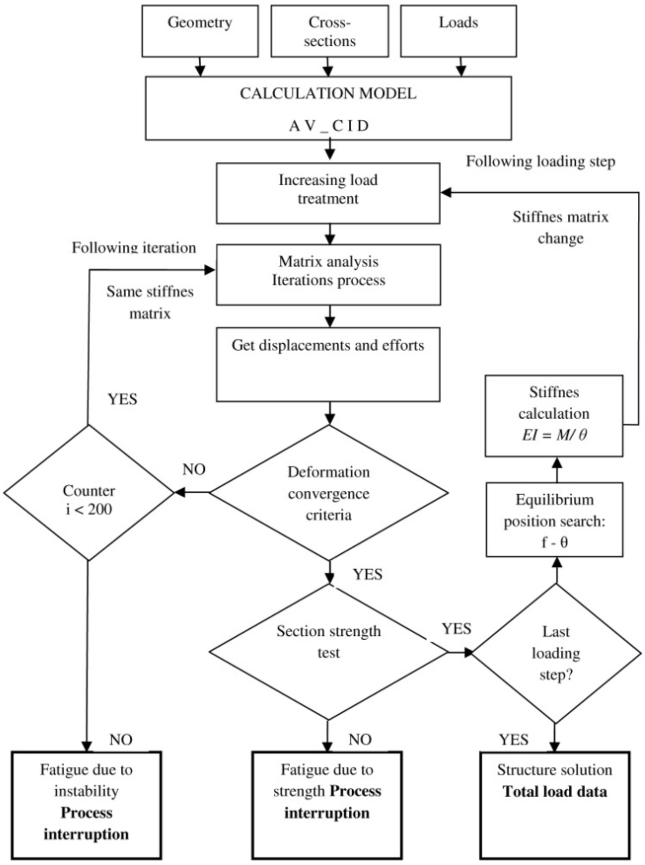


Figure 5 Section response assessment

5.3 Flow chart



6. Structure analyzed

In order to check the necessity of carrying out a second-order analysis, depending on the Eurocodes criteria, for unbraced frames, a structural typology related to "long spans" has been analysed.

It has been modelled on the structure of a 40 m length industrial building by means of single-storey flat frames and a single 18 m span between columns 5.00 m in height (Figure 6). The whole nave is made up of 6 frames separated by 8 m and over them prestressed concrete hollow core slabs have been arranged. Any bracing system has been placed and the roof slabs stiffness is neglected, horizontal displacements are only restricted by the frame stiffness itself.

Figure 6 "Solid" model of the structure

The columns and beams section take part as structural model variables, providing different relative stiffness between them. In any case, dimensions adopted for both elements take into consideration the stresses acting over them, in order to be acceptable under resistant criteria for a linear analysis. Among the multiple configurations, in the present paper the results for the frame composed of the following elements:

Columns: composite section formed by a WF 6X25 steel profile, arranged with its strong axis perpendicular to the structure plane, encased in concrete (25x25 cm). The reinforcement is set up by 8 bars of ϕ 20 mm organized with a 30 mm thickness coating.

Beam: reinforced concrete section of 25x180 cm with an upper longitudinal reinforcement of 2 bars of ϕ 16 and lower of 4 bars of ϕ 20 mm, both of them with a 30 mm thickness coating.

At this configuration with a beam edge equals to 1/10 of the span, the frame´s behaviour is similar to the supported beam. Their dimensions can be relatively small, due to the fact that they are under an axial load mainly, together with a small bending moment.

6.1. Characteristics of materials: σ-ε relationship

In the concrete case, the named as structural analysis diagram by EC2 has been used, it is shown at figure 7 a) and which stress-strain relationship can be expressed by means of the function:

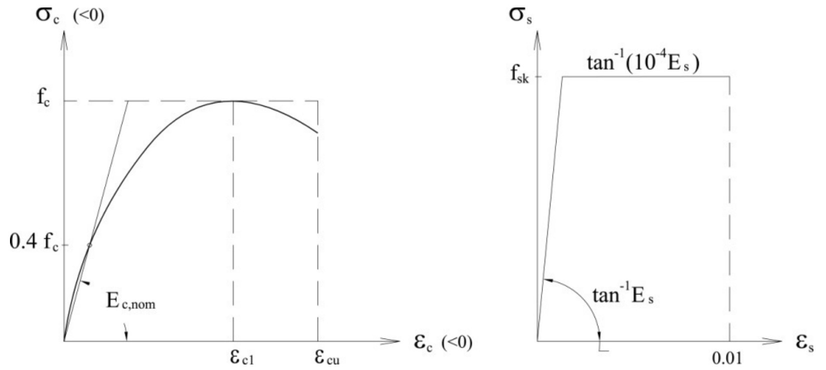
$$\frac{\sigma_c}{f_{cm}} = \frac{\kappa \cdot \eta - \eta^2}{1 + (\kappa - 2)\eta} \qquad (4)$$

where:

$f_{cm}=f_{ck}+8$	concrete average value compressive strength
$\eta = \varepsilon_c / \varepsilon_{c1}$	being ε_c and ε_{c1} both lower than zero
$\varepsilon_{c1}=0,7\cdot f_{cm}^{0,31} \leq 2,8$	strain due to maximum compression stress f_c
$\kappa =(1,05\cdot E_{c,nom})\cdot \varepsilon_{c1}/f_{cm}$	with f_{cm} introduced with a negative sign
$E_{c,nom}=22\cdot[(f_{cm})/10]^{0,3}$	secant modulus of elasticity of concrete
$\varepsilon_{cu1}=2,8+27\cdot[(98-f_{cm})/100]^4$	ultimate strain of concrete

Rheological effects have not been taken into account.

For the steel case, according to EC3, a stress-strain simplified diagram formed by two branches will be adopted (figure 7 b). First part of the origin with a slope equals to *Es* (210 kN/mm^2) until a characteristic yield limit is reached, fsk, which depends on the type of steel, and a second branch with an assigned slope of *Es*/10000. Unitary strain is limited for a 1% maximum value.



a) b)

Figure 7 Stress-strain curve a) concrete and b) steel

Table 1 compiles the characteristic strength of materials and the reduction factors used:

Table 1 Characteristic strength and safety factor

	Concrete	Structural steel	Reinforcing steel
Strength (MPa)	30	355	500
Safety factor	1,5	1,10	1,15

6.2. Loads actions and combinations

Dead loads (*P*), correspond to a roof made up of prestressed concrete hollow core slabs with an 8 m span between beams plus the roof coating, they are applied by distributed loads over the beams. Live loads caused by the wind over the façade (*W*), are applied by punctual forces over the top ending of the column.

For the checking purposes at the Ultimate Limit States and the aim of simplify the results, the following unique actions combination has been used:

$$\gamma \cdot P + \gamma \cdot W = 1{,}35 \cdot P + 1{,}50 \cdot W \tag{5}$$

6.3. Computational models

At the next section the results for two frame geometric models will be shown. For the first one the straight directrix bars have been considered, while for the second model the initial imperfections at the columns have been considered. These have been obtained by following the Eurocode 4 criteria (EC4) [20], starting from the deflection at the bar midpoint, $e_{0,d}$, whose value for the section strong axis can be obtained by equation 6.

$$e_{0,d} = 0{,}34 \cdot (\lambda - 0{,}2) \cdot (0{,}85 + 0{,}30 \cdot \lambda) \cdot W_{pl} / A \tag{6}$$

Deflected line resembles to a sinusoidal curve and after the columns discretization in 10 stretches of 0.50 m length each, the deformed directrix coordinates nodes are obtained (Table 2).

Table 2 Nodes coordinates of columns (bar local axis)

Nudo	0	1	2	3	4	5	6	7	8	9	10
y (mm)	0,0	7,6	14,4	19,8	23,3	24,5	23,3	19,8	14,4	7,6	0,0
z (m)	0,00	0,50	1,00	1,50	2,00	2,50	3,00	3,50	4,00	4,50	5,00

Both structural models have been subjected to the type of analysis indicated at Table 3.

Table 3 Type of analysis carried out in the structural models

	Model without imperfections	Model with imperfections
Linear analysis	YES	-
Non-linear analysis*	YES	YES

* Non-linear analysis includes the geometric and material non-linearity. In this case the loads have been progressively applied at 10 loading steps.

7. Results

The deformations obtained by two types of calculations carried out over the model without imperfections are shown in Figure 8. When a linear analysis is carried out a horizontal displacement of the right column top ending of 16.82 mm is obtained. If the second-order effects are taken into account the displacement of the same point is 36.00 mm, more than double of the previous value.

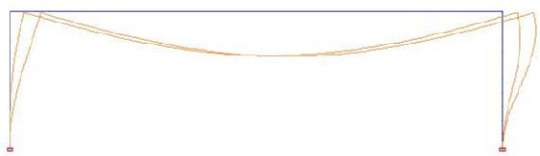


Figure 8 Deformations obtained in the frame without imperfections.

In the figure 9 the three bending moments diagrams of right support are shown with their values in the nodes: A and B diagrams belong respectively to the linear and the non-linear analysis, both from the model without imperfections and C diagram belongs to the non-linear analysis of the model with initial imperfections on the column.

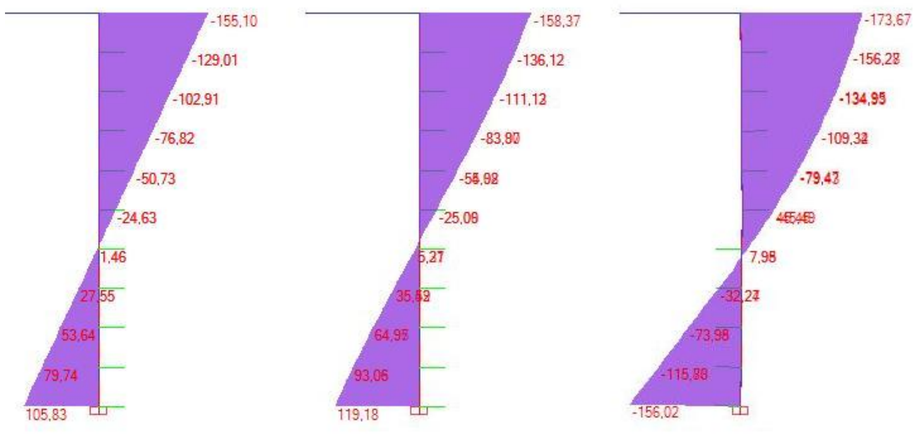


Figure 9 Right column biaxial bending moments diagram.

The axial load is noticeably the same for the three cases with a 1.150 kN value. The absolute value for the maximum moments and its increase in the second-order analysis regarding to the linear analysis is shown in Table 4:

In the Figure 10 the section column interaction diagram are represented that was obtained through the composite sections design program [15]. On the diagram, the effort values (N_d, M_d) obtained in the three analyses carried out for the most unfavourable section (column top ending) have been pointed out.

It can be observed that the three points kept inside the surface enclosed by the curve, this is the reason why at every case the section strength capacity it is not exceeded.

Table 4 Comparative bending moments at the column ends

		Low ending		Top Ending	
Model	Analysis	M_d (kN·m)	Increase %	M_d (kN·m)	Increase %
Without imperfections	Linear	105,83		155,10	
	Non-linear	119,18	12,61	158,37	2,11
With imperfections	Non-linear	156,02	47,42	173,67	11,97

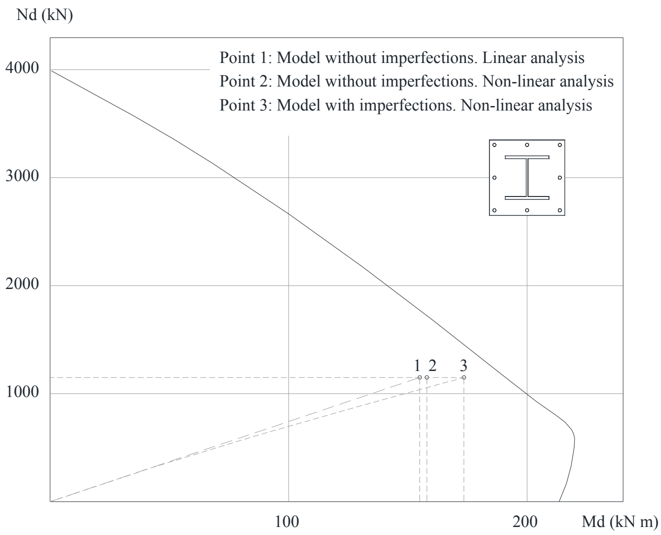


Figure 10 Section and solicitations interaction diagram.

However, the additional strength, understood as the relationship between the failure and calculus efforts, proportional to each other, varies at the three cases, due to the fact that the distances from the three points represented regarding to the interaction curve are different. The additional strength value for the three cases, together with the section solicitations are pointed out at Table 5:

Table 5 Stresses at the critical section

Model	Analysis		M_d (kN·m)	N_d (kN)	M_u (kN·m)	N_u (kN)	γ
Without imperfect.	Linear	Point 1	155,10	1.150	180,23	1.336,30	1,162
	Non linear	Point 2	158,37	1.150	181,33	1.316,75	1,145
With imperfect	Non linear	Point 3	173,67	1.150	186,17	1.232,80	1,072

Note: The pair of values (N_u, M_u) they are values over the interaction curvature and linear with (N_d, M_d)

The three coefficients obtained are superior to the unit; this fact indicates that an additional strength of the bar exists, besides the safety factors assumed.

8. Conclusions

A procedure that allows to carry out a non-linear geometric and mechanic analysis of the framed structures has been shown. The method has been applied on a single span frame and the results for one of the configurations have been displayed.

Regarding the frame exposed, it is made up of a large depth beam and slender columns, and one may conclude that:

- For the model without imperfections, the bending moments increasing at the most unfavourable column cross-section, is obtained at the non-linear analysis it exceeded the 10% (the Eurocode reference value), with regard to the linear-analysis.
- The increase of the bending moments at the columns is significantly higher, up to 47%, starting with a model where initial imperfections are taken into account.
- The additional strength is noticeably reduced in the model with initial imperfections regarding the model without imperfections

Even for a low slenderness structure, the interaction between the axial load and horizontal displacements increase the maximum bending moment up to 10%. Due to this, and according to the Eurocode´s criteria, its second-order analysis will be unavoidable.

About the results obtained from the different configurations studied it must be pointed out that the P-D effect is more important as the column stiffness decreases regarding the beam. This is due to the bending moment transferred to the columns by means of the beam smaller, making it possible to reduce its dimensions by resistant criteria. Its greater slenderness provides higher horizontal displacements.

To entrust the frame bracing only to the bars stiffness leads to an over dimensioning by stability conditions. In order to avoid the laborious and unusual second-order analysis and taking into account good constructive practices, to have a suitable bracing system in order to avoid or at least reduce the horizontal displacements is recommendable. For the frame typology analysed is particularly necessary when the stiffness of the columns is significantly less than that of the beam.

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