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Mohr’s circle for plane stress

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1 Summary

This document shows how to draw Mohr's stress circle for a planar state of stress.

Instructions, step by step are given and applied to a worked example.

A proposed exercise is included in order to involve the student in practising the procedure.

2 Introduction

When working with stress distributions, it must be pointed out that only one state of stress is intrinsically unique, regardless of the orientation of the coordinates system used to represent that state of stress. Therefore, when we have two elements at the same point in a body with different orientations, the stresses acting on the faces of the two elements are different, but they still represent the same state of stress.

Transformation equations for plane stress show how normal stresses σ_x and shear stresses τ_{xy} values vary when the axes are rotated through the angle θ.

It is known that there exists one set of axes, named principal axes, which respect to them, shear stresses are zero and normal stresses have their maximum and minimum values.

The maximum and minimum values for normal stresses are called principal stresses, being denoted by σ_1 the larger of the two principal stresses and σ_2 the smaller of the principal stresses.

Otto Christian Mohr (1835-1918) German civil engineer, developed the circle of stress in 1882 as a graphic method of solution for principal stresses. This graphical representation, is known as circle of stress of Mohr's circle of stress, shows the relationship between normal at shear stresses acting at a point of a stressed element, on different inclined planes.

3 Aims

At the end of this document, the student will be able to draw the Mohr's circle of stress knowing one initial state of stress of a biaxial elastic element.

Students will learn how to check that their graphic results are right, comparing the graphical values with the numerical ones calculated using the principal stresses equation.

4 Mohr's circle for plane stress

4.1 State of stress

Considering the state of stress of the biaxial elastic element in Figure 1, where signs of normal stresses σ_x and σ_y are positive (tension stress, going out of the surfaces) and sign[1] of shear stress τ_{xy} is negative, the steps to be followed, and the corresponding sketches are shown in the following epigraph.

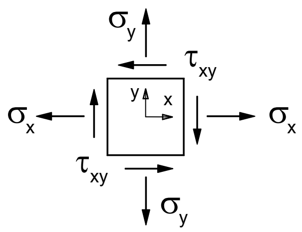

Figure 1

4.2 Steps to be followed

1. Draw horizontal and vertical axes.

2. Measure σ_x and σ_y along the horizontal axis considering their signs, and mark A (σ_x) and B (σ_y) as shown in Figure 2.

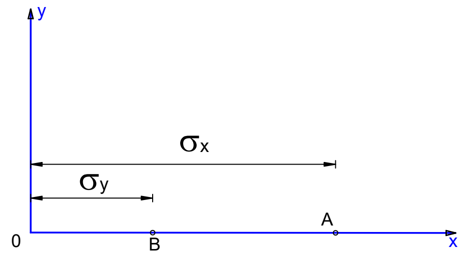

Figure 2

3. Draw τ_{xy} up at A (if it is positive, or down if it is negative) and mark point C as shown in Figure 3.

4. Draw τ_{xy} down at B (if it is positive and up if it is negative) and mark D, as shown in Figure 3.

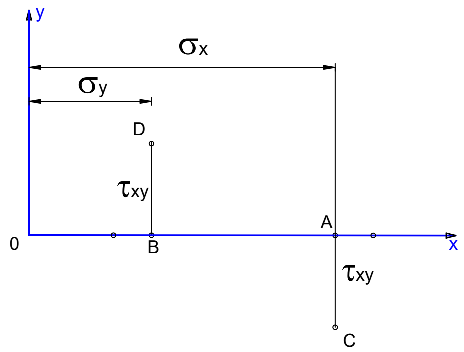

Figure 3

[1] the sign convention for shear stresses is as follows: A shear stress is positive when the direction of the stress in the first quadrant is the same than positive axes. In this case, as the direction of shear stress is opposite to positive axes, it is negative.

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5. Find the centre (M) halfway between points C and D, as shown in Figure 4.

6. Draw the diagonal CD which should pass through the centre M.

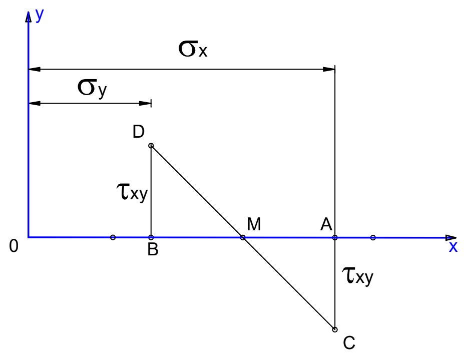


Figure 4

7. Draw the circle with centre at M and radius MC.
   The circle should also pass through D as can be seen in Figure 5.

8. Measure OE and this is principal stress σ_2 (the smallest value) as can be seen in Figure 6.

9. Measure OF and this is principal stress σ_1 (the biggest value) as can be seen in Figure 6.

10. The angle 2θ is shown in Figure 7.

11. τ_{max} is the radius of the circle.

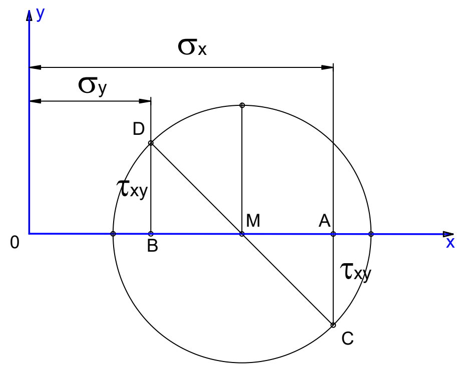


Figure 5

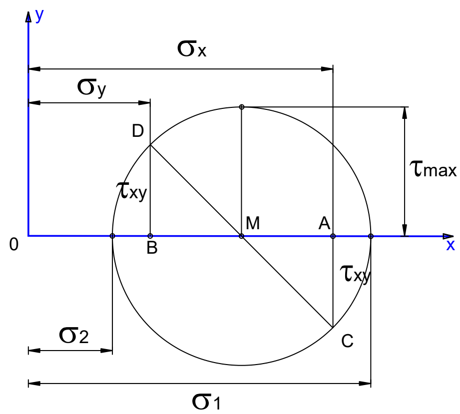


Figure 6

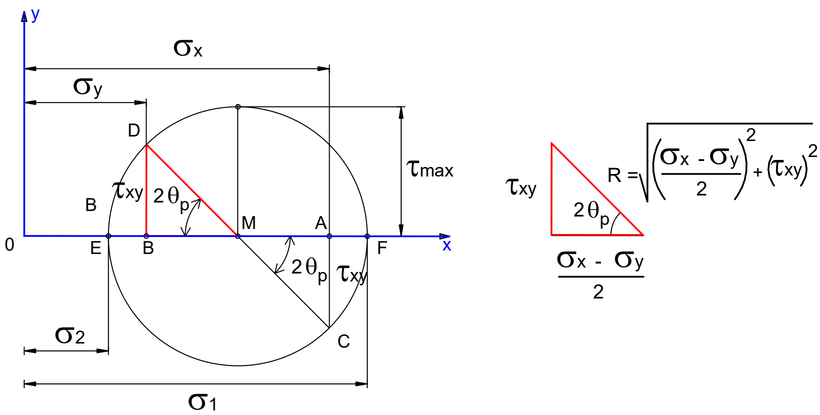


Figure 7

4.3 Principal stress equation

The principal stresses expression is shown in Equation 1

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} \qquad \text{Equation 1}$$

Equation 1 corresponds to two equations, one for the biggest value or principal stress 1 (σ_1) and the other for the smallest value or principal stress 2 (σ_2).

The planes on which the principal stresses act are called principal planes being the principal angles that define the principal planes θ_p given by Equation 2.

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \text{Equation 2}$$

4.4 Worked example

Given the elastic element in **¡Error! No se encuentra el origen de la referencia.**, it is requested to draw Mohr's circle of stress.

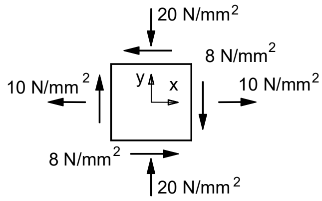


Figure 8

Defining the stresses in terms of established sign convention:

$$\sigma_x = 10\ N/mm^2$$

$$\sigma_y = -20\ N/mm^2$$

$$\tau_{xy} = -8\ N/mm^2$$

Principal stresses are, as can be seen in Figure 9

$$\sigma_1 = 12\ N/mm^2$$
$$\sigma_2 = -22\ N/mm^2$$

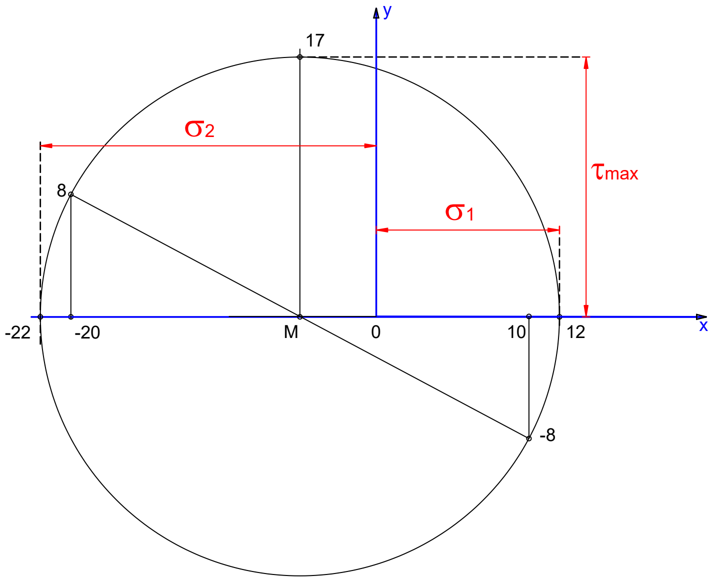


Figure 9

In order to check if the obtained results are right, principal stresses al calculated with equation 1

$$\sigma_{1,2} = \frac{10+(-20)}{2} \pm \sqrt{\left(\frac{10-(-20)}{2}\right)^2 + (-8)^2}$$

$$\sigma_{1,2} = -5 \pm \sqrt{(15)^2 + (-8)^2} = -5 \pm 17 = \left| \begin{array}{l} \sigma_1 = 12 \\ \sigma_2 = -22 \end{array} \right.$$

Giving the same values than the stress circle.

4.5 Proposed exercise

Given the elastic element in, it is requested to draw Mohr's circle of stress.

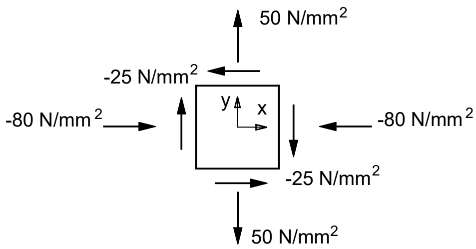


Figure 10

5 Conclusion

This document explains how to draw the circle of stress or Mohr's circle of stress

The method has been shown with a worked example done step by step.

The obtained results have been checked considering the principal stresses equation

Finally, a practical exercise has been proposed in order to encourage the student to practice about this topic. The solution of this exercise can be found at the end of this document

6 Bibliography

Gere, James M: "Mechanics of materials" Thomson Learning, Inc. 2004

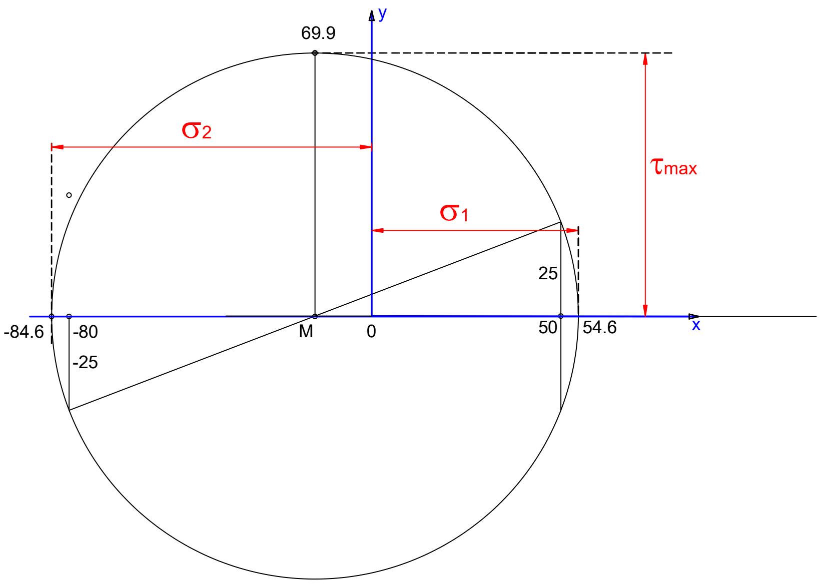
7 Proposed exercise solution

Defining the stresses in terms of established sign convention:

$$\sigma_x = -80\ N/mm^2$$
$$\sigma_y = 50\ N/mm^2$$
$$\tau_{xy} = -25\ N/mm^2$$

being

$$\sigma_1 = 54.6\ N/mm^2$$
$$\sigma_2 = -84.6\ N/mm^2$$



Former results are compared with those obtained with principal stresses equation:

$$\sigma_{1,2} = \frac{(-80)+(50)}{2} \pm \sqrt{\left(\frac{(-80)-(50)}{2}\right)^2 + (-25)^2}$$

$$\sigma_{1,2} = -15 \pm \sqrt{(-65)^2 + (-25)^2} = -15 \pm 69.6 = \left| \begin{array}{l} \sigma_1 = 54.64 \\ \sigma_2 = -84.64 \end{array} \right.$$

Giving the same values.