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ARTICLE TYPE

Online score-informed source separation in polyphonic mixtures using instrument spectral patterns

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[1], , ,
[2], , ,
[3], , ,

Correspondence
Email:

Abstract

Soundprism is a real-time algorithm to separate polyphonic music audio into source signals, given the musical score of the audio in advance. This paper presents a framework for a *Soundprism* implementation. A study of the sound quality of the online score-informed source separation is shown, although a real-time implementation is not carried out. The system is compound of two stages: 1) a score follower that matches a MIDI score position to each time frame of the musical performance; 2) a source separator based on a Non-negative Matrix Factorization (NMF) approach guided by the score. Real audio mixtures composed of an instrumental quartets were employed to obtain preliminary results of the proposed system.

KEYWORDS:
Sound source separation, NMF, Dynamic Time Warping, Online, Score-informed, Score-alignment

1 | INTRODUCTION

Currently, Internet stores a huge amount of musical content which are freely accessible in platforms such as Youtube, iTunes, Spotify, International Music Score Library Project†, etc. Different signal processing techniques could be employed to change the ways people enjoy this content [1]. In this sense, audio source separation seeks to identify and segregate individual signal components in a polyphonic mixture. Often, this technique relies on assumptions, such as the statistical independence of the source signals or the availability of multiple channels (recorded using several microphones). In the context of audio signal processing, sound sources often outnumber the information channels and are typically highly correlated in time and frequency. Consequently, typical statistical methods such as independent component analysis (ICA) [2][3] or nonnegative matrix factorization (NMF) [4] often fail to completely recover individual sound objects from music mixtures [5].

Over the last years, this field has been widely addressed with novel ideas and application scenarios [6] such as instrument-wise equalization [7], personal music remixing [8], music information retrieval [9] and intelligent audio editing [10]. In the context of classical music, source separation could be applied for different interesting scenarios. One of them is the improvement of the sound quality of the existing monophonic or stereo recordings [11], employing two approaches called offline or online. In offline case, the audio performance must be available as a whole from start to end. However, in the online approach, the input audio is processed piece-by-piece in a serial fashion without having the entire audio available from the start, which makes it suitable for streaming scenarios.

Furthermore, music source separation plays an essential role for education applications such as Minus One, which consists in removing one voice (i.e. instrument) from the mixture. Similarly to a karaoke, a musician can interpret a music piece with an accompaniment composed by all the source but the played one. Regarding the entertainment industry, some platforms of classical music live broadcast, such as "Palco Digital"‡, could benefit from source separation in order to enrich their contents with further features.

† http://imslp.org
‡ https://www.palcodigital.com

In the mentioned applications, the quality of the user experience is intimately related to the quality of the separation. Classical source separation methods exploit known spectro-temporal properties of the sources [5][12] and/or the annotations of the recording material as additional prior knowledge [13][14][15][16]. For instance, the musical score (in the form of MIDI) is the most common tool used to guide source separation process. In this way, a large amount of MIDI scores are available on the web at sources such as Classical Midi Connection[§]. Source separation guided by a musical score is called score-informed source separation [8][9][11]. In this scenario, a prior alignment between the score and audio is required.

In this paper, we propose an online score-informed method to separate polyphonic music recordings. We consider an online method when the factorization of the input signal is computed without any future information, so the system generates the output just after receiving the input. Thus, it is not necessary that the entire input is available from the start. Furthermore, our proposal allows it implementation as a low latency system, understanding latency as the time delay between the signal is received and the separation processed. This latency will depend on the computer and the optimization of programming. In fact, with a suitable implementation and architecture, the proposed system can be computed in real-time and the music source separation is referred as Soundprism. Note that in this work we focus on the description of the separation framework whereas the real-time implementation of the proposed system is not covered here.

In this context, we decompose our proposal into two stages (see Fig. 1): 1) the score alignment and 2) the factorization stage.

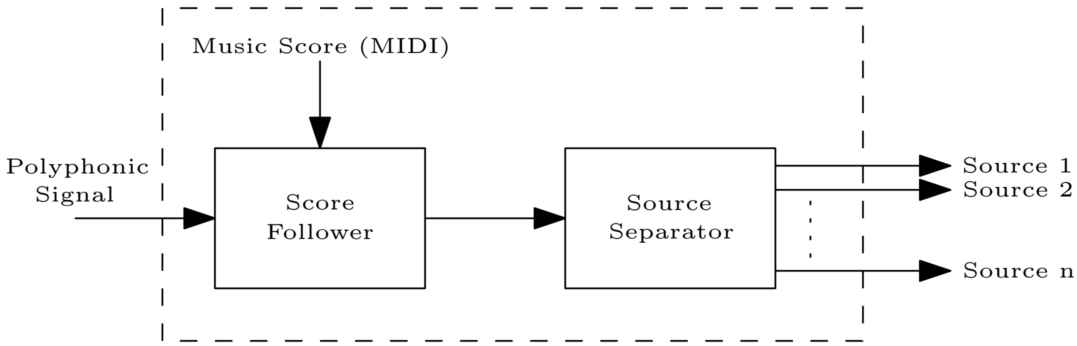


FIGURE 1 System overview.

In the first stage, each real audio frame is associated with a score position. For this purpose, we use an alignment kernel based on our robust system proposed in [17][18]. On the other hand, the aligned score position are used to guide source separation in the second stage. The segregation of the different sources is relied on a NMF framework using parametric instrument models.

2 | NON-NEGATIVE MATRIX FACTORIZATION FOR SOURCE SEPARATION

Non-Negative Matrix Factorization (NMF) [19] is one of the most successful technique used for audio source separation. The principle is to model the magnitude or power spectrogram of a music recording $X \in \mathbb{R}_+^{FxT}$ as a linear combination of K elementary nonnegative spectra called basis function. Therefore, NMF can factorize the spectrogram matrix as follow:

$$X \approx \hat{X} = WH \tag{1}$$

where $W \in \mathbb{R}_+^{MxK}$ is the basis matrix, $H \in \mathbb{R}_+^{KxN}$ is a matrix of component gains, $F \in \mathbb{N}$ and $T \in \mathbb{N}$ denote the number of frequency bins and number of time frames and $\hat{X}$ is the estimated spectrogram.

The procedure to compute the factorization in Eq. (1) is to minimize a cost function defined as:

$$D(X|\hat{X}) = \sum_F \sum_T d(X|\hat{X}) \tag{2}$$

[§]http://www.classicalmidiconnection.com where $d(a|b)$ is a function of two scalar variables. The β-divergence [19] [20] [21] is the most popular cost function and its definition includes Euclidean (EUC) distance ($\beta = 2$), Kullback-Leibler (KL) divergence ($\beta = 1$) and the Itakura-Saito (IS) divergence ($\beta = 0$).

Standard NMF used the gradient descend algorithm to estimate the model parameters during the factorization (see [19] for further details). Using this algorithm, the matrices W and H are initialized and then iteratively updated using the following multiplicative rules as follows:

$$H \leftarrow H \odot \left(\frac{\nabla_H^- D(X|\hat{X})}{\nabla_H^+ D(X|\hat{X})} \right) \tag{3}$$

$$W \leftarrow W \odot \left(\frac{\nabla_W^- D(X|\hat{X})}{\nabla_W^+ D(X|\hat{X})} \right) \tag{4}$$

After this process, each column of W represents a certain sound component and the corresponding row of H encodes the activation of these components.

3 | PROPOSED FRAMEWORK

The proposed framework for our source separation method is presented in Fig. 2. As can be seen, the framework is composed of two main stages. First, the audio-to-score alignment is computed. Then, the signal factorization is performed to learn all the parameters needed for the reconstruction of all sound sources. As can be observed, the MIDI score, the real performance signal and instruments models are the inputs of our proposal. These instrument models contains the spectral patterns of each note and instrument [22][23].

3.1 | Alignment stage

The aim of this stage is to synchronize the audio recording of the musical piece with the corresponding MIDI score. We propose the framework presented in [17], where the alignment is divided in two steps: MIDI feature extraction and alignment. As can be seen in Fig. 2, once the alignment is performed, the output parameter $A_{p,j}(t)$ is used for guiding the factorization. This parameter indicates the active notes p in MIDI scale of the instruments j in the time-frame t of the audio performance.

Firstly, the input MIDI score is represented by a binary matrix $GT(n, \tau)$ denoting the active notes n in MIDI scale at each time-frame τ referenced to the score (MIDI time). Each unique occurrence of individual or concurrent notes will be denoted as a *score unit*. In terms of score units, the score matrix $GT(n, \tau)$ can be decomposed as follows:

$$GT(n, \tau) = Q(n, k)R(k, \tau); \tag{5}$$

where $Q(n, k)$ is the binary notes-to-units matrix, k the index of each unique unit and $R(k, \tau)$ represents the binary activation of each unit. Observe that $Q(n, k)$ informs about the notes belonging to each unit, whereas $R(k, \tau)$ retains the MIDI time activation per unit. Fig. 3 displays an example of the score decomposition presented in Eq. 5.

Afterwards, a synthetic signal is generated from the score using a MIDI synthesizer in order to learn a single spectral pattern for each score unit (*spectral pattern learning block*). Expressing the magnitude spectrogram of the synthetic signal as $Y(f, \tau)$, where f being the frequency bin index, it can be decomposed according to the following model:

$$Y(f, \tau) \approx \hat{Y}(f, \tau) = B(f, k)G(k, \tau) \tag{6}$$

where $\hat{Y}(f, \tau)$ is the estimated spectrogram, $G(k, \tau)$ matrix represents the gain of the spectral pattern for unit k at frame τ, and $B(f, k)$ matrix represents the spectral patterns for all the units defined in the score. The parameters are estimated using NMF with β-divergence and multiplicative update rules, where $G(k, \tau)$ is initialized to $R(k, \tau)$, and $B(f, k)$ to random positive numbers.

At the *cost matrix computation block*, the distortion between the frequency transform of the input signal and the spectral patterns learned per unit is computed to measure the similarity between the audio and the different units defined by the score. In that sense, we denote the frequency-domain input signal vector at time t as $x_t(f)$ and the k-th unit spectral pattern as $b_k(f)$.

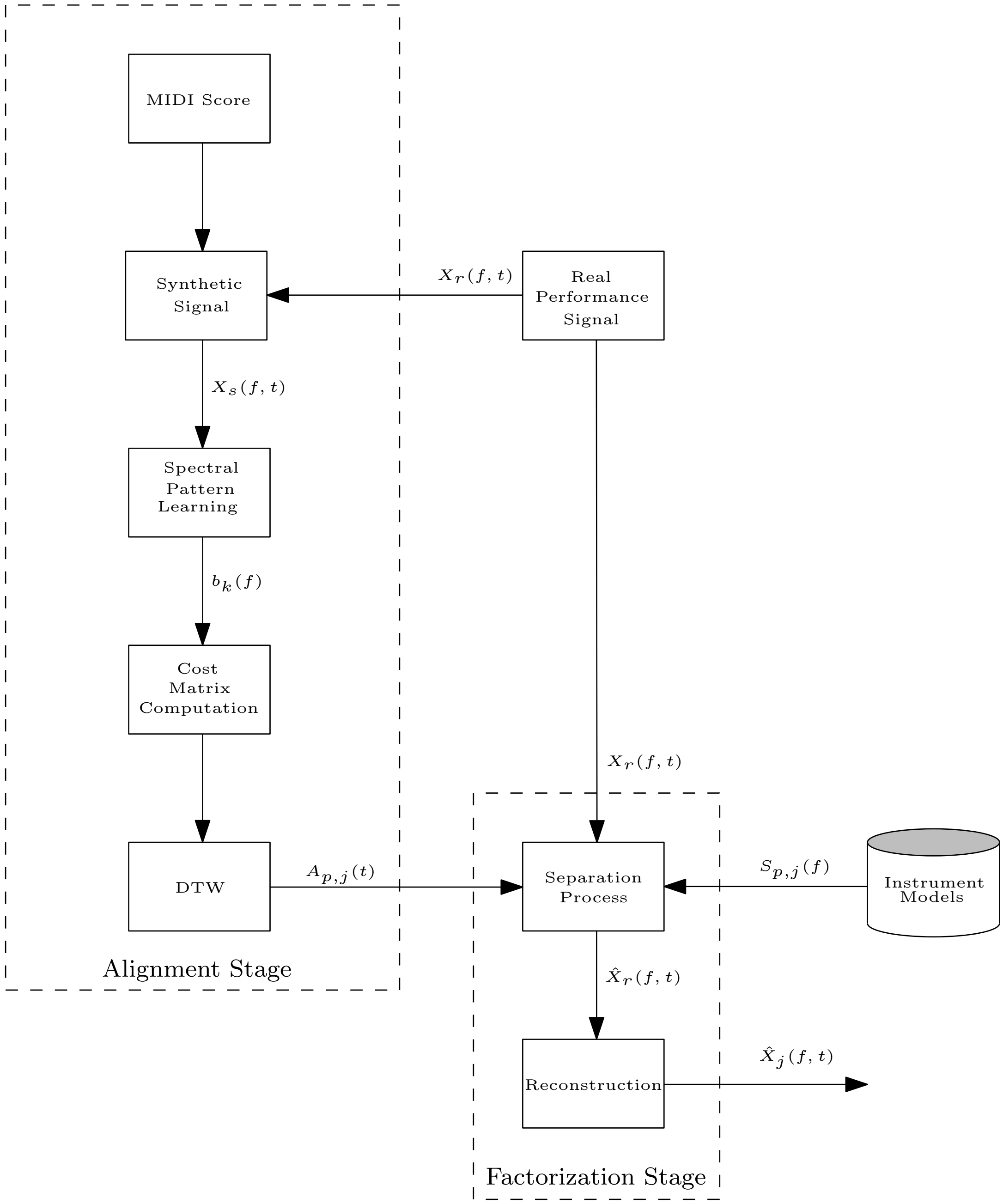


FIGURE 2 Block diagram of the proposed framework.

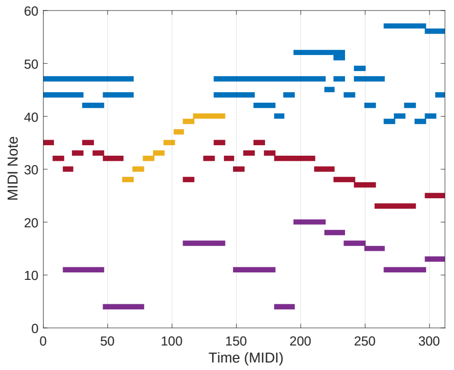
Assuming a signal model in which only a single pattern can be active at t, Eq. 7 computes the time varying gain for each unit $g_{k,t}$ as the projection of the pre-learned basis functions over the observed mixture signal spectrogram.

$$g_{k,t} = \frac{\sum_f x_t(f) b_k(f)^{(\beta-1)}}{\sum_f b_k(f)^{\beta}} \tag{7}$$

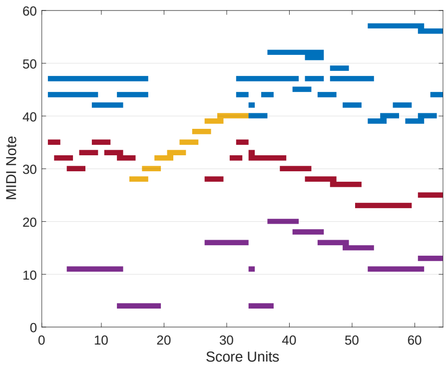
Finally, the distortion matrix for each unit at each frame is defined by:

$$\Phi(k,t) = D_{\beta}(x_t(f)) | g_{k,t} b_k(f)) \tag{8}$$

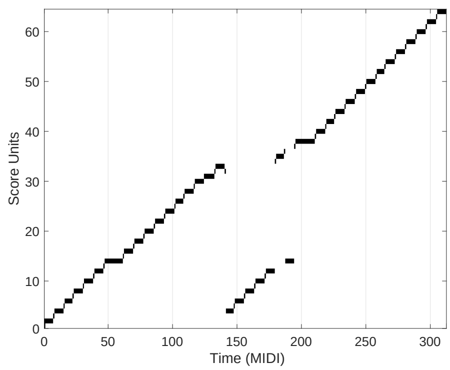
where $D(\cdot)$ is the β-divergence function and β can take values in the range $\in [0, 2]$.



(a) MIDI Ground-Truth Transcription $GT(n, \tau)$.



(b) Note-to-units matrix $Q(n, k)$.



(c) Units activation matrix $R(k, \tau)$.

FIGURE 3 Example of score matrix $GT(n, \tau)$ decomposition. (a) MIDI Ground-Truth Transcription $GT(n, \tau)$. (b) Note-to-units matrix $Q(n, k)$. (c) Units activation matrix $R(k, \tau)$. The instruments in (a) and (b) are displayed using different colors.

As can be inferred, the distortion matrix $\Phi(k, t)$ provides us information about the similitude of each k-th unit spectral pattern with the real signal spectrum at each frame t. Using this information, we can directly compute the cost matrix between the MIDI time τ and the time of the input signal t as:

$$D(\tau, t) = R^T(\tau, k)\Phi(k, t), \tag{9}$$

where superscript "T" stands for matrix transposition. This matrix has dimensions $T_m \times T_r$, where T_m is the number of frames of the MIDI score and T_r is the number of frames of the input signal.

Finally, at the *DTW block*, the optimum path across matrix $D(\tau, t)$ is computed using DTW algorithm to provide the alignment between the score and performance times. More information about the alignment described above can be found in [17][18].

The software ReMAS (Real-time Musical Accompaniment System), designed to track the reproduction of a musical piece with the aim to match the score position into its symbolic representation on a digital sheet, was presented in [24]. ReMAS shows that it is possible to exploit efficiently several cores of an ARM® processor, or a GPU accelerator, reducing the processing time per frame in a few milliseconds in most of the cases. On the other hand, a parallel online DTW solution based on a client-server architecture implemented for multi-core architectures (86, 64 and ARM®) is proposed in [25].

Both softwares have a high degree of similarity to the alignment stage. Thereby the framework proposed for the alignment in this work aims to adapt, extend and test the parallel heterogeneous algorithms of [24] and [25] to the new problem.

3.2 | Factorization stage

For the factorization of the audio signal, we proposed the following signal model presented in [26]:

$$X(f,t) \approx \hat{X}(f,t) = \sum_{p,j} S_{p,j}(f) A_{p,j}(t) \tag{10}$$

where $X(f,t)$ is the magnitude spectrogram of the audio signal, $\hat{X}(f,t)$ is the estimation of this magnitude spectrogram, $S_{p,j}(f)$ is the basis matrix and contains the spectral patterns of each note of each instrument, $A_{p,j}(t)$ is the time varying gains matrix that shows the temporal intervals in which the previous basis functions are active, p are the notes in MIDI scale and j is the number of instruments.

In this work, the basis matrix $S_{p,j}(f)$ is trained using the Real World Computing (RWC) music database [27][28] as in [22], where it is shown that it is profitable to learn bases in advance and fix them during the separation process. Besides, the coefficients of the gains matrix $A_{p,j}(t)$ are initialized with the optimum path across matrix $D(\tau, t)$ provided by DTW.

Subsequently, an standard NMF with the following multiplicative update rules which minimize the β-divergence between the observed spectrogram $X(f,t)$ and the model $\hat{X}(f,t)$ can be used for the factorization:

$$A_{p,j}(t) \leftarrow A_{p,j}(t) \odot \left(\frac{\sum_{f,p,j} S_{p,j}(f) [\hat{X}(f,t)^{\beta-2} \odot X(f,t)]}{\sum_{f,p,j} S_{p,j}(f) \hat{X}(f,t)^{\beta-1}} \right) \tag{11}$$

Finally, we proposed to apply a Wiener filter method to compute the relative energy of the mixed signal as in [23].

4 | EXPERIMENTS AND RESULTS

For testing our method, we have used the database proposed in [29]. This database is compounded of 10 J.S. Bach four-part chorales and each music excerpt consists of an instrumental quartet (violin, clarinet, tenor saxophone and bassoon). The audio files are approximately $30s$ long and are sampled at $44.1kHz$ from real performances.

We are going to compare different configurations of the proposed method and a baseline score-informed source separation method proposed in [29], denoted as Duan's *Soundprism*. Duan et al. proposed an online method without instrument models. Its algorithm separates sources using harmonic masking where the energy of overlapping harmonics are distributed according to the harmonic indixes of the sources.

In the comparison, four configurations of our method are presented. *GT* denotes the variant of our proposed method in which the perfect annotation score is used to guide the factorization stage, *ScoreFree* denotes the variant in which no alignment stage is used and, therefore the gains matrix $A_{p,j}(t)$ is initialized with random values, and, *Offline* and *Online* make reference to variants whose alignment stages have been implemented following the *Offline* and *Online* approaches described in [18]. Furthermore, we are also going to contrast our proposal with *Oracle* version, which uses the individual sources to establish the best separation that can be obtained using the proposed softmask reconstruction strategy. It sets an upper bound of all the configurations of the proposed method.

For obtain numerical results of this comparison, we use the metrics *Source to Distortion Ratio* (SDR), the *Source to Interference Ratio* (SIR) and the *Source to Artifacts Ratio* (SAR), implemented in [30] (BSSEVAL Toolbox 2.1). These metrics are widely used by the research community in source separation, and therefore facilitate a fair evaluation of the method.

Fig. 4, Fig. 5 and Fig. 6 show the comparison results on the 10 excerpts. Each box represents 40 data points, one for each individual instrument of the ten mixtures test database. The lower and upper lines of each box show 25th and 75th percentiles of the sample. The line in the middle of each box is the median. The lines extending above and below each box show the extent of the rest of the samples. As can be observed, *ScoreFree* method obtain the worst results because of not employing the alignment stage. *GT*, Duan's *Soundprism*, *Offline* and *Online* obtain results very similar. The reason of this behaviour is because the alignment scheme used in these methods obtained a high percent of precision, as can be observed in the MIREX task of Real-time Audio to Score Alignment (MIREX 2010[¶]). Therefore, the initialization of the parameter $A_{p,j}(t)$ is practically the same for this database with a low polyphonic complexity. The slight outperforming of the *Offline* and *Online* methods over the Duan's *Soundprism* system is due to the introduction of the instrument models learned with real instruments (using RWC database), instead of using fixed timbrical models of Duan's *Soundprism* which are independent from the instruments.

[¶]https://www.music-ir.org/mirex/wiki/2010:Real-time_Audio_to_Score_Alignment_(a.k.a._Score_Following)_Results

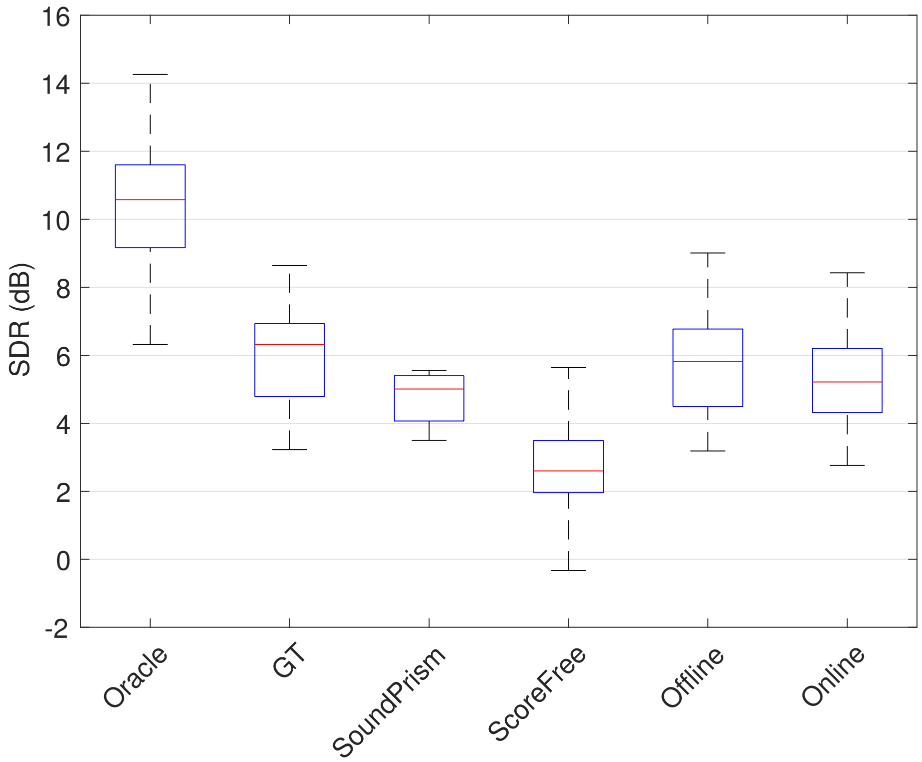


FIGURE 4 Comparison of the source separation methods in terms of SDR.

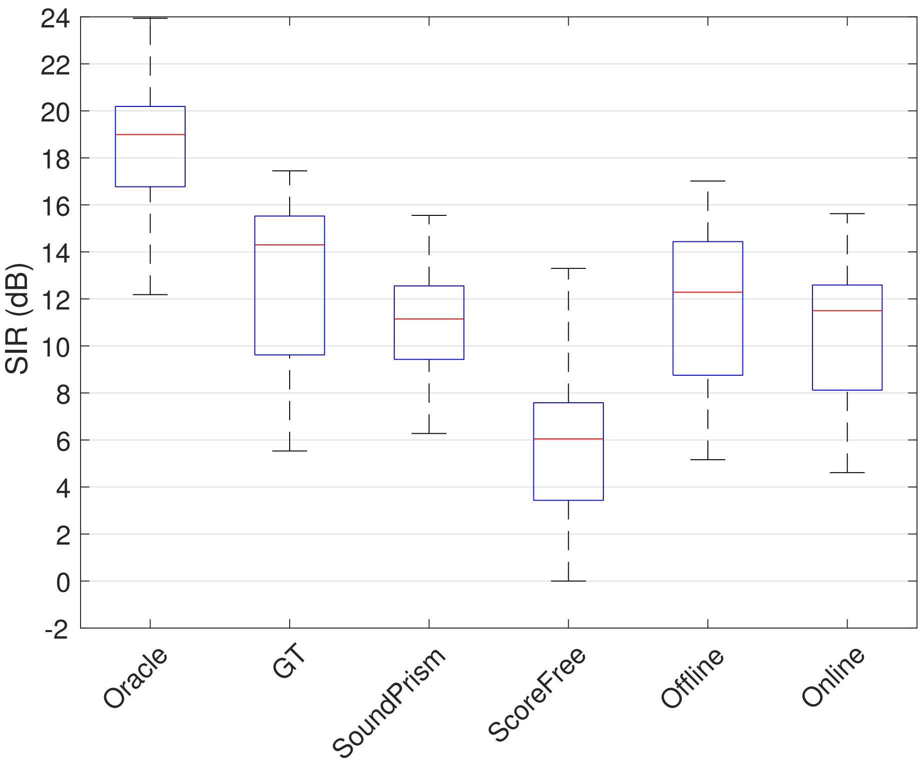


FIGURE 5 Comparison of the source separation methods in terms of SIR.

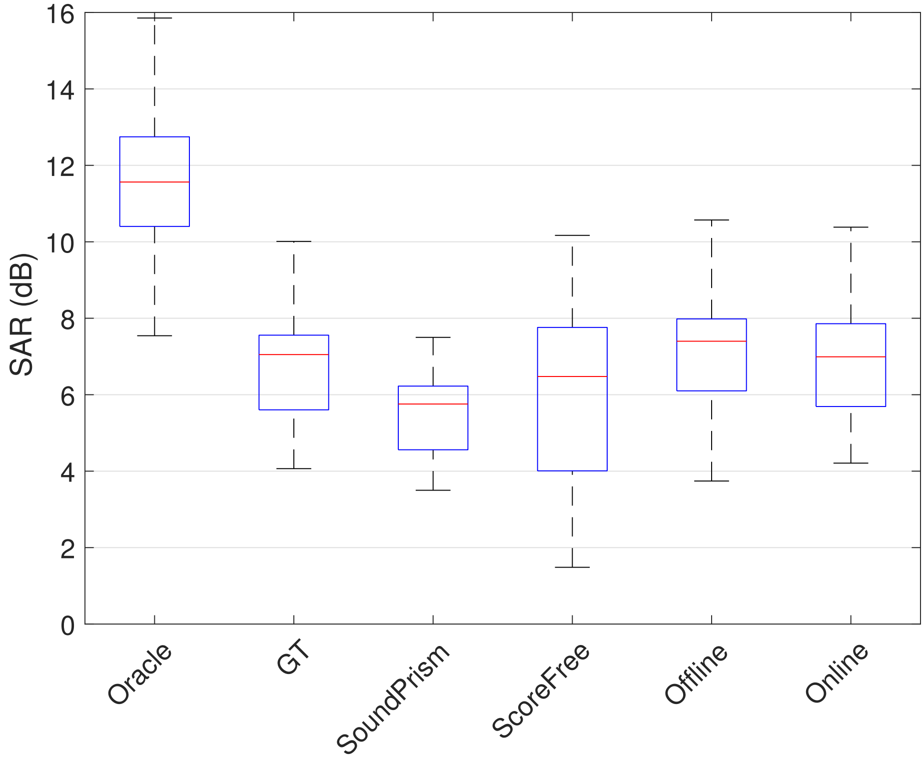


FIGURE 6 Comparison of the source separation methods in terms of SAR.

5 | CONCLUSION

In this paper, we have presented a framework for a *Soundprism* implementation. The proposed system aims to segregate an input polyphonic mixture into source signals and is compound by two stages: a score follower and a factorization stage. The first one is based on DTW to synchronize the audio recording of the musical piece with the corresponding score. Then, the factorization is computed guided by the score follower and using NMF approach. Finally the reconstruction of the source signals is performed by a Wiener filter method to obtain the energy contribution of each source.

We have tested our proposal with real audio mixtures composed of an instrumental quartets. For the evaluated dataset, our method has demonstrated superior results in terms of separation metrics among the compared methods. The evaluation has revealed the robustness of the proposed method for online scenarios.

6 | FUTURE WORK

Regarding future work, we propose the following lines of work to improve the system:

- Test new databases with more polyphonic complexity in order to study the behavior of the proposed method in orchestra performances.
- Study alternatives to the proposed signal model with less dependent of the alignment errors. Modifying the output of the *DTW block* (see Fig. 2) for activating more notes in each frame could solve these alignment errors.
- Make a real-time implementation using parallel programming. Duan's *Soundprism* (proposed in [29]) presents a high computational cost that makes it impossible to implement in real-time with the current technology. In their experiments, the algorithm runs about three times slower than real time. However, our proposal has a lower computational cost that combined with efficient techniques to solve the NMF problem, such as [31], could make its implementation possible.

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