Real-time high-rise building monitoring system by GNSS technologies

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Abstract

The continuous assessment of the displacement of a high-rise building enables veri cation that meets the structural assumptions calculated in the project, which provide an additional useful feedback for structural engineers. Unexpected structural responses during the phases of construction or once the building is in use can help detect current and future problems as well as providing cost savings. It also contributes to an improved risk management for natural phenomena; it can be used to verify the stability of the structure providing an additional safety item for the building.

The technology improvement of GNSS techniques has allowed us important advancements of Gaussian methodology applied to control the dynamics of building structures in real time, especially for calculating, controlling and interpreting of satellite survey measurements based on Gaussian Analysis and least squares adjustment.

The Real-Time monitoring system works by implementing a local geodetic network with GNSS technology on the structure to be monitored. Algorithms are then applied that improve the compensated network solution and this is integrated into original software. The system allows us to achieve a high level of security and e ective risk management in real time because for each building a unique mathematical model is designed that allows GNSS position errors to be reduced. In the case of Torre Espacio the overall mathematical adjustment model reduces the maximum error by 40%. The system has been installed on a high-rise building called Torre Espacio, in Madrid, Spain and is fully operational.

Keywords: Monitoring Control, Local Geodetic Network, Gaussian Analysis, Least Squares Adjustment, Satellite Surveys Measurements, Torre Espacio

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1. Introduction

The last decade has seen a notable increment in the demand for increasingly flexible structures and of complex architectural forms which can help to solve the problem of rapid urbanization. In fact, today as of mid-2017, nearly 7.6 billion people inhabit the planet and, since 2016, more than 4.1 billion of the world's population lives in urban areas that barely cover 5% of the earth's surface [1]. It is increasingly evident that cities need to grow vertically, in the form of high-rise buildings.

The control of the structure positions with GNSS techniques, is a fact nowadays. To determine the exact position of these structures (which varies due to the natural phenomena such as wind and earthquakes) reduces the risk of failure, improves the structures level of safety and it can also provide a cost saving in the construction process.

The technological improvement of GNSS techniques and the progress of computer data processing, combined with the development of the Internet and mobile telephony, has allowed us to upgrade and advance the Gaussian methodologies applied to control the dynamics of building structures in real time.

The coordinates obtained with real-time GNSS techniques have an accuracy which can vary between 0.2 and 3 cm in a short time. Sometimes the demands of the project require superior accuracy. The proposed system allows instant quality control of precision, corrects errors of GNSS coordinates using Gaussian algorithms and determines the accuracy and reliability of the errors that could not be corrected.

In short, through implementing a local geodetic network with GNSS antennas and receivers on the structure to be monitored, and through applying mathematical and statistical algorithms it is possible to improve the compensated network solution. Software has been designed which implements all the algorithms that are used in the Real-Time Monitoring System. It is precisely the treatment of data, adjustment and analysis of partial and final results that allow us to achieve a high level of security and effective risk management in real time.

Specifically, monitoring and the dynamic deformation behaviour of civil engineering structures and building structures has been a subject of concern of engineers for many years [2]. It is also becoming a research field of great interest academic [3], [4] with an understanding that tolerance levels, sometimes to millimeter accuracy, are a vital detail.

The advancement of technology and communications has allowed the development of methods that provide these positions with millimeter accuracy. In this sense, the adoption of GPS technology for monitoring of civil and architectural structures has already been validated in other preliminary research verifying its adoption as a standard technique for measuring structural vibrations. Example of this include the Calgary Tower, in Canada [5], the "Central Business District" in Singapore [6] and in the UK, the Humber suspension bridge [7], in Nottingham [8].

Later, due to the evolution of GPS technology an improved method using differential positioning real-time kinematic (RTK) was developed. Several studies have demonstrated the feasibility of RTK-GPS such as the observed full-scale performance of three tall buildings in downtown Chicago (USA). The wind-induced responses of the buildings were measured and compared to wind tunnel tests and nite element models by installing force balance accelerometers and a GPS receiver [9], [10]. Another study with the same aim used full-scale measurement for the displacement of a tower in Japan [11]. And in Singapore, from 2001 to 2005, a GPS system was installed to monitor a high-rise building and test the capability of GPS to resolve the relatively small de ections expected. In addition, GPS technology has been able to detect and resolve dynamic responses to both wind and seismic e ects. [12].

Other preliminary studies in line with what is discussed in this paper were conducted in Slovakia, Singapore and Korea. In the rst example in Slovakia a 24-storey building [13] was monitored by applying electronic distance measurement technology and GNSS techniques, using network Slovak GNSS permanent stations (SKPOS). Similarly, in \Republic Square" in the Central Business District of Singapore [14], the implementation of a reliable and robust monitoring system of high-rise buildings was studied. This used the Singapore Satellite Positioning Reference Network (SIReNT) and incorporated VRS-RTK methodology. Finally, in order to verify the feasibility of a GPS method for monitoring high-rise building an investigation was carried out to compare acceleration measurement from GPS and accelerometers in a 66-storey high-rise building in Korea [15].

In Hong Kong, China, wind engineering studies on the e ect of typhoons on tall buildings through full-scale monitoring with accelerometers, anemometers and pressure sensors have been carried out [16]. There have been further preliminary studies undertaken about the technical feasibility of GNSS observations such as Kijewski-Correa, T. [17], Li, Xiaojing [18] and Li, Hong-Nan [19].

The authors Yi, T.H., Li, H.N. and Gu, M. [20] present an interesting review of current research and development activities in the eld of high-rise structure health monitoring using the Global Positioning System (GPS).

What makes our system di erent from others is the methods of calculation used, the interpretation and the control of the results, through the original sotware designed speci cally to control high- rise buildings. This system has been operating in the 236 meters high building, Torre Espacio (Figure 1), located on Paseo de la Castellana in Madrid, Spain.

The software of the Real-Time Monitoring System achieves robust results for the instantaneous position of high-rise buildings. A speci c constraint is inbuilt into the mathematical model, which de nes the inner constraints required. For each structure to be monitored we add a unique condition equation to the mathematical model of the Observation Equations. The system can be adapted to other structures by changing the speci c conditioning and the inner constraints of the mathematical model.

2. Methodology

The proposed real time monitoring system is composed of a local geodetic network with four GNSS antennas and receivers positioned on the rooftop of the structure to be monitored, in this case the Torre Espacio high-rise building. In general terms, the system strictly determines the position of a local geodetic network observed with VRS-RTK techniques [21].

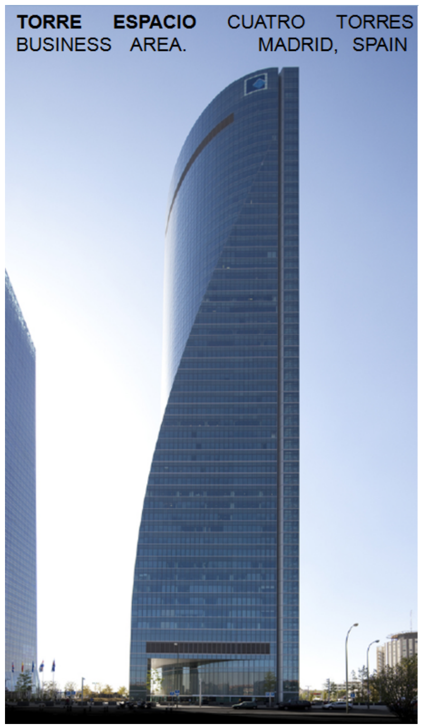

Figure 1: Building Torre Espacio, Madrid

The physical location of the four permanent GNSS antennas is shown in figure 2. In principle it would be enough to utilize a line defined by two GNSS antennas [13] [22] in order to define the torsional and translation response of the rooftop, but the more secure option is constituted by a quadrangle that keeps its shape.

Based on previous studies and the project background, it is essential to have detailed structural information about any structure to be monitored in order to provide a reliable assessment of its dynamic response. In this case, we know from previous reports that the Tower can move around 20 cm in 5 minutes, with wind speeds close to or exceeding 90 km/h, so, in order to detect Torre Espacio's wind-induced response we use kinematic GNSS observation using measurement intervals of 5 seconds. The IGS (International GNSS Service) advises that

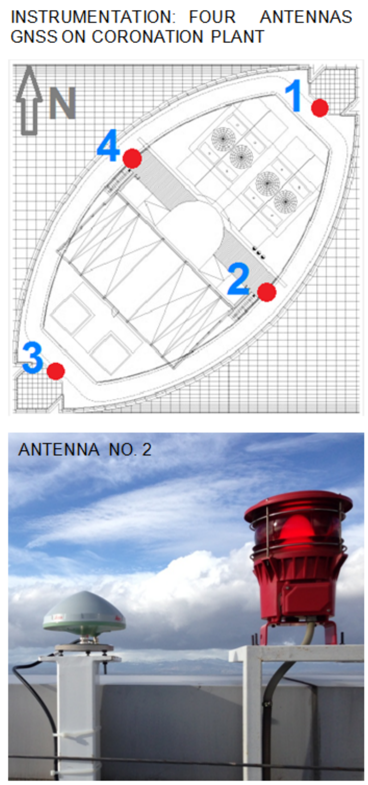

Figure 2: Building Torre Espacio, Madrid. Four GNSS antennas on rooftop

observations should be at 30 second intervals to guarantee independence between them as long as the receiver does not move and remain static, but this interval had not to use in our case. In general, the precision of displacement results of any structure will be directly proportional to the GNSS observation period.

In this way, we can determine the position of the building every 25 seconds, according to client needs, by means of Least Squares Adjustment of 5 GNSS observations, as will be explained later.

In this investigation we distinguish between \observation intervals" and \intervals of determination of the position of the structure" to be monitored. The rst one refers to the time that elapses between one observation and the next, which in this case is 5 seconds.

On the other hand, the intervals of determination of the position of the structure to be monitored, indicate the time that passes between the calculation of the position of the structure adjusted mathematically according to the algorithms developed. This is done with 5 observation intervals, that is every 25 seconds.

Altitude monitoring is not necessary considering that a rooftop oscillation of 25 cm represents an altitude increment of 012 mm, which in this case is insignificant. For this reason the study of displacement dynamics of points 1; 2; 3 and 4 are accepted as two-dimensional.

2.1. Preadjustment data analysis

Prior to adjusting GPS networks, a series of procedures should be followed to analyze the data for internal consistency and to eliminate possible outliers. No control points are needed for these analyses [23]. Depending on the actual observations taken, the network geometry, GNSS system implementation and initial adjustment mathematical model these procedures consist of analyzing (1) the GNSS observation using the Outliers Detection Test and (2) the previous errors by determinating the sensitivity of the network.

2.1.1. Analyzing the GNSS observation

Once the observations using GNSS began, the instantaneous quality data was continuously checked with a filter which we have called the Outliers Detection Test.

The aim of this test was to evaluate the precision of the observation during the 25 seconds that are used to determine the position of the structure to be monitored. This test guarantees that all of the observations that we were processing for the calculation are below the threshold established by the user, therefore our weights (which according to their definition are closely related to accuracy) remain homogeneous while the system is operational.

The outliers detection test allows the user to define an a priori error threshold. When the data does not reach the required threshold set by the users, the system alerts them and they can decide whether to accept or reject the lower accuracy at this particular moment.

Depending on the configuration of the GNSS network implemented in the structure to be monitored, the distances measured by GNSS observation are compared with those same distances that are known in advance. In this case, these distances correspond to the sides and diagonals of the quadrilateral that forms the GNSS network located on the rooftop of Torre Espacio.

The implemented application allows the user to know what precision interval is found in the GNSS observations in this moment.

Park, H.S. et al. [15] used a similar error detection procedure and slated that in general a slab in a building is assumed to be a rigid diaphragm with infinite stiffness. Therefore, the distance between two GPS stations may remain constant during the measurement period. The error distance can be computed through the difference between two GPS stations during the measurement period and the known distance of these stations. Using this distance error we can indirectly evaluate the accuracy of the GPS displacement measurement system.

2.1.2. Analyzing the previous errors

When the GNSS system has been implemented in the structure, and before begining to adjust the GNSS network observation, it is necessary to estimate the previous errors. GPS observations contain errors, such as multipath errors and miscentering errors of the receiver antenna over the ground station and receiver height-measuring errors [23]. It is important to know the errors, so the accuracy of the GNSS implementation and initial adjustment mathematical model can be con rmed. To account for these and other errors together with the methodology it is necessary to analyze the measurements from a period when there was no movement of the building.

In this way we can de ne and understand the sensitivity of the network through an accuracy threshold; below this threshold gure we cannot be certain about the building deformation indicated by the network data.

In order to do this, it is recommended that continuous GNSS observation of the structure take place over an extended observation period of stability, in the case of tall building this stable period needs to be a time of no appreciable wind. Considering what we know already the displacement of the structure will be zero.

Based on the calibration of one point made by Hofmann-Wellenhof [21], which consists of measuring a zero-length baseline with a single antenna in a normal 60-minute session, in this research, we have extended it to all points of the GNSS network, denominating sensitivity of the network.

The statistical procedure to determine the sensitivity of the network, which is de ned by a vector, consists of several complementary methods. In short, by using the Baarda test to obtained displacements from the external reliability [24] it is possible to determine the theoretical sensitivity vector.

First step: calculating the external reliability

The in uence of each of the marginally detectable errors on the parameters of the adjustment, or on functions of the parameters, is called external reliability. This is used in order to quantify marginally detectable blunders and to determine their potential in uence on the estimated parameters [25]. The e ects of undetectable blunders on the coordinates is evaluated by the below equation:

$$r_{di} = (N)^{1} A^T P e_i r_i = (A^T P A)^{1} A^T P e_i r_i$$

where
A is the design matrix
P is the diagonal weight matrix
r_i is the marginally detectable blunder
nally, r_{di} is the e ect of the blunder r_i in observation d_i the number of observations is n

The length of vector r_d will be $\|r_d\| = \sqrt{r_{d1}^2 + r_{d2}^2 + + r_{di}^2 + r_{dn}^2}$

Second step: calculating the real deformation vector d in order to compare it with the external reliability values r_d.

It is obtained from the system:

$$A\ d \quad L_d = v_d \tag{1}$$

And the solution of the system is

$$d = (A^T A)^{-1} L_d \qquad (2)$$

where
A is the design matrix
$L_d = w = L_{x1} - L_{x2}$
v_d are the residuals

The length of vector d will be $\|d\| = \sqrt{d_1^2 + d_2^2 + \cdots + d_i^2 + \cdots + d_n^2}$
The length of $\|r_d\|$ should be similar to the length of $\|d\|$, and also between the mean value $\frac{\|r_d\|}{\sqrt{n}}$ and $\frac{\|d\|}{\sqrt{n}}$, if so, then these values are an objective verification using least-squares simulation.

This provides the d vector. Both the theoretical and practical values calculated have to be very similar and always, for maximum security, we adopt the highest value which defines the sensitivity of the network.

2.2. Adjustment GNSS network

After making the previous analyses, including the outliers detection test and determining the sensitivity value of the network, we can continue with the adjustment of the GNSS network. In this research we considered the deformation experienced by the network in the monitored structure d to be a vector which is a random variable and has a multivariate normal distribution.

This deformation vector d will be the result of the differences in the position of the measuring points of the network in different GNSS measurement epochs:

$$d = x_2 - x_1 \qquad (3)$$

being
$d = x_2 - x_1$ = deformation experienced by the network
x_1 = coordinates UTM-ETRS89 at epoch 1
x_2 = coordinates UTM-ETRS89 at epoch 2

Based on the method of Sequential Solution [25] for the particular case of the Observation Equation Model, and assuming that observations are made in two groups, with the second group consisting of the addition of the function of variables or parameters. Both groups have a common set of parameters. The two mixed adjustment models can be written as:

$$f_1(l_{1a}; x_a) = 0$$
$$f_2(l_{2a}; x_a) = 0 \qquad (4)$$

Both sets of observations should be uncorrelated, and the *a priori* variance of unit weight should be the same for both groups, so:

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} = \sigma_0^2 \begin{bmatrix} \Sigma_1^{-1} & 0 \\ 0 & \Sigma_2^{-1} \end{bmatrix} \quad (5)$$

The number of observations in l_{1a} and l_{2a} are n_1 and n_2, respectively; and r_1 and r_2 are the number of equations in the models f_1 and f_2, respectively. The linearization of (4) yields

$$\begin{aligned} B_1 v_1 + A_1 x + W_1 &= 0 \\ B_2 v_2 + A_2 x + W_2 &= 0 \end{aligned} \quad (6)$$

where

$$\begin{aligned} B_1 &= \left.\frac{\partial f_1}{\partial l_1}\right|_{l_{1b};x_0} & A_1 &= \left.\frac{\partial f_1}{\partial x}\right|_{l_{1b};x_0} & W_1 &= f_1(l_{1b}; x_0) \\ B_2 &= \left.\frac{\partial f_2}{\partial l_2}\right|_{l_{2b};x_0} & A_2 &= \left.\frac{\partial f_2}{\partial x}\right|_{l_{2b};x_0} & W_2 &= f_2(l_{2b}; x_0) \end{aligned} \quad (7)$$

and the subscript b is to be read as "observed" and the subscript a is to be read as "adjusted".

A solution is obtained by introducing vectors of Lagrange multipliers, k_1 and k_2, and minimizing the function:

$$\phi(v_1; v_2; k_1; k_2; x) = v_1^T P_1 v_1 + v_2^T P_2 v_2 - 2k_1^T (B_1 v_1 + A_1 x + W_1) - 2k_2^T (B_2 v_2 + A_2 x + W_2) \quad (8)$$

The solution is obtained by setting the partial derivatives of (8) to zero,

$$\frac{1}{2}\frac{\partial \phi}{\partial v_1} = P_1 v_1 - B_1^T k_1 = 0 \quad (9)$$

$$\frac{1}{2}\frac{\partial \phi}{\partial v_2} = P_2 v_2 - B_2^T k_2 = 0 \quad (10)$$

$$\frac{1}{2}\frac{\partial \phi}{\partial x} = -A_1^T k_1 - A_2^T k_2 = 0 \quad (11)$$

$$\frac{1}{2}\frac{\partial \phi}{\partial k_1} = B_1 v_1 + A_1 x + W_1 = 0 \quad (12)$$

$$\frac{1}{2}\frac{\partial \phi}{\partial k_2} = B_2 v_2 + A_2 x + W_2 = 0 \quad (13)$$

and solving for v_1, v_2, k_1, k_2, and x.[25]

The normal equations can be written as

$$\begin{aligned} A_1^T M_1^{-1} A_1 x + A_1^T M_1^{-1} W_1 - A_2^T k_2 &= 0 \\ -M_2 k_2 + A_2 x + W_2 &= 0 \end{aligned} \quad (14)$$

By using

$$M_1 = B_1 P_1{}^{1} B_1^T \quad M_2 = B_2 P_2{}^{1} B_2^T \tag{15}$$

We can establish a overall mathematical model adjustment for which the expression, in matrix form, is

$$\begin{array}{cc} A_1{}^T M_1{}^{1} A_1 & A_2^T \\ A_2 & M_2 \end{array} \quad \begin{array}{c} x \\ k_2 \end{array} = \begin{array}{c} A_1^T M_1{}^{1} W_1 \\ W_2 \end{array} \tag{16}$$

Equation (16) shows how the normal matrix of the first group must be augmented in order to find the solution of both groups. The whole matrix can be inverted in one step to give the solution for x and k_2.

For the Observation Equation Model, there is an explicit relationship between the observations and the parameters, such as

$$\begin{array}{c} l_{1a} = f_1(x_a) \\ l_{2a} = f_2(x_a) \end{array} \tag{17}$$

Using model specification

$$\begin{array}{c} B \quad l \\ l \quad W = f(x_0) \quad l_b = l_0 \quad l_b \end{array} \tag{18}$$

The symbol l_0 equals the value of the observations as computed from the approximate parameters x_0. It is customary to denote the discrepancy by l instead of W when dealing with the observation equation model.

The algorithms developed can be used to incorporate exterior information about parameters. So in assuming the addition of the function of variables or parameters, the observation equation model is

$$\begin{array}{c} l_{1a} = f(x_a) \\ g(x_a) = 0 \end{array} \tag{19}$$

with

$$\begin{array}{c} v_1 = A_1 x + l_1 \\ A_2 x + l_2 = 0 \end{array} \tag{20}$$

Similar to the previous reasoning, for the Observation Equation Model and in assuming the addition of the function of variables or parameters, an adjustment overall mathematical model, in matrix form, is

$$\begin{array}{cc} A_1{}^T P_1 A_1 & A_2^T \\ A_2 & 0 \end{array} \quad \begin{array}{c} x \\ K_2 \end{array} = \begin{array}{c} A_1^T P_1 L_1 \\ L_2 \end{array} \tag{21}$$

The deformation d is a random variable so it can be analyzed statistically. Therefore, the maximum error in the determination of the position of the points of the control network can be determined and evaluated.

So, in particular for the deformation d of the structure that we have to monitor, we obtain this expression:

$$\begin{matrix} A_{d1}{}^{T} P_1 A_{d1} & A_{d2}^{T} \\ A_{d2} & 0 \end{matrix} \quad \begin{matrix} d \\ K_2 \end{matrix} = \begin{matrix} A_{d1}^{T} P_1 L_{d1} \\ L_{d2} \end{matrix} \tag{22}$$

being
A_{d1} = design matrix
$P_1 \quad I$ = weigth matrix (all the weights are homogeneous and have been checked in the test)
A_{d2} = matrix used to incorporate exterior information about parameters
$d = x_2 \quad x_1$ = deformation experienced by the network
K_2 = Lagrange multiplier

From the preceding expression, we know the displacement of the four points on the rooftop according to the unknown vector d from the result of the overall mathematical model, which is formed by [25]:

Mathematical model-initial adjustment, that de nes the basic geometric observational network.

Through the application of least squares to the problem of adjusting baselines in GPS networks, observation equations are written that relate the station coordinates to the di erent coordinates that have been observed and their residual errors [23].

Assume that station i is a reference point (control point) and that station j is a point of unknown position. For line ij, an observation equation can be written for each baseline (vector distances between stations) component observed as

$$\begin{aligned} X_{a_j} &= X_{a_i} + 4\, X_{b_{ij}} + {}_{X\,ij} \\ Y_{a_j} &= Y_{a_i} + 4\, Y_{b_{ij}} + {}_{Y\,ij} \end{aligned} \tag{23}$$

Note that the subscript b is to be read as \observed" and the subscript a is to be read as \adjusted".

These observation equations can be written, in an alternative way, that relate the approximate values of the parameters and their parameter corrections.

$$\begin{aligned} X_{a_j} &= X_{0_j} + x_j \\ Y_{a_j} &= Y_{0_j} + y_j \end{aligned} \tag{24}$$

$$\begin{aligned} X_{a_i} &= X_{0_i} \\ Y_{a_i} &= Y_{0_i} \end{aligned} \tag{25}$$

as noted earlier, station i is a reference point, so $x_i = 0$ and $y_i = 0$ Substituting (24), (25) into (23) yields

$$\begin{aligned} X_{0_j} + x_j &= X_{0_i} + 4\,X_{b_{ij}} + \ _{Xij} \\ Y_{0_j} + y_j &= Y_{0_i} + 4\,Y_{b_{ij}} + \ _{Y\,ij} \end{aligned} \qquad (26)$$

The symbols X_b and X_0 denote the observed parameters and approximate parameters.

$$\begin{aligned} (X_{0_j} \quad X_{0_i}) + x_j &= 4\,X_{b_{ij}} + \ _{Xij} \\ (Y_{0_j} \quad Y_{0_i}) + y_j &= 4\,Y_{b_{ij}} + \ _{Y\,ij} \end{aligned} \qquad (27)$$

Considering that $(X_{0_j} \quad X_{0_i})$ and $(Y_{0_j} \quad Y_{0_i})$ are the approximate coordinate di erences, it follows that

$$\begin{aligned} x_j &= 4\,X_{b_{ij}} \quad (X_{0_j} \quad X_{0_i}) + \ _{Xij} \\ y_j &= 4\,Y_{b_{ij}} \quad (Y_{0_j} \quad Y_{0_i}) + \ _{Y\,ij} \end{aligned} \qquad (28)$$

$$\begin{aligned} x_j &= 4\,X_{b_{ij}} \quad 4\ X_{0_{ij}} + \ _{Xij} \\ y_j &= 4\,Y_{b_{ij}} \quad 4\ Y_{0_{ij}} + \ _{Y\,ij} \end{aligned} \qquad (29)$$

Observation equations of the foregoing paragraph would be written for all GNSS baselines observed in any gure.

Due to the de nition of deformation in this research, $d = x_2 \quad x_1$, the observation equation model have to be di erence between two di erent GNSS measurement epochs. It follows that:

$$\begin{aligned} x_{j\,1} &= 4\,X_{b_{ij}\,1} \quad 4\ X_{0_{ij}\,1} + \ _{Xij\ 1} \\ y_{j\,1} &= 4\,Y_{b_{ij}\,1} \quad 4\ Y_{0_{ij}\,1} + \ _{Y\,ij\ 1} \end{aligned} \qquad (30)$$

$$\begin{aligned} x_{j\,2} &= 4\,X_{b_{ij}\,2} \quad 4\ X_{0_{ij}\,2} + \ _{Xij\ 2} \\ y_{j\,2} &= 4\,Y_{b_{ij}\,2} \quad 4\ Y_{0_{ij}\,2} + \ _{Y\,ij\ 2} \end{aligned} \qquad (31)$$

Assuming that the approximate parameters, X_0, obtained by the equation observed model are the same along the GNSS measurement epochs.

$$\begin{aligned} x_{j\,2} \quad x_{j\,1} &= 4\,X_{b_{ij}\,2} \quad 4\ X_{b_{ij}\,1} + \ _{Xij\ 2} \quad \ _{Xij\ 1} \\ y_{j\,2} \quad y_{j\,1} &= 4\,Y_{b_{ij}\,2} \quad 4\ Y_{b_{ij}\,1} + \ _{Y\,ij\ 2} \quad \ _{Y\,ij\ 1} \end{aligned} \qquad (32)$$

There is one equation for each observation and, in this case, the number utilized is 5 GNNS observations in order to determine the adjustment position of the structure.

The observation equations can be expressed in matrix form as:

$$A \;\; x = L + V \tag{33}$$

It follows that in deformation term,

$$\begin{aligned} d_{xj} &= x_{j\,2} \quad x_{j\,1} = 4\,X_{b_{ij}\,2} \;\; 4 \;\; X_{b_{ij}\,1} + \;\; _{X ij\;\;2} \quad\; _{X ij\;\;1} \\ d_{yj} &= y_{j\,2} \quad y_{j\,1} = 4\,Y_{b_{ij}\,2} \;\; 4 \;\; Y_{b_{ij}\,1} + \;\; _{Y ij\;2} \quad\; _{Y ij\;1} \end{aligned} \tag{34}$$

$$A_{d1} \;\; d_{xj} = L_{d1} + V_{d1} \tag{35}$$

The numerical values of the elements of the L matrix are determined by rearranging the observation equations. Those elements are calculated as follows:

$$\begin{aligned} L_X &= 4\,X_{b_{ij}\,2} \;\; 4 \;\; X_{b_{ij}\,1} \\ L_Y &= 4\,Y_{b_{ij}\,2} \;\; 4 \;\; Y_{b_{ij}\,1} \end{aligned} \tag{36}$$

Note that the observation equations for GPS network adjustment are linear and that the only non-zero elements of the matrix A_{d1} are either 1 and 1.

The matrix A_{d1} has been created and is derived from the GNSS observation network, taken every 5 seconds. As noted earlier, in order to process the adjustment of the network we consider 5 observation equations, in this way we can determine the adjustment position of the structure every 25 seconds (Figure 3).

We recommend that users always have to de ne observational data as an Observation equations Model between reference points (in this case we have used the IGNE belonging to a permanent GNSS station network located 4.17 km from Torre Espacio) and non- xed points (1; 2; 3; and 4).

Another mathematical model that de nes the speci c conditions of the network located in the monitored structure. In this case, the second observations group of the mathematical model consisting the additional of function of variables or parameters that de nes the speci c conditions of the network located in the monitored structure will be developed for the new structure to be monitored. The linear forms that de ne the inner constrains of this mathematical model allow us to form the design matrix A_{d2}.

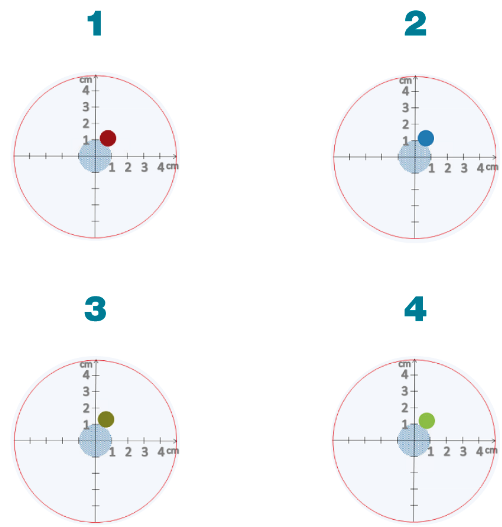

Figure 3: Real-Time Monitoring System: every 25 seconds it is determined the four antennas displacement

With the aim of giving more rigour to the results, we added an enforceable condition suitable for this speci c application case.

In this case, we know that the deformation of the quadrilateral which is de ned by the non- xed points of the network situated on the rooftop of Torre Espacio is null. Therefore it is possible to use ve equations from this quadrilateral to de ne the inner constraints of the mathematical model.

So the vector d calculated comes from the initial adjustment mathematical model and complies with the inner constraints which are laid down by the design matrix A_{d2}. This re ectes the physical reality of the movement of the structure to be monitored with con dence since at all times the imposed conditions are met.

Once the system has determined the position of the non- xed points we have to evaluate the con dence regions of calculated displacement d of the GNSS antennas. For that purpose, from the maximum value of the diagonal $_{dd}$ of the a posteriori matrix variance covariance of the variables $_{dd} = {}^{2}_{0}Q_{dd}$ [25], we can set an error surface for each GNSS antenna (extendable at any point of the rigid surface of the rooftop), which in this case will be a circle. As the columns move in both abscissa and ordinate, the composition of both displacements constitutes the radius of the enclosure error r:

$$r = \sqrt{\sigma_{dx}^2 + \sigma_{dy}^2} = \sqrt{\sigma_{ddi}^2 + \sigma_{ddi}^2} = \sqrt{2\sigma_{ddi}^2} = \sqrt{2}\,\sigma_{ddi} \tag{37}$$

Consequently, when these confidence regions has been determined, we can apply standard multivariate analysis to obtain the simultaneous reliability of the confidence regions of the 8 variables (4 for x and 4 for y), so that can obtain the instantaneous and simultaneous precision of each antenna at any given time. Understanding by precision: the error surface and simultaneously and the individual reliability of each GNSS receiver

For any group of normally distributed measurements, the probability of one normal random variable can be computed by analyzing the integration of the distribution function[23]:

$$N(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \tag{38}$$

Being the case of multidimensional normal random variable:

$$P(z) = 2^{n}[N(0;1)^{k}]^{n} \tag{39}$$

Then we can establish this figure with a maximum reliability of 92.27% simultaneous in all the vertices of the quadrilateral we will have to multiply by $k = 2.5$.

2.3. Designing an intergraph's software

All the processes and algorithms described above are implemented in the original application we have designed, an intergraph's software for Real-Time high-rise building monitoring system by GNSS technologies.

As shown in the following figures, the application every 25 seconds calculates the displacement, with its associated maximum error circle, of each of the GNSS antennas of the quadrilateral formed on the rooftop of the Tower, as a result of a Gaussian adjustment.

As we can see Figure 4 that shows the position and the instantaneous error surface antenna 1 network located on the roof of the Torre Espacio has a reliability of 0.99.

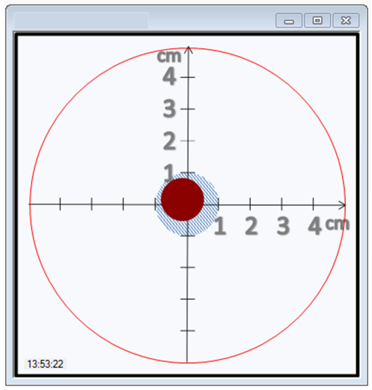

Figure 4: The red circle is the instant error surface of the antenna number 1, with 0 :99 reliability

Figure 5 shows the graph of the dynamics of movement of antenna 1 of the Torre Espacio in the last 5 minutes recorded.

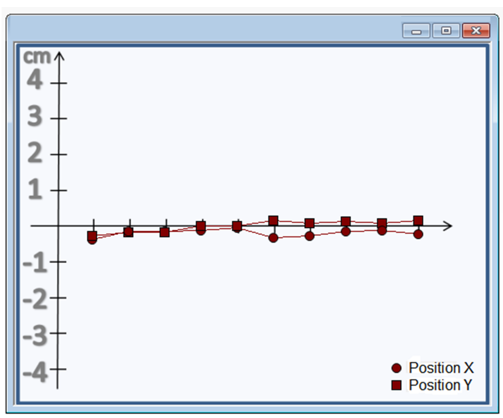

Figure 5: Antenna number 1: movements of the last ve minutes. The red circle is the displacement x, red square is the displacement y

Also monitoring and remote monitoring of the proposed system continuously keeps the customer informed of the status of the structure in real time with high accuracy and reliability and sends automatic alerts about potential danger.

In the case of Torre Espacio, the Real-Time Monitoring System sends automatic alerts when the displacement is greater than 5 cm, as is shown in the gure 6 with a red circumference. This threshold is de ned by the user.

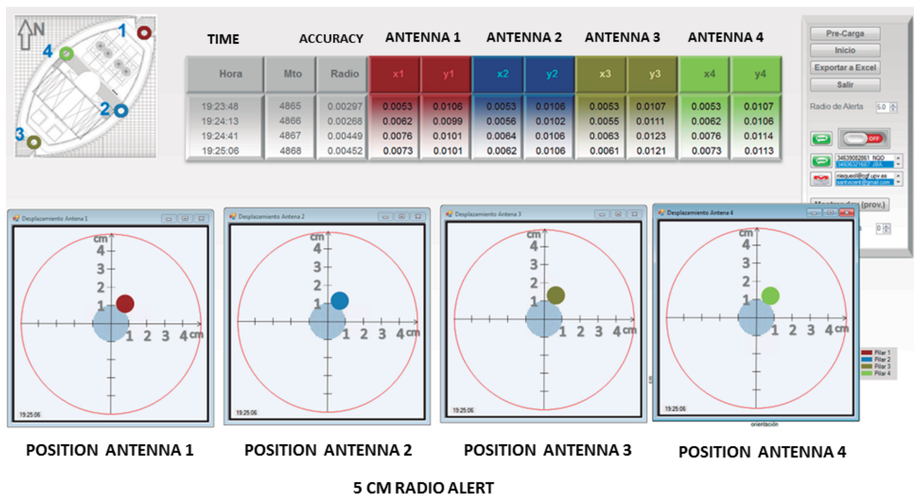

Figure 6: Software Graphical User Interface: The red, blue and green circles represent the position of the antennas 1, 2, 3 and 4.

This Real-Time Monitoring System could be developed for installation in another type of structure [26] [27] (for example a bridge or and o shore platform). In this case we will develop the conditioned mathematical model that characterizes the new structure to be monitored and should allow the system to reduce the position errors.

These algorithms must be transferred to the structure you want to control so that it will be di erent and unique for each structure. Then it is possible to say that the Real-Time Monitoring System we have developed is a system that can be adapted to provide an exclusive "tailor-made" monitoring system for each speci c structure.

Also the Real-Time Monitoring System that we have developed incorporates a system which allows detection if one or more of the GNSS receivers is abnormal, warns the user and dispenses with the receiver in the calculation of the adjusted position of the network.

The developed software has an interesting innovation, which is able to calculate displacement of the building monitored every 5 seconds instead of every 25 seconds, if the speed of displacement of the building is greater than:21 mm=second

Intergraph's software can also be installed on any mobile because we have developed our own compatible app which allows us to access information about the same control parameters calculated on a mobile phone, as illustrated in the following gures (7,8 and 9).

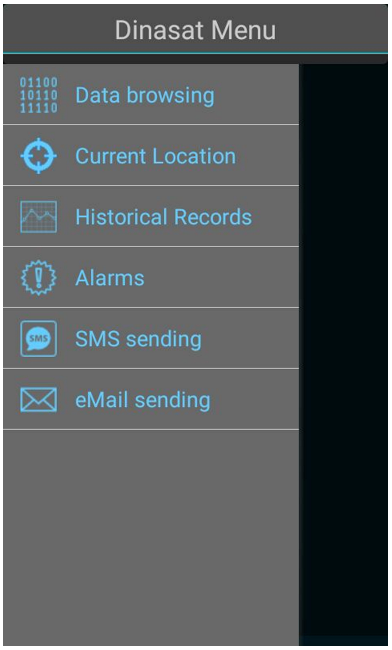

Figure 7: APP Menu: review data, current position, position graphics, alarms, send an alarm via SMS or email.

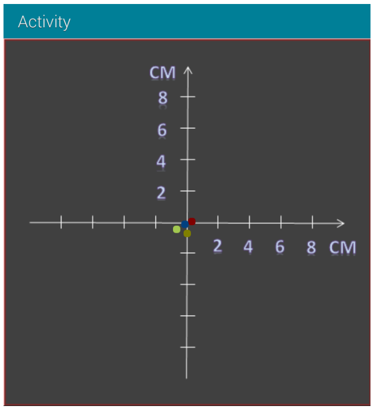

Figure 8: APP screen: instantaneous positioning of four GNSS antennas.

Figure 9: APP screen: Graphics of last minutes of movements antenna number 1. Blue line is the displacement x, red line is the displacement y.

3. Discussion

Global Navigation Satellite Systems GNSS have revolutionized the science of positioning and Earth measurement. One part of that revolution is accuracy, another part is speed and simplicity [28].

In this paper we introduced a method of monitoring horizontal displacement using di erential techniques GNSS. We focused on one major advantage of GNSS: accuracy. The inherent accuracy of a GNSS receiver can be enhanced or degraded. The range could be 2 mm to 3 cm GNSS coordinates in real time with an accuracy that can vary between 0.2 and 3cm in a short time. In some cases the requirements of the project require superior accuracy.

Torre Espacio is a high-rise building in Madrid. Two years ago we installed four GNSS receivers on Torre Espacio to measure and analyze the building's response to wind speed. For data GNSS receivers specially developed software was used in order obtain two-dimensional displacements every 25 seconds, according to OHL's R& D need.

In this investigation a preadjustment data analysis has been takend. In order to analyze the GNSS observation we have designed the outliers detection test. This test has helped us to verify that the strategic position of the GNSS antennas was optimal, since the GNSS observations horizon in Torre Espacio is very clear so the geometric dilution of precision (GDOP), which is a composite measure re ecting the geometry of the position and the time estimate of the observation [25], is approppriate. Sometimes there was abnormal disturbance