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Research Article

Computing the Mean-Variance-Sustainability Nondominated Surface by ev-MOGA

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Despite the widespread use of the classical bicriteria Markowitz mean-variance framework, a broad consensus is emerging on the need to include more criteria for complex portfolio selection problems. Sustainable investing, also called socially responsible investment, is becoming a mainstream investment practice. In recent years, some scholars have attempted to include sustainability as a third criterion to better reflect the individual preferences of those ethical or green investors who are willing to combine strong financial performance with social benefits. For this purpose, new computational methods for optimizing this complex multiobjective problem are needed. Multiobjective evolutionary algorithms (MOEAs) have been recently used for portfolio selection, thus extending the mean-variance methodology to obtain a mean-variance-sustainability nondominated surface. In this paper, we apply a recent multiobjective genetic algorithm based on the concept of ε-dominance called ev-MOGA. This algorithm tries to ensure convergence towards the Pareto set in a smart distributed manner with limited memory resources. It also adjusts the limits of the Pareto front dynamically and prevents solutions belonging to the ends of the front from being lost. Moreover, the individual preferences of socially responsible investors could be visualised using a novel tool, known as level diagrams, which helps investors better understand the range of values attainable and the tradeoff between return, risk, and sustainability.

1. Introduction

Financial markets are a clear example of complexity in action [1–3]. As stated by Brunnermeier and Oehmke [4], complexity is a relevant concept in finance and, in particular, when building an optimization model for portfolio selection. Some authors have been recently concerned about adapting and extending the classical bicriteria Markowitz's mean-variance [5] methodology to integrate additional linear criteria such as dividends, liquidity, or sustainability for a suitable portfolio investor [6–11]. In these pioneering works, the above researchers propose exact optimization techniques to solve complex portfolio selection problems although they cannot deal efficiently with nonlinear objectives.

These works estimate risk, return, and additional criteria using historical data which are inevitably subject to estimation error. According to Nathaphan and Chunhachinda [12], three groups of studies can be identified dealing with this problem. The first group of contributions is based on historical data and ignores the estimation error, the second group studies estimation risk and proposes a Bayesian resample efficient frontier approach, and finally, the third group focuses on the asset pricing approach by incorporating a factor model such as the Capital Asset Pricing Model or Arbitrage Pricing Theory. One potential way to solve this problem is to use a resampling approach [13, 14].

Within the methodological framework in complex portfolio selection problems, researchers have started to propose advanced computational techniques such as multiobjective evolutionary algorithms (MOEAs) and multiobjective genetic algorithms (MOGAs) to handle two or more conflicting goals subject to several constraints [15]. A literature review of recent contributions of MOEAs and MOGAs for portfolio management is conducted in [16] where the authors highlight the sharp increase in the number of contributions focused on MOEAs and MOGAs compared with the moderate growth of specific applications for portfolio management, thus revealing that this area of research is still in its early stages. Moreover, this study shows that the majority of scholars applied MOEAs and MOGAs to portfolio selection only in the case of two objectives and only ten percent of the contributions dealt with three objectives. The portfolio expected returns to measure profitability and variance as a risk measure were the most common objectives among authors (see, for example, [17–25]). However, other objectives such as VaR, annual dividends expected shortfall, skewness, or social responsibility have appeared, albeit to a lesser extent [26–29]. Regarding the number of constraints, most models basically make use of two constraints in the problem formulation, namely, cardinality constraints and lower and upper bounds, and transaction round was the most popular.

There are a few academic studies on the application of MOEAs and MOGAs to tackle tricriterion portfolio selection. One of these studies is reference [30], and this has provided a multi-integer multiobjective optimization approach comparing a nondominated sorting genetic algorithm II (NSGA-II), a Pareto envelope-based selection algorithm (PESA), and a strength Pareto evolutionary algorithm II (SPEA2) to find the best possible tradeoffs between profitability, risk, and cardinality of the portfolio. Nonetheless, the purpose of our paper is to propose a MOGAs approach for a tricriterion mean-variance portfolio selection problem as a triobjective optimization problem whose third criterion is sustainability.

In a context of global awareness about climate change and sustainable growth, ethical investing is making inroads into the financial community. In recent years, sustainable investing, which is an approach that considers environmental, social, and governance (ESG) factors in portfolio selection and management, has become a mainstream investment practice. Professional investors, financial institutions, and the research community are working together to propose new quantitative methods to quantify the impact of including sustainability concerns in standard financial analysis. According to the Global Sustainable Investment Review (GSIA [31]), sustainable investments, or socially responsible investments (SRI), have risen to an average of 25% from 2014 to 2016 and have increased to 61% over the previous two-year period. It is remarkable that more than half of total professionally managed assets in Europe use some SRI strategies. For papers that widen the traditional risk-return tradeoff to integrate sustainability criteria in the portfolio selection formulation through different multicriteria decision-making (MCDM) approaches, we can cite a string of contributions such as references [6, 32–38].

This research aims to provide a MOGAs approach to obtain a mean-variance-sustainability nondominated surface. In particular, we apply an elitist multiobjective evolutionary algorithm based on the concept of ε-dominance called ev-MOGA. This algorithm tries to ensure convergence towards the Pareto set in a smart distributed manner along the Pareto front with limited memory resources. It also adjusts the limits of the Pareto front dynamically and prevents the solutions belonging to the ends of the front from being lost. Once the Pareto front and the Pareto set have been obtained, the individual preferences of socially responsible investors could be considered using a novel tool known as level diagrams. The level diagram of a Pareto front is a collection of 2D graphical representations synchronized by the y-axis and expressing the x-axis in the units of the objective. This helps the investor better understand the range of values attainable and the tradeoff between the different solutions in physical units. A second important characteristic is that the y-axis synchronizes the different plots and provides a way to show a particular property of each possible portfolio.

Although we rely on historical data to estimate risk and return, our proposal can be applied to the three groups of studies dealing with the estimation error problem described in Nathaphan and Chunhachinda [12].

The paper is organized as follows. In Section 2, we review the problem of tricriterion portfolio selection, including sustainability as a third objective. In Section 3, we formulate the ev-MOGA algorithm to analytically derive the mean-variance-sustainability nondominated surface. The use of the level diagrams tool to include particular preferences of the decision maker and represent in 2D the Pareto front and the Pareto set is explained in Section 4 with an illustrative six-stock example. A real-world empirical application is presented in Section 5 using data from Morningstar open-end funds for the period 2009–2019. The paper closes with conclusions and further research proposals.

2. Tricriterion Multiobjective Portfolio Selection from a Sustainable-Financial Perspective

The standard bicriterion portfolio selection problem assumes that investors are only concerned about achieving a certain level of profitability for specific levels of risk. Since the early 1970s, several authors have attempted to include an additional criterion beyond the expected return and variance when constructing a portfolio [39, 40], but it was not until the 2000s that the idea of additional objectives was further boosted from the methodological framework.

In [41], liquidity is introduced as a third criterion into the standard mean-variance portfolio optimization model. By defining several measures of liquidity, the authors constructed a three-dimensional mean-variance-liquidity frontier.

A triobjective optimization problem is proposed in [30] to find the tradeoff between risk, return, and the number of securities in the portfolio. The authors apply and compare three evolutionary multiobjective optimization techniques, namely, the nondominated sorting genetic algorithm II (NSGA-II); the Pareto envelope-based selection algorithm (PESA); and the strength Pareto multievolutionary algorithm 2 (SPEA2) to find the best tradeoff between risk, return, and cardinality of the portfolio.

In [6], a general framework for computing the nondominated surface in a tricriterion portfolio selection that extends the Markowitz portfolio selection approach to an additional linear criterion (dividends, liquidity, or sustainability) is addressed. By solving a quad-lin-lin program, they provide an exact method for computing the nondominated surface that can outperform standard portfolio strategies for multicriteria decision makers. An empirical application where the third criterion is sustainability is developed to illustrate how to compose the nondominated surface.

In [7], sustainability is included as the third criterion to obtain the variance-expected return-sustainability efficient frontier to explain how the sustainable mutual fund industry can increase levels of sustainability. The tricriterion nondominated surface is computed through the quadratic constrained linear program (QCLP) approach, and from the experimental results, it can be concluded that there was room to expand the sustainability levels without hampering the levels of risk and return.

After reviewing the most significant contributions dealing with tricriterion portfolio selection, we have to take into account that to select portfolios from a purely financial perspective generally requires a two-stage process. The first stage is to define the opportunity set and narrow down the larger pool to a more workable number of securities. The second stage is asset allocation, which is to decide how to distribute the wealth of an investor between the different asset classes. As our scenario consists in making investment decisions from a sustainable-financial perspective, the difference between the way in which assets are managed from purely financial criteria and the way a sustainable portfolio is managed only takes place in the first stage [8]. The stages in this process are as follows.

2.1. First Stage. To define the opportunity set from a sustainable point of view, two types of screening techniques are used [42], thus obtaining an approved list of securities. To this end, one of the earliest methods used by socially responsible investors is negative screening (NS), in which the investors establish a kind of "red line" to rule out companies that do not develop sustainable strategies, and deal in, for example, controversial weapons, tobacco, gambling, pornography, nuclear energy, or animal testing. The other is positive screening (PS), in which investors select companies that set positive examples of environmentally friendly products and socially responsible business practices, for instance, renewable energy and sustainable transport companies.

2.2. Second Stage. The decision maker defines the asset allocation of portfolio from the approved list in the first stage. Thus, the investment decisions are made in terms of profitability and risk, but there is no evidence proving that sustainability is taken into account in the asset allocation stage.

In this second stage, a tricriterion portfolio selection problem including sustainability could be mathematically formulated as follows:

$$\min f_1 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}, \quad (1)$$

$$\max f_2 = \sum_{i=1}^{N} w_i \mu_i, \quad (2)$$

$$\max f_3 = \sum_{i=1}^{N} w_i s_i, \quad (3)$$

$$\text{subject to} \sum_{i=1}^{N} w_i = 1, \quad (4)$$

$$w_{i_{\min}} \leq w_i \leq w_{i_{\max}}, \quad (5)$$

where N is the number of available securities, μ_i is the expected return on security i ($i = 1, 2, \ldots, N$), σ_{ij} is the covariance between security i and j. In addition, s_i is the portfolio sustainability score, and w_i is the investment proportion. Constraint (4) is called the budget constraint and implies that 100% of the budget will be invested in the portfolio. Furthermore, a minimum and maximum investment rule is considered in the constraint (5).

The above stated model could be solved by obtaining a Pareto optimal front that represents the best tradeoffs between mean return, variance, and sustainability. When including a third criterion, the nondominated frontier in the two-dimensional space becomes a nondominated surface in a three-dimensional space. In [6], an exact method for computing the nondominated surface in a tricriterion portfolio selection problem was proposed to extend the Markowitz approach to an additional linear criterion. The previous method for computing the nondominated set when the third criterion was sustainability is applied in [7]. The authors used the CIOS (custom investment objective solver) code to derive the nondominated surface from quad-lin-lin programs, which is composed of a connected collection of parabolic segments called "platelets". A quadratic constrained ε-constraint linear program to derive a nondominated surface is proposed in [8] to prove that investors could increase the sustainability levels of their portfolios without undermining risk or return.

Hereafter, we propose a more-integrated second-stage approach to approximate the Pareto front by applying an elitist multiobjective evolutionary algorithm based on the concept of ε-dominance called ev-MOGA [43]. Existing multiobjective techniques for extending mean-variance portfolio selection problems have limited capabilities if the new objectives are nonlinear. The proposed ev-MOGA algorithm adjusts the Pareto front dynamically, ensuring convergence and uniform distribution of solutions with no conditions related to the type of the objective function (quadratic, linear, or nonlinear).

3. Deriving the Nondominated Mean-Variance-Sustainability Surface

Generally speaking, multiobjective programming (MOP) approaches face the simultaneous optimization of multiple objective functions subject to a set of constraints. As no single solution can achieve all the objectives, in most real problems, MOP tries to find the Pareto efficient solution or the Pareto front. Hence, a set of solutions is called Pareto efficient (or nondominated or noninferior), when no other feasible solution can achieve the same or better performance for all the objectives and it is strictly better for at least one criterion.

According to Miettinen [44], the mathematical model of a multiobjective problem with n objectives can be formulated as follows:

$$\min \mathbf{F}(\theta) = \min\left[F_1(\mathbf{X}), F_2(\mathbf{X}), \ldots, F_n(\mathbf{X})\right], \quad (6)$$

$$\begin{aligned} &\text{s.t.:} \\ &g_q(\mathbf{X}) \le 0, \qquad (1 \le q \le r), \\ &h_k(\mathbf{X}) = 0, \qquad (1 \le k \le m), \\ &x_{li} \le x_i \le x_{ui}, \qquad (1 \le i \le L), \end{aligned} \quad (7)$$

where $F_j(\mathbf{X})$ $j = 1, \ldots, n$ are the n objectives, $g_q(\mathbf{X})$ and $h_k(\mathbf{X})$ are the r inequality and m equality problem constraints, respectively, x_{li} and x_{ui} are the lower and upper constraints which define the solution space, and $\mathbf{X} = (x_1, \ldots, x_p)^T$ are the independent variables. Constraint (7) defines a set called searching space D.

The tricriterion multiobjective portfolio model described by equations (1)–(5) can be recast in the form of the MOP, taking $\mathbf{X} = (w_1, \ldots, w_n)$ as the portfolio weights and the number of objectives $n = 3$, $p = L = N$, $r = 0$, and $m = 1$. Equations (6) and (7) then become

$$F_1(\mathbf{X}) = f_1 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij}, \quad (8)$$

$$F_2(\mathbf{X}) = -f_2 = -\sum_{i=1}^{N} w_i \mu_i, \quad (9)$$

$$F_3(\mathbf{X}) = -f_3 = -\sum_{i=1}^{N} w_i s_i, \quad (10)$$

$$h_1(\mathbf{X}) = \sum_{i=1}^{N} w_i - 1 = 0, \quad (11)$$

$$\begin{aligned} x_{li} &= w_{i_{\min}}, \\ x_{ui} &= w_{i_{\max}}. \end{aligned} \quad (12)$$

According to [16], evolutionary algorithms provide a suitable framework for solving multiobjective problems as they are too complex to be solved using deterministic techniques. Over the last 20 years, MOEAs and MOGAs have demonstrated their effectiveness in solving MOP problems and approximating their corresponding Pareto optimal front using the concept of dominance [45].

Definition 1 (dominance [43]). Given two feasible solutions, a solution $\mathbf{X}^u$ is said to dominate $\mathbf{X}^v$, denoted by $\mathbf{X}_u \prec \mathbf{X}_v$, if and only if

$$\begin{aligned} F_i(\mathbf{X}^u) &\le F_i(\mathbf{X}^v), \quad \forall i \in n, \\ F_k(\mathbf{X}^u) &< F_k(\mathbf{X}^v), \quad \exists k \in n. \end{aligned} \quad (13)$$

Definition 2 (Pareto optimal set and the Pareto front [43]). The Pareto optimal set $\mathbf{X}_P$, which includes the solutions that are not dominated by any other solutions, is given by

$$\mathbf{X}_P = \{\mathbf{X} \in D \mid \nexists \widetilde{\mathbf{X}} \in D : \widetilde{\mathbf{X}} \prec \mathbf{X}\}. \quad (14)$$

The Pareto front $\mathbf{F}(\mathbf{X}_P)$ is the plot of the objective functions whose nondominated vectors are in the Pareto optimal set.

The ev-MOGA [43] is an elitist MOGA based on the concept of ε-dominance [46]. In ev-MOGA, the objective function space is split into a fixed number of boxes forming a grid taking ε as the length of edges. The concept of ε-dominance is based on the idea that a particular solution inside a given box dominates the remaining solutions belonging to this box.

For each dimension $i \in n$, n_box_i cells of ε_i width are created where

$$\begin{aligned} \epsilon_i &= \frac{\left(F_i^{\max} - F_i^{\min}\right)}{n\_\text{box}_i}, \\ F_i^{\max} &= \max_{\mathbf{X} \in \mathbf{X}_P} F_i(\mathbf{X}), \\ F_i^{\min} &= \min_{\mathbf{X} \in \mathbf{X}_P} F_i(\mathbf{X}). \end{aligned} \quad (15)$$

For a solution $\mathbf{X}$, $\text{box}_i(\mathbf{X})$ is defined by

$$\text{box}_i(\mathbf{X}) = \left\lceil \frac{F_i(\mathbf{X}) - F_i^{\min}}{F_i^{\max} - F_i^{\min}} \cdot n\_\text{box}_i \right\rceil, \quad \forall i \in n, \quad (16)$$

where $\lceil a \rceil$ rounds a to the nearest integer towards infinity.

Remark 1. $\text{box}_i(\mathbf{X})$ is always an integer belonging to the set $\{0, 1, 2, \ldots, n\_\text{box}_i\}$.

Definition 3 (ε-dominance [43]). Given $\mathbf{box}(\mathbf{X}) = \{\text{box}_1(\mathbf{X}), \ldots, \text{box}_n(\mathbf{X})\}$, a solution $\mathbf{X}^u$ with value $\mathbf{F}(\mathbf{X}^u)$ ε-dominates the solution $\mathbf{X}^v$ with value $\mathbf{F}(\mathbf{X}^v)$, denoted by $\mathbf{X}^u \prec_\epsilon \mathbf{X}^v$, if and only if

$$\begin{aligned} &\mathbf{box}(\mathbf{X}^u) \prec \mathbf{box}(\mathbf{X}^v) \\ &\text{or } \left(\mathbf{box}(\mathbf{X}^u) = \mathbf{box}(\mathbf{X}^v), \mathbf{X}^u \prec \mathbf{X}^v\right). \end{aligned} \quad (17)$$

This grid preserves the diversity of the Pareto front $\mathbf{F}(\mathbf{X}_P)$ as each box can be occupied by only one solution at the same time and produces a smart distribution since the algorithm only checks occupied boxes, rather than all boxes. This content management avoids the need to use other clustering techniques to obtain adequate distributions, which leads to a considerable reduction in the computational burden [46].

Definition 4 (ε-Pareto set [43]). A set $\mathbf{X}^*_{P_\varepsilon} \subseteq \mathbf{X}_P$ is ε-Pareto if and only if

$$\forall \mathbf{X}^u, \mathbf{X}^v \in \mathbf{X}^*_{P\varepsilon}, \mathbf{X}^u \neq \mathbf{X}^v, \mathbf{box}(\mathbf{X}^u) \neq \mathbf{box}(\mathbf{X}^v), \quad \mathbf{box}(\mathbf{X}^u) \nprec_{\epsilon} \mathbf{box}(\mathbf{X}^v). \tag{18}$$

Remark 2. $\mathbf{X}_P$ is unique and normally includes infinite solutions. Hence, a set $\mathbf{X}^*_{P_\varepsilon}$, with a finite number of elements from $\mathbf{X}_P$, should be obtained. Notice that $\mathbf{X}^*_{P_\varepsilon}$ is not unique.

For the implementation of the ev-MOGA algorithm, three types of populations are defined as follows:

(1) $P(t)$ (t represents the actual iteration or generation of the algorithm) is the main population, which explores the searching space D (defined by constraints (7)) during the algorithm iterations (t). The main population size is denoted by Nind_P.

(2) $A(t)$ Archive, which stores the solution $\mathbf{X}^*_{P_\varepsilon}$. Its size is denoted by Nind_A, which is variable but bounded (see justification below in equation (19)).

(3) $\text{GA}(t)$ is the auxiliary population. Its size is denoted by Nind_{GA}, which must be an even number. This population is formed by new individuals obtained by crossover or mutation from individuals belonging to $P(t)$ and $A(t)$. This procedure is explained later in detail.

A uniform distribution of solutions is achieved by only including in the archive population $A(t)$, the ε-dominant solutions allocated in different boxes. If two solutions are shared in the same box, the solution that prevails can be established using different criteria which can be set by the user. For instance, it is possible to choose the closest solution to the centre of the box or the closest to the origin of the searching space D.

The aim of ev-MOGA is to achieve a ε-Pareto set $\mathbf{X}^*_{P\varepsilon}$ with the greatest possible number of solutions in order to characterize the Pareto front adequately. Although the number of possible solutions will depend on the shape of the front and on n_box_i, it will not exceed the following bound:

$$\text{card}\left(\mathbf{X}^*_{P_\epsilon}\right) \leq \frac{\prod_{i=1}^{n} n\_\text{box}_i + 1}{n\_\text{box}_{\max} + 1}, \quad n\_\text{box}_{\max} = \max_i n\_\text{box}_i, \tag{19}$$

where $\text{card}(X)$ is the number of elements of set X. With this bound, it is possible to control the maximum number of solutions that will characterize the Pareto front.

The step-by-step process applying the ev-MOGA algorithm to obtain the nondominated mean-variance-sustainability surface is as follows:

Step 1. Initialize $t = 0$. Create a uniformly distributed initial population of portfolio weights $P(0)$ with Nind_P individuals (portfolios) randomly selected from the searching space D, and create an empty archive population $A_0 = \varnothing$.

Step 2. Conduct the multiobjective evaluation of the main population of portfolios $P(0)$ using equations (8)–(10).

Step 3. Detect the ε-nondominated portfolios ($\mathbf{X}_{\text{ND}}$) from $P(0)$, which are stored in $A(0)$. The Pareto front limits $F_i^{\max}$ and $F_i^{\min}$ are calculated from population $A(0)$.

Step 4. Generate a new auxiliary population $\text{GA}(t)$ from the main population $P(t)$ and the archive population $A(t)$ following this procedure:

(1) Set $j = 0$.

(2) Two portfolios are randomly selected, $\mathbf{X}^P$ from $P(t)$ and $\mathbf{X}^A$ from $A(t)$.

(3) A random number $u \in [0, \ldots, 1]$ is generated.

(4) If $u > P_{c/m}$ (probability of crossing/mutation. It has to be prefixed in advance by the user. It usually equals 0.2), $\mathbf{X}^P$ and $\mathbf{X}^A$ are crossed over by means of the extended linear recombination technique, generate two new portfolios for $\text{GA}(t)$.

(5) If $u \leq P_{c/m}$, $\mathbf{X}^P$ and $\mathbf{X}^A$ are mutated using random mutation with Gaussian distribution and then included in $\text{GA}(t)$.

(6) $j = j + 1$.

(7) If $j < \text{Nind}_{\text{GA}}/2$, go to (2). Otherwise, the procedure finishes.

Step 5. Evaluate population $\text{GA}(t)$ using the multiobjective approach defined by equations (8)–(10).

Step 6. Check which portfolios in $\text{GA}(t)$ must be included in $A(t+1)$ on the basis of their location in the objective space. $A(t+1)$ will contain all the portfolios from $A(t)$ that are not ϵ-dominated by elements of $\text{GA}(t)$, and all the portfolios from $\text{GA}(t)$ which are not ε-dominated by elements of $A(t)$.

Step 7. Update population $P(t+1)$ with portfolios from $\text{GA}(t)$. Every portfolio $\mathbf{X}^{\text{GA}}$ from $\text{GA}(t)$ is compared with a portfolio $\mathbf{X}^P$ that is randomly selected from the portfolios in $P(t)$. $\mathbf{X}^{\text{GA}}$ will replace $\mathbf{X}^P$ in $P(t+1)$ if it ε-dominates $\mathbf{X}^P$. Otherwise, $\mathbf{X}^P$ will not be replaced.

Step 8. In $t = t + 1$, check if $t < t_{\max}$, then go to Step 4, otherwise stop.

Portfolios from $A(t)$ will belong to $\mathbf{X}^*_{P\epsilon}$, the smart and efficient approximation of the Pareto set.

The ev-MOGA algorithm used in this article is a modified version of the algorithm published in Matlab Central [47].

The original ev-MOGA algorithm was unable to cope simultaneously with the weighting constraints stated in equations (11) and (12). Thus, it has been necessary to implement some changes related to the random generation of a uniformly distributed initial population and the crossover and random mutation of individuals.

Example 1 (six-stock portfolio selection). To show the applicability of the ev-MOGA algorithm to solve a tricriterion multiobjective portfolio selection from a sustainable-financial perspective, we have developed a six-stock example based on equations (8)–(12). In this example, the following vector of expected returns, matrix of covariances, and vector of portfolio sustainability scores have been used:

$$\boldsymbol{\mu} = \begin{pmatrix} 1.8426 \\ 1.4060 \\ 0.8346 \\ 1.5745 \\ 1.4133 \\ -0.4145 \end{pmatrix} \cdot 10^{-2};$$

$$\boldsymbol{\sigma} = \begin{bmatrix} 21.6368 & 3.7021 & -0.9104 & 3.4893 & 3.2989 & 5.1921 \\ 3.7021 & 5.4762 & 0.5264 & -0.0284 & 2.3047 & 1.0302 \\ -0.9104 & 0.5264 & 3.4024 & 0.5005 & 1.0575 & 0.8166 \\ 3.4893 & -0.0284 & 0.5005 & 5.2031 & 0.8882 & 1.9082 \\ 3.2989 & 2.3047 & 1.0575 & 0.8882 & 8.3785 & 1.7110 \\ 5.1921 & 1.0302 & 0.8166 & 1.9082 & 1.7110 & 6.349 \end{bmatrix} \cdot 10^{-3};$$

$$\mathbf{s} = \begin{pmatrix} 45 \\ 58 \\ 55 \\ 46 \\ 52 \\ 60 \end{pmatrix}. \quad (20)$$

The ev-MOGA algorithm has been executed with the following parameters: $\text{Nind}_P = 10^4$, $\text{Nind}_{\text{GA}} = 8$, $t_{\max} = 10^5$, $P_{m/c} = 0.2$, and $n\_\text{box}_i = 1000$ $i = 1, \ldots, 6$. After 10^5 iterations, population A has 40414 individuals. Figure 1 shows the ε-Pareto front obtained by applying the ev-MOGA algorithm.

Figure 1 depicts the three-dimensional representation of the approximated ε-Pareto front, thus providing the nondominated mean-variance-sustainability surface. This ε-Pareto frontier is made up of 40414 uniformly distributed points representing nondominated portfolios for which none of the three objectives can be improved without sacrificing any others. Notice that, the north-west boundary of the plotted surface is the standard Markowitz's mean-variance frontier. For this example, when the risk goes down, the return becomes smaller and the sustainability becomes bigger. Moreover, the points that are performing well in sustainability are plotted in green in the bottom left corner.

4. Representing the Nondominated Mean-Variance-Sustainability Surface with 2D Figures Using Level Diagrams

It is widely recognized that as the number of dimensions increases, it is increasingly difficult to analyse the graphical information provided by the Pareto front. According to

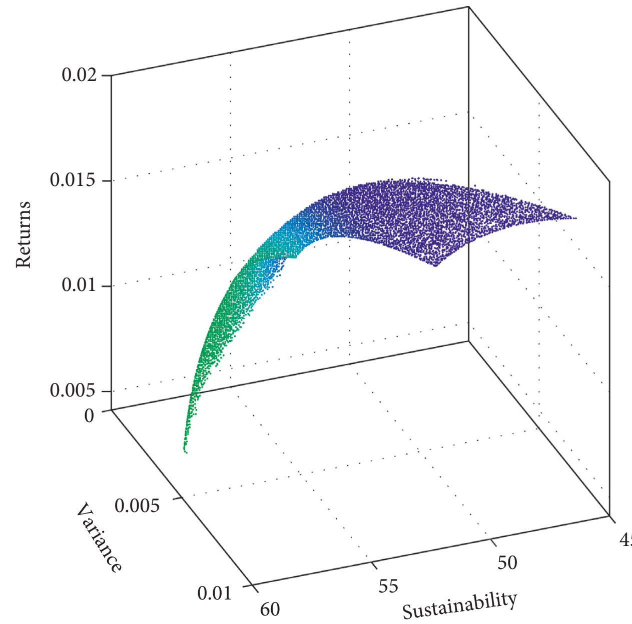

Figure 1: 3D ε-Pareto front corresponding to the six-stock example.

Miettinen [44], approximating the Pareto front is an open research field in which a broad array of techniques have been proposed.

In [48], a new visualization tool of n-dimensional Pareto fronts called level diagrams (LDs) is developed. LD is shown to be a useful analysis tool to help decision makers face with large sets of Pareto points obtained from multiobjective optimization problems. With LD, each objective is represented on the X-axis of a separate 2D diagram, and each diagram is synchronized with the others because all share the same Y-axis. To represent the points of the Pareto front in LDs, every objective is normalized with respect to minimum and maximum values by applying a norm, such as, the 1-norm, the Euclidean norm (2-norm), or the infinity norm (∞-norm) (it is also possible to apply any user-defined function to perform this normalization procedure). In each diagram, the Y-axis corresponds to the sum of the normalized objectives, while the values of a particular objective are represented on the X-axis. There will be as many LDs as objectives. Consequently, LD methodology consists in replacing a n-dimensional Pareto front by n LDs that share the same Y-axis.

For the proposed mean-variance-sustainability problem, the 3D-Pareto front will be replaced with three LDs in two dimensions. In the first LD, the X-axis will be the values of variance (risk), the return values will be represented in the second LD, and the third LD refers to the values of sustainability. LD is available from Matlab Central Reynoso-Meza [49].

Example 2 (six-stock portfolio selection (continued)). According to the previous statement, from Example 1, the 3D-Pareto front of Figure 1 can be represented by three two-dimensional LDs in Figure 2.

In each LD, the Y-axis corresponds to the normalized value of the three objectives using the 2-norm. The green points in the first LD are situated at the lower levels in the X-axis (risk or variance), and they correspond to the zones of the Pareto front nearer to the ideal point which represent the minimum risk. These green points in the LD of risk correspond to the green points in the LD of return and in the LD of sustainability. It is worth noting that all the graphs are synchronized because they share the same Y-axis. This synchronization can be used to check where a particular group of points is located in different LDs.

Imagine that an investor determines a sustainability benchmark hoping to make a positive impact through his/her responsible investing. This sustainability aspiration level can be coloured in purple in the sustainability LD (see the last LD in Figure 3). Furthermore, all the objectives are coordinated and by selecting a given level of sustainability, the decision maker can visualize the corresponding value of risk and return in the remaining LDs and also in the 3D-Pareto front. In this way, the level diagram information on the objective values of returns, variance, and sustainability can be represented in a clearer form than a three-dimensional graph.

LD tool also works with the Pareto set. In this example, the Pareto set is a 6-dimensional set, so it is impossible to represent it using a standard graph. Fortunately, following the same idea used for the Pareto front, it is possible to display this 6-dimensional set with 6 LDs, each one corresponding to each portfolio weight (w_i).

From Example 1, the Pareto set can be equivalently represented by the 6 LDs that are shown in Figure 4. The green points in these new LDs also correspond to the zones of the Pareto set nearer to the ideal point which represents the maximum sustainability. Due to the synchronization of all the graphs, when the decision maker sets a level for one objective in the LDs of the Pareto front, the range of the budget that could be invested in each asset appears in purple in the corresponding LDs of the Pareto set.

Remark 3 (about n-dimensional Pareto sets). Generalising the results of Example 2, it is possible to conclude that a n-dimensional Pareto set can be replaced by n LDs that share the same Y-axis, each one corresponding to each weight (w_i).

Overall, this interactive tool could be interesting for the visualization of the Pareto fronts involving more than three objectives and helping users of a posteriori methods find the best solutions in multiobjective optimization problems.

5. An Application to the European SRI Open-End Funds

In this section, we report the experimental results obtained with the application of ev-MOGA to a real-world empirical study using a data set of institutional SRI open-end funds from Morningstar to explore the variance-expected return-sustainability tradeoff. The data from Morningstar cover an opportunity set that includes 22 institutional SRI open-end funds offered in Spain, and the base currency is the Euro. For each SRI open-end fund, we have the monthly returns for 120 months for the period 2009–2019. Monthly data for the period were downloaded to compute the expected return vector $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_{22})^T$ and the covariance matrix $\boldsymbol{\Sigma} = [\sigma_{ij}]$, $i, j = 1, \ldots, 22$.

As for the model sustainability vector $\mathbf{s} = (s_1, \ldots, s_{22})^T$, we downloaded the historical portfolio sustainability score that provides reliable information about how well the holdings in a portfolio perform on environmental, social, and governance (ESG) issues. For more information about the sustainability scoring method, see Morningstar [50].

We have applied the ev-MOGA algorithm with these parameters: $\text{Nind}_P = 5 \cdot 10^4$, $\text{Nind}_{\text{GA}} = 20$, $t_{\max} = 3 \cdot 10^5$, $P_{m/c} = 0.2$, and $n\_\text{box}_i = 500$ $i = 1, \ldots, 22$.

The set of points belonging to ε-Pareto front in Figure 5 represents the set of nondominated (or efficient) portfolios for which none of the three objectives, risk, return, or sustainability, can be improved without sacrificing any others. In this case, it is shown that improving the level of sustainability in a second stage of the portfolio multi-objective optimization undermines the financial goal in terms of return but improves the risk. In fact, the area of the surface for the highest level of sustainability coloured in green corresponds to low level of returns and low values for risk.

Figure 6 shows the three LDs in two dimensions for return, variance (risk), and sustainability related to the previous 3D ε-Pareto front. If a particular investor sets a level of sustainability using the purple marker, the LD tool immediately offers the corresponding values of risk and return. Moreover, Figure 7 plots the 22 LDs of the Pareto set, thus providing the corresponding portfolio weights.

When analysing Figure 7, we should draw attention to the following groups of SRI open-end funds: (i) a first group including funds 1-2-5-6-15-17-18-20 that does not (or only marginally) contribute to achieving any nondominated point of the ε-Pareto front; (ii) a second group composed of funds 8-11-12-13-19 generating efficient solutions, but not providing the required sustainability benchmark; and (iii) a third group made up of funds 3-4-7-9-10-14-16-21-22 providing efficient solutions in which a high level of sustainability is also ensured.

6. Conclusions

The Markowitz mean-variance approach has been the prevailing model for portfolios for over 60 years and is often viewed as a basic model to represent the complexity of real-world portfolio selection problems, especially when investors are concerned about additional criteria such as sustainability. Therefore, a new ev-MOGA approach has been applied in this paper to approximate the non-dominated mean-variance-sustainability surface by providing a well-distributed Pareto front. We have reviewed the main contributions in the literature addressing the problem of including additional criteria to the classical mean-variance Markowitz portfolio selection approach. Scholars started to use MOGAs for portfolio selection,

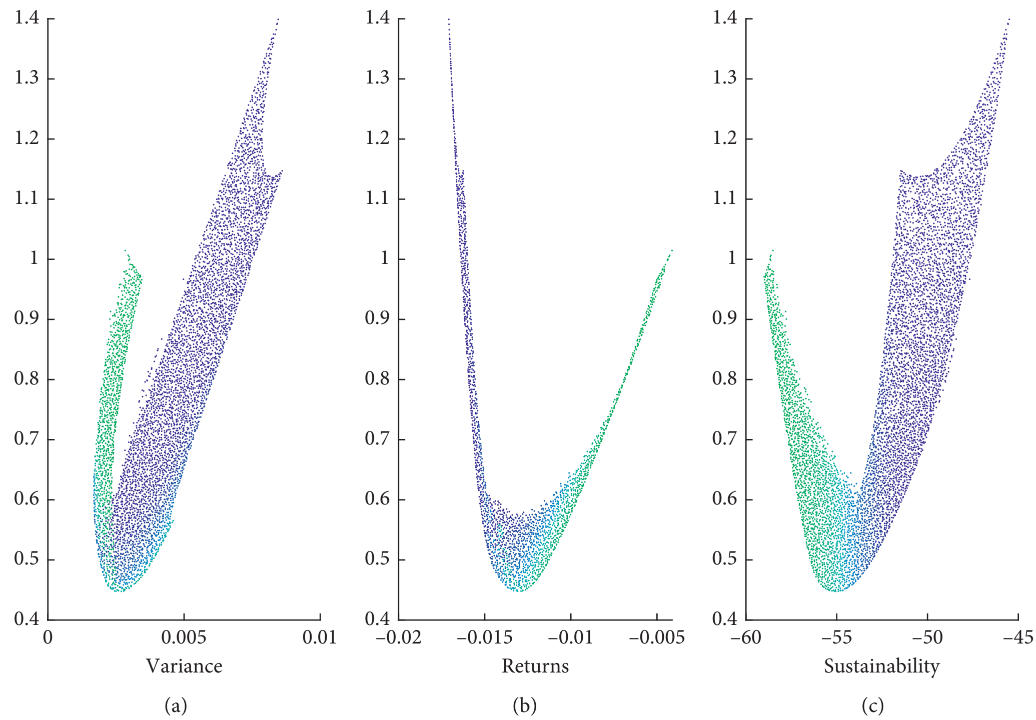

Figure 2: 2D-level diagrams corresponding to the Pareto front of the six-stock example.

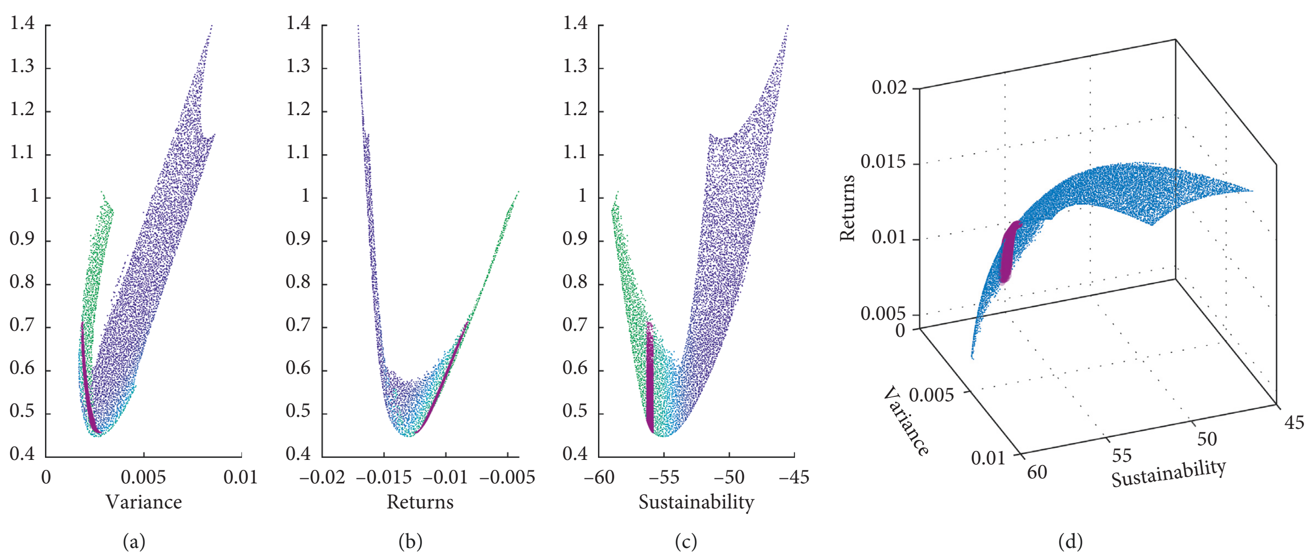

Figure 3: 2D LDs and the corresponding 3D-Pareto front of the six-stock example with a coordinated purple set for a sustainability benchmark.

especially to the two-objective case, but few studies have dealt with three or more objectives.

When more dimensions are added to the problem, the complexity increases and graphical analysis tools are needed for the visualization of the Pareto front to facilitate the decision-making process. In our proposal, the level diagrams tool has been used to consider sustainability preferences in the portfolio selection problem to better understand the tradeoffs between risk, return, and sustainability in a 2D graphical representation. To illustrate the methodology, a retrospective case of portfolio selection in a European stock exchange is developed. By starting with an opportunity set of 22 institutional SRI open-end funds, we derive the nondominated surface from information of historical returns and sustainability indices.

Our approach has several advantages over previous work on tricriteria portfolio selection because (i) existing multiobjective techniques for extending mean-variance portfolio selection problems have limited capabilities if the new objectives are nonlinear, and our approach could provide a

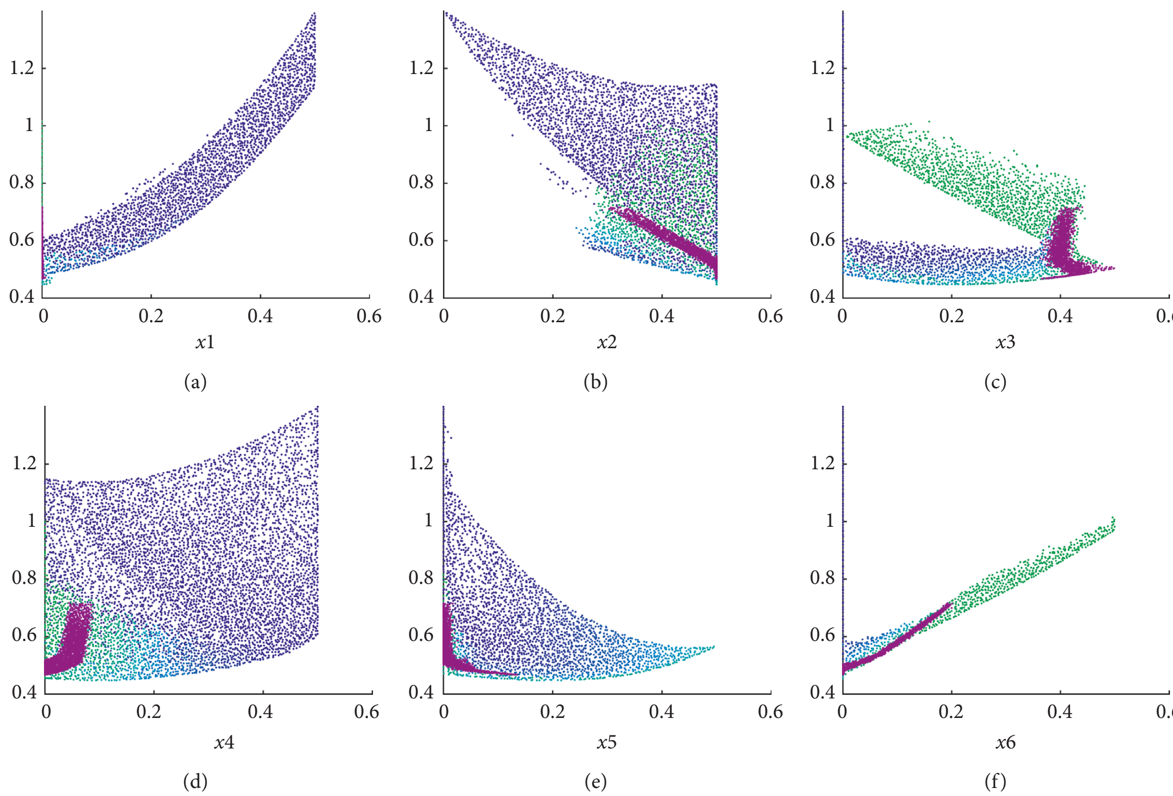

Figure 4: 2D LDs corresponding to the Pareto set of the six-stock example.

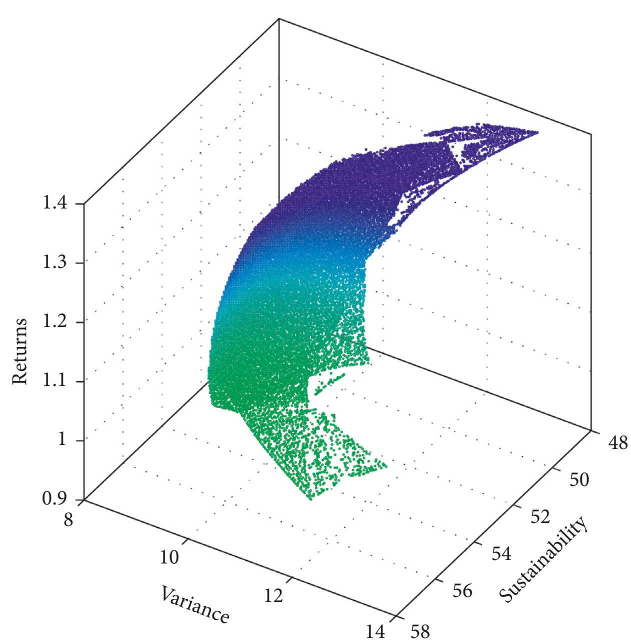

Figure 5: 3D ε-Pareto front corresponding to the European SRI open-end funds.

general framework for n objectives with no conditions related to the type of the objective function (quadratic, linear, or nonlinear); (ii) the proposed ev-MOGA algorithm adjusts the Pareto front dynamically, ensuring convergence and uniform distribution of solutions; (iii) level diagrams provide a new tool of visualization to better understand the tradeoff between objectives and give a two-dimensional representation of high-dimensional Pareto fronts and Pareto sets.

Finally, there are several future lines of research for overcoming the limitation of this study. Firstly, to validate the use of our proposed ev-MOGA approach to multi-objective portfolio selection, it would be interesting to conduct a computational comparison of the exact methods

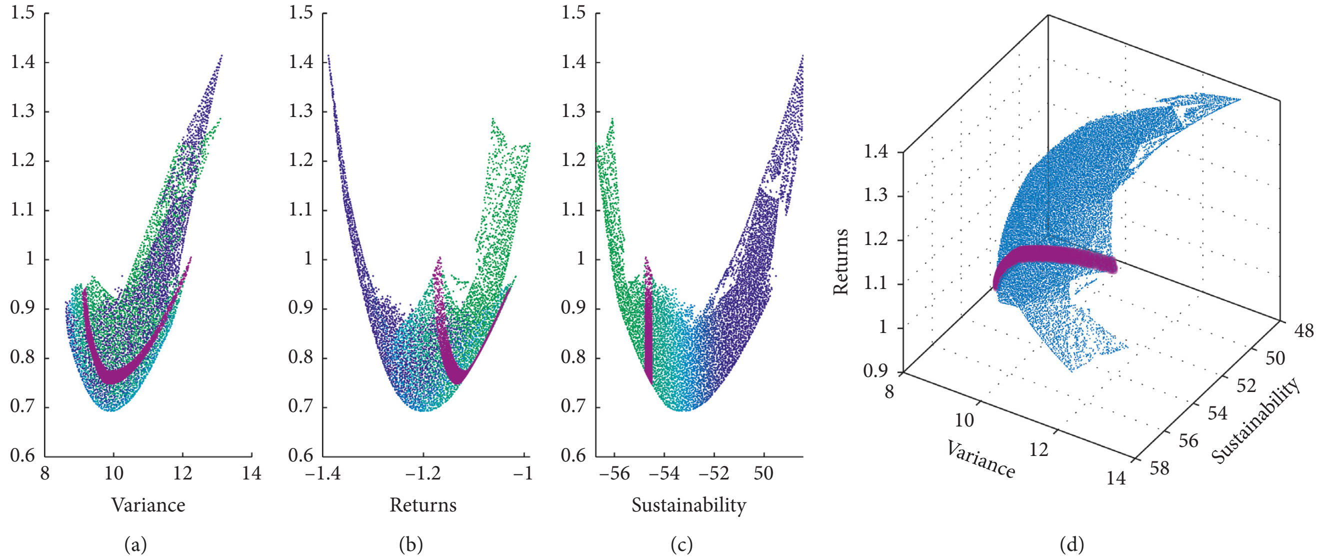

Figure 6: 2D LDs corresponding to the Pareto front of the European SRI open-end funds with a coordinated purple set for a sustainability benchmark.

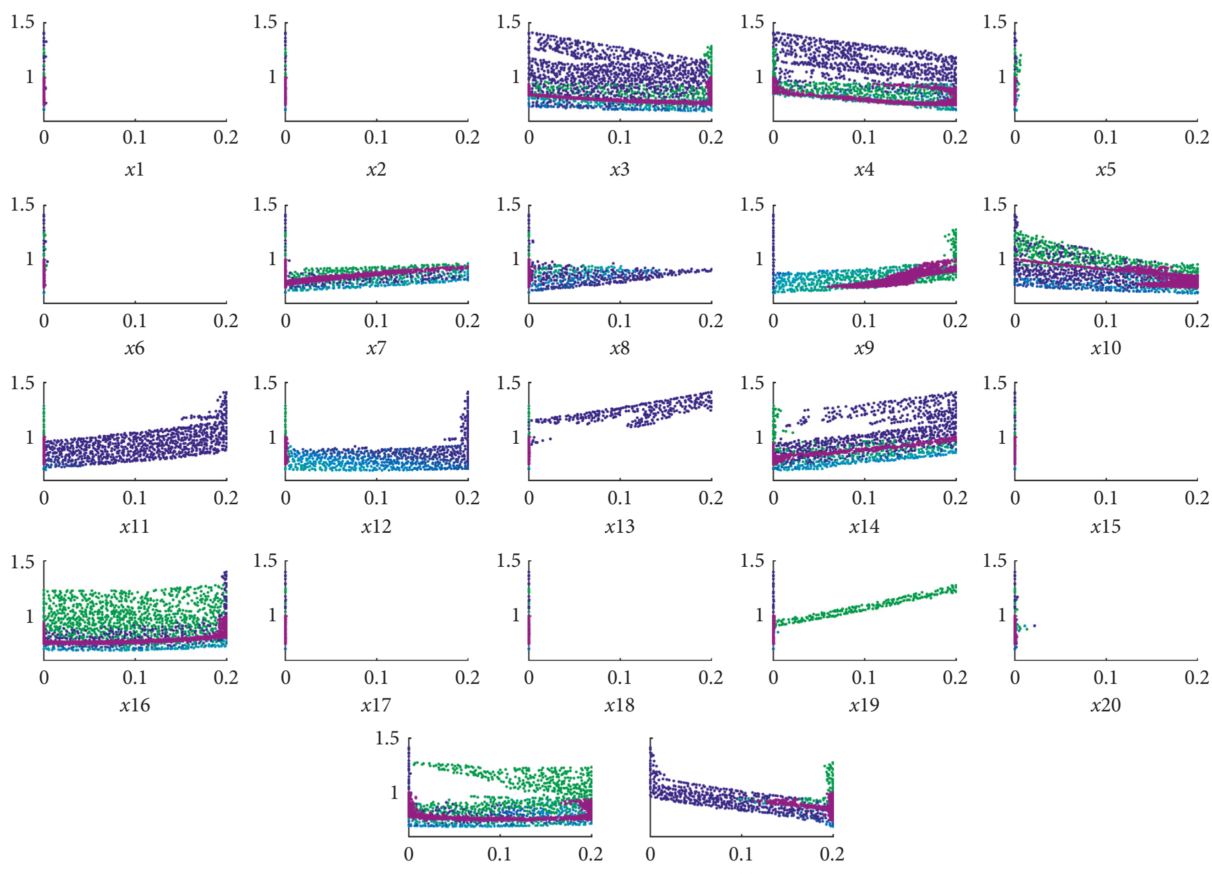

Figure 7: 2D LDs corresponding to the Pareto set of the European SRI open-end funds.

previously proposed in the literature, as well as study the advantages and disadvantages of both approaches. Secondly, a future research opportunity would be to extend the proposed model and incorporate other realistic objectives (such as liquidity, number of securities in the portfolio, or turnover), thus providing a general framework for n objectives. Finally, the impact of estimation errors on mean-variance-sustainability portfolio optimization will be incorporated in future works to compare the optimal portfolio performance by using resampling approaches and others.

Data Availability

The SRI data from Morningstar of the application used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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