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Unbalanced and Reactive Currents Compensation in Three-Phase Four-Wire Sinusoidal Power Systems

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Abstract: In an unbalanced linear three-phase electrical system, there are inefficient powers that increase the apparent power supplied by the network, line losses, machine malfunctions, etc. These inefficiencies are mainly due to the use of unbalanced loads. Unlike a three-wire unbalanced system, a four-wire system has zero sequence currents that circulate through the neutral wire and can be compensated by means of compensation equipment, which prevents it from being delivered by the network. To design a compensator that works with unbalanced voltages, it is necessary to consider the interactions between it and the other compensators used to compensate for negative-sequence currents and positive-sequence reactive currents. In this paper, through passive compensation, a new method is proposed to develop the zero sequence current compensation equipment. The method does not require iteration algorithms and is valid for unbalanced voltages. In addition, the interactions between all compensators are analyzed, and the necessary modifications in the calculations are proposed to obtain a total compensation. To facilitate the application of the method and demonstrate its validity, a case study is developed from a three-phase linear four-wire system with unbalanced voltages and loads. The results obtained are compared with other compensation methods that also use passive elements.

Keywords: unbalanced power; reactive power; negative-sequence current; zero-sequence current; compensation

1. Introduction

Low voltage electrical systems usually operate unbalanced, especially in four-wire supply networks where three-phase and single-phase loads coexist. These imbalances are attributable to both the loads and the voltages; this imbalance manifests as an increase of the total apparent power with respect to the ideal power of a balanced system, which is characterised by the positive sequence active power. The difference with three-phase three-wire systems is that there exists a zero sequence of the line current or current flowing through the neutral wire. These unbalanced powers that appear can be studied from Buchholz's apparent power [1] expressed as a function of its symmetric components [2].

The first reactive power compensator was developed by Steinmetz in 1917 [3]. Subsequently research continued, until relatively recently, in the investigation of reactive compensation giving rise to several research papers (those worth mentioning include [4–6]). However, imbalances are responsible for increases in the total apparent power that the generator must deliver, increased line losses [7], heating [8], machine malfunctions [9], protection malfunctions [10], etc. These unbalanced apparent powers have already been established in the IEEE-Std 1459-2010 [11].

A three-phase four-wire system powered by an infinite short-circuit power generator would have negative- and zero-sequence voltages that generate unbalanced powers, which cannot be efficiently compensated for. However, there are regulatory standards on the quality of the electricity supply that limit the unbalanced voltages [12,13].

The unbalanced currents are directly attributable to the characteristics of the load, although in systems with unbalanced voltages and balanced loads, unbalanced currents are also generated. From the above, the need to study the mechanisms to eliminate these inefficiencies is clear.

There have been more papers aimed at eliminating inefficient apparent powers in three-phase three-wire systems than in four-wire systems. In this work, the three-phase four-wire systems are analyzed and a new methodology for the elimination of said powers is proposed, since in a previous work [14] the authors already resolved this problem for three-phase three-wire systems through passive compensators.

It is clear that active filters are more effective than passive compensators. However, active filters are much more expensive and less robust than passive compensators. Nevertheless, the reactive power compensators remain valid solutions for applications in consumer and electricity distribution in those situations when the criterion regarding the costs of installing and operating the equipment is more important than the ones related to the reaction speed or the control accuracy. This is also the case of the equipment for power factor improvement and load balancing in a three-phase distribution network. Of the works aimed at three-phase four-wire systems, it is worth mentioning the ones discussed below.

Lee and Wu [15] developed three-phase three-wire systems based on an earlier article by Gyugyi [16]. At first, they assumed that the voltage system was unbalanced, but it really used balanced voltage, much like Gyugyi.

De Oliveira et al. [17], also working off Gyugyi, extended the method to four-wire systems. They added the desired power factor to the equations of the compensators. However, their method was only valid for balanced voltages and presented infinite solutions.

Li and Wang [18] presented a methodology to obtain a compensator for negative- and zero-sequence currents, and positive-sequence reactive currents. The compensator is only formed by capacitors. For this, they assumed the load to be very inductive. They based this on the theory of instantaneous reactive power (Akagi) and used Clarke's transform matrix. However, it is only applicable to three-phase systems with balanced voltages. It is also necessary to know the value and characteristics of the load.

León [19] presented an equivalent circuit that represented the inefficiency of the zero-sequence current which also becomes a compensator of said current when the sign is changed to that of the reactances. Like the previous studies, it is also only valid for balanced systems.

In 2015, Czarnecki and Haley [20] extended their previous work for three-wire systems [21,22] to four-wire systems. They showed that the unbalanced power in four-wire systems is due to a negative-sequence current and a zero-sequence current. To compensate for these currents, it was necessary to use at least two compensators. This conclusion was also reached earlier by León-Martinez and Montañana-Romeu [19] and in the current study.

Finally, in 2018, Pana et al. [23] developed a mathematical model that they call the balanced capacitive compensator (BCC). The algorithm eliminated the positive reactive power, as well as the negative-sequence and zero-sequence currents. They used single-phase capacitor banks. To implement the mathematical model, the nature and values of the loads need to be known and several computer programs are necessary. However, as in all previous papers, it is only valid for systems with balanced voltages.

From the analysis of the methods mentioned above it was observed that they have limitations. This is mainly because they are designed to work with balanced voltages. In addition, they consider that the imbalances of the voltages are due to the load of the analyzed system and not to the characteristics of the network itself. These types of limitations are resolved in the compensation proposal presented in this work. That is, it is valid for unbalanced voltages and independent of the values and characteristics of the load.

The present work shows the development of a compensating circuit for the zero-sequence current (ZSCC) that consumes the load in a three-phase four-wire system with unbalanced voltages and unbalanced currents. This compensator, together with the compensating circuits of the negative-sequence currents (NSCC) and the positive-sequence reactive component (SVC) for three-wire networks, developed by Blasco and colleagues [14], constitute a procedure of total compensation for the inefficient currents. Therefore, the present study is a continuation of said work, and gives a solution to the interactions between the sequence currents caused by each compensator in four-wire electrical systems.

The compensating circuits are connected in parallel to the load in the bus, so that the network in the bus only provides a positive-sequence active currents system. The current flowing through the neutral wire is zero. This procedure provides maximum efficiency downstream of the bus (i.e., the unit for the power factor).

Thus, a procedure of total optimization of an electrical system is presented, which, given the characteristics of passive compensators, makes it effective when the loads are stationary or of low variability over time. The described procedure allows adaptability to hybrid compensation systems, thus increasing their versatility. In addition, it provides great robustness and low cost compared to active or hybrid systems.

This entails great benefits, since the network will always provide a balanced system of positive-sequence currents in the bus, even if the load is single-phase. No current will flow through the neutral wire, and the losses in the network will be minimal.

To determine the values of the reactances that make up the compensation circuits, it is enough to know the line-to-neutral voltages and the line currents in the bus, not the characteristics and the way of connecting the load. It is also not necessary to know the state of the network upstream of the bus, since the compensators adapt to the voltage and current values obtained in the bus at all times.

The remainder of this article is structured as follows. In Section 2, the expressions of the powers to be used in this paper are established. In Section 3, the expressions that will give rise to the values of the reactances to be placed in the zero-sequence current compensator are developed. Three cases of three-phase four-wire systems fed with different voltage systems are analyzed: balanced, unbalanced only in modules, and unbalanced in modules and angles. In addition, the interaction between the different compensators is analyzed to eliminate the different inefficient apparent powers and a calculation sequence of the different compensators is proposed. An example with unbalanced loads and voltages is presented in Section 4, where the compensators are applied to verify the proposed method. In addition, the results obtained are compared with other existing compensation methods. Finally, in Section 5 the results obtained are analyzed and conclusions are established.

2. Analysis of the Unbalanced Powers in a Three-Phase Four-Wire Linear System

Figure 1 shows a linear unbalanced load that is connected to a three-phase four-wire system with unbalanced voltages.

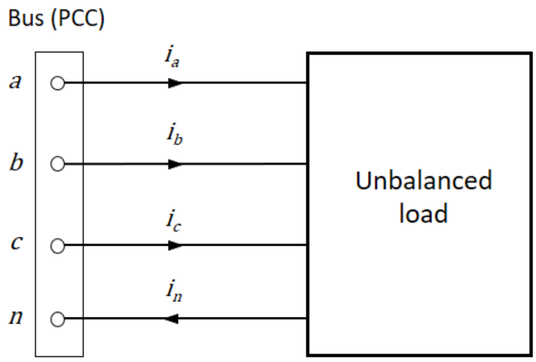

Figure 1. Three-phase four-wire system with an unbalanced load.

The apparent power in the bus is determined by Equation (1).

$$\underline{S} = \underline{S_a} + \underline{S_b} + \underline{S_c} \tag{1}$$

where

- $\underline{S}$ is the complex apparent power of the system defined in the classical theories.
- $\underline{S_a}$, $\underline{S_b}$ and $\underline{S_c}$ are the complex powers in each of the phases.

In contrast, the total apparent power of Buchholz S_T of a three-phase four-wire system, which is expressed in symmetrical components and related to the apparent power S, is determined by Equation (2).

$$S_T = 3\sqrt{\left(V_+^2 + V_-^2 + V_0^2\right)\left(I_+^2 + I_-^2 + I_0^2\right)} = \sqrt{S^2 + S_{uT}^2} = \sqrt{P^2 + Q^2 + S_{uT}^2} \tag{2}$$

where S_{uT} is the unbalanced apparent power caused by voltages and currents of different sequences. Its value is obtained from Equation (3).

$$\begin{aligned} S_{uT}^2 = {S_{u+}^{-}}^2 + {S_{u+}^{0}}^2 + {S_{u-}^{+}}^2 + {S_{u-}^{0}}^2 + {S_{u0}^{+}}^2 + {S_{u0}^{-}}^2 - 2(P_+P_- + P_+P_0 + P_-P_0) - \\ 2(Q_+Q_- + Q_+Q_0 + Q_-Q_0) \end{aligned} \tag{3}$$

Here,

- P_+, P_- and P_0 are the positive-, negative- and zero-sequence active power, respectively.
- Q_+, Q_- and Q_0 are the positive-, negative- and zero-sequence reactive power, respectively.

The values of these unbalanced apparent powers are calculated from Equations (4)–(9), for $z = \{a, b, c\}$.

$$S_{u+}^{-} = 3V_+I_- \tag{4}$$

$$S_{u+}^{0} = 3V_+I_0 \tag{5}$$

$$S_{u-}^{+} = 3V_-I_+ \tag{6}$$

$$S_{u-}^{0} = 3V_-I_0 \tag{7}$$

$$S_{u0}^{+} = 3V_0I_+ \tag{8}$$

$$S_{u0}^{-} = 3V_0I_- \tag{9}$$

The powers S_{u+}^{-}, S_{u+}^{0}, S_{u-}^{0}, S_{u0}^{-}, Q_+, Q_- and Q_0 can be cancelled in the bus, if we compensate for negative-sequence currents, zero-sequence currents, and the imaginary part of the positive sequence current that consumes the load. In contrast, P_- and P_0 will become P_+ to keep the total active power consumed by the load constant. S^{+}_{u-} and S^{+}_{u0} cannot be compensated, since in an infinite short-circuit power network, negative-sequence voltages and zero-sequence voltages are imposed by the network itself, although its value will change depending on the new values of positive-sequence currents.

3. Compensation of Unbalanced Three-Phase 4-Wire Linear Systems

In a three-phase four-wire system with unbalanced loads connected to an infinite short-circuit power network, the voltages are fixed and imposed by the network. The currents are unbalanced; therefore, there are positive-sequence currents, negative-sequence currents, and zero-sequence currents. Figure 2 shows our compensation proposal for an unbalanced four-wire system. We have used three passive compensators constituted from capacitors and/or coils. The SVC compensator (static VAR Compensator) and the NSCC compensator (negative sequence current compensator) allow us to compensate for the imaginary part of the positive-sequence current and the negative-sequence current provided by the network to the bus; therefore, these devices will compensate for the reactive powers and the unbalanced powers caused by these currents. These two compensators can be unified into one, as proposed in the previous work of the same authors [14]. Finally, the zero-sequence current compensator (ZSCC compensator) allows us to cancel the zero-sequence current provided by the network to the bus, which will compensate for the unbalanced powers caused by zero-sequence currents.

The main objective of this study is the design of the ZSCC compensator, because the design of the NSCC and SVC compensators was addressed by Blasco and colleagues [14]. However, as will be shown below, the ZSCC compensator consumes negative-sequence currents and positive-sequence currents that depend on whether the bus voltages are balanced or unbalanced. For this reason, the expressions of the NSCC compensator and SVC compensator proposed by Blasco and co-worker [14] must be modified to include these negative sequence currents and the positive sequence currents caused by the ZSCC compensator.

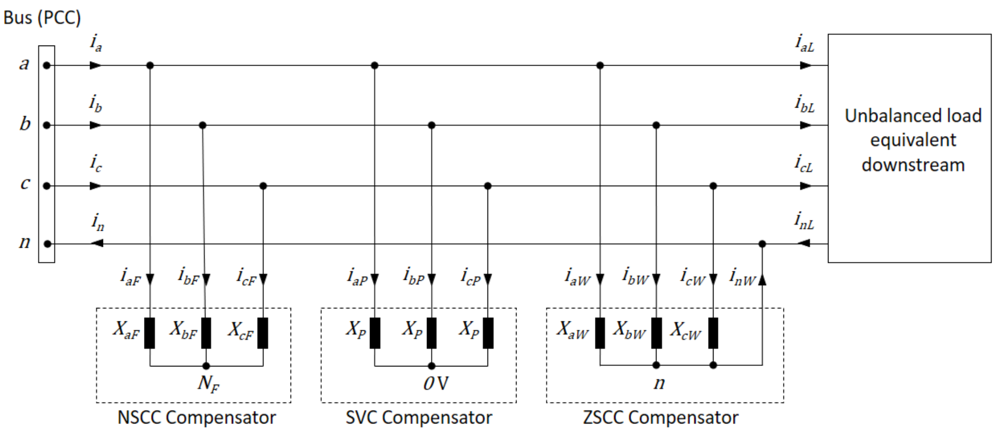

Figure 2. Proposal for compensation of an unbalanced three-phase four-wire linear system.

3.1. ZSCC Compensator with Balanced Voltages

Figure 3 shows the four-wire system proposed in Figure 2, but with only the ZSCC compensator and the equivalent network load downstream of the bus. Supposing that the voltages in the bus are balanced, the line-to-neutral voltages coincide with the positive-sequence voltages in Equation (10), where $\underline{a} = e^{j120}$.

$$\underline{V_{an}} = \underline{V_{a+}} = \underline{V_{+}} \qquad \underline{V_{bn}} = \underline{V_{b+}} = \underline{a}^2\, \underline{V_{+}} \qquad \underline{V_{cn}} = \underline{V_{c+}} = \underline{a}\, \underline{V_{+}} \tag{10}$$

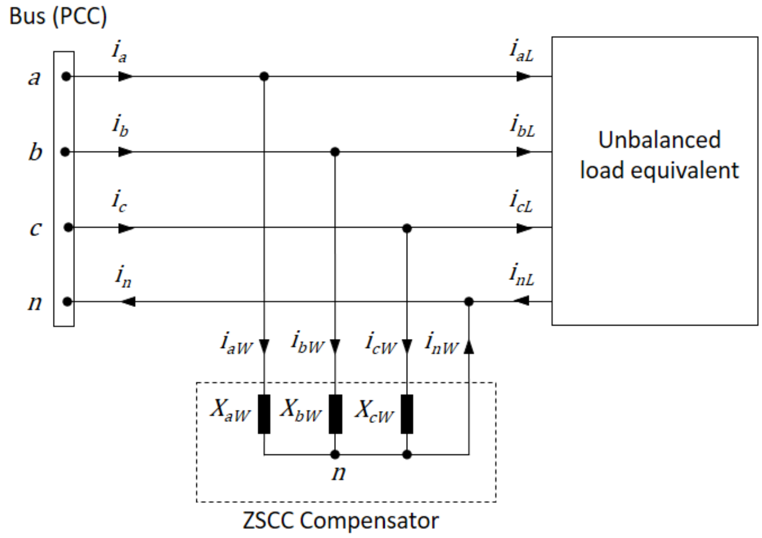

Figure 3. Compensation of zero-sequence currents.

Using the Fortescue transformation matrix, the line currents of the load expressed in symmetric components are determined by Equation (11).

$$\underline{I_{aL}} = \underline{I_{aL+}} + \underline{I_{aL-}} + \underline{I_{aL0}} \quad \underline{I_{bL}} = \underline{I_{bL+}} + \underline{I_{bL-}} + \underline{I_{bL0}} \quad \underline{I_{cL}} = \underline{I_{cL+}} + \underline{I_{cL-}} + \underline{I_{cL0}} \tag{11}$$

In addition, it is known that $\underline{I_{aL0}} = \underline{I_{bL0}} = \underline{I_{cL0}}$.

The objective of the ZSCC compensator is to compensate for $\underline{I_{aL0}}$, $\underline{I_{bL0}}$ and $\underline{I_{cL0}}$, which are the zero-sequence currents of the respective loads. Figure 4a shows the vector diagram corresponding to phase A, where it is considered that the line-to-neutral voltage is $\underline{V_{an}} = V_{an}e^{j0}$ and that the zero-sequence current of the load is $\underline{I_{aL0}} = I_{aL0}e^{-j\beta_{aL0}}$. Therefore, the zero-sequence current to compensate will be equal to the zero-sequence current of the load but with the opposite sign. As $\underline{I_{aL0}}$ is delayed with respect to $\underline{V_{an}}$, the reactance X_{aw} of the compensator must be capacitive (capacitor), hence the current of the compensator $\underline{I_{aW}}$ is advanced $+\frac{\pi}{2}$ with respect to $\underline{V_{an}}$. Obviously, it is impossible to comply with $\underline{I_{aW}} = -\underline{I_{aL0}}$, therefore, the compensator will consume an additional current $\underline{I'_{aW}}$, such that $\underline{I'_{aW}} = I_{aL0}\, e^{j\beta_{aL0}}$ (see Figure 4b). It follows that $\underline{I_{aW}}$ will be determined by Equation (12).

$$\underline{I_{aW}} = \underline{I'_{aW}} - \underline{I_{aL0}} \tag{12}$$

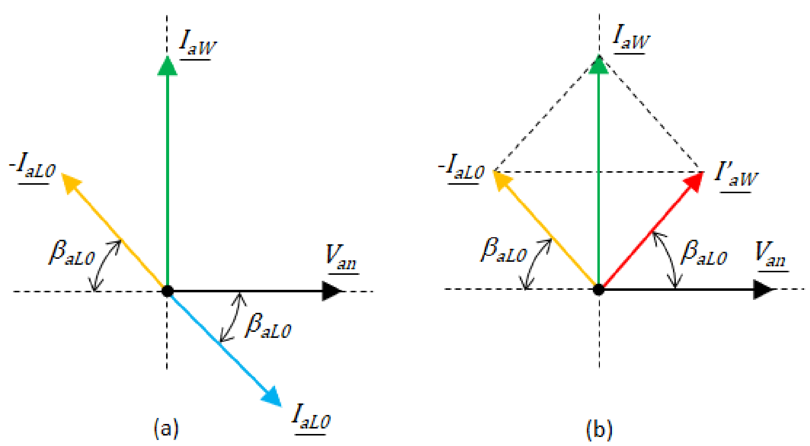

Figure 4. Vector diagram referring to phase A. (**a**) Initial situation and (**b**) Compensated situation.

If we consider phases B and C, taking into account that $\underline{V_{bn}} = V_{bn}e^{-j120}$ and $\underline{V_{cn}} = V_{cn}e^{j120}$ and performing the same procedure as in phase A, it is observed that $\underline{I_{bL0}}$ is advanced with respect to $\underline{V_{bn}}$, therefore, the reactance X_{bw} to be placed in phase B of the ZSCC compensator is inductive (coil), as seen in Figure 5a. On the other hand, $\underline{I_{cL0}}$ is delayed with respect to $\underline{V_{cn}}$, therefore, the reactance X_{cw} to be placed in phase C of the ZSCC compensator is capacitive (capacitor), as shown Figure 5b. That X_{aw}, X_{bw} and X_{cw} do not have the same sign is evident since the arithmetic sum of the unbalanced powers of the compensator must be zero.

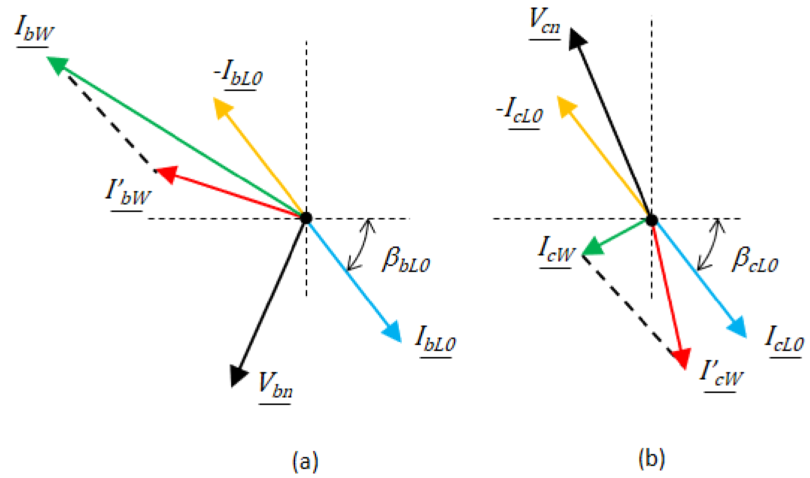

Figure 5. Vector diagram referring to phases B and C. (**a**) for phase B and (**b**) for phase C.

In the same way as Equation (12), $\underline{I_{bW}}$ and $\underline{I_{cW}}$ are determined by Equation (13) and (14), respectively.

$$\underline{I_{bW}} = \underline{I'_{bW}} - \underline{I_{bL0}} \quad (13)$$

$$\underline{I_{cW}} = \underline{I'_{cW}} - \underline{I_{cL0}} \quad (14)$$

If we join the vector diagrams of the three phases, the diagram in Figure 6 is obtained. It shows that currents $\underline{I'_{aW}}$, $\underline{I'_{bW}}$ and $\underline{I'_{cW}}$ form a system of negative-sequence currents. Therefore, at balanced line-to-neutral voltages, currents $\underline{I_{aW}}$, $\underline{I_{bW}}$ and $\underline{I_{cW}}$ consumed by the ZSCC compensator are broken down into a system of zero-sequence currents in the direction opposite to the zero-sequence currents of the load, and a balanced system of negative-sequence currents is created, whose values are obtained from Equations (15)–(17); where $\underline{I_{aW-}}$, $\underline{I_{bW-}}$ and $\underline{I_{cW-}}$ are the negative sequence currents consumed by the compensator in phases A, B and C, respectively.

$$\underline{I_{aW-}} = \underline{I'_{aW}} = \left(\underline{I_{aL0}}\right)^{*} \quad (15)$$

$$\underline{I_{bW-}} = \underline{I'_{bW}} = \underline{a}\ \underline{I_{aW-}} \quad (16)$$

$$\underline{I_{cW-}} = \underline{I'_{cW}} = \underline{a}^{2}\ \underline{I_{aW-}} \quad (17)$$

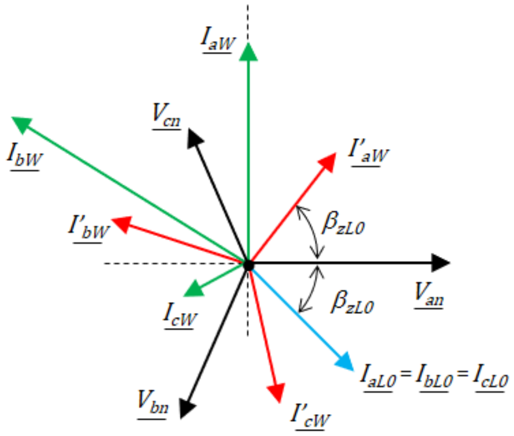

Figure 6. Joint vector diagram of phases A, B and C.

Substituting Equations (15)–(17) in Equations (12)–(14) and generalizing for $z = \{a, b, c\}$, Equation (18) is obtained.

$$\underline{I}_{zW} = \underline{I}_{zW-} - \underline{I}_{zL0} \tag{18}$$

Considering Equations (15)–(17) and Equations (11)–(14), the neutral current $\underline{I}_{nW}$ of the ZSCC compensator is determined by Equation (19) since $\underline{I}_{aW-} + \underline{I}_{bW-} + \underline{I}_{cW-} = 0$. This result is logical since the objective of the ZSCC compensator is to compensate for the neutral current consumed by the load. Therefore, in the bus, $\underline{I}_n = \underline{I}_{nW} + \underline{I}_{nL} = 0$.

$$\underline{I}_{nW} = \underline{I}_{aW} + \underline{I}_{bW} + \underline{I}_{cW} = -\left(\underline{I}_{aL0} + \underline{I}_{bL0} + \underline{I}_{cL0}\right) \tag{19}$$

If we re-analyze the vector diagram in Figure 4, it can be seen that the angle between $\underline{I}_{aW}$ and $\underline{V}_{an}$ is $\frac{\pi}{2}$ and the RMS value of $\underline{I}_{aW}$ is determined by double the imaginary part of the zero-sequence current of the load $\underline{I}_{aL0}$. In terms of power, these statements translate into the fact that the only power consumed by the ZSCC compensator in phase A is reactive power Q_{an}^{aW}. Therefore, we affirm that the reactive power consumed by the compensator in phase A is equal to twice the unbalanced reactive power caused by the line-to-neutral voltage and the zero-sequence current of the load, obviously with the opposite sign. Its value is determined by Equation (20).

$$Q_{an}^{aW} = -2\ V_{an} I_{aL0}\ \sin(\alpha_{an} - \beta_{aL0}) = -2\ Q_{an}^{aL0} \tag{20}$$

Similarly, for phases B and C from the vector diagrams of Figure 5, Equations (21) and (22) are obtained.

$$Q_{bn}^{bW} = -2\ V_{bn} I_{bL0}\ \sin(\alpha_{bn} - \beta_{bL0}) = -2\ Q_{bn}^{bL0} \tag{21}$$

$$Q_{cn}^{cW} = -2\ V_{cn} I_{cL0}\ \sin(\alpha_{cn} - \beta_{cL0}) = -2\ Q_{cn}^{cL0} \tag{22}$$

If we generalize Equations (20)–(22) for $z = \{a, b, c\}$, we get Equation (23).

$$Q_{zn}^{zW} = -2\ V_{zn} I_{zL0}\ \sin(\alpha_{zn} - \beta_{zL0}) = -2\ Q_{zn}^{zL0} \tag{23}$$

For each phase of the compensator, the reactances are given by

$$X_{zW} = \frac{Q_{zn}^{zW}}{{I_{zW}}^2} \tag{24}$$

Considering Equations (23) and (24), the reactance value of the ZSCC compensator in each phase can also be expressed from Equation (25); where, coil for $X_{zW} > 0$ and capacitor for $X_{zW} < 0$.

$$X_{zW} = -2\frac{Q_{zn}^{zL0}}{I_{zW}{}^2} = \frac{-(V_{zn})^2}{2\,Q_{zn}^{zL0}} \tag{25}$$

3.2. ZSCC Compensator with Unbalanced Voltages

If the voltages in the bus in Figure 3 are unbalanced, then in this case, two situations with different behaviors are possible:

- When the modules of the line-to-neutral voltages are different, but their phase angles are offset ±120 degrees from each other.
- When the phase angles are not offset ±120 degrees from each other.

When the voltage imbalance is only because the modules of the line-to-neutral voltages in each phase are not equal, the behavior of the ZSCC compensator is identical to that set out above for balanced voltages. The decomposition of currents $\underline{I_{aW}}$, $\underline{I_{bW}}$ and $\underline{I_{cW}}$ into symmetric components, gives rise to a system of zero-sequence currents and a system of negative-sequence currents. As with balanced voltages, the ZSCC compensator does not consume positive-sequence currents. Therefore, the reactive power in each phase consumed by the ZSCC compensator will also be equal to twice the reactive power caused by the line-to-neutral voltage and zero-sequence current of the load, but of the opposite sign. Hence, the values of the compensator reactances calculated from Equation (25) are perfectly valid for this type of situation.

On the contrary, when the phase angles of the line-to-neutral voltages are not offset ±120 degrees, regardless of the RMS value of the voltages, when decomposing the currents $\underline{I_{aW}}$, $\underline{I_{bW}}$ and $\underline{I_{cW}}$, in addition to the system of zero-sequence currents and from the negative-sequence current system, a positive-sequence current system is obtained that is not null. Therefore, the currents $\underline{I_{zW}}$ consumed by the compensator are determined by Equation (26). All this implies that Equation (18) is not valid when the phase angles of the voltages are not compensated ±120 degrees from each other, and as a consequence, Equation (25) is also not valid. Moreover, if we calculate the compensator reactances at unbalanced voltages using Equation (25) and analyze the system, we observe that the value of the zero-sequence current consumed by the ZSCC compensator is different from that necessary to compensate for the load. A new zero-sequence current different from the desired one appears and must be taken into account to perform the calculation again. This procedure will be performed iterating indefinitely until a valid solution is obtained. The number of iterations required will depend on the degree of voltage imbalance and the accuracy that is desired.

$$\underline{I_{zW}} = \underline{I_{zW+}} + \underline{I_{zW-}} + \underline{I_{zW0}} \tag{26}$$

To solve this problem, the authors present an exact calculation procedure that makes it unnecessary to use iterative methods. To do this, we will use the zero-sequence currents consumed by the load with the opposite sign $-\underline{I_{zL0}}$ and the zero-sequence currents consumed by the ZSCC compensator obtained from the values of the reactances according to Equation (25), which we will call $\underline{I_{zW0(25)}}$ for $z = \{a, b, c\}$. If we define the phasor of deviation of the zero-sequence current $\underline{f_{W0}}$ as the quotient between the phasors of both currents, Equation (27) is obtained; where, A is the real part of $\underline{f_{W0}}$ and B is the imaginary part of $\underline{f_{W0}}$. Obviously, $\underline{f_{W0}}$ is identical in each of the phases. When the angles of the line-to-neutral voltages are ±120 degrees out of phase, then A = 1 and B = 0.

$$\underline{f_{W0}} = \frac{-\underline{I_{aL0}}}{\underline{I_{aW0(25)}}} = \frac{-\underline{I_{bL0}}}{\underline{I_{bW0(25)}}} = \frac{-\underline{I_{cL0}}}{\underline{I_{cW0(25)}}} = A + jB \tag{27}$$

where $\underline{I_{aW0(25)}} = \underline{I_{bW0(25)}} = \underline{I_{cW0(25)}}$ is given by Equation (28). Here, $\underline{X_{aW(25)}}$, $\underline{X_{aW(25)}}$ and $\underline{X_{aW(25)}}$ are the ZSCC compensator reactances calculated from Equation (25).

$$\underline{I_{zW0(25)}} = \frac{1}{3}\left[\frac{\underline{V_{an}}}{\underline{X_{aW(25)}}} + \frac{\underline{V_{bn}}}{\underline{X_{bW(25)}}} + \frac{\underline{V_{cn}}}{\underline{X_{cW(25)}}}\right] \tag{28}$$

Performing the same procedure for the positive-sequence current system and for the negative sequence current system consumed by the ZSCC compensator, we define the phasor of deviation of the positive-sequence current $\underline{f_{W+}}$ and the phasor of deviation of the negative-sequence current $\underline{f_{W-}}$ according to Equation (29). These phasors are equal to each other and in each of the phases; where, C is the real part of both phasors and D is the imaginary part. When the angles of the line-to-neutral voltages are ±120 degrees out of phase, then $C = 1$ and $D = 0$.

$$\underline{f_{W+}} = \underline{f_{W-}} = \frac{\underline{I_{zW+}}}{\underline{I_{zW+(25)}}} = \frac{\underline{I_{zW-}}}{\underline{I_{zW-(25)}}} = C + jD \tag{29}$$

Here

$$\underline{I_{aW+(25)}} = \underline{I_{W+(25)}} \qquad \underline{I_{bW+(25)}} = \underline{a}^2\,\underline{I_{W+(25)}} \qquad \underline{I_{bW+(25)}} = \underline{a}\,\underline{I_{W+(25)}} \tag{30}$$

$$\underline{I_{aW-(25)}} = \underline{I_{W-(25)}} \qquad \underline{I_{bW-(25)}} = \underline{a}\,\underline{I_{W-(25)}} \qquad \underline{I_{bW-(25)}} = \underline{a}^2\,\underline{I_{W-(25)}} \tag{31}$$

where

$$\underline{I_{W+(25)}} = \frac{1}{3}\left[\frac{\underline{V_{an}}}{\underline{X_{aW(25)}}} + \frac{\underline{V_{bn}}}{\underline{X_{bW(25)}}}\underline{a} + \frac{\underline{V_{cn}}}{\underline{X_{cW(25)}}}\underline{a}^2\right] \tag{32}$$

$$\underline{I_{W-(25)}} = \frac{1}{3}\left[\frac{\underline{V_{an}}}{\underline{X_{aW(25)}}} + \frac{\underline{V_{bn}}}{\underline{X_{bW(25)}}}\underline{a}^2 + \frac{\underline{V_{cn}}}{\underline{X_{cW(25)}}}\underline{a}\right] \tag{33}$$

If we define the global phasor of deviation of currents $\underline{f_{WG}}$ consumed by the ZSCC compensator as the ratio between any of the phasors ($\underline{f_{W+}}$ or $\underline{f_{W-}}$) and the phasor $\underline{f_{W0}}$, Equation (34) is obtained. It is observed that

- The real part of $\underline{f_{WG}}$ is unity. This is because the values of the actual components of the sequence currents that are obtained with the application of Equation (25) generate an active power of null value. This must be so since the elements that make up the ZSCC compensator are reactive (coils and capacitors). Then the real part of $\underline{f_{WG}}$ must be the unit to maintain this proportion between the currents of different sequences.
- The imaginary part of $\underline{f_{WG}}$ is twice the imaginary part of $\underline{f_{W+}}$ or $\underline{f_{W-}}$. This is because in Equation (25) the imaginary part of the zero-sequence component is multiplied by two, then to maintain the same proportion they must also multiply for two the imaginary parts of $\underline{f_{W+}}$ and $\underline{f_{W-}}$.

$$\underline{f_{WG}} = \frac{\underline{f_{W+}}}{\underline{f_{W0}}} = \frac{\underline{f_{W-}}}{\underline{f_{W0}}} = 1 + \mathrm{j}2D \tag{34}$$

Knowing that the values of $\underline{f_{W+}}$ and $\underline{f_{W-}}$ are calculated from known currents and considering Equation (37), it is easy to determine A and B. Using trigonometric operations, the values of C and D are determined from Equations (35) and (36), respectively.

$$C = A - \frac{2B^2}{(1 - 2A)} \tag{35}$$

$$D = \frac{B}{(1-2A)} \tag{36}$$

Therefore, $\underline{f_{W+}}$ or $\underline{f_{W-}}$ are determined by Equation (37).

$$\underline{f_{W+}} = \underline{f_{W-}} = \left\{A - \frac{2B^2}{(1-2A)}\right\} + \mathrm{j}\left\{2\frac{B}{(1-2A)}\right\} \tag{37}$$

From Equations (37) and (34), we will determine $\underline{I_{zW+}}$ and $\underline{I_{zW-}}$ from Equations (38) and (39). The zero-sequence currents consumed by the ZSCC compensator are the same as the zero sequence currents of the load but with the opposite sign. Therefore, the currents per phase of the optimal solution $\underline{I_{zW}}$ consumed by the ZSCC compensator are given by Equation (40).

$$\underline{I_{zW+}} = \underline{f_{W+}}\,\underline{I_{zW+(25)}} \tag{38}$$

$$\underline{I_{zW-}} = \underline{f_{W-}}\,\underline{I_{zW-(25)}} \tag{39}$$

$$\underline{I_{zW}} = \underline{I_{zW+}} + \underline{I_{zW-}} - \underline{I_{zL0}} \tag{40}$$

From Equation (40) and applying Ohm's law, the values of the ZSCC compensator reactances are determined by Equation (41).

$$\underline{X_{zW}} = \frac{\underline{V_{zn}}}{\underline{I_{zW}}} \tag{41}$$

Considering Equation (24), Equation (41) can be expressed in terms of reactive power according to Equation (42); where, coil for $X_{zW} > 0$ and capacitor for $X_{zW} < 0$.

$$X_{zW} = \frac{Q_{zn}^{zW}}{I_{zW}{}^2} = \frac{Q_{zn}^{zW+} + Q_{zn}^{zW-} - Q_{zn}^{zL0}}{I_{zW}{}^2} = \frac{(V_{zn})^2}{Q_{zn}^{zW+} + Q_{zn}^{zW-} - Q_{zn}^{zL0}} \tag{42}$$

As expected, when the voltages are balanced, Equation (42) coincides with Equation (25), since $Q_{zn}^{zW+} = 0$ and $Q_{zn}^{zW-} = -Q_{zn}^{zL0}$.

Analyzing the active powers and reactive powers once the ZSCC compensator is included, we highlight that:

- The total active system power P consumed by the network is the same with or without the ZSCC compensator. The compensator does not consume active power, therefore, P will always be constant. Since the compensator compensates for zero-sequence currents, the value of the zero-sequence active power P_0 is equal to zero. Therefore, the values of P_+ and P_- will be modified so that P is constant.
- The total reactive power of the system Q will change its value. Now the zero-sequence reactive power will be zero, $Q_0 = 0$; on the other hand, the values of Q_+ and Q_- will be different from the initial ones. Depending on the characteristics of the system, the power factor of the system will improve or worsen.

In conclusion, to design the ZSCC compensator, it is only necessary to know the values of the line-to-neutral voltages and the zero-sequence current in the bus.

In the next section, we will use the SVC compensator to compensate for the positive-sequence reactive currents and the NSCC compensator to compensate for the negative-sequence currents.

3.3. Analysis and Application of SVC Compensator and NSCC Compensator to a Four-Wire System

As mentioned in the previous sections, by including the ZSCC compensator we know that:

- When the angles of the line-to-neutral voltages are offset ±120 degrees from each other, the ZSCC compensator consumes a negative-sequence current, in addition to the zero-sequence current.

- When the angles of the line-to-neutral voltages are not offset ±120 degrees from each other, the ZSCC compensator consumes a positive-sequence current and a negative-sequence current, in addition to the zero-sequence current.

Since the ZSCC compensator compensates for the zero-sequence currents of the load, the line currents in the bus were modified and consist only of positive-sequence currents and negative-sequence currents. In both cases, the new line currents will be the result of adding the currents consumed by the load and the currents consumed by the ZSCC compensator.

These changes must be taken into account in the calculation expressions of the SVC and NSCC compensators, since we must consider the set formed by the load plus the ZSCC compensator instead of the initial load. Therefore, the sequence of calculation of the compensators is important to obtain a total compensation of the inefficient currents. Thus, the ZSCC compensator must be designed first, followed by the SVC compensator and then the NSCC compensator.

From Equation (43), the values of the SVC compensator reactances are obtained, where Q_{zP+} is the positive-sequence reactive power consumed by the SVC compensator and is given by Equation (44). From Equation (45), the values of the reactances of the NSCC compensator are obtained, where Q^{-}_{zF+} is the unbalanced reactive power consumed by the NSCC compensator caused by the positive-sequence voltage $\underline{V_{z+}}$ and $\underline{I_{zF-}}$ which is the negative-sequence current which consumes the NSCC compensator. $\underline{I_{zF-}}$ is determined by Equation (46) and calculated from the sum of the negative-sequence currents of load $\underline{I_{zL-}}$, the ZSCC compensator $\underline{I_{zW-}}$ and the SVC compensator $\underline{I_{zP-}}$.

$$X_{zP} = \frac{{V_{z+}}^2}{Q_{zP+}} \tag{43}$$

$$Q_{zP+} = \frac{(Q_{zL-} + Q_{zW-}) - (Q_{zL+} + Q_{zW+})}{1 - \delta_-^2} \qquad \text{where } \delta_- = \frac{V_-}{V_+} \tag{44}$$

$$X_{zF} = -2\frac{Q^{-}_{zF+}}{{I_{zF-}}^2} \tag{45}$$

$$\underline{I_{zF-}} = \underline{I_{zL-}} + \underline{I_{zW-}} + \underline{I_{zP-}} \tag{46}$$

In the same way as in another work [14], using the Kennelly–Rosen transformation, the compensators (SVC and NSCC) can be joined in a single delta connected compensator. Figure 7 shows our compensation proposal using this delta connected compensator.

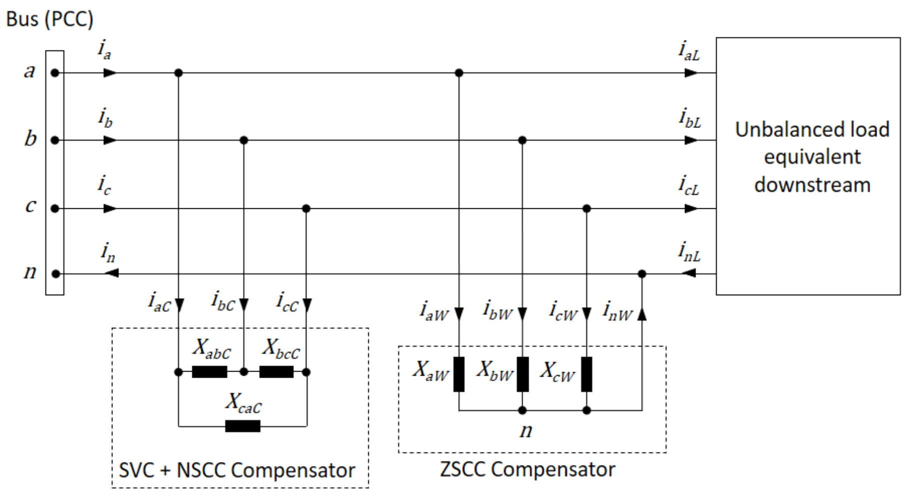

Figure 7. Unified proposal with compensator "SVC + NSCC" connected in delta. SVC = positive-sequence reactive component; NSCC = negative-sequence currents.

4. Practical Application

In this section, a practical case study to verify all of the concepts discussed in the previous sections is developed. Consider the scheme in Figure 8 that shows an unbalanced four-wire three-phase linear system connected to an infinite short-circuit power network with unbalanced voltages. The load is modeled at a constant impedance and its values are indicated in the same figure. The voltages are unbalanced and sinusoidal in PCC (Point of Common Coupling), in which

$$\underline{V_{an}} = 231.00\ e^{j0} \qquad \underline{V_{bn}} = 240.00\ e^{-j110} \qquad \underline{V_{cn}} = 195.00\ e^{j90}$$

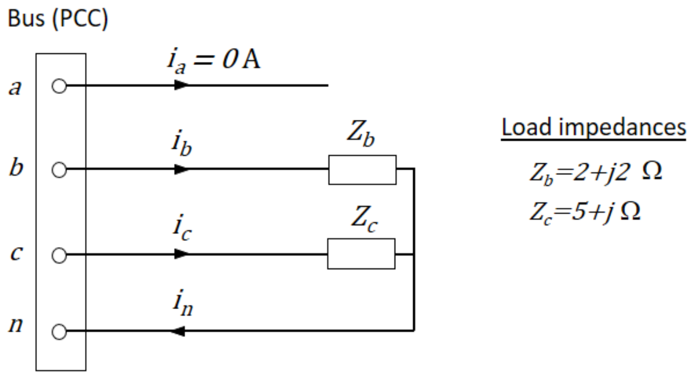

Figure 8. Proposed system for practical application.

Using the 'PSPICE V.9.2' analysis software, the currents circulating in the load are obtained, in which

$$\underline{I_a} = 0.00\ e^{-j0.00} \qquad \underline{I_b} = 84.853\ e^{-j155.00} \qquad \underline{I_c} = 38.243\ e^{j76.69}$$

Figure 9 displays the waveform of line currents in each phase obtained from the simulation of the system of Figure 8.

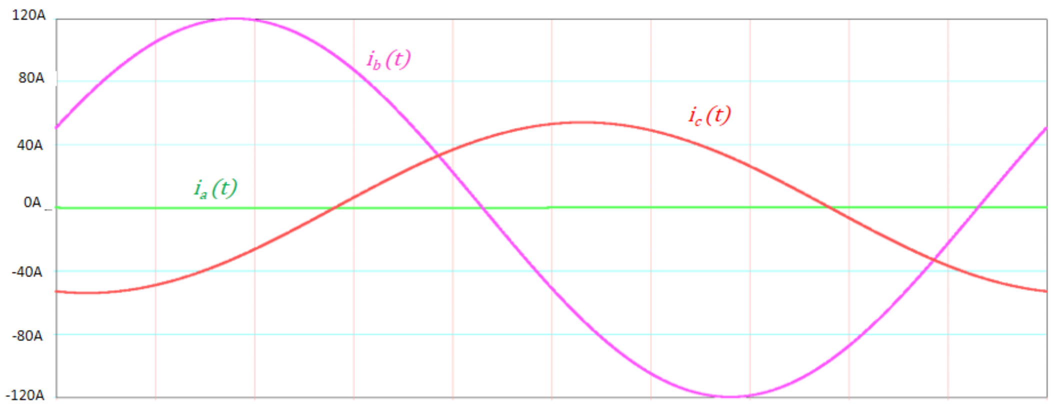

Figure 9. Line currents before compensation in PCC.

Considering the values of line-to-neutral voltages and line currents, Tables 1 and 2 show these values in symmetric components. These values have been determined from the Fortescue transformation matrix.

Table 1. Positive-, negative-, and zero-sequence voltages.

Phases	V_{zn+} (V)		V_{zn-} (V)		V_{zn0} (V)	
z	**Mod.**	**Ang.**	**Mod.**	**Ang.**	**Mod.**	**Ang.**
a	212.891	−5.014	42.094	136.859	50.671	−11.585
b	212.891	−125.014	42.094	−103.141	50.671	−11.585
c	212.891	114.986	42.094	16.859	50.671	−11.585

Table 2. Positive-, negative-, and zero-sequence line currents.

Phases	I_{z+} (A)		I_{z-} (A)		I_{z0} (A)	
z	**Mod.**	**Ang.**	**Mod.**	**Ang.**	**Mod.**	**Ang.**
a	40.979	−36.959	25.938	111.747	23.141	178.647
b	40.979	−156.959	25.938	−128.253	23.141	178.647
c	40.979	83.041	25.938	−8.253	23.141	178.647

The total apparent power of Buchholz that supplied the network in the bus was determined by Equation (2) and its value is as follows:

$$S_T = 3\sqrt{(212.891^2 + 42.094^2 + 50.67^2)\,(40.979^2 + 25.938^2 + 23.141^2)} = 35924.80 \text{ VA}$$

Then, we proceeded to calculate the reactance values in each phase of the ZSCC, the SVC and NSCC compensators as per the scheme of Figure 2. As described in the previous sections, it is very important to calculate the compensators in the following order: first the ZSCC compensator, then the SVC compensator, and finally the NSCC compensator.

4.1. Calculation of the ZSCC Compensator

The unbalanced reactive powers that are caused by line-to-neutral voltages and zero-sequence currents in the load are given by:

$$Q_{zn}^{zL0} = V_{zn} I_{zL0}\ \sin(\alpha_{zn} - \beta_{zL0})$$

For $z = \{a, b, c\}$, their values are as follows:

$$Q_{an}^{aL0} = -126.253 \text{ var } Q_{bn}^{bL0} = 5262.245 \text{ var } Q_{cn}^{cL0} = -4511.18 \text{ var}$$

Substituting these values in Equation (25), $X_{aW(25)}$, $X_{bW(25)}$ and $X_{cW(25)}$ were obtained as follows:

$$X_{aW(25)} = 211.325\ \Omega\ \underline{X_{aW(25)}} = 211.325e^{j90.00}\ \Omega$$
$$X_{bW(25)} = -5.473\ \Omega\ \underline{X_{bW(25)}} = 5.473e^{-j90.00}\ \Omega$$
$$X_{cW(25)} = 4.215\ \Omega\ \underline{X_{cW(25)}} = 4.215e^{j90.00}\ \Omega$$

Using Equation (28), we determined the zero sequence current $\underline{I_{zW0(25)}}$ given by:

$$\underline{I_{aW0(25)}} = \underline{I_{bW0(25)}} = \underline{I_{cW0(25)}} = 29.648e^{-j10.423} \text{ A}$$

Comparing these values with those shown in Table 2, it is observed that this zero-sequence current does not coincide with the zero-sequence current of the load with the opposite sign. This is because the line-to-neutral voltages are not offset ±120 degrees from each other, as indicated in Section 3.2. Thus, considering Equation (27), the phasor of deviation of the zero-sequence current $\underline{f_{W0}}$ is given by:

$$\underline{f_{W0}} = 0.771 + \mathrm{j}0.123$$

Substituting $\underline{f_{W0}}$ in Equation (37), $\underline{f_{W+}}$ and $\underline{f_{W-}}$ are given by:

$$\underline{f_{W+}} = \underline{f_{W-}} = 0.827 - \mathrm{j}0.227$$

Therefore, considering Equations (38)–(40), the currents consumed by the ZSCC compensator are as follows:

$$\underline{I_{aW}} = 9.61e^{j90.00}\ \mathrm{A}\ \underline{I_{bW}} = 32.889e^{-j20.00}\ \mathrm{A}\ \underline{I_{cW}} = 38.497e^{j0.00}\ \mathrm{A}$$

Table 3 shows currents $\underline{I_{aW}}$, $\underline{I_{bW}}$ and $\underline{I_{cW}}$ in sequence values. Positive- and negative-sequence currents are necessary in subsequent sections to calculate SVC and NSCC compensators. As expected, the zero-sequence current is equal to the zero-sequence current of the load with the opposite sign.

Table 3. Positive-, negative-, and zero-sequence currents that consume the ZSCC compensator.

Phases	I_{zW+} (A)		I_{zW-} (A)		I_{zW0} (A)	
z	**Mod.**	**Ang.**	**Mod.**	**Ang.**	**Mod.**	**Ang.**
a	8.806	160.868	16.502	153.863	23.141	−1.353
b	8.806	40.868	16.502	−86.137	23.141	−1.353
c	8.806	−79.132	16.502	33.863	23.141	−1.353

Substituting the values of $\underline{I_{aW}}$, $\underline{I_{bW}}$ and $\underline{I_{cW}}$ in Equation (24), the values of reactances in the ZSCC compensator were obtained:

$$X_{aW} = \frac{231.00e^{j0.00}}{9.61e^{j90.00}} = -24.04\ \Omega,$$

$$X_{bW} = \frac{240.00e^{-j110.00}}{32.889e^{-j20.00}} = -7.297\ \Omega,$$

$$X_{cW} = \frac{195.00e^{j90.00}}{38.497e^{j0.00}} = 5.065\ \Omega,$$

From their sign, X_{aW} and X_{bW} are capacitors and X_{cW} is a coil.

Adding the line currents that consume the load and the ZSCC compensator, the new line currents supplied by the network in the bus were obtained after including the ZSCC compensator. Their values are given by:

$$\underline{I_a} + \underline{I_{aW}} = 9.609e^{j90.00}\ \mathrm{A}\ \underline{I_b} + \underline{I_{bW}} = 65.841e^{-j134.316}\ \mathrm{A}\ \underline{I_c} + \underline{I_{cW}} = 59.346e^{j39.189}\ \mathrm{A}$$

Table 4 shows these currents expressed in symmetric components. It is observed that the zero-sequence current is null, therefore, the current in the neutral wire is also null.

Table 4. Positive-, negative-, and zero-sequence line currents in the bus after ZSCC collocation.

Phases	I_{z+} (A)		I_{z-} (A)		I_{z0} (A)	
z	**Mod.**	**Ang.**	**Mod.**	**Ang.**	**Mod.**	**Ang.**
a	32.706	−41.688	39.75	127.912	0.00	−26.565
b	32.706	−161.688	39.75	−112.088	0.00	−26.565
c	32.706	78.312	39.75	7.912	0.00	−26.565

Table 5 compares the values of the active and reactive powers, before and after using the ZSCC compensator. As expected, the active power P remains constant and P_0 and Q_0 are null.

Table 5. Comparison of powers, before and after including the ZSCC compensator.

	P	Q	P_+	Q_+	P_-	Q_-	P_0	Q_0
Compen.	**(W)**	**(var)**	**(W)**	**(var)**	**(W)**	**(var)**	**(W)**	**(var)**
Before	21,712.50	15826.50	22,208.37	13,847.63	2965.86	1390.06	−3461.73	624.81
After	21,712.50	13256.39	16,753.88	12,475.74	4958.62	780.65	0	0

Considering Equation (2), Buchholz's apparent power in the bus was determined by:

$$S_T = 3\sqrt{\left(212.891^2 + 42.094^2 + 50.67^2\right)\left(32.706^2 + 39.75^2 + 0.00^2\right)} = 34414.22 \text{ VA}$$

If we compare this value of S_T with the initial value, it is observed that it is slightly lower.

4.2. Star-Connected SVC Compensator Calculation

The unbalanced factor δ_- is given by:

$$\delta_- = \frac{V_-}{V_+} = 0.1977$$

The reactive powers Q_{zL+} and Q_{zL-} in the load are determined by the following expressions:

$$Q_{zL+} = V_{zn+} I_{zL+} \sin(\propto_{zn+} - \beta_{zL+}) = Q_{aL+} = Q_{bL+} = Q_{cL+}$$

$$Q_{zL-} = V_{zn-} I_{zL-} \sin(\propto_{zn-} - \beta_{zL-}) = Q_{aL-} = Q_{bL-} = Q_{cL-}$$

If we substitute the values of Tables 1 and 2 in the above expressions, we obtain:

$$Q_{aL+} = Q_{bL+} = Q_{cL+} = 4615.876 \text{ var}$$

$$Q_{aL-} = Q_{bL-} = Q_{cL-} = 463.353 \text{ var}$$

The reactive powers Q_{zW+} and Q_{zW-} in the ZSCC are determined by the following expressions:

$$Q_{zW+} = V_{zn+} I_{zW+} \sin(\propto_{zn+} - \beta_{zW+}) = Q_{aW+} = Q_{bW+} = Q_{cW+}$$

$$Q_{zW-} = V_{zn-} I_{zW-} \sin(\propto_{zn-} - \beta_{zW-}) = Q_{aW-} = Q_{bW-} = Q_{cW-}$$

If we substitute the values of Tables 2 and 4 in the above expressions, we obtain:

$$Q_{aW+} = Q_{bW+} = Q_{cW+} = -457.297 \text{ var.}$$

$$Q_{aW-} = Q_{bW-} = Q_{cW-} = -203.135 \text{ var.}$$

Equation (44) determines the positive-sequence reactive power, which the SVC compensator consumes, and its value is as follows:

$$Q_{zP+} = \frac{(463.353 - 203.135) - (4615.876 - 457.297)}{1 - 0.1977^2} = -4056.968 \text{ var.}$$

Considering Q_{zP+} in Equation (43), the reactances that the SVC compensator must have are determined by:

$$X_P = X_{aP} = X_{bP} = X_{cP} = \frac{212.891^2}{-4056.968} = -11.172\ \Omega.$$

Therefore, $\underline{X_P} = 11.172e^{-j90.00}$ Ω.

Considering the line-to-line voltages V_{zz}, where $zz = \{ab, bc, ca\}$, and applying the mesh currents method, we determined the currents consumed by the SVC compensator:

$$\underline{I_{aP}} = 16.26e^{j93.211} \text{ A } \underline{I_{bP}} = 22.597e^{-j31.453} \text{ A } \underline{I_{cP}} = 18.986e^{-j166.40} \text{ A}$$

Therefore, the new line currents supplied by the network in the bus after including the ZSCC and SVC compensators are determined by:

$$\begin{aligned} \underline{I_a} + \underline{I_{aW}} + \underline{I_{aP}} &= 25.859e^{-j5.014} \text{ A} \\ \underline{I_b} + \underline{I_{bW}} + \underline{I_{bP}} &= 64.678e^{-j125.014} \text{ A} \\ \underline{I_c} + \underline{I_{cW}} + \underline{I_{cP}} &= 43.084e^{j114.986} \text{ A} \end{aligned}$$

Table 6 shows these currents expressed in symmetric components.

Table 6. Positive-, negative-, and zero-sequence line currents in the bus after ZSCC and SVC collocation.

Phases	I_{z+} **(A)**		I_{z-} **(A)**		I_{z0} **(A)**	
z	**Mod.**	**Ang.**	**Mod.**	**Ang.**	**Mod.**	**Ang.**
a	26.237	−6.057	39.341	133.341	0.00	0.00
b	26.237	−126.057	39.341	−106.659	0.00	0.00
c	26.237	113.943	39.341	13.341	0.00	0.00

Considering Equation (2), the total apparent power of Buchholz that supplies the network in the bus is given by:

$$S_T = 3\sqrt{(212.891^2 + 42.094^2 + 50.67^2)\ (26.237^2 + 39.341^2 + 0.00^2)} = 31613.58 \text{ VA}$$

Comparing this value of S_T with the previous values, it is observed that its value continues to decrease.

4.3. Star-Connected NSCC Compensator Calculation

The values of Q_{zF+}^{-} are determined by the following expressions:

$$Q_{zF+}^{-} = V_{zn+} I_{zF-} \sin(\propto_{zn+} - \beta_{zF-})$$

If we substitute the values of Tables 1 and 6 in the above expressions, we obtain:

$$Q_{aF+}^{-} = -5565.414 \qquad Q_{bF+}^{-} = -2637.477 \qquad Q_{cF+}^{-} = 8202.891$$

Substituting the values of $\underline{I_{zF-}}$ and Q_{zF+}^{-} in Equation (45), the values of X_{zF} are given by:

$$X_{aF} = -2\frac{-5565.414}{39.341^2} = 7.192\ \Omega \qquad \underline{X_{aF}} = 7.192e^{j90.00}\ \Omega$$

$$X_{bF} = -2\frac{-2637.477}{39.341^2} = 3.408\ \Omega \qquad \underline{X_{bF}} = 3.408e^{j90.00}\ \Omega$$

$$X_{cF} = -2\frac{8202.891}{39.341^2} = -10.60\ \Omega \qquad \underline{X_{cF}} = 10.60e^{-j90.00}\ \Omega$$

Therefore, X_{aF} is a capacitor, X_{bF} is a coil, and X_{cF} is a capacitor.

Considering the line-to-line voltages V_{zz}, where $zz = \{ab, bc, ca\}$, and applying the mesh currents method, we determined the currents consumed by the NSCC compensator:

$$\underline{I_{aF}} = 45.163e^{-j39.644}\ \text{A}\ \underline{I_{bF}} = 31.884e^{j76.923}\ \text{A}\ \underline{I_{cF}} = 42.051e^{-j176.944}\ \text{A}$$

Therefore, the new line currents supplied by the network in the bus after including the ZSCC, SVC, and NSCC compensators are determined by:

$$\underline{I_a} + \underline{I_{aW}} + \underline{I_{aP}} + \underline{I_{aF}} = 33.996e^{-j5.014}\ \text{A}$$
$$\underline{I_b} + \underline{I_{bW}} + \underline{I_{bP}} + \underline{I_{bF}} = 33.996e^{-j125.014}\ \text{A}$$
$$\underline{I_c} + \underline{I_{cW}} + \underline{I_{cP}} + \underline{I_{cF}} = 33.996e^{j114.986}\ \text{A}$$

Figure 10 displays the waveform of the line currents in each phase obtained from the simulation after include compensation.

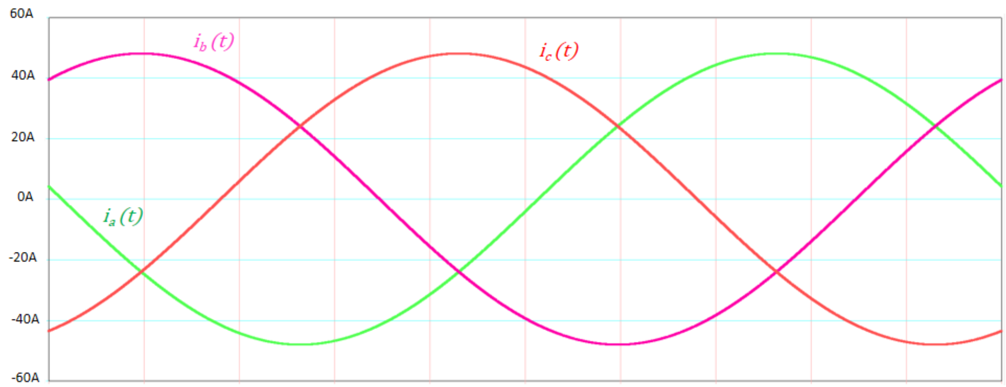

Figure 10. Line currents after compensation in PCC.

It is observed that after compensation we obtained a system of balanced line currents. These same results can be obtained by joining the SVC and NSCC compensators in delta connection according to Figure 7. Using the star-delta transformations, the reactance values of the compensator "NSCC + SVC" were obtained as follows:

$$X_{abC} = 11.011\ \Omega \qquad \underline{X_{abC}} = 11.011e^{j90.00}\ \Omega$$
$$X_{bcC} = -8.952\ \Omega \qquad \underline{X_{bcC}} = 8.952e^{-j90.00}\ \Omega$$
$$X_{caC} = -14.57\ \Omega \qquad \underline{X_{caC}} = 14.57e^{-j90.00}\ \Omega$$

Table 7 compares the values of active and reactive powers before and after compensation. It is observed that the network supplies a positive-sequence active power equal to the active power that the load demands. This achieves maximum efficiency in the transfer of electrical energy between the network and the load in the bus.

Table 7. Comparison of powers, before and after total compensation.

	P	Q	P_+	Q_+	P_-	Q_-	P_0	Q_0
Compen.	**(W)**	**(var)**	**(W)**	**(var)**	**(W)**	**(var)**	**(W)**	**(var)**
Before	21,712.50	15,826.50	22,208.37	13,847.63	2965.86	1390.06	−3461.73	624.81
After	21,712.50	0	21,712.50	0	0	0	0	0

Considering Equation (2), the total apparent power of Buchholz that supplies the network in the bus is given by:

$$S_T = 3\sqrt{(212.891^2 + 42.094^2 + 50.67^2)\,(34.996^2 + 0.00^2 + 0.00^2)} = 22728.17 \text{ VA}$$

If we compare the final value of S_T with the previous values, it is observed that the total apparent power is practically equal to the total active power and the positive-sequence active power. This small difference is due to the negative- and zero-sequence voltages that are imposed by the infinite short-circuit power network.

Table 8 compares the results of the line currents that have been obtained from our proposal with other existing methods and that have been cited in this work. It is observed that the line currents that circulate in the bus or PCC after compensation, are only balanced when the proposed compensation method is used. In addition, it can be seen that these currents are in phase with the positive-sequence voltage (see Table 1). This implies that its sum is zero. Then, the negative-sequence and zero-sequence currents and the reactive component of the positive-sequence current are removed. In the rest of the methods, these line currents are still unbalanced, therefore they do not compensate for inefficient currents. Obviously, these results are logical since the expressions used in each of the other methods are only valid for three-phase systems with balanced voltages.

Table 8. Comparison of the proposed method with other existing methods.

Method	I_a (A)		I_b (A)		I_c (A)	
	Mod.	**Ang.**	**Mod.**	**Ang.**	**Mod.**	**Ang.**
Proposed method	33.996	−5.014	33.996	−125.014	33.996	114.986
Classical method	36.438	7.389	37.814	−115.626	29.150	49.677
Leon [19]	39.527	−4.442	43.804	−102.403	11.226	91.381
Pana [23]	37.212	−0.121	32.645	−101.964	28.015	78.780

Table 9 compares the values of the line currents of the system in Figure 8, considering that the voltages in the bus are balanced, where the RMS value of the voltage is V = 230 V. Under these conditions, it is observed that the resulting line currents system is balanced with any of the methods used, including the method proposed by us. Therefore, it is demonstrated that the proposed compensation method is valid for three-phase systems with balanced and unbalanced voltages.

Table 9. Comparison of the proposed method with other methods with unbalanced voltages.

Method	I_a (A)		I_b (A)		I_c (A)	
	Mod.	**Ang.**	**Mod.**	**Ang.**	**Mod.**	**Ang.**
Proposed method	33.910	0	33.910	−120	33.910	120
Classical method	33.910	0	33.910	−120	33.910	120
Leon [19]	33.910	0	33.910	−120	33.910	120
Pana [23]	33.910	0	33.910	−120	33.910	120

5. Conclusions

Electrical systems with unbalanced loads and voltages give rise to inefficient currents: negative-sequence currents, zero-sequence currents, and positive-sequence reactive currents. This produces inefficient powers, which causes an increase in the total apparent power that the generator or network must supply in the bus.

In this work, a three-phase four-wire system connected to an infinite short-circuit power network with unbalanced voltages and unbalanced currents was considered. A methodology for compensating zero-sequence currents through the parallel connection of a compensating circuit (ZSCC) was developed.

This compensator is formed by reactances. A simple analytical equation is provided to determine these elements. This ensures that the network does not provide a zero sequence current in the bus. That is, no current will flow through the neutral wire.

With this compensator (ZSCC), together with those developed by the authors in a previous work for three-wire systems, the negative sequence current compensator (NSCC), and the positive sequence reactive current compensator (SVC), we have formulated a procedure for the total compensation of inefficient currents produced by the unbalanced load. This achieves maximum efficiency in the transfer of electrical energy between the network and the downstream load of the bus.

It becomes clear that the active power consumed by the load remains constant, while the reactive power has a null value. The value of Buchholz's total apparent power is greatly diminished in the bus. Only the inefficiencies due to the negative- and zero-sequence voltages that are not compensable efficiently (since they are inherent in the network) will remain.

This is achieved without knowledge of the nature and characteristics of the load, only of the voltages and currents measured in the bus.

The proposed equations can be used with active or hybrid filters in order to overcome the inconvenience of passive filters. These are only useful in installations with a fairly stationary consumption which need a staggering of the compensation reactances just like the current SVC.

Finally, a case study is developed and compared with three of the methods analyzed to show the simplicity of the application of the proposed compensation methodology. The results obtained in each step and the final result are analyzed. This shows that the classic models or the capacitors-based model are not intended to work correctly in three-phase four-wire systems with unbalanced voltages.

On the other hand, the model proposed in this article does respond correctly. In addition, it is not affected by the type and characteristics of the load connected downstream or those connected upstream. This is because the model presented is adaptable, at all times, to the voltages and currents that the network presents in the PCC. This validates the proposed compensation model, highlighting the contributions of this work.

From the results obtained in this work, it is worth highlighting, as future research lines, the extension of the model to non-linear systems and the integration with active or hybrid filters in order to overcome the inconveniences of passive filters.

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Nomenclature

The following nomenclature is used in this manuscript:

f_{WG}	global phasor of deviation of currents
f_{W+}	phasor of deviation of the positive sequence current
f_{W-}	phasor of deviation of the negative sequence current
f_{W0}	phasor of deviation of the zero sequence current
I_z	line current in each phase, $z = \{a, b, c\}$
I_+	positive sequence current
I_-	negative sequence current
I_0	zero sequence current

I_{zL} current consumed by the load in each phase, $z = \{a, b, c\}$
I_{zL+} positive sequence current consumed by the load in each phase, $z = \{a, b, c\}$
I_{zL-} negative sequence current consumed by the load in each phase, $z = \{a, b, c\}$
I_{zL0} zero sequence current consumed by the load in each phase, $z = \{a, b, c\}$
I_{zW} current consumed by the ZSCC compensator in each phase, $z = \{a, b, c\}$
I_{zW+} positive sequence current consumed by the ZSCC compensator in each phase, $z = \{a, b, c\}$
I_{zW-} negative sequence current consumed by the ZSCC compensator in each phase, $z = \{a, b, c\}$
I_{zW0} zero sequence current consumed by the ZSCC compensator in each phase, $z = \{a, b, c\}$
I_{zF-} negative sequence current consumed by the NSCC compensator in each phase, $z = \{a, b, c\}$
I_{zP-} negative sequence current consumed by the SVC compensator in each phase, $z = \{a, b, c\}$
P active power
P_+ positive sequence active power
P_- negative sequence active power
P_0 zero sequence active power
Q reactive power
Q_+ positive sequence reactive power
Q_- negative sequence reactive power
Q_0 zero sequence reactive power
Q_{zL+} positive sequence reactive power of the load in each phase, $z = \{a, b, c\}$
Q_{zL-} negative sequence reactive power of the load in each phase, $z = \{a, b, c\}$
Q_{zW+} positive sequence reactive power of the ZSCC compensator in each phase, $z = \{a, b, c\}$
Q_{zW-} negative sequence reactive power of the ZSCC compensator in each phase, $z = \{a, b, c\}$
Q_{zF+} positive sequence reactive power of the NSCC compensator in each phase, $z = \{a, b, c\}$
Q_{zF-} negative sequence reactive power of the NSCC compensator in each phase, $z = \{a, b, c\}$
Q_{zP+} positive sequence reactive power of the SVC compensator in each phase, $z = \{a, b, c\}$
Q_{zn}^{zL} reactive power of the load caused by the line-to-neutral voltage and the current in each phase, $z = \{a, b, c\}$
Q_{zn}^{zL0} reactive power of the load caused by the line-to-neutral voltage and the zero sequence current in each phase, $z = \{a, b, c\}$
Q_{zn}^{zW} reactive power of the ZSCC compensator caused by the line-to-neutral voltage and the current in each phase, $z = \{a, b, c\}$
Q_{zn}^{zW+} reactive power of the ZSCC compensator caused by the line-to-neutral voltage and the positive sequence current in each phase, $z = \{a, b, c\}$
Q_{zn}^{zW-} reactive power of the ZSCC compensator caused by the line-to-neutral voltage and the negative sequence current in each phase, $z = \{a, b, c\}$
Q_{zF+}^{-} unbalanced reactive power of the NSCC compensator caused by the positive sequence line-to-neutral voltage and the negative sequence current in each phase, $z = \{a, b, c\}$
S apparent power in the classical theories
S_z apparent power in the classical theories in each phase, $z = \{a, b, c\}$
S_T apparent power of Buchholz
S_+^- apparent power due to positive sequence voltage and the negative sequence current
S_+^0 apparent power due to positive sequence voltage and the zero sequence current
S_-^+ apparent power due to negative sequence voltage and the positive sequence current
S_-^0 apparent power due to negative sequence voltage and the zero sequence current
S_0^+ apparent power due to zero sequence voltage and the positive sequence current
S_0^- apparent power due to by zero sequence voltage and the negative sequence current
V_{zn} line-to-neutral voltage in each phase, $z = \{a, b, c\}$
V_{z+} positive sequence line-to-neutral voltage in each phase, $z = \{a, b, c\}$
X_{zW} ZSCC compensator reactance in each phase, $z = \{a, b, c\}$
X_{zF} NSCC compensator reactance in each phase, $z = \{a, b, c\}$
X_{zP} SVC compensator reactance in each phase, $z = \{a, b, c\}$

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