Triangulation Network of 1929-1944 of the First 1:500 Urban Map of València

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Abstract

Triangulation is a surveying method on which earlier maps made were based. Although the origins of the method can be traced back to the 16^{th} century, it is still used today, with minor changes, to adjust networks observed with modern geodetic techniques. In this paper we present the geodetic survey work that was carried out for the primary triangulation network of the first 1:500 urban map of the city of València (Spain). It spanned from 1929 to 1944 and resulted in 421 maps covering about 174 square kilometres. We focus on four key elements to define the geometric framework of a map: (1) the geodetic network, (2) the cartographic projection, (3) the baseline measurements, and (4) the primary triangulation. The paper is based on the interpretation of original documents and field books recovered from the archives of the València City Council. In order to check the accuracy and consistency of the survey work, we recomputed all calculations directly from the field data, following the mathematical procedures of the time. We obtained a set of transformation parameters to convert the coordinates of 1929 to current coordinates based on the European Terrestrial Reference System of 1989 (ETRS89). Results showed that the 1929 primary triangulation angles and coordinates are accurate to 8” and 35 cm respectively, and that the coordinates transform well into the current reference system with average residuals of 26 cm across nine control points, demonstrating the high quality of the 1929 work.

Keywords: urban mapping, triangulation, cartographic heritage, quality control, geodetic surveying, ETRS89

1. Introduction

In 1929, the València City Council commissioned the Instituto Geográfico y Catastral (IGC), the former Spanish National Mapping Agency, to make the first accurate urban map at a scale of 1:500. This was quite a challenging technical endeavour at the time. The project took 15 years to complete, with an intervening civil war. València is located on the Mediterranean coast of Spain next to the mouth of the river Turia (Fig. 1) and has been historically the third city in Spain, temporarily becoming the capital of the country during the civil war. In the beginning of the 20th century, social and economic forces demanded further signs of development and modernity that had already been started with the celebration of the Regional Expo in 1909 and the inauguration of the new railway station in 1917. One key element that had to support those developments was a new urban map of the city, whose backbone was the triangulation network.

[Figure 1 near here]

The first use of the triangulation method in geodetic networks is usually attributed to the Dutch astronomer Willebrord Snel van Royen (Haasbroek 1968, Murdin 2009, Shank 2012). Snel, sometimes spelled as Snell after his latinised name Snellius, carried out a triangulation in 1615 with the purpose of finding the diameter of the earth. However, there are earlier references that include either theoretical definitions or practical applications of the method of triangulation. Indeed, it seems that Snel learned this method from publications by the Dutch cartographer and mathematician Gemma Frisius (Haasbroek 1968, Hewitt 2011), among whose students was the great cartographer Gerardus Mercator. The main reference by Frisius regarding triangulation was *Cosmographia Petri Apiani*, published in 1533, which contained an appendix defining the method of triangulation.

Much less known are the early works of the Spaniard Jerónimo Muñoz who used a triangulation sketch on the Valencian coast (Fig. 2) in his university lectures. It is thought that this graphical triangulation, made in 1568 without the help of trigonometric or logarithmic calculations, and known afterwards through Snel, was the basis for the creation of the Ortelian Valencian Map in 1585. It is considered a modern map with correct geodetic references, at the same technological level as other contemporary maps from nearby European countries (Navarro 2004, Roselló 2000, Roselló 2008).

In 1929 the method of triangulation had evolved since its inception four centuries earlier, but was still in the pre-computer era. This means that computations had to be done by manual methods, with the use of logarithm tables, thus eliminating any possibility of automation. The standard procedure was to establish two baselines at the limits of the surveyed area. Once the baselines were measured and orientated, a chain of triangles, the triangulation network proper, had to be designed to connect both baselines. After computing all triangles from the starting baseline, the computed length and azimuth for the closing baseline were available. The comparison of the measured and the computed values of this baseline determined the quality of the network and, most importantly, the scale and the orientation of the resulting map.

[Figure 2 near here]

It should be noted that the map obtained from the network of 1929 was a high class product of its time. This map was routinely used for urban planning and public information purposes until the 1990s, and even later, in several city council services. The 1929 map is still the most reliable source for graphical information on older real estate properties, roads, railways, sewer lines and other public facilities.

Therefore, this map must be considered as a valuable cartographic heritage item to be preserved. It should be made available to interested users, including general public users as well as researchers. In fact, urban development studies are potential candidates to extract new insights from such documents (Gatta 2010). In this line of research, the International Cartographic Association (ICA) created in 2007 the Commission on

Digital Technologies in Cartographic Heritage, whose aim is to encourage digital approaches to cartographic heritage (Bitelli et al. 2014).

The main purpose of this paper is to report on field and mathematical procedures that defined the geodetic triangulation network of the map. A second, though not less important, goal of the paper is to keep a record of the standard surveying procedures existing in the first half of the 20th century, and preserve that information for future generations of cartographers and scholars.

2. Description and analysis of the triangulation

In this section we provide the basic elements, formulas and terminology to conveniently follow the calculations given below. Triangulation has been used to define national mapping programmes in many countries (Ogilvie 1921, Adams 1940, Culley 1940, Staack 1940, Schofield & Breach 2007) using a hierarchical structure based on a primary network that is densified into several lower order networks, typically until the third or fourth order (Blachut et al. 1979). The network discussed herein must be considered as a fourth order or local network according to this classic approach. Modern techniques, especially space geodesy, have made this approach obsolete.

Triangulation is a well-known surveying technique that relies on the measurement of the inner angles of a triangle network with the aim of determining the distances between the stations by trigonometry. This method requires that one or more baselines with known lengths and azimuths be measured separately to define the scale and orientation of the network (Gorse et al. 2012). In the end, the triangulation method provides spatial locations for every station in a plane coordinate system. Moreover, the three interior angles in each triangle allow for the checking of measurement errors (Brinker & Minnick 1987). In summary, a triangulation project requires a number of interconnected triangles covering the mapping area whose angles are observed using typical surveying equipment.

2.1. Coordinate reference system

The lack of standards in 1929 led each country to adopt different local coordinate reference systems. In Spain, there were indeed several coordinate systems that were used simultaneously. Although the International Meridian Conference that defined Greenwich as the standard prime meridian had been held in 1884 in Washington, the use of a different prime meridian in each country was still common practice. In Spain, the geographic coordinates of the national geodetic network were computed using the so-called Madrid datum (Mugnier 2000), which was based on the prime meridian defined in the Madrid Astronomical Observatory (3º 41´15.45´´ west of Greenwich) and the Struve ellipsoid (IGC, 1928). The fieldwork reported in this paper was still based on the Spanish datum, even though the Greenwich meridian had officially been adopted in Spain in 1901.

In geodetic terms, an urban network is a lower order (4th order) or local network that needs to be geometrically connected to higher order networks to be consistent with national reference systems. Such a connection is achieved by including several high order stations in the urban network design, observation and computation. According to the documentation located for our study, the connecting network, which was executed prior to 1929, comprised 12 stations (Fig. 3), two of which (Miguelete and Almàcera) were also used in the urban network.

We did not find specific information on the geodetic coordinate reference system used in the project, although it was not difficult to guess. The hint was a listing containing three geodetic stations located outside of the working area with geographical and plane coordinates. We assumed that the geographical coordinates were in the

Madrid datum and tried several projections. It turned out that the projected coordinates were computed using the Tissot projection (Cebrián & Los Arcos 1895, Tissot 1881) still used in 1929.

It is not clear why this connecting network (see Fig. 3) was included in the files of the 1929 map. In theory, those networks were intended to transfer the coordinate reference system (specifically origin, scale and orientation) from the higher to the lower order network. However, the final decision was to use a local reference system with an arbitrary origin and astronomical orientation (see details in Section 2.3 and Section 4). Maybe, the information on this geodetic network was collected by the engineers from an early IGC project for the definition of the new urban coordinate system. But for some reason it was finally dismissed.

[Figure 3 near here]

2.2. Baseline length

There were two baseline measurements in the triangulation project of 1929. The measurement stage comprised the length and the orientation determinations of the baselines which were conducted independently from the angle observations in the network. In the beginning of the 20th century the use of rigid bars was common for baseline measuring apparatus until the advent of the invar wires technology. Invar devices were introduced in Spain by 1924 (de la Puente, 1925) and used in the 1929 project. Invar is an alloy made of nickel (36%) and iron (64%) that has a uniquely low thermal expansion coefficient. Its invention dates back to the experiments conducted by Benoît and Guillaume in 1896 (Benoît & Guillaume 1917).

Previously in 1880, Jadörin established a new methodology to stretch metal wires which was then used to manufacture invar measurement equipment. Both inventions allowed geodesists to dramatically reduce the time required for distance measurements while increasing the accuracy. The paramount role of the invar measurement technique was explained in the lecture entitled ‘Invar and elinvar’, given by Guillaume when he was awarded the Nobel Prize in 1920 (Nobel Foundation 1998). The prize was awarded ‘in recognition of the service he has rendered to precision measurements in Physics by his discovery of anomalies in nickel steel alloys.’

The invar measurement technique requires a division of the baseline in sections that are measured sequentially and added up to obtain the total baseline length. All the sections must be perfectly aligned with auxiliary equipment to give reliable results. The nominal length of the invar wire is 24 m, its diameter is 1.65 mm and its circular cross section is 2.14 mm^2 (Bomford 1952). The auxiliary equipment included a clinometer to read the slope angles, a thermometer to compute the thermal coefficient of the wires, target devices mounted on tripods to make readings against an index, a spring balance and tension poles.

The measurement procedure gives the length of each section in 3D space. The raw measurement (L_0) is first corrected for the observation temperature (IGE 1907):

$$L = L_O \cdot [1 + (0.0618 \cdot (t - t_{REF}) - 0.00065 \cdot (t - t_{REF})^2) \cdot 10^{-6}] \quad [1]$$

where L is the corrected measurement, Lo is the field observed measurement, t is the observation temperature and t_{REF} is the reference calibration temperature (here 15ºC). This length should then be projected onto the horizontal plane using the formula:

$$L' = L \cdot cos\ C_t \quad [2]$$

where L is the corrected slope length and C_t is the slope angle. The sum of all the values of L' amounts to the total baseline length. In order to avoid gross errors, each section was measured a number of times (five times in the project of 1929). The sample of observations provides a residual (v_i) per individual measurement:

$$v_i = L_m - L_i \qquad [3]$$

where L_m is the average value of the series and L_i is the ith measurement. The standard procedure also gives a formula to compute the standard deviation (s_{Li}) of a single distance measurement for every section of the baseline:

$$s_{L_i} = \sqrt{\frac{\sum_{i=1}^{n} v_i^2}{n-1}} \qquad [4]$$

The standard deviation of the mean of five measurements for each section computes as:

$$s_{\bar{L}_i} = \frac{s_{L_i}}{\sqrt{5}} \qquad [5]$$

2.3 Baseline orientation

The angular orientation with respect to a standard reference line is one of the basic operations in any topographic survey. Historically, map orientation tasks have been made by astronomical observations that provide the azimuth, that is the clockwise angle from geographic north, of one or more baselines. Then, the orientation was carried forward towards the rest of the map elements through the field survey work (Bennett & Freislich 1979).

In the project of 1929, the orientation was conducted by the method of Polar star observations which allows obtaining the azimuth of the baseline with respect to the local meridian, or south-north line, and therefore the orientation of the map. We recreated the procedure recommended by the astronomic branch of the IGC back in 1929. The procedure requires an astronomic almanac containing star ephemerides in tabular format. After some bibliographic research, we found a copy of the almanac used in 1929 published by the IGC (1928).

The computation consists basically of solving a spherical triangle whose corners are the astronomic pole (P), the local zenith (Z) and the observed star (S). The sides of the PZS triangle are the Polar star zenith distance $z = 90 - h$, colatitude $c = 90 - \phi$, and Polar distance $p = 90 - \delta$, where h is the altitude, ϕ is the latitude of the station point, and δ is the Polar star declination, which are all positive numbers when the station point and the observed star are on the same astronomic hemisphere.

The azimuth angle θ is calculated for each polar observation in the field observation series with the following formula:

$$\tan\theta = \frac{\sin H}{\sin\delta \cdot \cos H + \tan\delta \cdot \cos\phi} \qquad [6]$$

Thus, the determination of the azimuth requires observations of the hour angle (H) of the Polar star, together with the values of the Polar star declination δ and the latitude ϕ of the station point. H is the inner angle at the Pole in triangle *ZPS,* also called the local time angle or hour angle, and provides the time difference of the star position at observation time with respect to the local meridian of the observer. The *H* angle value to be taken for computational purposes in Eq. (6) is that corresponding to the mean of the corrected chronometer times of the *n* observations forming a set (Clark 1948).
The other two parameters in Eq. (6) are known beforehand. The polar declination value used in the computations was $\delta = 88^{o}\ 55'12''$ as published in the almanac (IGC, 1928) and the latitude was $\phi = 39^{o}28'30''$ N known from previous national geodetic campaigns. The uncertainty in the measurement of the hour angle propagates into the value of the computed azimuth. Assuming that the observed hour angle (H) is in error by ΔH, the error in the azimuth angle (θ) (both quantities in the same units, for instance

seconds of arc), may be easily obtained by differentiating the formula in Eq. (6) with respect to *H*. After some simplifications the error formula reduces to (Clark 1948):

$$\Delta\theta = -(\sin\phi - \cos\phi \cdot \cos\theta \cdot \tan h) \cdot \Delta H \quad [7]$$

2.4. Triangulation

The 1929 primary triangulation comprised 28 triangles and 23 triangulation stations (Fig. 4). The observation was carefully planned to achieve quasi-equilateral triangles following the instructions of the IGE (1907). The computation was done using plane surveying procedures given the extent and the topography of the area.

The location of the stations was a key issue in 1929 since the instruments used back then needed clear lines of sight. The engineers of the time selected a number of elevated sites as triangulation points, mainly building rooftops and bell towers, to achieve good visibility. All stations were described and identified in specific forms (Fig. 5). Those documents are very accurate and contain text and graphical information to locate the exact station point; however, most of the marks have been lost over the years. After an exhaustive search, we found nine stations which were then used to conduct further geometric analyses (see Sections 3 and 4).

The angular observation procedure used in the 1929 field work is described in the instruction manual of the *Instituto Geográfico y Estadístico* (IGE 1907). The field notes indicate that two sets (arcs) of directions were measured at the stations, with each set in two faces, following the standard procedure of 'direction measurements' (Kahmen & Faig 1988). For our computations, we extracted the required angles from the mean directions of all sets measured at stations that were reported in field notes.

[Figure 4 near here]

[Figure 5 near here]

The computations were done originally by hand with the help of logarithm tables (Schrön 1893). In order to check for errors in the computations, we wrote a computer program that simulates each step of the manual procedure and found no errors. As for the accuracy of the primary triangulation, we recomputed the direction measurements directly from field data and adjusted the angles using the least squares method. Although the least squares technique is most often associated with high precision surveying, it can be used for quality control by processing sets of redundant observations according to mathematically well-defined rules (Kennie & Petrie 2010, Leick et al. 2015).

Theoretically, all angles of a triangulation should be processed together by least squares to yield simultaneously their most probable values (Clark 1948). We processed the angle observations from 13 selected stations and created a redundant equation system of dimensions 30x20, where the number of rows (30) equals the number of angles and the number of columns (20) is the number of coordinates to be adjusted.

Each angle generates an independent equation comprising three points i, j and k as follows (Teunissen 2006):

$$d\alpha_{ijk} = \frac{y_j^o - y_i^o}{(l_{ij}^o)^2}\mathrm{d}x_j - \frac{y_j^o - y_i^o}{(l_{ij}^o)^2}\mathrm{d}x_i - \frac{x_j^o - x_i^o}{(l_{ij}^o)^2}\mathrm{d}y_j + \frac{x_j^o - x_i^o}{(l_{ij}^o)^2}\mathrm{d}y_i - \frac{y_k^o - y_j^o}{(l_{ij}^o)^2}\mathrm{d}x_k + \frac{y_k^o - y_j^o}{(l_{ij}^o)^2}\mathrm{d}x_j + \frac{x_k^o - x_j^o}{(l_{jk}^o)^2}\mathrm{d}y_k - \frac{x_k^o - x_j^o}{(l_{jk}^o)^2}\mathrm{d}y_j = \alpha_o - \alpha_{ca} \quad [8]$$

where i denotes the left target, j is the instrument station, k is the right target, dx_i, dx_j, dx_k, dy_i, dy_j, dy_k are the unknowns (corrections to approximate coordinates), x_i^o, x_j^o, x_k^o, y_i^o, y_j^o, y_k^o are the approximate coordinates, and α_o, α_{ca} are the observed and calculated angle values respectively.

The least squares adjustment requires approximate coordinate values for each unknown station to calculate all parameters in Eq. (8). We used the coordinate values reported in the original documentation as approximations. The coordinates were computed using a set of 'corrected angles' that were obtained for every triangle by adding up the values of the three inner angles, then calculating the difference with respect to 180º, and finally distributing the angular misclosure equally to all three angles.

The matrix form of the equation system and its solution is well-known (Leick et al. 2015, Strang & Borre 1997):

$$A \cdot x = b + \upsilon \qquad [9]$$

where A is the coefficient matrix, x is the vector of unknowns, b is the vector of independent terms, and υ is the vector of residuals. The least squares method allows specific weighting for every observation equation. Since we did not find any suggestions for a weighting, we computed the adjustment with equally weighted observations:

$$x = (A^T \cdot A)^{-1} \cdot A^T \cdot b \qquad [10]$$

The vector x of the unknowns gives the corrections to the initial approximations of the coordinates. The most interesting point of the least squares method with respect to the approximate methods used in 1929 is the calculation of the variance-covariance matrix which contains the precision information of the variables. The expression of the variance-covariance matrix (Σ_x) is:

$$\Sigma_x = \sigma_o^2 \cdot (A^T \cdot A)^{-1} \qquad [11]$$

where σ_o^2 is the a posteriori variance of unit weight which is computed using the following formula:

$$\sigma_o^2 = \frac{v^T \cdot v}{n - u} \quad [12]$$

where v is again the vector of residuals, n is the number of equations (observations), and u is the number of unknowns. In network theory, the expression $n - u$ is usually referred to as the degrees of freedom of the network which equals the number of redundant equations in the model.

As mentioned in Section 1, there was no least squares adjustment to calculate the variances of the unknowns in the project of 1929. However, the triangle misclosures in the network may be used to estimate the overall angular precision of triangulations such as that of 1929. The classical literature provides the Ferrero equation as a means to compute the accuracy s_α of the observed angles in triangulation projects (Bomford 1952):

$$s_\alpha = \sqrt{\frac{\sum \varepsilon_i^2}{3 \cdot n}} \quad [13]$$

where n is the number of triangles, and ε_i is the misclosure in triangle *i*. It is worth noting here that this experimental formula is intended to calculate approximate probable errors in unadjusted triangulations from the angular measurements (Clark 1948). The value of the s_α value will be discussed later in relation to the precision of the theodolite used and the a posteriori variance of unit weight σ_o^2 in Section 4.2.

3. Transformation of the 1929 network to ETRS89

An interesting and challenging task of the present study was how to transform the 1929 network into a modern coordinate reference system such as the European Terrestrial Reference System of 1989 (ETRS89), which is the official system in Spain since 2012. We had some cues, such as the relative error of the original baseline data and the least squares adjustment results, which suggested the high quality of the data, and thereby promised good transformation results (Section 4.4). However, there was a practical limitation when selecting the control points for the transformation. After preliminary field work, we found that all stations of the southern half of the area, and some others in the central (urban area) and north area were lost. We were able to find nine points that have survived almost 90 years (Fig. 6), most of them pertaining to the primary network.

[Figure 6 near here]

The input data of a coordinate transformation consists of several pairs of coordinates in the source (1929) and target (ETRS89) coordinate systems, each pair representing some sort of transformation vector between the two spaces. While we had the source data from the published coordinates of 1929 (X_{1929} and Y_{1929} in Table 4), we did not have any information on the target system. In consequence, we had to do fieldwork to survey the target coordinates for every control point. Eight original marks were easily located following the descriptions in the project documentation. Seven marks were stations of the primary network (Grao, Castellar, Mislata, Almàcera, Benimàmet, Sancho, and Miguelete II), whereas the other two were stations of second order traverses (Puente del Mar and Petxina).

We collected ETRS89 coordinates using global navigation satellite system (GNSS) equipment. Specifically, we used the virtual reference station (VRS) technique because it allows short observation lengths and requires no post-processing. VRS provides instant access to real-time kinematic (RTK) corrections utilising a network of permanent (fixed), continuously operating reference stations (Leick et al. 2015, Seeber 2003).

GNSS provides geographical coordinates (ϕ, λ) of the station points in the ETRS89 coordinate reference system. In order to be more compatible with the original

Euclidean 2D system of 1929, the geographical coordinates were transformed to linear coordinates expressed in metres. Using well-known formulae (Snyder 1987, Wolf et al. 2014) we converted the geographical coordinates into two coordinate systems: (1) the Universal Transverse Mercator (UTM) projection, which is the official map projection in Spain, and (2) a local 3D three dimensional vertical coordinate (LVC) system which is geometrically defined in very similar terms to those of the original 1929 triangulation, namely a 3D rectangular system with the *z*- axis parallel to the local vertical and the *y*-axis pointing North.

The subsequent conversion of the 1929 coordinates to the two contemporary coordinate sets (UTM and LVC) was conducted with an affine transformation. We chose the affine transformation because it is very flexible and allows a detailed analysis of the conversion. The formulas of the six-parameter affine transformation are well-known (Wolf et al. 2014):

$$X_2 = A \cdot X_1 + B \cdot Y_1 + C \qquad [14]$$

$$Y_2 = D \cdot X_1 + E \cdot Y_1 + F \qquad [15]$$

where (X_1, Y_1) and (X_2, Y_2) are coordinates in the source and target systems respectively. Two of the six parameters A, B, C, D, E and F have a direct geometrical meaning (C and F represent coordinate shifts or translations). The other four parameters can be expressed in terms of scale factors and rotations of the axes. The formulas to obtain the scales and rotations are (Wolf et al. 2014):

$$\theta = \arctan\left(\frac{D}{A}\right) \qquad [16]$$

$$\in = \arctan\left(\frac{B}{E}\right) + \theta \qquad [17]$$

$$SF_x = \frac{A}{\cos\theta} \qquad [18]$$

$$SF_y = E \cdot \frac{\cos\epsilon}{\cos(\epsilon - \theta)} \qquad [19]$$

where SF_x and SF_y are the scale factors in X and Y directions, $\in$ is the correction for non-orthogonality between the *x*- and *y*-axes, and θ is the rotation angle of the x-axis. The $(\epsilon - \theta)$ difference may be interpreted as the rotation angle of the *y*-axis.

4. Results and discussion

In this section we argue that the map of 1929, backed by a number of precise geodetic operations described in this paper, was a first class surveying project. We base our view on careful analyses of the original fieldwork records, as well as on exhaustive recomputations relating to the geodetic reference system, baseline measurements, triangulation adjustment, and geometric transformations.

4.1. Geodetic coordinate system

As reported above, the only reference to a proper geodetic reference system in this project was a list of coordinates in an unidentified system found in the project files. After trying several projection formulae sets, we found out that they were Tissot coordinates. Table 1 contains the coordinates listed in the 1929 project documentation together with our own Tissot computations from the original data and the differences between the two datasets. Although the computed coordinates are close to those reported in the 1929 dossier, there is a systematic shift in the *x*-axis, probably owing to computation procedures of the time. While we are pretty sure that the original listing contains Tissot coordinates, the reference system adopted finally was a different one as discussed below.

[Table 1 near here]

4.2. Baselines

The baselines (*AB* and *CD* in Fig. 4) gave orientation and scale to the 1929 urban map. Astronomic azimuth measurements were conducted according to the instructions defined by the IGC (1928). Each azimuth was determined twice, in opposite directions to avoid gross errors. The values reported in the project files were $\theta_A^B = 139^o$ 03’17’’, $\theta_B^A = 319^o$ 03’29’’ for the baseline *AB* and $\theta_C^D = 351^o$ 11’ 35’’, $\theta_D^C = 171^o$ 11’30’’ for the baseline *CD*. The differences between the reverse observations were 12’’ and 5’’ for the two baselines and demonstrate the accuracy of the method. The computation report of the project also contains the difference between the observed (astronomic) azimuth and the computed (carried forward from *AB*) azimuth of the baseline *CD*. That difference was 3’17’’ which is another indicator of the quality of the observational scheme of the 1929 network.

As for the baseline measurements, the execution of the invar wire measurements provided an estimation of the precision of each section. The project files contain observation data for the *CD* baseline only, although similar precision values should be expected for the baseline *AB*. The length values reported in the documentation were 508888.627 mm for *AB* and 524362.716 mm for *CD*. The final coordinates of the terminals *A*, *B*, *C* and *D* are given in Table 2. We computed the standard deviations of each single measurement as well as of the mean of every group of five observations according to the instructions published by the IGE (1907). The standard deviations for the mean values of all sections of the baseline *CD* ranged from 0.0045 mm to 0.0656 mm. Using the propagation law of variances we derived the precision of the whole length of 524362.716 mm as 0.17 mm (0.32 ppm). It is worth noting that this computation covers the precision of the invar measurements only. Classic references in the geodetic literature introduce the concept of ‘probable errors’ as a measure of the influence of all other measurement and reduction errors. For instance, Bomford (1952) quotes typical values of 1-2 ppm for the total error budget of baselines for higher order networks and gives the following formula for the probable error of measurements:

$$pe = \sigma \cdot 0.6745 \qquad [20]$$

where σ is the standard deviation of the invar measurement. Given the precision of the 1929 invar measurements (0.17 mm), the probable error of the raw distance measurements equates only 0.114 mm. Other error sources are likely larger (Bomford, 1952).

[Table 2 near here]

The primary triangulation network (Fig. 4) may be considered as a triangulation chain of 16.94 km length from the baseline *AB* to the baseline *CD*. In consequence, there are two length values for *CD*, the direct invar measurement (524.363 m) and the computed value (524.510 m) carried forward from AB. The difference between the measured and the computed lengths is 0.147 m, which is quite good considering the instruments of the time.

The ratio of the baseline *CD* difference (0.147 m) to the distance (16.94 km) between the two baselines gives some sort of 'kilometric error' of the method. This error in the 1929 network amounts to 8.7 mm per km, or 8.7 parts per million (ppm). Some contemporary triangulation projects report relative errors ranging from 1 ppm to 13.1 ppm (Hotine, 1939). Therefore the resulting value of 8.7 ppm in the 1929 project conforms to common practice of the time and must be considered a satisfactory result.

4.3 Triangulation network

The least squares adjustment of the network shown in Fig. 4 gave very good results (Table 3). The original 1929 network extends 13.4 km NS and 7.7 km EW. Due to practical constrains, the network used to conduct the adjustment covers a reduced area of 8.9 km NS and 7.7 km EW.

We fixed the stations Benimàmet, Miguelete I and Miguelete II based on a previous field study. The vector of unknowns gave very small coordinate corrections and the residuals of the angles were also small. Probably, the most interesting output of the least squares adjustment were the standard deviations of the coordinates and the angles. It is clear from Table 3 that the standard deviations s_X and s_Y of the coordinates increase from the first station Burriel (0.023 m) to the last station Castellar (0.354 m). This increase of the standard deviation of the coordinates from North to South is expected since the three fixed stations are in the North. We based our choice of the three fixed points on previous work. In consequence, the southern part lacks geometric control leading to higher standard deviations. Still, the adjustment results agree well with the standards of 1929 for the geodetic control of large scale urban maps. .

[Table 3 near here]

The least squares analysis also gives the *a posteriori* variance of an observation of unit weight σ_o^2 as 63.97 (Eq. (12)). Thus, the *a posteriori* standard deviation σ_o of the measured angles was 8 seconds of arc. The triangle misclosures computed in 1929 from triangles 1 to 10 (Fig. 4) were: 7, 8, 9, 1, 17, 3, 9, 21, 29 and 11 seconds of arc respectively. Substituting these values into Eq. (13) provides an estimate of the overall angular precision s_α in the network:

$$s_\alpha = \sqrt{\frac{\sum(7^2+8^2+9^2+1^2+17^2+3^2+9^2+21^2+29^2+11^2)}{3\cdot 10}} = 8.12\ seconds \qquad [21]$$

This value (8.1 seconds of arc) is very close to the *a posteriori* standard deviation of a measurement of unit weight (8 seconds of arc for the measured angles) of the least squares adjustment, which clearly indicates that the surveying methods of 1929 were appropriate and the quality of the angular observations (i.e. directions) was high.

The coordinate values in Tables 1 and 3 deserve an explanation with regard to the geodetic reference system. The coordinates in Table 3 differ from those of the Tissot projection system (Table 1) that was common in Spain in 1929. Instead, the station *A* was given arbitrary, local coordinates (X = 20000, Y = 40000) ensuring that the origin of the coordinate system falls southwest of the entire surveyed area (Table 2). A major drawback of this project, was that the project files contained information on a geodetic network that was eventually not used by the engineers. The choice of setting an arbitrary coordinate origin, however, agrees with recommendations for the set-up of urban grid systems in the absence of reliable higher order networks. For instance, Blachut et al. (1979) state that 'the plain coordinate system for each urban community, or group of communities, should be designed so as to fulfill the particular needs of that community, even if this means a departure from an otherwise accepted regional coordinate system.' The coordinates in the false origin system are referred to as X_{1929} and Y_{1929}.

4.4 Coordinate transformations

The result of the transformation of the adjusted coordinates (X, Y) of Table 3 to ETRS89 is another interesting finding. It actually provides an insight into the geometric quality of the 1929 survey. As we progressed in our research, we realised that the original observations gave very consistent results and we expected a good agreement in rotation and scale when transforming the data of Table 3 into the ETRS89 system.

We carried out the transformation with two points in mind. First, we chose the affine transformation (Eqs. (14)-(19)) among many possible candidates because it does not constrain scale and rotation, which facilitates posterior interpretation. This transformation uses six parameters that can be converted to a differential rotation representing the lack of perpendicularity between the axes ($\in$), a rotation angle (θ), and two scale factors, one for each of the two axes (SF_X, SF_Y).

The second point refers to the definition of the source and target coordinate systems and the agreement between them. As stated above, the 1929 coordinate reference system was defined as a plane, rectangular 2D system with an arbitrary origin and an orientation by astronomic methods. The ETRS89 was defined using GNSS techniques and geographic coordinates that are not compatible with plane coordinates. Therefore, we converted the geographical coordinates into two different target coordinate systems.

[Table 4 near here]

The first target system was the so-called local vertical coordinate (LVC) system, whose origin can be arbitrarily selected by the user (Wolf et al. 2014). It is worth noting here that the horizontal component (X, Y) of this coordinate system is, in many aspects, quite the same as the 2D plane system of 1929. The LVC coordinates are given in Table 4.

The second system is the well-known Universal Transverse Mercator (UTM) cylindrical projection which is widely used in urban and large scale mapping (Blachut et al. 1979). The UTM grid defines a 2D rectangular system but the coordinates are affected locally by several elements of the projection such as the grid convergence and the point scale factor (Iliffe & Lott 2008, Snyder 1987). The UTM coordinates of Table 4 are in Zone 30.

Table 5 shows the residuals of the affine transformations of the 1929 coordinates (Y_{1929} and Y_{1929} in Table 4) for all nine control points. The figures in both transformations are very similar. There are seven control points with residuals equal to or less than 30 cm in the LVC transformation and six in the UTM transformation. We found residual vectors ranging from 4 cm to 65 cm (LVC) and from 3 cm to 63 cm (UTM) respectively. These large differences suggest that the use of more localised transformations would be beneficial to reduce the residuals of the transformation. Although this would be interesting, it is outside of the scope of this study.

[Table 5 near here]

The average residual s_0 for each transformation is another precision criterion which can be calculated from the residuals of the least squares solution of the affine transformation (Table 5):

$$s_0 = \sqrt{\frac{r_X^2 + r_Y^2}{2 \cdot n - p}} \qquad [22]$$

where n is the number of control points (n=9) and p the number of parameters (p=6) of the transformation.

The average values are 0.260 m and 0.255 m for the 1929-LVC and 1929-UTM transformations respectively, showing very similar residual behaviour in both cases. It is worth noting that these average values include the larger error vectors of 0.65 m for 'Grao' in the two transformations under study.

Some geometric parameters of the transformations, namely scales and rotations in X and Y, are shown in Table 6. For the LVC transformation, a perfect fit would give a value of 1.0 for the scale factors and a value of 0.0 degrees for the rotation and perpendicularity angles. Regarding the LVC system, the scale factors are very close to the unit value, the rotation angle θ of the X axis is -0.024253 degrees (1'27''), and the perpendicularity angle $\in$ is 0.007595 degrees (27 seconds of arc). These values prove the great performance of the invar wire and astronomical orientation techniques.

[Table 6 near here]

When considering the UTM parameters, it is clear that the UTM scale factor and the UTM grid convergence mask the fit between the 1929 and the UTM spaces. In order to account for the influence of the projection on the transformation parameters we computed the nominal UTM values (point scale factor and grid convergence) of the central point Miguelete II of the study area and corrected the raw UTM transformation parameters.

The UTM nominal values for Miguelete II are:

Latitude:	39º 26’ 33.79’’ N
Longitude:	0º 20’ 07.36’’ W
Point scale factor:	1.00024744
Grid convergence:	-1º 41’ 37’’ ~ -1.693611 deg

Regarding the scale factors, it is worth noting that in the 1929-LVC transformation, they can be considered ‘true scale factors’ in the sense that they connect two pure Cartesian systems. In the 1929-UTM transformation, however, the nominal UTM scale factor affects the experimental scale factors so that they cannot be directly compared to their 1929-LVC counterparts. In order to do that comparison properly, we applied the UTM nominal scale factor as follows:

$$SF'_X = \frac{1.000143}{1.00024744} = 0.999896 \quad [23]$$

$$SF'_Y = \frac{1.000205}{1.00024744} = 0.999957 \quad [24]$$

where SF'_X and SF'_Y are the corrected scale factors which are similar to those of the 1929-LVC transformation. Note that scale factors may also be computed in units of ppm by subtracting the computed and nominal values which gives -104 ppm (1.000143-1.000247) for SF'_X and -42 ppm (1.000205-1.000247) for SF'_Y.

The influence of the grid convergence angle is larger since it is 'embedded' in the target reference system. Furthermore, the affine transformation formulas do not provide mechanisms to separate convergence from true rotation either, so that we have to correct for that convergence after computing the transformation. The result is a corrected rotation angle for the *x*-axis after a summative operation:

$$\theta' = 1.645097 - 1.693611 = -0.048514 \text{ degrees} = -2'55'' \quad [25]$$

This corrected value is now much smaller and about double the θ angle in the 1929-LVC transformation.

The results of the scale and rotation analyses suggest that the LVC system works slightly better than the UTM system in geometric terms, even after correcting the UTM parameters. However, in common practice, the LVC system is rarely used in large scale and urban mapping, and the UTM projection system is preferred. Be that as it may, the UTM results are suitable to transform the 1929 map into modern reference systems and integrate the 1929 map with digital databases. The relevant parameters (*A* to *F*) of the affine transformation are listed in Table 6.

Another finding of the study was the exact location of the Miguelete I station used in the 1929 project. In the current Spanish national geodetic network there is a first order station called Miguelete. It was not clear whether those two stations were identical or in two close, but different locations. We confirmed that they were two different stations when we calculated the transformation between the 1929 local and the ETRS89 coordinates.

Figure 7 shows three different points on the rooftop of the tower. The station Miguelete I is shown by an empty triangle with a central dot and the current first order station Miguelete with a red, solid triangle on the western half of the tower (above the ‘e’ of ‘del’). The other solid triangle on the eastern half is Miguelete II (above the ‘t’ of ‘Micalet’), which was also used in the 1929 project and is still marked on the tower roof. The resolution of the issue of the Miguelete I station was important for the determination of the geodetic coordinate reference system for the project.

[Figure 7 near here]

5. Conclusions

In this paper we examined the geometric and cartographic characteristics of the triangulation network used to make the first 1:500 urban map of València. The study involved bibliographic work, field trips, and computer programming in a demanding research effort. As a result, we gained considerable insight into the fundamentals of the observational and computational processes of the network that were originally conducted almost one century ago.

The quality and detail of the original survey documentation allowed us to reprocess the original data. The least squares processing of the original angular observations showed that the standard deviations of the measured angles and the adjusted coordinates were very satisfactory.

The affine transformations based on a set of points of the original network and GNSS observations were used to convert the 1929 data into modern coordinate reference systems. The scale and rotation parameters of the affine transformations demonstrated the accuracy of the invar wire length measurements and the astronomic orientation of the baselines.

Although the plane coordinate system of the 1929 network is local, the fieldwork and computations conducted in this study allow us to integrate the urban map with modern spatial databases stored in global coordinate systems. There are many applications of such data integration in the regular operation of City Survey Offices as well as in cadastral and urban planning services. In the case of the València City Council, the 1929 map has been used in several legal matters. In summary, the map of 1929 is a cartographic gem that can now be integrated with other municipal spatial databases for urban planning, cadastral and even legal purposes.

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Table 1. Computed Tissot coordinates and differences from the published Tissot coordinates of 1929

Station	**X_{COMP} (m)**	**Y_{COMP} (m)**	**X_{PUBL} (m)**	**Y_{PUBL} (m)**	**ΔX (m)**	**ΔY (m)**
Rebalsadores	877312.65	571584.17	877312.30	571584.14	0.35	0.03
Cullera	896560.14	514338.09	896559.61	514338.07	0.53	0.02
Faro	898140.52	569714.64	898140.20	569714.62	0.32	0.02

Table 2. Coordinates of the terminals *A*, *B*, *C* and *D* of the two baselines *AB* and *CD* defining the 1929 local coordinate system

Station	X_{1929} (m)	Y_{1929} (m)
A	20000.00	40000.00
B	19666.57	40384.44
C	27659.42	24889.63
D	27740.30	24371.39

Table 3. Approximate coordinates (X_A, Y_A), adjusted coordinates (X, Y), and standard deviations of the adjusted coordinates s_X and s_Y. Coordinates in the 1929 local coordinate system

Station	**X_A(m)**	**Y_A(m)**	**X (m)**	**Y (m)**	**s_X (m)**	**s_Y (m)**
Benimamet	20225.56	37946.63	--------	--------	--------	--------
Burriel	21930.63	38069.11	21930.67	38069.09	0.023	0.071
Mislata	20310.29	35452.40	20310.27	35452.41	0.071	0.093
Miguelete I	23908.07	35473.72	--------	--------	--------	--------
Tormo	22563.03	33613.49	22563.02	33613.46	0.098	0.083
S. Luis M	24720.81	32034.10	24720.89	32034.07	0.187	0.162
Almácera	25616.76	39589.87	25616.86	39589.83	0.171	0.162
Miguelete II	23915.46	35480.59	--------	--------	--------	--------
Malvarrosa	27994.72	36417.30	27994.73	36417.15	0.179	0.202
Grao	27488.37	33846.34	27488.30	33846.17	0.237	0.166
S. Luis M II	24723.88	32034.15	24723.73	32034.20	0.237	0.166
Sancho	27377.67	31860.15	27377.41	31860.07	0.289	0.259
Castellar	24961.98	30377.44	24961.90	30377.43	0.355	0.232

Table 4. Coordinates of the stations used in coordinate transformations (UTM coordinates in zone 30). X_{1929}, Y_{1929} are the 1929 published coordinates in the local coordinate system. The UTM and LVC target coordinates were converted from the ϕ, λ of the new GPS survey

Station	X_{1929} (m)	Y_{1929} (m)	X^a_{UTM} (m)	Y^a_{UTM}(m)	X_{LVC} (m)	Y_{LVC} (m)
Benimamet	20225.56	37946.63	721973.321	4375175.925	-3688.540	2467.358
Mislata	20310.29	35452.40	722139.615	4372684.928	-3604.900	-26.661
Almácera	25616.76	39589.87	727326.543	4376970.213	1703.719	4108.799
Miguelete 2	23915.46	35480.59	725740.934	4372816.608	0.000	0.000
Sancho	27377.67	31860.15	729318.691	4369282.395	3459.953	-3621.696
Grao	27488.37	33846.34	729361.254	4371284.690	3571.367	-1636.282
Castellar	24961.98	30377.44	726935.213	4367744.983	1043.888	-5103.069
Puente del Mar	25029.35	34925.50	726870.327	4372293.579	1113.434	-555.580
Pechina	22514.06	35662.39	724350.668	4372923.164	-1401.247	182.420

Table 5. Residuals of the coordinate transformations of the 1929 coordinates (X_{1929}, Y_{1929} of Table 3) to the LVC (X_{LVC}, Y_{LVC} in Table 4) and UTM spaces (X_{UTM}, Y_{UTM} in Table 4)

Station	1929 to LVC			1929 to UTM (Zone 30)		
	r_X (m)	r_X (m)	$\sqrt{r_X^2 + r_Y^2}$ (m)	r_X (m)	r_X (m)	$\sqrt{r_X^2 + r_Y^2}$ (m)
Benimamet	-0.2242	-0.1985	0.2994	-0.2040	-0.2615	0.3317
Mislata	0.0684	0.0148	0.0700	0.0927	0.0410	0.1013
Almácera	0.2707	0.3129	0.4138	0.2196	0.3361	0.4015
Miguelete 2	0.0628	0.0024	0.0628	0.0581	0.0213	0.0619
Grao	-0.3641	-0.5353	0.6474	-0.3038	-0.5504	0.6287
Sancho	0.0191	0.1590	0.1602	0.0423	0.1187	0.1260
Castellar	0.2164	0.2178	0.3071	0.1478	0.2382	0.2804
Puente del Mar	-0.0406	-0.0158	0.0436	-0.0327	-0.0019	0.0327
Pechina	-0.0087	0.0427	0.0435	-0.0200	0.0585	0.0618

Table 6. Geometrical parameters of the affine transformations from the published 1929 coordinates in the local coordinate system (columns 2, 3 in Table 4) to the GPS derived UTM (columns 4, 5 in Table 4) and LVC (columns 6, 7 in Table 4) systems

Parameter	**LVC**	**UTM**
SF_X	0.999933	1.000143
SF_Y	0.999983	1.000205
$\in$ (degrees)	0.007595	0.007705
θ (degrees)	-0.024253	1.644985
A	0.999933	0.999731
B	0.000556	-0.028578
D	-0.000423	0.028711
E	0.999983	0.999797
C (m)	-23933.622	702845.845
F (m)	-35469.863	4336656.581

Figure 1

Figure 2

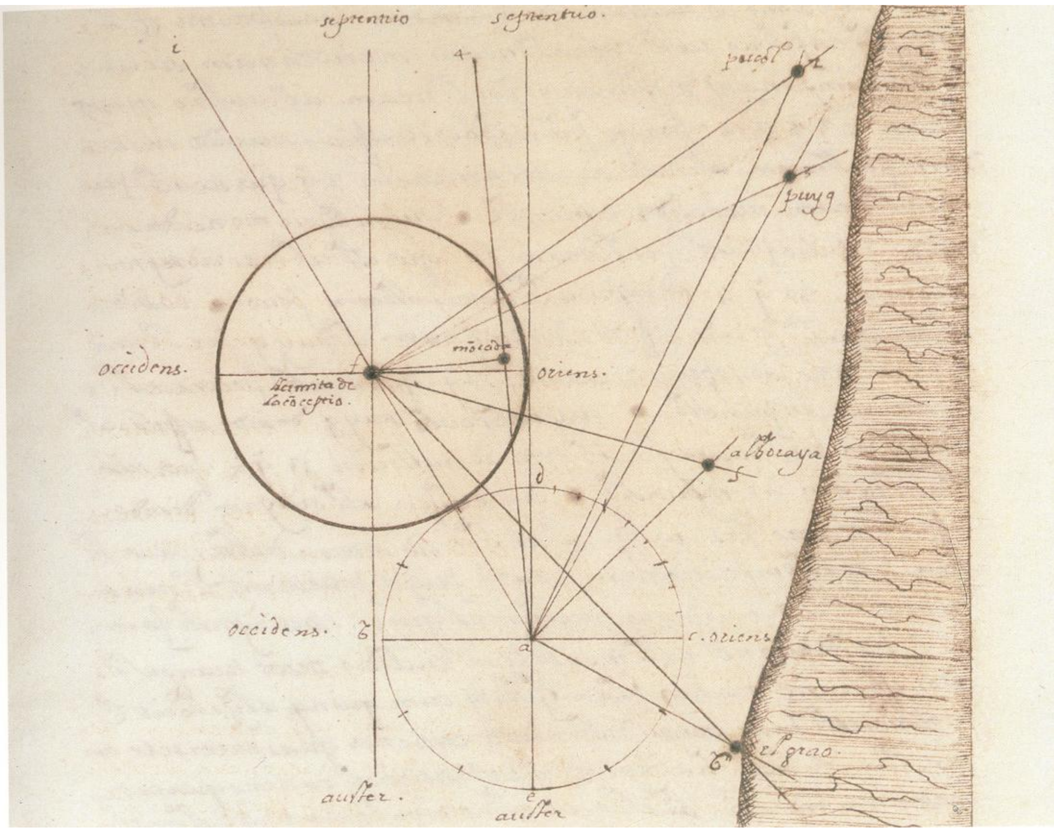


Figure 3

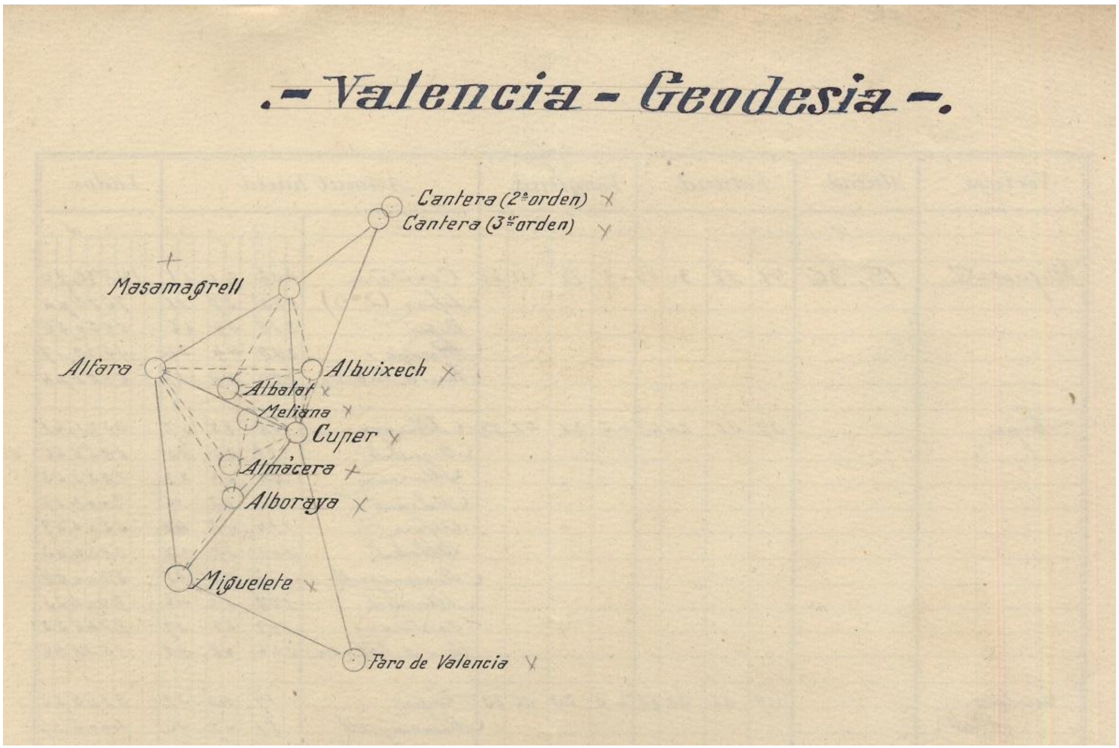

.- Valencia - Geodesia -.
Cantera (2ª orden)
Cantera (3ª orden)
Masamagrell
Alfara
Albuixech
Albalat
Meliana
Cuper
Almàcera
Alboraya
Miguelete
Faro de Valencia

Figure 4

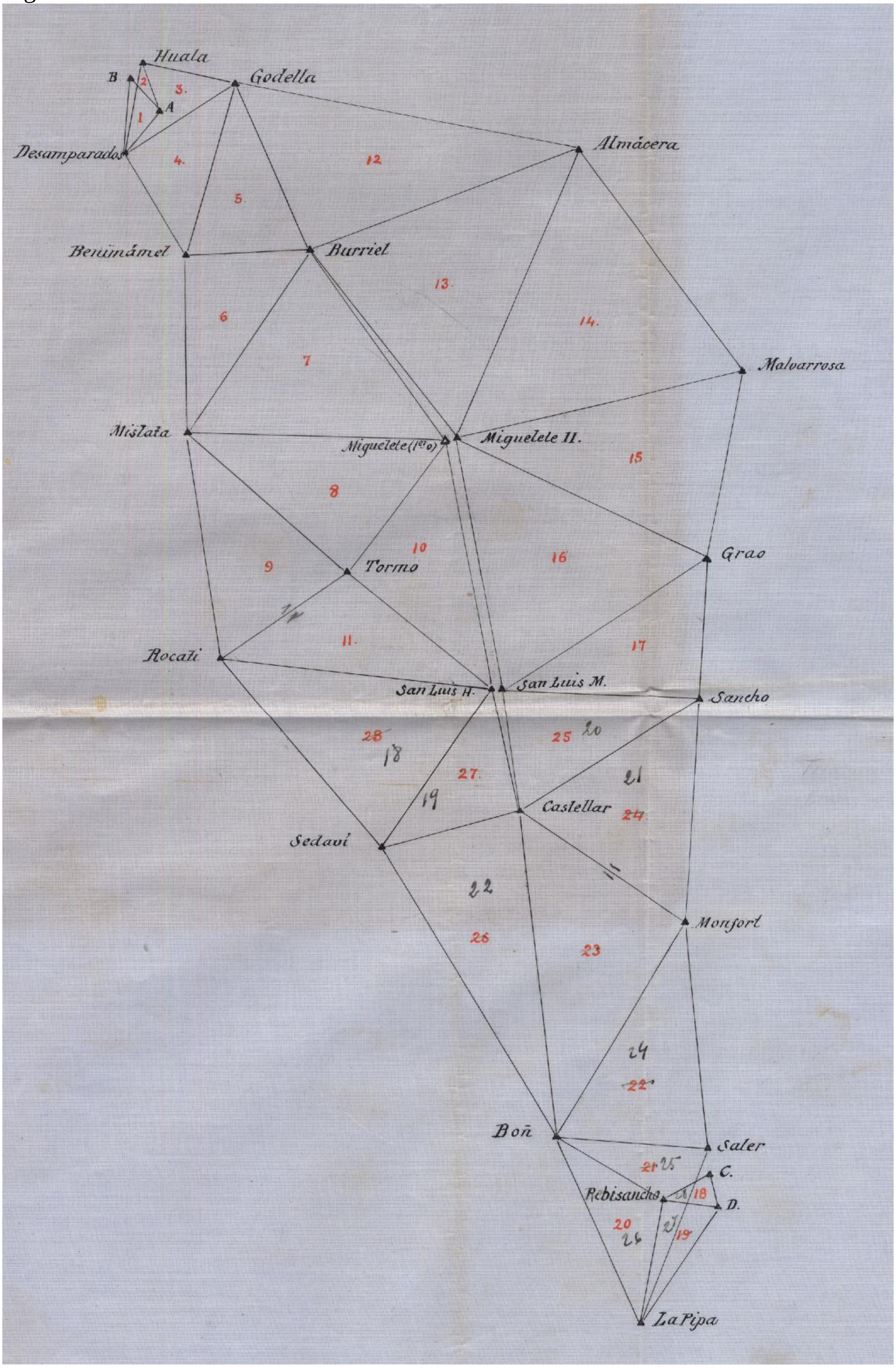
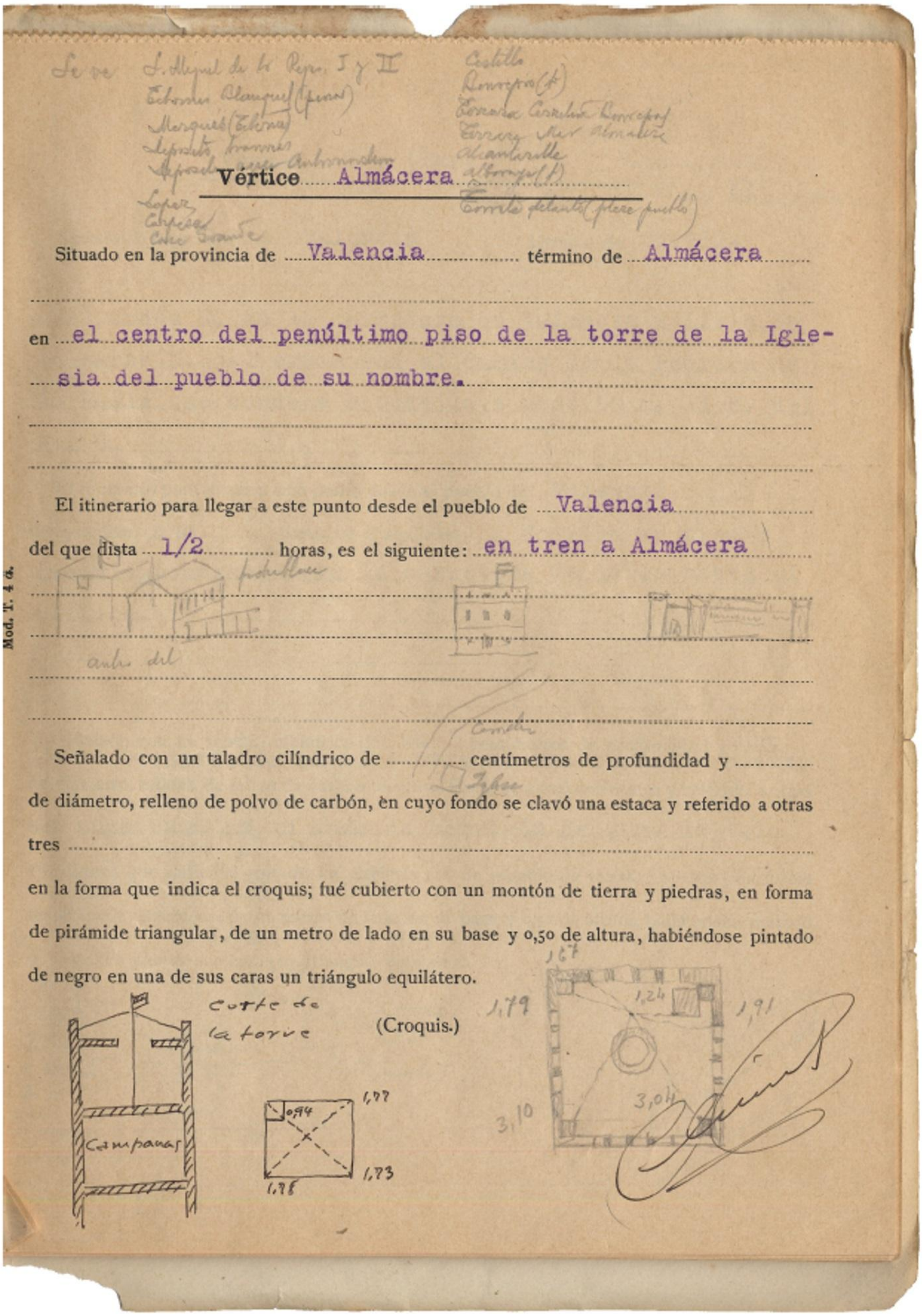

Huala
Godella
B
A
Desamparados
Almácera
Benimámel
Burriel
Malvarrosa
Mislata
Miguelete (1ero)
Miguelete II.
Grao
Tormo
Rocafi
San Luis H.
San Luis M.
Sancho
Castellar
Sedaví
Monfort
Boñ
Saler
C.
Rebisancho
D.
La Pipa

Figure 5



Mod. T. 4 d.

Vértice Almácera

Situado en la provincia de Valencia término de Almácera

en el centro del penúltimo piso de la torre de la Iglesia del pueblo de su nombre.

El itinerario para llegar a este punto desde el pueblo de Valencia

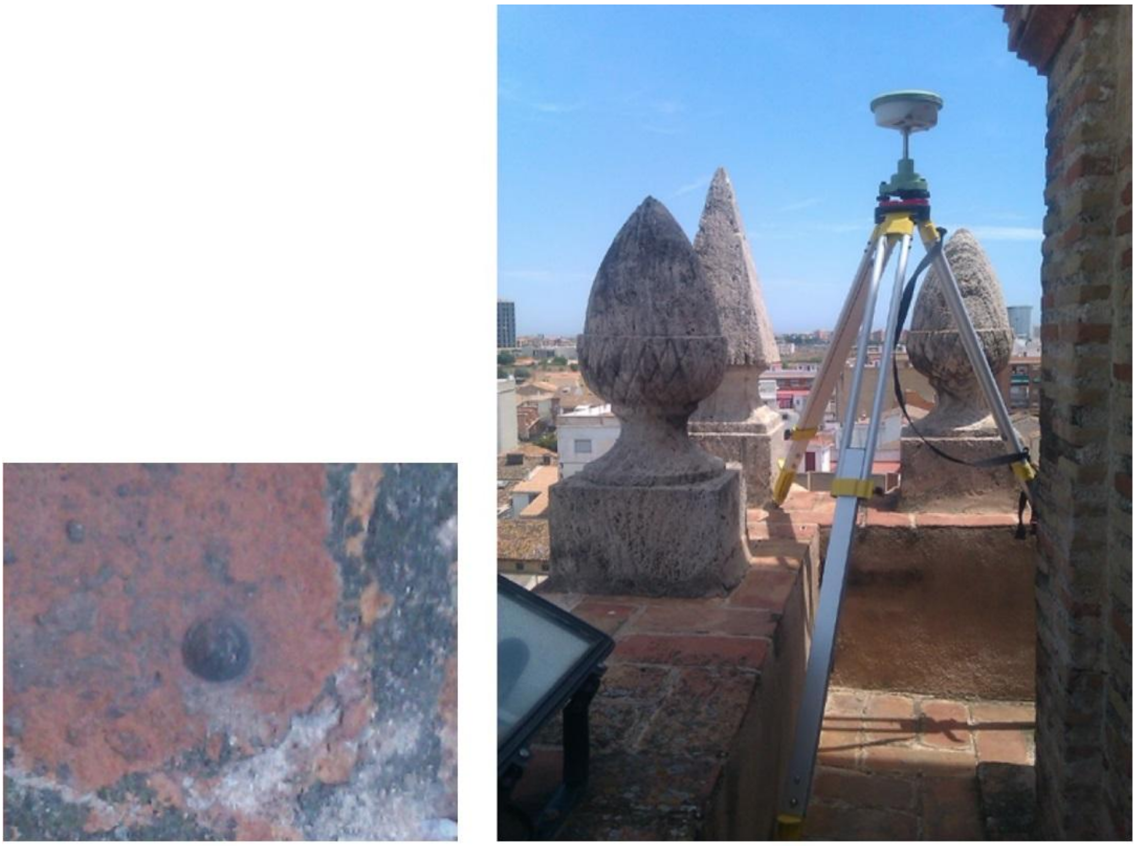
del que dista 1/2 horas, es el siguiente: en tren a Almácera

Señalado con un taladro cilíndrico de ... centímetros de profundidad y ... de diámetro, relleno de polvo de carbón, èn cuyo fondo se clavó una estaca y referido a otras tres ...

en la forma que indica el croquis; fué cubierto con un montón de tierra y piedras, en forma de pirámide triangular, de un metro de lado en su base y 0,50 de altura, habiéndose pintado de negro en una de sus caras un triángulo equilátero.

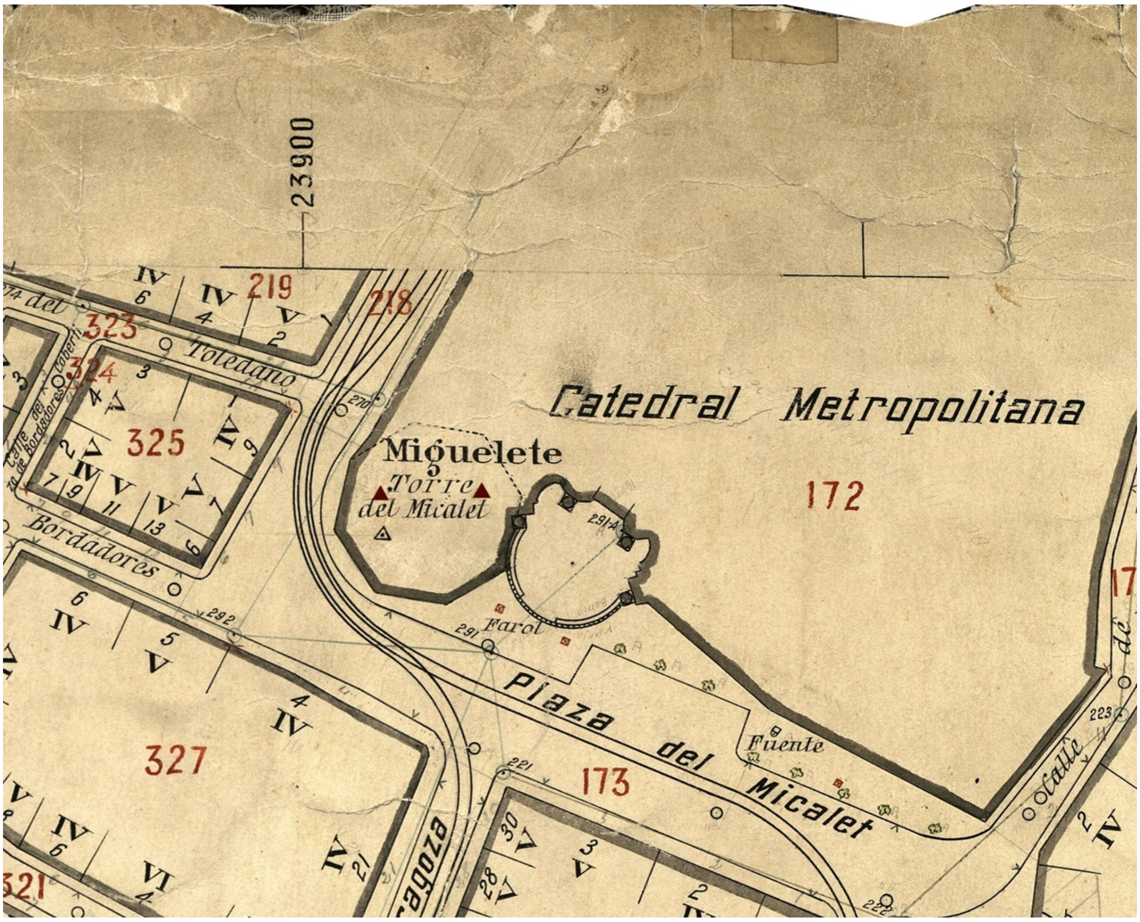
(Croquis.)

Figure 6



(a) (b)

Figure 7



23900
Catedral Metropolitana
172
Miguelete
Torre del Micalet
Farol
Fuente
Plaza del Micalet
173
Toledano
Bordadores
219
218
323
324
325
327
321

Figure 1. Location of the city of València in Spain (North towards top of figure).

Figure 2. Early triangulation draft by J. Muñoz on the Valencian coast (Navarro 2004). Note the use of latin words for the cardinal points (*septentrio* [north], *auster* [south], *oriens* [east], and *occidens* [west]) as usual at the time (North towards top of figure).

Figure 3. Scketch of the network used to connect the 1929 urban network with the Spanish national geodetic network. North is upwards. The distance from 'Cantera' to 'Faro de Valencia' is about 15 km.

Figure 4. Original plan of the urban primary triangulation of 1929 (North-South: 13.4 km, East-West: 7.7km). Note the strategic locations of the baselines *AB* and *CD* at the limits (S, N) of the surveyed area (North upwards).

Figure 5. Point description of the station Almácera. The recovery scketches at the bottom of the form allow the relocation of the point, if necessary.

Figure 6. Original mark of the station Benimàmet (a) and the GNSS antenna during our resurvey (b).

Figure 7. Sample of the 1929 map with the location of the points Miguelete (upper left, red triangle), Miguelete I (bottom left, empty triangle with central dot) and Miguelete II (upper right, red triangle). Note that the unlabelled grid line in the upper-right area (over the 'r' in 'Metropolitana') that corresponds to the X coordinate 23950.