A methodology for modeling and optimizing social systems

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Abstract:

A system methodology to modeling and optimizing social systems is presented. It allows constructing dynamical models formulated stochastically, i.e., their results are given by confidence intervals. The models provide optimal intervention ways to reach the stated objectives. Two optimization methods are used: (1) testing strategies and scenarios and (2) optimizing with a genetic algorithm. The application case presented is a small non-formal education Spanish business. First, the model is validated in the 2008-2012 period, and subsequently, the optimal way to obtain a maximum profit in the 2013-2025 period is obtained using the two methods.

Keywords: Social systems; simulation; sensitivity analysis; stochastic model; genetic algorithm; strategies and scenarios; economical model.

1. Introduction

In the world in which we live, wrapped with a global crisis, we must give hopes to new generations. A new general methodology is the way to solve this. The methodology will allow to model and optimize a social system, and optimal goals can be obtained to solve a specific case. The General Systems Theory provides the tool to create this, because it proposes the use of transdisciplinary methodologies that allow to build mathematical models which solve problems in the field of complex systems.

Forrester (1961) developed in the 50s the System Dynamics at the *Massachusetts Institute of Technology* (MIT) as transdisciplinary methodology with which to build dynamic models of complex systems and use them as a tool for intervention in them.

A continuation of the Forrester's methodology is proposed by Caselles (1994, 2008), the Methodology of General Modeling (MMG), which is used in this work, it can be implemented using intelligent generator models of complex systems SIGEM (Caselles, 1992, 1993, 1994, 2008). Caselles et al. (1998) present an application to a complex real case (control unemployment in a country). But with this same methodology have already been solved problems such as the environment in Spain (Sanz et al., 2016), to get a demographical stable society in Austria (Sanz et al. 2013) or Spain (Sanz et al., 2014) or increase the life expectancy in Spain (Caselles et al., 2014).

As an example of this methodology, in this paper is presented an economic-mathematical model to optimize a small non-Spanish-formal education business.

The main advantage of the methodology presented in this paper is that it is a generic methodology applicable to any social system, holding the notion of Gutierrez et al. (2012). Shannon et al. (1976) support the use of a model-simulation methodology; they say: "*Simulation is the process of designing a model for a real system and carry experiences with him, in order to understand the system behavior and evaluate new strategies within the limits imposed by a determined set of criteria to operate the system*".

In addition, the present methodology produces dynamical models and permits their stochastic formulation (Caselles, 1992, 2008). The stochastic formulation allows determining the reliability of the results for each calculation instant in two equivalent ways: (1) by presenting them with confidence intervals (for a given confidence level); (2) by their respective average values and their corresponding standard deviations.

The rest of the article is divided as follows:

- Section 2: Used methodology.
- Section 3: Presentation and explanation of the application case: the mathematical/economic model, its input and output variables and equations which related these variables.
- Section 4: Results. It is divided in two subsections: (1) the validation of the model obtained for a small business in the 2008-2012 period; (2) Those results related with simulations of the model, i.e., the sensitivity analysis, the design of future scenarios and strategies, and the genetic algorithm to determine the best actions on the input variables to maximize the objective variable.
- Section 5: Conclusions and suggestions for the future.

2. METHODOLOGY

The methodology suggested to design the model is the General Modeling Methodology (*GMM*) proposed by Caselles (1994, 2008) which is a widening of the methodology proposed by Forrester (1961).

The *GMM* is a hypothetical-deductive methodology to build complex system models. It includes some steps and its corresponding methods to obtain the objective. These steps belong to a life-circle when if a bug is detected you have to return to the previous step to correct it and follow with the process.

These steps can be synthesized as:

- Description of the real problem.
- Conceptual model (list of variables and functional relationships among them).
- Verification and Validation.
- Simulation or optimization.
- Decision making.

Some helping tools such as the automatic programming tool *SIGEM* (intelligent generator of models of complex systems) and the best-function finder *REGINT*, are free available in http://www.uv.es/~caselles/. In this webpage more information about them can be found.

3. APPLICATION CASE

In the beginning, the methodology is detailed for the application case.

3.1 Description of the real problem

The objective consists in optimizing the benefits of a small/medium Spanish company dedicated to the non-formal education in the 2013-2025 period. Some variables are specific of this activity but the essential structure of the model can be extrapolated to any small/medium.

3.2 Conceptual model

The first part of this section explains how the variables of the model are identified and in the second part how the relationships among them are obtained.

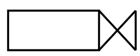
3.2.1 Model Variables

The method to identify model variables in this case is the “Brainstorming” (see for instance Caselles, 2008). All variables have been codified with a name of four letters. Therefore, in the variable list appears three items: the code, its meaning and its unit of measurement (bracketed) as follow,

XXXX Variable explanation [unit of measurement]

In this case, the selected variables are classified into five groups (not required by the methodology but useful for presentation), **Heritage, Income, Expense, Profit and Tax Agency.** Note that inside these groups the variables are alphabetically ordered (only useful for presentation). You can find more details in Appendix I.

The influence relation between the model variables is shown in the Forrester Diagram (Forrester, 1961) (Figure 1, Appendix I). In addition, the five groups of variables are differenced by color in this diagram, (Heritage: black; Income: red; Expense: orange, Profit: blue, Tax Agency: green). This is the characteristic diagram of System Dynamics, and is a hydrodynamic simile (it resembles a network of tanks connected by pipes with flows regulated by valves) that facilitates writing equations when programming a model for a computer. In this diagram the variables are classified into the following types:

- **Level variables**: require an initial value, which is an input variable, whose values are updated constantly. They are represented by a square or rectangle, and its meaning can be compared with a tank in which a fluid is stored.
- **Flow variables**: they compare with valves that regulate the flow to or from a fluid reservoir. These are represented by a characteristic icon: 
- **Auxiliary variables**: they are intermediate variables used to calculate flows, or are strictly output variables (which are not used in further calculations, and usually are optimizing variables). They are represented by a circle or an ellipse.

- **Control variables**: input variables whose value can be assigned by the person using the model or decision maker.
- **Exogenous or “scenario” variables**: input variables that are non controllable.
- **Constants**: system parameters with a known and fixed value.

All input variables and constants are represented by a double line ellipse or circle. Sources or sinks are represented by a cloud.

3.2.2 Functional relations

Once all variables which are involved in the model are defined, we study the relationships (functions) among them to present them as equations or/and logical rules. Appendix II shows the equations.

3.3 Verification and Validation

Synthetically, verification consists in testing for errors and validation consists in testing for suitability. Good reproduction of the past values after fitting the model to the same past data is verification of the model. Good reproduction of past data after fitting the model with different past data is the commonly used and called “ex post” validation (which is not the only validation method). In the present case the method used for validation is “revision by experts”.

3.3.1 Fitting input variables

As a step to verify the model with its stochastic formulation, all input variables are fitted respect to time. The fit has been obtained by a process of trial and error with two software tools: Mathematica 9.0 (Wolfram Mathematica, http://www.wolfram.com/) and Regint (Caselles, 1998, 2008). The last one allows us to obtain the best fitting function and provides the necessary information to write the input variables with the specific format (stochastic formulation) that Caselles (1992, 2008) suggests.

The first part consists in calculating the mean value (represented by h in Appendix V) using the corresponding fitted function. The second part consists in calculating the corresponding standard deviation (represented by s in Appendix II). A generic variable like Y is obtained as so: $Y = h + s \cdot \varepsilon(t).$, where $\varepsilon(t)$ is $N(0,1)$, h takes the following structure: $h = a + b_1T_1 + b_2T_2 + \ldots + b_mT_m$, and a, $b_1, \ldots, b_m$ are parameters, $T_1, \ldots, T_m$ are the functions transformed from the independent variables, and s is calculated according to the following formula:

$$s = s_{yx}\sqrt{1 + \frac{1}{n} + \tau' C\tau} \qquad [1]$$

Here s_{yx} is the regression standard deviation, n is the number of available data, the components of vector τ are the differences between the T_i functions of the independent variables and their respective means, and C is the inverse of the variances-covariances numerators matrix that corresponds to the transformed functions. For instance, Table 1 in Appendix II shows the following equation:

$$\text{TQSW}(t) = -1.42632 + \frac{52}{1 + 0.474899e^{-1.02205(-2008+t)}} \qquad term\ 1$$

In this case, the transformed functions are only one:

$$T_1 = \frac{1}{1 + 0.474899e^{-1.02205(-2008+t)}}$$

Matrix C is:

$$(0.0055061)$$

The mean of the transformed function is:

$$T_1 = 46.223568$$

The standard deviation is:

$$s = 1.573375$$

This information has been used in the model as follow,

h=-1.42632+52/(1+0.474899*Exp(-1.02205*(-2008+tems)))

A=1/(1+0.474899*Exp(-1.02205*(-2008+tems)))- 46.223568

s=1.573375*sqr(1+1/48+0.0055061*A*A)

Historical data have been obtained from the quarterly results for the 2008-2012 period of the business used as an example.

To verify the fitted functions, we calculate the coefficient of determination (R^2). The residuals have been tested for normality (using the Kolmogorov-Smirnov test).

Appendix III presents these results for all input variables.

3.3.2 Model Verification

Verification using the model's deterministic formulation

The model has been written as a set of finite difference equations and the solutions have been computed with Euler approximation, following Djidjeli et al (1998), where they explain that Euler's method is most appropriate to solve these equations in similar situations.

Verification is graphically displayed and the results for each quarter and the corresponding historical data have been superimposed. Also, the model has been verified numerically in two ways: calculating the coefficients of determination and testing the randomness of the residuals (Figures 43 and 44).

(Please insert Figure 43 about here)

(Please insert Figure 44 about here)

The result is satisfactory because:

- The graphic overlay is very good.
- The coefficient of determination (calculated with equation [2]) is very high (R^2= 0.996831)

$$R^2 = \frac{(\sum_i (x_i - \mu_x)(y_i - \mu_y))^2}{\sum_i (x_i - \mu_x)^2 \sum_i (y_i - \mu_y)^2}, \qquad [2]$$

where (x_i, y_i) are data and μ_x y μ_y are the respective averages.

- In addition, the maximum relative error is lower than 5%.

Figure 43 shows that the maximum income corresponds to the third quarter of each year. During July, August and September, a greater influx of students occurs in order to prepare the September exam.

Verification using the model's stochastic formulation

The stochastic formulation of the model allows us to determine the reliability of the results (each result is obtained with its respective confidence interval or with its own mean and standard deviation).

The central equations of the stochastic formulation of the model of this business are the same as in its deterministic model. The difference between both models is in the input variables, which in this case are written as stochastic functions of time (explained in 3.3.1).

The procedure to verify that the stochastic formulation of the model is correct is the following:

- Noting that all results are normally distributed (for this purpose, *SIGEM* programs automatically a $\chi 2$ test).
- Calculating a confidence interval of 95% (for instance) for each outcome and checking that all historical data is within this range.

The data corresponding to the stochastic formulation verification are presented in Figure 44, confirming that the model is valid for the small business case that has served as an example.

(Please insert Figure 43 about here)

(Please insert Figure 44 about here)

3.4 Simulation and Decision Making

The methodology presented in this article allows using two different ways to optimize the results of these models. On the one hand the design and test of strategies and scenarios is set up and, on the other hand, the optimization in each by means of a genetic algorithm is put in practice. Both processes are explained with detail in the following, taking into account that the objective-variable to optimize must be defined.

In the study case the objective-variable is the *benefits to end of year*, trying to maximize it in the 2013-2025 period.

3.4.1 Strategies and scenarios

A previous step to design and to test strategies and scenarios is discovering what variables have more influence on the objective-variable. In order to determine them a sensitivity analysis is made. Sensitivity Analysis is here considered as the study of the impact that a small change in an input variable has on an output variable considering the model as of black-box type. But obviously, this output variable could be also affected by the rest of input variables. Therefore, to see the real effect of each input variable on the given output variable, the other input variables must be considered as constants or a random sample of all their possible combined values must be taken. In other cases the analysis would be only valid for the considered concrete situation. An instance of this approach can be found in a book of Caselles et al. (1999).

Some methodological remarks:

- The input variables are assumed to be independent.
- The values of the input variables are recomputed as the following relative values:

$$rel = \frac{value - min_{variable}}{max_{variable} - min_{variable}} \quad [3]$$

- The R^2 coefficient determines the fraction of the variability of the output variable that is explained by the function of the corresponding input variables.

- If the data corresponding to input variables do not come from a random sample, the conclusions are only valid for the considered situation. In this paper a random sample is considered, because the simulator generated by *SIGEM* is able to do this.
- The data are fitted by linear or quadratic functions: $y = mx + n$, where m gives the increasing or decreasing rate, or $y = ax^2 + bx + c$, where the increasing or decreasing rate is determined by the derivative function $2ax+b$.

In our example, determining what input variables are the most influencing on the objective variable *PATQ* (the quarterly year after tax) has been tried. In Appendix IV these influences are shown (Table 41 and 42). Table 41 shows the relations between *PATQ* and the model input variables in the 2008-2012 period. Table 42 is equivalent to Table 41 but with the simulated data for the 2013-2025 period.

The determination coefficient indicates the proportion of the observed variability that is explained by the function that has been fitted.

According to Table 42, the most relevant input variables are expenses in water (*TQSW*), light (*TQSL*) and others (*OQEX*), as well as the number of students of type 1 (*QST1*) and the number of students of type 4 (*QST4*). To interpret these relationships, for example, in the case of the first variable, *TQSW*, an increase of one euro in a quarter year in the 2013-2025 period, *TQSW*, corresponds with an increase of 20.32 Euros in *PATQ*. We want to emphasize that this relationship is not a causal relation but a correlation.

Regarding the variables involved in incomes, we have that type 1 students, *QST1*, (students who attend the center 3 hours weekly) and type 4 students, *QST4*, (tutorials) are the most relevant. Note (Table 42) that the increase of one person in tutorials (*QST4*) in the 2013-2025 period is associated with a decrease of approximately 157 Euros in *PATQ*. This fact has an obvious explanation: benefit is greater with group classes (group classes have usually 5-6 people, equivalent to about 40 € / h, while tutorials are about 15 € / h).

We have observed with the sensitivity analysis what variables are most correlated with the main variable. In the beginning, we are going to make simulated predictions with the model. To do this, we extrapolate all input variables. *EXTRAPOL* is the tool that allows extrapolating the trend of the input variables with its confidence interval from a function previously obtained with *REGINT*. This tool can be found free in http://www.uv.es/~caselles/.

From this point, input variables must be classified into control variables (those controllable by decision makers) and scenario variables (those non controllable).

On the one hand, as the sensitivity analysis reveals, the variables most correlated with the objective variable are *TQSW*, *TQSL* and *OQEX*, which will be assumed as scenario variables due to the difficulty to its control. Note that the use of light, water and others can be controlled but not its price.

On the other hand, the variables related with the number of students, specifically those of type 1 and type 4, *QST4* and *QST1*, are assumed as control variables, because the business may carry out mechanisms that empower one or the other.

The considered tentative strategies and scenarios are:

• Strategy 1: strengthen the number of students of type 1, *QST1*, increase its trend and restrict the number of students of type 4, *QST4*. To do this, the extrapolated values of *QST1* will be increased by 2% (provisional value) and in *QST4* their values will be decreased by 1%.

• Strategy 2: strengthen the number of students of type 1, *QST1*, above their trend and keep the trend of the number of students of type 4, *QST4*. To do this, the extrapolated values of *QST1* will be increased by 2% (provisional value) and in *QST4* their extrapolated values will keep its trend.

• Strategy 3: keep the trend of the number of students of type 1, *QST1*, and restrict the number of students of type 4 *QST4*. For this case, the values of *QST1* keep their trend and those of *QST4* decrease 1%.

• Strategy 4: keep the trend of the number of students of type 1, *QST1*, and that of the number of students of type 4, *QST4*. For this case, both, *QST1* and *QST4* will take the values of their extrapolated trend.

• Scenario 1, trend: to keep the trend in variables related with expenses.

• Scenario 2, pessimistic: to increase expenses by 2% above its trend.

• Scenario 3, optimistic: to reduce expenses by 2% below its trend.

The hypothetical probability assigned to each scenario is:

- Scenario 1: p_1=0.3
- Scenario 2: p_2=0.2
- Scenario 3: p_3=0.5

In this case, the objective of the business is to maximize the cumulative quarterly results after applying the tax of the 2013-2025 period. To accumulate the values of *PATQ*, we define $ACUM_{ij}$, where index *i* represents a possible strategy and index *j* represents a possible scenario. The variable that characterizes each strategy is $zopt_i$ calculated as follows:

$$zopt1_i = \sum_j ACUM_{ij} * p_j \qquad [3]$$

where p_j is the probability that experts assign to scenario *j*. The maximum value of this variable indicates which strategy should be selected.

The corresponding calculations are performed with the simulator generated by *SIGEM*.

The optimal strategy to achieve the goal is chosen watching the results of $zopt1_i$, $i=1,\dots,4$:

$zopt1_1$: € 105834

$zopt1_2$: € 105499

$zopt1_3$: € 102841

$zopt1_4$: € 102700

According to these results, Strategy 1 produces a greater profit in the 2013-2025 period.

In Figure 45 the annual evolution of *PATQ* can be observed as well as that the greatest benefit after tax in 2025 corresponds to the Strategy 1, noticing, as expected, higher increase in scenario 3 than in the other scenarios.

(Please insert Figure 45 about here)

After determining the optimal strategy for maximum benefit of the business, let see in this specific optimal situation, what will be the Active, Equity and Passive during the study period.

Note that the existing active at the start of the activity (2008) amounted to €175,000 and passive of that year was worth €150,000. With these two values the equity of the business was € 25,000.

In Tables 43, 44 and Figure 46 the Active, Passive and Equity at the beginning of each year are shown. The maximum and minimum values are presented, between these values the true value can be found with 95% confidence.

(Please insert Tables 43, 44 and 46 about here)

3.4.2 Optimization with a Genetic Algorithm

A genetic algorithm can be automatically programmed by SIGEM inside the simulator. The genetic algorithm (GA) allows optimizing, in each time step, an objective variable previously defined from other variables included in the model (it is one of them). This variable is named OBJE, and is the one that the program tries to minimize. In the application case the objective is to maximize PATQ, for that reason the equation that calculates OBJE will be:

$$OBJE= -PATQ$$

The difference between the optimization obtained with the GA and the quasi-optimization obtained with the method of strategies and scenarios displayed in 3.4.1 consists in that the input variables which were fitted to time and its trend evolution were tentatively increased or decreased to design feasible strategies (the control variables), now are calculated by the model by means of the GA. The GA looks for the optimal strategy to reach the objective. The GA needs that the names of these control variables are introduced in a vector named as CROM (chromosome). This variable has an initial-value-variable, CROI, in which the initial values of the variables inside the chromosome corresponding to the period that is desired to simulate are placed. In the present case CROI contains the data of the last fourth month period of the year 2012, that is, the feasible starting point. Also another new variable is needed, PCRO, in which the user will mark the minimum and maximum values between which the variables located in CROM will be able to take value (in opinion of experts). The values that have been assigned to variables CROM, CROI and PCRO can be seen in Table 45.

(Please insert Tables 45 about here)

Once the simulator is generated by SIGEM and at the beginning to use it, the following data are asked to the user: Nº of individuals of the population (N); % of reproducers (R);

Nº of immigrants in each generation (I); Rate of mutants (per thousand) (M); Nº of generations (G); Nº of iterations of the genetic algorithm (updates of the initial values). (IT).

How the GA works?

The GA obtains random values (within the rank marked in PCRO) for each one of the CROM variables, obtaining therefore an individual of the population, and proceeding of this manner until completing the size of the population (N). Variable OBJE is calculated for each one of the individuals of the population. They are ordered of minor to greater value (notice than the program minimizes OBJE). The first R ones are kept and the rest is eliminated. Once finalized this process, immigrants (I) are defined (randomly) and are written after the R ones. The resting population is created by random cross-over within the R+I ones considering M. The cycle is repeated until completing N. The new values that now CROM has are taken like initial values to make the following iteration of the GA until completing IT. Observe that this procedure is designed to guarantee not to fall into local minima and to be as simple (and quick) as possible.

The results obtained in the application case have been the following ones

(Please insert Tables 46 to 49 about here)

In Tables 46, 47, 48 and 49 the Profit, Active, Passive and Equity at the beginning of each year are shown. The maximum and minimum values of each variable are presented. Between these values the true value can be found with 95% confidence. We can see that its tendency is the same than the observed one with the scenarios and strategies method. We also show the values of the control variables that must be used to reach the optimal ones. Note that *QST1* and *QST4* follow the same tendency that was determined by the strategies and scenarios method, that is to say, to increase the first and to diminish the second. This fact makes the results of both methods consistent.

(Please insert Tables 50 to 58 about here)

4. CONCLUSIONS

A methodology to construct dynamic mathematical models to model and optimize some social system has been presented. The application case has been validated for a study center, where non-formal education is imparted, in 2008-2012 period. The variables used in the model are taken from the Spanish General Accounting Plan for Small and Medium Enterprises (Official State Bulletin (BOE), num. 279). It is a general methodology, i.e., it can be used by any other business, saving the differences of the special activity.

All the variables and relationships between them are described. The model has been written in its deterministic and stochastic formulations, and a positive validation has been obtained for both formulations. We have used the results of the quarterly financial year after tax as a variable to validate the model. In the deterministic formulation the determination coefficient is higher than 0.99 and a relative error do not exceed 5%. Similar is the situation in the stochastic formulation, where real data are between maximum and minimum values of the 95% simulated confidence interval.

Simulation of the future has been made by means of two different methods: testing strategies and scenarios and optimizing by means of a genetic algorithm.

In the strategies and scenarios method a sensitivity analysis has been performed to observe which input variables are more closely correlated with the objective variable (result for the quarterly year after tax). Costs of water, electricity and other expenses, as well as the number of students of types 1 and 4 are variables with greater correlation with profit. These most correlated variables with the goal variable have been used to propose different possible strategies and scenarios. Concretely, four strategies and three scenarios have been proposed to determine which strategy would maximize the cumulative result of the quarterly financial year after tax for the 2013-2025 period. It is concluded that an increase of 2% (provisional value) on the trend in the number of students in Type 1 and a decrease of 1% (provisional value) of students in Type 4, and a decrease in expenses, causes the maximum benefit in this period.

All the control variables have been used to optimize with the genetic algorithm and the obtained results are equivalent to those obtained using the strategies and scenarios method. For future work we intend to apply this methodology to other economic and social problems in cities and regions.

References

Caselles, A., 1992. Simulation of Large Scale Stochastic Systems. *In Cybernetics and Systems'92. R. Trappl (ed.)*. Singapore. World Scientific. 221-228.

Caselles, A., 1994. Improvements in the Systems Based Program Generator SIGEM. *Cybernetics and Systems: An International Journal*. 25, 81-103.

Caselles, A., 1998. A tool for discovery by complex function fitting. *In Cybernetics and Systems Research'98. R. Trappl (ed.)*. Vienna. Austrian Society for Cybernetic Studies, 787-792.

Caselles, A., 2008. *Modelización y simulación de sistemas complejos*. Editorial: Universitat de València.

Caselles, A., Soler, D., Sanz, M.T., Mico, J.C. 2014. Simulating Demography ans Human Developments Dynamics. *Cybernetics and Systems: An International Journal.* 45, *465-485.*

Djidjeli A, W.G. Price A, P. Temarel A, & E.H. Twizell B., 1998. Partially implicit schemes for the numerical solutions of some non-linear differential equations. *Applied Mathematics and Computation*, 96, 177-207.

Forrester, J. W., 1961. *Industrial dynamics*. Cambridge: MIT Press.

Garcia M., Serrano F., 1990. La Modelización del comportamiento de sistemas económicos (A través de la dinámica de sistemas y la contabilidad circulatoria). *Revista Española de Financiación y contabilidad,* XX, 62, 45-114.

Micó, J.C., Caselles, A., Soler, D., 2006. Age-Structured Human Population Dynamics. *Journal of Mathematical Sociology*, 30, 1-31.

Sanz, M.T., Micó, J.C., Caselles, A., Soler, D. 2013. Welfare Human and Population in Austria. *Systems. Connecting matter, life, culture and technology,* 1 (2), 60-82.

Sanz, M.T., Micó, J.C., Caselles, A., Soler, D. 2014. A Stochastic model for population and well-being dynamics. *Journal of Mathematical Sociology*, 38, 75-94.

Sanz, M.T., Caselles, A., Micó, J.C., Soler, D. (2016). Including an environmental quality index in a demographic model. *International Journal in the Global Warming, 9(3), 362-396.*

Shannon, R., Johannes, J., 1976. Systems simulation: the art and science. *IEEE Transactions on Systems, Man and Cybernetics*, 6(10), 7234-724.

http://www.boe.es/buscar/doc.php?id=BOE-A-2007-19966. Official State Bulletin (BOE), num.279.

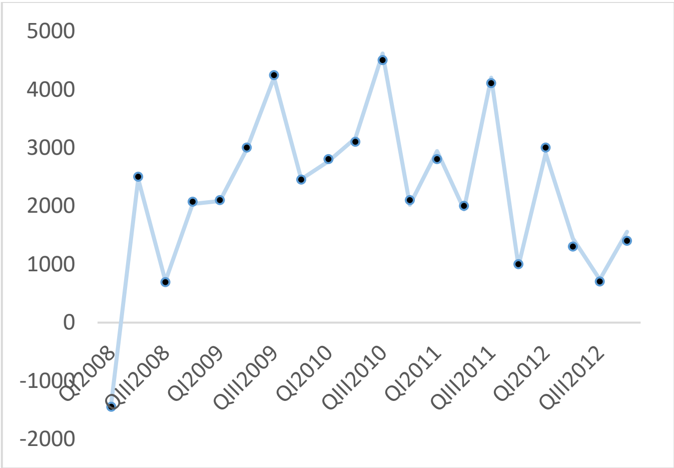


Figure 43. Simulated data (solid line) and real data (points) for the quarterly result after tax of a study center. The time period considered corresponds to 2008-2012. R^2 = 0.996831. Maximum relative error is 3.84%.

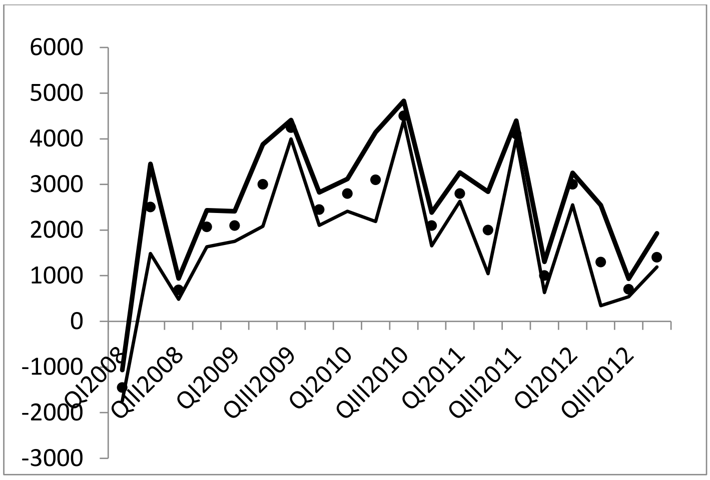


Figure 44. Simulated data, maximum and minimum (solid line) and real data (points) for the quarterly result after tax of a study center. The time period considered corresponds to 2008-2012.

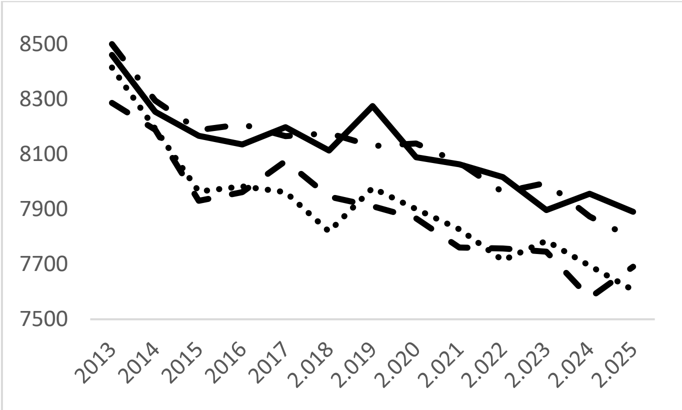


Figure 45. Results of the simulation. Variable *PATQ*. Strategy 1, line, Strategy 2, line-dots, Strategy 3, dot, Strategy 4, disline. 2013-2025 period.

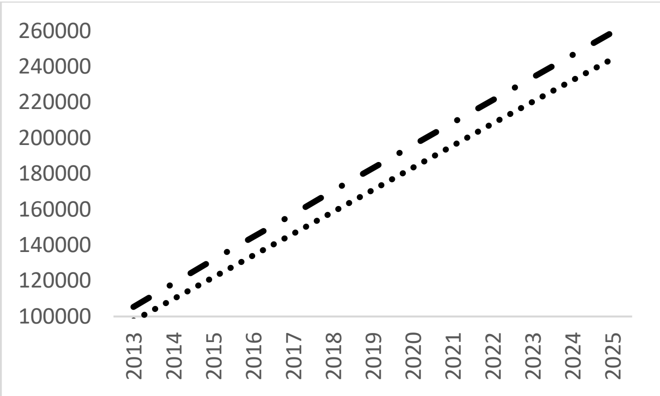


Figure 46. Equity results (*EQUI*), with Strategy 1 and Scenario 3. Minimum value (dots), maximum value (line-dots). 95% Confidence interval. 2013-2025 period.

Year	Minimum	Maximum
2013	127657.5	128404.7
2014	123308	124004.2
2015	118964.5	119597.7
2016	114563.1	115249.1
2017	110112.6	110949.6
2018	105747.6	106564.6
2019	101522.4	102039.8
2020	97085.5	97726.69
2021	92719.81	93342.39
2022	88364.54	88947.66
2023	84052.96	84509.26
2024	79580.15	80232.07
2025	75252.86	75809.38

Table 43. Passive results (*LIAB*), with Strategy 1 and Scenario 3. 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	225676.3	233385.5
2014	233464	242258.6
2015	241161.6	251045.4
2016	249019.5	259512.5
2017	256968.4	267898.2
2018	264776.5	276602.5
2019	272626	284883.3
2020	280783.3	293244.4
2021	288776.9	301584.1
2022	296660.1	309858.8
2023	304474	318140.6
2024	312107.6	326310.4
2025	319788.2	334696.2

Table 44. Active results (*ASST*), with Strategy 1 and Scenario 3. 95% Confidence Interval. 2013-2025 period.

CROM	CROI	PCRO	
QST1	10	1	100
QST2	7	1	100
QST3	3	1	100
QST4	2	1	100
PBMR	0.7	0.25	0.9
DRFU	0.025	0.01	0.05
DRCO	0.1	0.05	0.2
DRRE	0.005	0.0025	0.0075
RCQU	0.99	0.7	1

Table 45. Values of the control variables for the genetic algorithm: CROI (initial ones); PCRO (tentative minimum and maximum ones).

Year	Minimum	Maximum
2013	1446.9847	4487.5657
2014	865.8411	3911.835
2015	584.4804	3589.00796
2016	452.6224	3398.54859
2017	395.5638	3288.1947
2018	399.0916	3207.665
2019	358.5355	3195.8564
2020	313.4034	3135.1336
2021	298.01885	3099.1768
2022	299.26352	3097.2343
2023	300.54109	3104.1628
2024	303.7415	3111.6876
2025	305.5456	3111.742

Table 46. Profit after tax (*PATQ*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	270106.26	318944.52
2014	293904.96	350021.36
2015	315842.54	379819.68
2016	336649.57	409091.06
2017	356841.26	438110.9
2018	376839.56	466884.3
2019	396581.14	495719.1
2020	416149.6	524322.8
2021	435538.6	552813
2022	454762.8	581409
2023	473854.2	610157.1
2024	492848.1	639049
2025	511758.1	668046.4

Table 47. Equity results (*EQUI*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	518642.4	519371.2
2014	501441.7	502218.1
2015	484272.7	485080.3
2016	467117.7	467949.5
2017	449975.1	450824.7
2018	432835.6	433703.2
2019	415701.8	416587.4
2020	398571.99	399476.9
2021	381449.06	382361.79
2022	364330.92	365244.75
2023	347212.78	348127.76
2024	330093.16	331008.82
2025	312973.52	313892.05

Table 48. Passive results (*LIAB*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	789137.5	837926.5
2014	795750.1	851835.7
2015	800530	864485
2.016	804196	876611
2.017	807257	888494
2018	810129.8	900133
2019	812748.2	911841.7
2020	815196.5	923325
2.021	817466	934697
2.022	819573	946175
2023	821547.3	957804.5
2024	823422.3	969576.5
2025	825215	981455

Table 49. Active results (*ASST*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	80.51	88.12
2014	84.13	91.68
2015	87.25	94.23
2016	89.52	96.46
2017	91.20	98.00
2018	93.08	99.15
2019	94.63	99.94
2020	95.58	100.67
2021	96.35	101.41
2022	96.60	101.99
2023	97.02	102.61
2024	97.57	103.21
2025	97.85	103.34

Table 50. Quarterly students with type 1 fee results (*QST1*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	19.47	27.28
2014	17.27	24.76
2015	15.74	22.51
2016	14.05	20.89
2017	12.90	19.59
2018	12.04	17.96
2019	11.31	16.59
2020	10.58	15.72
2021	9.85	14.96
2022	9.26	14.72
2023	8.75	14.21
2024	8.14	13.70
2025	7.96	13.40

Table 51. Quarterly students with type 2 fee results (*QST2*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	6.59	9.13
2014	5.67	7.85
2015	4.87	6.82
2016	4.42	6.10
2017	4.16	5.62
2018	4.08	5.18
2019	4.08	4.93
2020	4.08	4.87
2021	4.08	4.84
2022	4.11	4.84
2023	4.09	4.82
2024	4.07	4.79
2025	4.11	4.84

Table 52. Quarterly students with type 3 fee results (*QST3*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	4.03	4.68
2014	4.03	4.40
2015	4.03	4.38
2016	4.03	4.35
2017	4.03	4.33
2018	4.02	4.32
2019	4.03	4.32
2020	4.02	4.31
2021	4.02	4.32
2022	4.02	4.31
2023	4.02	4.32
2024	4.03	4.33
2025	4.02	4.32

Table 53. Quarterly students with type 4 fee results (*QST4*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	2.01	3.17
2014	1.98	3.18
2015	1.92	3.11
2016	1.89	3.15
2017	1.87	3.16
2018	1.81	3.11
2019	1.75	3.09
2020	1.65	3.04
2021	1.58	3.06
2022	1.55	3.04
2023	1.53	3.00
2024	1.51	2.96
2025	1.51	2.90

Table 54. Percentage of benefits who moves to renewal results (*PBMR*). 95% Confidence Interval.

Year	Minimum	Maximum
2013	0.0501	0.0834
2014	0.0436	0.0780
2015	0.0411	0.0717
2016	0.0395	0.0652
2017	0.0389	0.0605
2018	0.0376	0.0574
2019	0.0366	0.0562
2020	0.0377	0.0522
2021	0.0405	0.0483
2022	0.0429	0.0451
2023	0.0429	0.0451
2024	0.0429	0.0451
2025	0.0439	0.0439

Table 55. Depreciation rate of furniture results (*DRFU*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	0.1994	0.2140
2014	0.2001	0.2121
2015	0.2001	0.2106
2016	0.1998	0.2106
2017	0.2000	0.2108
2018	0.1998	0.2110
2019	0.2009	0.2089
2020	0.1997	0.2098
2021	0.2009	0.2082
2022	0.2008	0.2085
2023	0.2001	0.2100
2024	0.2011	0.2086
2025	0.2020	0.2078

Table 56. Depreciation rate of computers results (*DRCO*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	0.011761	0.011761
2014	0.011034	0.011034
2015	0.010718	0.010718
2016	0.010627	0.010627
2017	0.010582	0.010582
2018	0.010566	0.010566
2019	0.010531	0.010531
2020	0.010539	0.010539
2021	0.010553	0.010553
2022	0.010517	0.010517
2023	0.010529	0.010529
2024	0.010588	0.010588
2025	0.010543	0.010543

Table 57. Depreciation rate of property results (*DRRE*). 95% Confidence Interval. 2013-2025 period.

Year	Minimum	Maximum
2013	3.02	3.74
2014	2.99	3.67
2015	3.01	3.70
2016	3.00	3.72
2017	2.99	3.71
2018	3.02	3.71
2019	2.98	3.68
2020	2.96	3.65
2021	2.97	3.66
2022	3.02	3.73
2023	3.02	3.70
2024	3.01	3.69
2025	3.00	3.72

Table 58. Rate cut quotas (*RCQU*). 95% Confidence Interval. 2013-2025 period.

Appendix I.
Model Variables and Forrester Diagram

Variables related with heritage
ASST Assets [€]
ASS0 Initial Assets [€]
EQUI Equity [€]
LIAB Liability [€]
LIA0 Initial Liability [€]

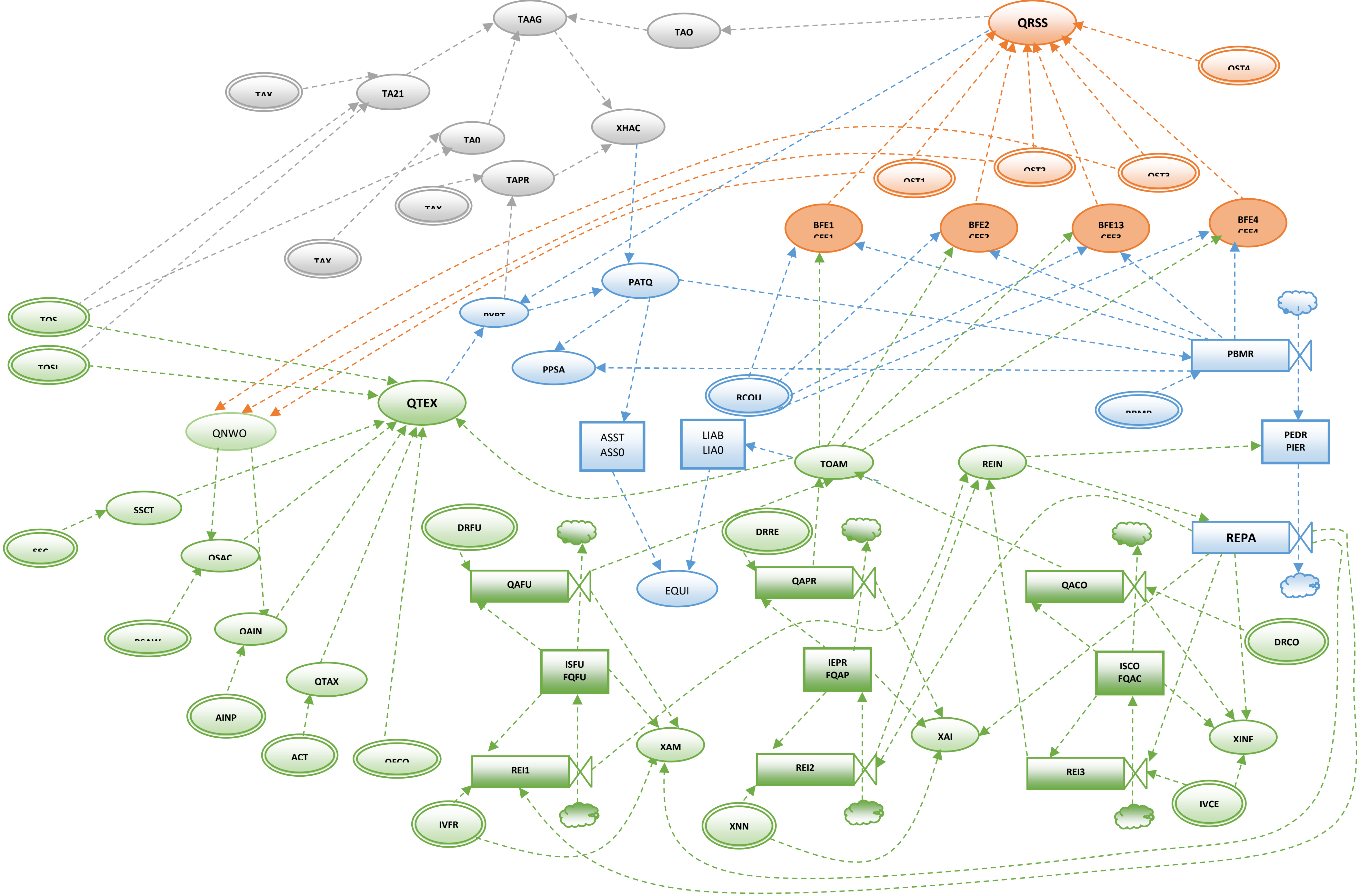
Variables related with the Income
BFE1 Basic fee for three days of class per week [€]
BFE2 Basic fee for two days of class per week [€]
BFE3 Basic fee for one day of class per week [€]
BFE4 Basic fee for tutorials [€]
CFE1 Current fee for three days of class per week [€]
CFE2 Current fee for two days of class per week [€]
CFE3 Current fee for one day of class per week [€]
CFE4 Current fee for tutorials [€]
QRSS Quarterly revenue from sales and services [€]
QST1 Quarterly students with type 1 fee [number]
QST2 Quarterly students with type 2 fee [number]
QST3 Quarterly students with type 3 fee [number]
QST4 Quarterly students with type 4 fee [number]

Variables related with the Expense
ACTE Annual council tax expense [€]
AINP Annual insurance premiums [€]
BSAW Base salary for worker [€]
DRFU Depreciation rate of furniture [ratio]
DRCO Depreciation rate of computers [ratio]
DRRE Depreciation rate of property [ratio]
FQAC Final quarterly Amortization of computers [€]
FQAP Final quarterly Amortization of property [€]
FQFU Final quarterly furniture value [€]
IEPR Initial expenditure on property [€]
ISCO Initial spending on computers [€]
ISFU Initial spending on furniture [€]
IVCE Initial value of computer equipment to renew [€]
IVFRI Initial value of the furniture to renew [€]
IVPR Initial value of the property to renew [€]
OQEX Other quarterly expenses [€]
QACO Quarterly Amortization of computer equipment [€]
QAFU Quarterly Amortization of furniture [€]
QAIN Quarterly amortization Insurance [€]
QAPR Quarterly Amortization of property [€]
QFCO Quarterly financial costs [€]
QSAC Quarterly salary costs [€]
QNWO Quarterly number of workers [people]
QPTM Quarterly purchases, training material [€]

QTAX Quarterly tax expense [€]
QTEX Quarterly total expenditure [€]
QWPC Quarterly Work performed by other companies [€]
REI1 Renewal of furniture [€]
REI2 Renewal of property [€]
REI3 Renewal of computers [€]
REIN Total quarterly Renewal [€]
SSCC Social security base (autonomous) quarterly [€]
SSCT Social security (autonomous) quarterly [€]
TQAM Total quarterly amortization [€]
TQSL Total quarterly spending on light [€]
TQSW Total quarterly spending on water [€]

Variables related with Profit
BRMR Benefits rate that moves to renewal [probability]
IPPS Initial part of the profit dedicated to creating savings [€]
PATQ Profit after tax for the year quarterly [€]
PBMR Percentage of benefits who moves to renewal [€]
PEDR Part of equity dedicated to renewal [€]
PIER Part of the initial equity dedicated to renewal [€]
PPSA Part of profit dedicated to creating savings [€]
PYBT Profit for the year before tax quarterly [€]
RCQU Rate cut quotas [probability]

Variables related with the Tax Agency
TA08 Input Quarterly Value-Added Tax 8% [€]
TA21 Input Quarterly Value-Added Tax 21% [€]
TAAG Tax Agency [€]
TAOU Output Quarterly Value-Added Tax [€]
TAPR Percentage 12% quarterly tax over profits [€]
TAX1 Percentage Tax type 1 for water [%]
TAX2 Percentage Tax type 2 for general values [%]
TAX3 Percentage Tax type 3 to apply to profits of an enterprise [%]



TAAG
TAO
QRSS
TA21
XHAC
TAPR
OST1
OST2
OST3
BFE1
BFE2
BFE13
BFE4
PATQ
PPSA
PBMR
RCOU
QTEX
QNWO
ASST
ASSO
LIAB
LIAO
TQAM
REIN
PEDR
PIER
SSCT
OSAC
REPA
DRFU
DRRE
QAFU
QAPR
QACO
EQUI
OAIN
DRCO
ISFU
FQFU
IEPR
FQAP
ISCO
FQAC
QTAX
AINP
XAM
XAI
XINF
ACT
REI1
REI2
REI3
IVFR
XNN
IVCE

Figure 1: Forrester Diagram for the economical/mathematical model

Appendix II

Equations of the Model

First the initial position of the business is observed.

The existing asset consists of the property, computer equipment and furniture, and has a total value of € 175,000 (value of *ASS0*), of which € 150,000 is a loan, which the liability is this amount (value of *LIA0*) and equity, *EQUI*, amounts at the beginning of the activity are € 25,000.

The following equations are included in the model to observe how the asset, liability and equity range in each quarterly period.

```
ASST asst = ass0 + patq
LIAB liab = lia0 - tqam
EQUI equi = asst – liab
```

Equations related with expenses

Water, light, shopping, work done by other companies, insurance, financial expenses, no state taxes, employee salaries, social security, depreciation of property, furniture and equipment are considered as expenses.

Note that the number of company employees is calculated as the integer part of the result of dividing the total number of students at the school by 30 because it is considered that a teacher can deal with 30 students.

```
TQAM tqam = qafu + qapr + qaco + 1000
QAIN qain = ainp
QTEX qtex = qptm + tqsw + tqsl + oqex + qwpc + ssct + qsac
      + qain + qtax + qfco + tqam - 1000
QTAX qtax = acte
QNWO qnwo = (qst1 + qst2 + qst3 + qst4) \ 30
SSCT ssct = sscc * qnwo
QSAC qsac = bsaw * qnwo
```

Also note that € 1000 is added in *TQAM* and then this amount is subtracted in *QTEX*. Such € 1000 refers to an upper limit mark after calculating quotas.

Equations related with Tax Agency

Here we look at the input VAT and output VAT.

We consider as input VAT that has been generated when spending (it is incorporated in the price of the purchased product). We use the VAT 6% (2008) which evolved to 8% (year 2011) also VAT at 16% (2008) which evolved to 18% (in 2011) and 21% (in the third quarter of 2012).

The output VAT which is generated by the company and has evolved as the VAT of 16%, 18% and 21%

Also the tax of 12%, which must be paid for profits, has been included in this section,

```
TA08 ta08 = tqsw * tax1
TA21 ta21 = (qptm + tqsw + tqsl + oqex + qwpc) * tax2
TAOU taou = qrss * tax2
TAAG taag = taou - (ta08 + ta21 + tapr)
```

TAAG is the result of output VAT minus input VAT. We should note that:

- If *TAAG* is positive, the company pays to Tax Agency.
- If *TAAG* is negative, Tax Agency returns paid taxes to the company.

Equations related with Income

In this small company there are four types of fees; it depends on the number of class hours that students hired. Besides, fees may be lowered by 0.1% on a quarterly if the percentage of quarterly profits which are moving into investment exceeds the total expense in amortization plus € 1000 (those that are reflected in the equation that calculates *TQAM*). Otherwise, the fee should not be modified. This is reflected in the following algorithms. Note that capital letters correspond to the name of the variable that is calculated with the corresponding algorithm:

```
CFE1 if pbmr > tqam then
        cfe1 = rcqu * bfe1
       else
        cfe1 = bfe1
      endif
CFE2 if pbmr > tqam then
       cfe2 = rcqu * bfe2
       else
        cfe2 = bfe2
      endif
CFE3 if  pbmr > tqam then
       cfe3 = rcqu * bfe3
      else
       cfe3 = bfe3
      endif
CFE4 if pbmr > tqam then
       cfe4 = rcqu * bfe4
      else
       cfe4 = bfe4
      endif
QRSS qrss = bfe1 * qst1 + bfe2 * qst2 + bfe3 * qst3 + bfe4
      * qst4
```

Equations related with quarterly amortizations of furniture, building and informatics equipment

Note that an amortization period of 10, 50 and 5 years has been programmed for each type of good. When these periods finish, the following amounts will be used to renovate the respective goods:

- For computers: € 1000.

- For furniture: € 500.
- For property: € 5000.

Such renewal will be financed with the existing capital for renovations. Thus, the depreciation and renewals are calculated as follows:

```
QAFU qafu = drfu * ivfri
FQFU fqfu = isfu - qafu + rei1
REI1 if isfu < qafu then
       rei1 = ivfri
      else
       rei1 = 0
      endif
QAPR qapr = drre * ivpr
FQAP fqap = iepr - qapr + rei2
REI2 if iepr < qapr then
       rei2 = ivpr
      else
       rei2 = 0
      endif
QACO qaco = drco * ivce
FQAC fqac = isco - qaco + rei3
REI3 if isco < qaco then
       rei3 = ivce
      else
       rei3 = 0
      endif
REIN rein = rei1 + rei2 + rei3
```

Equations related with Profit

Note that if there are some benefits in the company, 30% of them will be used to create a financial cushion and the remaining will be accumulated for possible future reinvestment in the business. Thus, capital for reinvestment will be distributed as follows:

- If reinvestment is needed in one of three considered chapters (property, furniture and computer equipment), the capital for reinvestment should go entirely to this one.
- If reinvestment is needed in two of them, such capital should be split fifty-fifty.
- If reinvestment is needed in all of them, a third should be assigned each one.

This should be written in programming language as follows:

```
PYBT pybt = qrss - (qtex + 1000)
TAPR if pybt > 0 then
       tapr = pybt * tax3
      else
       tapr=0
      endif
PATQ patq = pybt + (-taag)
PPSA if patq > 0 then
       ppsa = ipps + patq - pbmr
      else
       ppsa = ipps + patq
      endif
BRMR brmr = 0.7
PBMR if patq > 0 then
       pbmr = brmr*patq
      else
       pbmr = 0
      endif
PEDR if rein = 0 then
       pedr = pier + pbmr
      else
       if rei1 = ivfri then
         pedr = pier + pbmr - ivfri
        else
         if rei2 = ivpr then
           pedr = pier + pbmr - ivpr
          else
           if rei3 = ivce then
             pedr = pier + pbmr - ivce
           endif
          endif
        endif
      endif
```

Appendix III

Input variables in their stochastic formulation

TQSW Total quarterly spending on water

$$\mathrm{TQSW}(t) = \begin{cases} -1.42632 + \dfrac{52}{1+\ 0.474899e^{\ -1.02205(-2008+t)}} & term\ 1 \\ -5.99658 + \dfrac{23}{1+0.331495e^{\ -0.915049(-2008.25+t)}} & term\ 2 \\ -3.33509 + \dfrac{55}{1+\ 0.474495e^{\ -0.885117\ (-2008.5+t)}} & term\ 3 \\ -1.70267 - \dfrac{27}{1+\ 0.450443e^{\ -1.01076\ (-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
-1.42632	52	0.474899	-1.02205	2008
C Matrix		Mean of transformed function		Estimation Error
(0.0055061)		T1=46.223568		S=1.573375

Table 1. $TQSW_1$ Variable

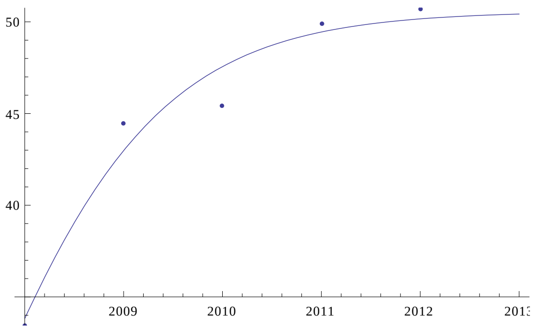


Figure 2. $TQSW_1$ as a function of time in 2008-2012 period. R^2=0.961032

Second term,

α	β	γ	δ	μ
-5.99658	23	0.331495	-0.915049	2008.25
C Matrix		Mean of transformed function		Estimation Error
(0.048472266)		T1=20.948577		S=0.523382

Table 2. $TQSW_2$ Variable

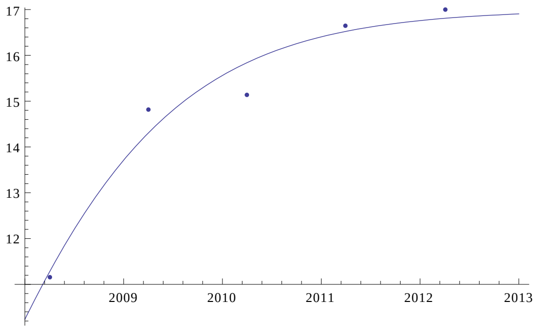


Figure 3. $TQSW_2$ as a function of time in 2008-2012 period. R^2=0.961899.

Third term,

α	β	γ	δ	μ
-3.33509	55	0.474495	-0.885117	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.005189633)		T1=48.332337		S=1.700310

Table 3. $TQSW_3$ Variable

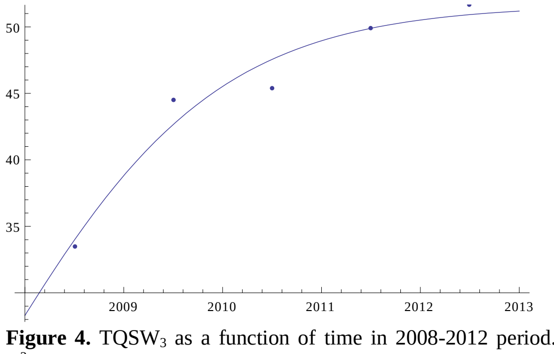


Figure 4. $TQSW_3$ as a function of time in 2008-2012 period. R^2=0.957316.

Fourth term,

α	β	γ	δ	μ
-1.70267	27	0.450443	-1.01076	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.0220059)		T1=24.101295		S=0.783738

Table 4. $TQSW_4$ Variable

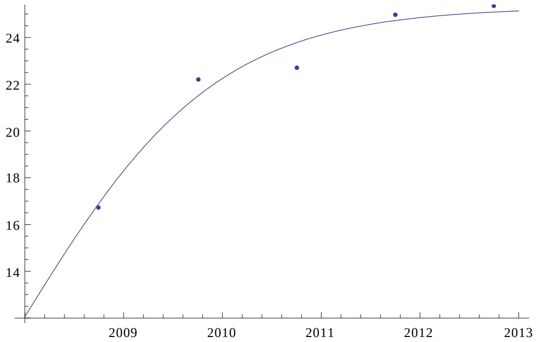


Figure 5. $TQSW_4$ as a function of time in 2008-2012 period. R^2=0.961323.

QFCO Quarterly financial costs

$$qfco(t) = -14.2903 + \frac{35}{1 + 2.65463e^{-1.06623\,(-2010.25+t)}} + \frac{25}{1 + 9.27027 \times 10 - 7e^{3.75051\,(-2008+t)}}$$

α	$β_1$	$γ_1$	$δ_1$	$μ_1$
-14.2903	35	2.65463	-1.06623	2010.25
	$β_2$	$γ_2$	$δ_2$	$μ_2$
	25	$9.27 \cdot 10^{-7}$	3.75051	2008
C Matrix		Mean of transformed function		Estimation Error
$\begin{pmatrix} 0.02771956 & 0.00268491 \\ 0.00268491 & 0.003230485 \end{pmatrix}$		T1=12.753452 T2=19.128360		S=4.665131

Table 5. QFCO Variable

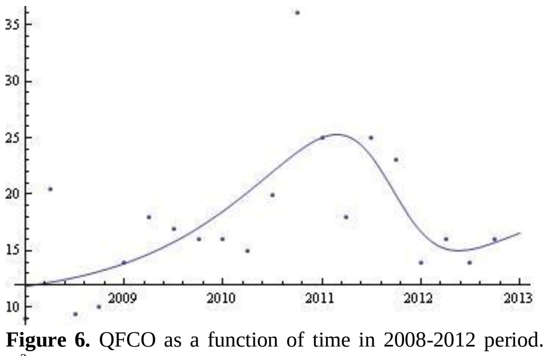


Figure 6. QFCO as a function of time in 2008-2012 period. R^2=0.504142.

ACTE Annual council tax expense

There is no annual trend observed. Thus, the trend is studied quarterly.

$$acte(t) = \begin{cases} 0 & term\ 1 \\ 31.8197 + \dfrac{100}{1 +\ 761.108e^{\ -2.11399\ (-2008.25+\mathrm{t})}} & term\ 2 \\ \dfrac{115}{1 +\ 2959.9e^{\ -7.55107\ (-2008.5+\mathrm{t})}} & term\ 3 \\ 9.4325 + \dfrac{110}{1 +\ 1.94299 \times 10 - 14e^{\ 29.336\ (-2008.75+\mathrm{t})}} & term\ 4 \end{cases}$$

Second term,

α	β	γ	δ	μ
31.8197	100	761.108	-2.11399	2008.25
C Matrix		Mean of transformed function		Estimation Error
(0.000182493598)		T1=27.655123		S=31.202546

Table 6. $ACTE_2$ Variable

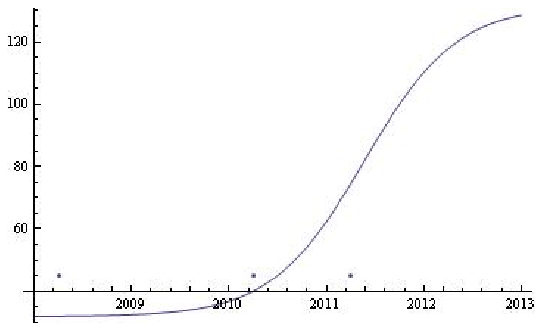


Figure 7. $ACTE_2$ as a function of time in 2008-2012 period. R^2=0.802051.

Third term,

α	β	γ	δ	μ
0	115	2956.89	-7.55006	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.000023993272)		T1=77.988927		S=0.052593

Table 7. $ACTE_3$ Variable

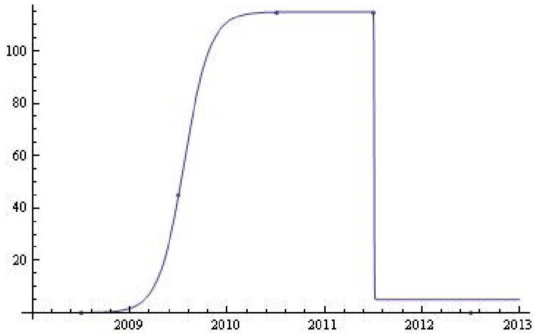


Figure 8. $ACTE_3$ as a function of time in 2008-2012 period. R^2=0.998892.

Fourth term,

α	β	γ	δ	μ
9.4325	110	$1.94299 \cdot 10^{-14}$	29.336	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.000075711117)		T1=41.875500		S=22.108790

Table 8. $ACTE_4$ Variable

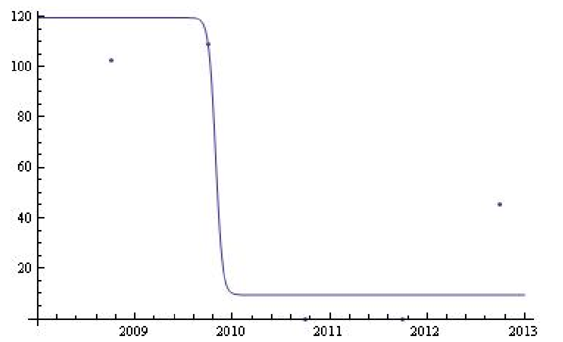


Figure 9. $ACTE_4$ as a function of time in 2008-2012 period. R^2=0.869718.

OQEX Other quarterly expenses

$$oqex(t) = \begin{cases} 427.56 + \dfrac{1100}{1 + 0.538236e^{1.74555\,(-2008+t)}} & term\ 1 \\ 144.348 + \dfrac{400}{1 + 0.687859e^{1.66073\,(-2008.25+t)}} & term\ 2 \\ 425.137 + \dfrac{1200}{1 + 0.671446e^{1.63431\,(-2008.5+t)}} & term\ 3 \\ 215.702 + \dfrac{580}{1 + 0.628745e^{1.69208\,(-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
427.56	1100	0.538236	1.74555	2008
C Matrix		Mean of transformed function		Estimation Error
(0.000002743680)		T1=211.212694		S=12.795513

Table 9. $OQEX_1$ Variable

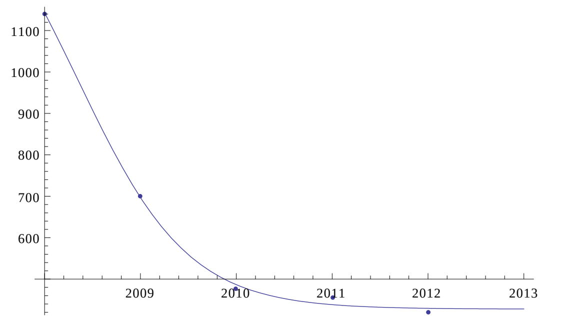


Figure 10. $OQEX_1$ as a function of time in 2008-2012 period. R^2=0.998652.

Second term,

α	β	γ	δ	μ
144.348	400	0.687859	1.66073	2008.25
Covariance Matrix		Mean of transformed function		Estimation Error
(0.000025112120)		T1=69.642562		S=4.162523

Table 10. $OQEX_2$ Variable

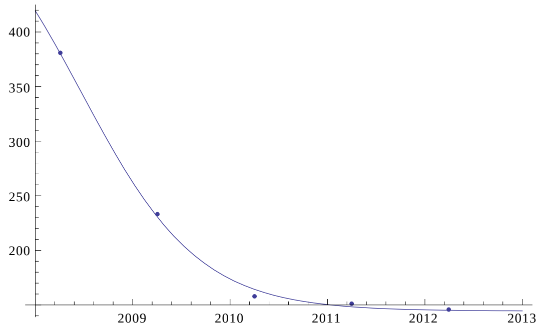


Figure 11. $OQEX_2$ as a function of time in 2008-2012 period. R^2=0.998692.

Third term,

α	β	γ	δ	μ
425.137	1200	0.671446	1.63431	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.000002743126)		T1=213.635989		S=13.605470

Table 11. $OQEX_3$ Variable

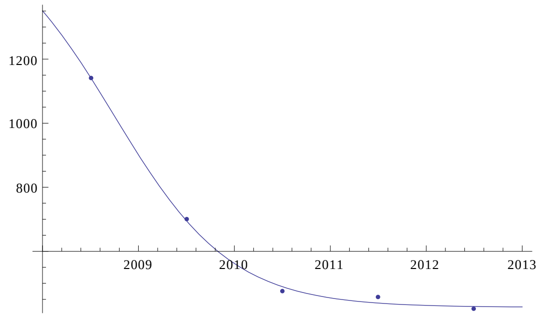


Figure 12. $OQEX_3$ as a function of time in 2008-2012 period. R^2=0.998476.

Fourth term,

α	β	γ	δ	μ
215.702	580	0.628745	1.69208	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.000011100548)		T1=104.784748		S=5.963226

Table 12. $OQEX_4$ Variable

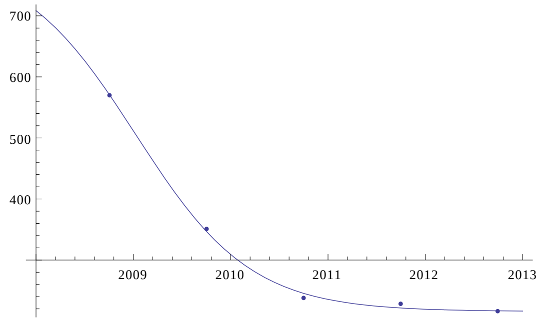


Figure 13. $OQEX_4$ as a function of time in 2008-2012 period. R^2=0.998813.

AINP Annual insurance premiums

$$ainp(t) = \begin{cases} \dfrac{150}{1+\ 2879.7e^{\ -2.55069\,(-2008+t)}} & term\ 1 \\ 23.5519 + \dfrac{500}{1+\ 39.7638e^{\ -2.38296\,(-2008.25+t)}} & term\ 2 \\ 285.069 + \dfrac{800}{1+\ 14.0268e^{\ -0.879185\,(-2008.5+t)}} & term\ 3 \\ 94.8596 - \dfrac{100}{1+\ 23317.5e^{\ -6.4834\,(-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
0	150	2879.7	2.55069	2008
C Matrix		Mean of transformed function		Estimation Error
(0.000044548553)		T1=41.537304		S=74.515881

Table 13. $AINP_1$ Variable

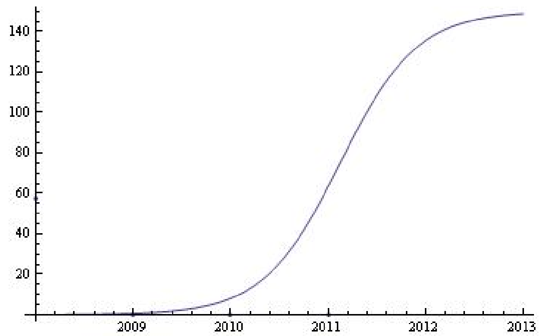


Figure 14. $AINP_1$ as a function of time in 2008-2012 period. R^2=0.698438

Second term,

α	β	γ	δ	μ
23.5519	500	39.7638	2.38296	2008.25
C Matrix		Mean of transformed function		Estimation Error
(0.000005027678)		T1=295.258053		S=71.174843

Table 14. $AINP_2$ Variable

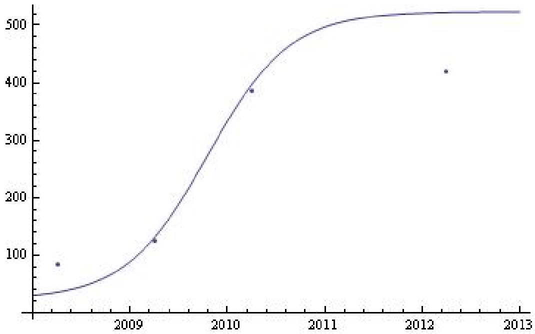


Figure 15. $AINP_2$ as a function of time in 2008-2012 period. R^2=0.912327.

Third term,

α	β	γ	δ	μ
285.069	800	14.0268	0.879185	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.000005709743)		T1=273.735162		S=63.530681

Table 15. $AINP_3$ Variable

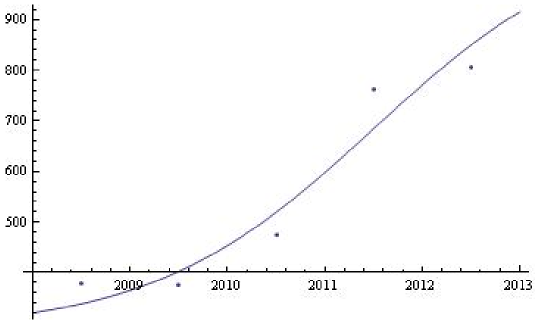


Figure 16. $AINP_3$ as a function of time in 2008-2012 period. R^2=0.931492.

Fourth term,

α	β	γ	δ	μ
94.8596	-100	23317.5	6.4834	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.000088566615)		T1=59.511622		S=4.681471

Table 16. $AINP_4$ Variable

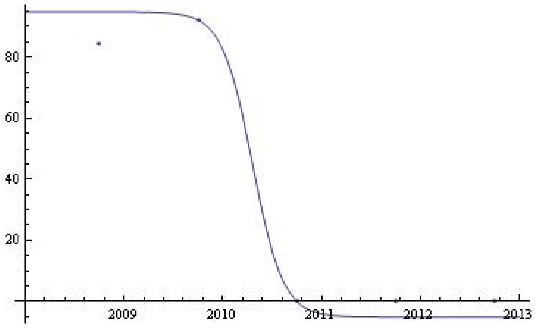


Figure 17. $AINP_4$ as a function of time in 2008-2012 period. R^2=0.993006.

SSCC Social security base (freelance) quarterly

$$sscc(t) = \frac{760}{1 + 0.000513795e^{-15.6765(-2009.75+t)}} + \frac{562}{1 + 1.5049 \cdot 10^{-9} e^{16.5377\,(-2008+t)}}$$

α	$β_1$	$γ_1$	$δ_1$	$μ_1$
0	760	0.000513795	-15.6765	2010.75
	$β_2$	$γ_2$	$δ_2$	$μ_2$
	562	1.5049	16.5377	2008
C Matrix		Mean of transformed function		Estimation Error
$\begin{pmatrix} 0.000000122529 & -0.000000006618 \\ -0.000000006618 & 0.000000618200 \end{pmatrix}$		T1=548105038 T2=151740993		S=4.903342

Table 17. SSCC Variable

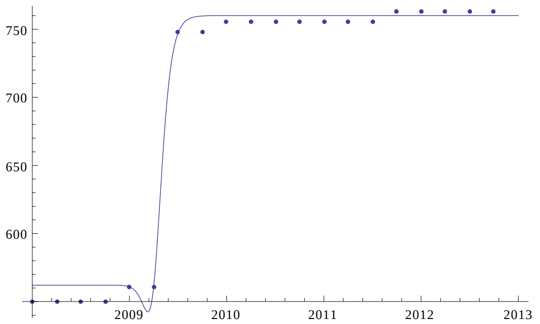


Figure 18. SSCC as a function of time in 2008-2012 period. R^2=0.997328.

BSAW Base salary (employed)

$$bsaw(t) = \frac{650}{1 + 30.4991e^{-0.0512281\,(-2010.75+t)}} + \frac{620}{1 + 0.0179502e^{-1.14956\,(-2009.75+t)}} + \frac{600}{1 + 15.6752e^{3.46454\,(-2008+t)}}$$

α	β_1	γ_1	δ_1	μ_1
0	650	30.4991	-0.0512281	2010.75
	β_2	γ_2	δ_2	μ_2
	620	0.0179502	-1.14956	2009.75
	β_3	γ_3	δ_3	μ_3
	600	15.6752	3.46454	2008
C Matrix		Mean of transformed function		Estimation Error
$\begin{pmatrix} 0.056794 & -0.001916 & 0.0014336 \\ -0.001916 & 0.000064 & -0.0000517 \\ 0.0014336 & -0.0000517 & 0.000737 \end{pmatrix}$		T1=20.305313 T2=604.558224 T3=3.162605		S=3.891826

Table 18. BSAW Variable

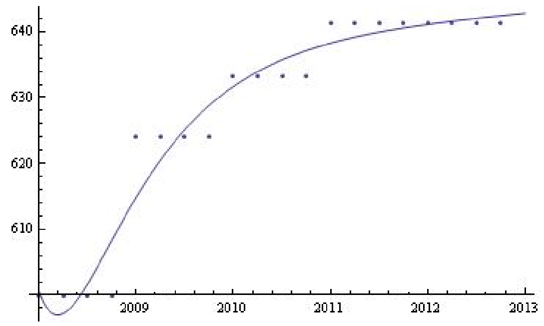


Figure 19. BSAW as a function of time in 2008-2012 period. R^2=0.948982.

TQSL Total quarterly spending on light

$$tqsl(t) = \begin{cases} -7.47817 + \dfrac{200}{1 + 0.296665e^{-1.39606\,(-2008+t)}} & term\ 1 \\ -2.43371 + \dfrac{67}{1 + 0.303852e^{-1.30708\,(-2008.25+t)}} & term\ 2 \\ -3.486 + \dfrac{196}{1 + 0.304488e^{-1.40082\,(-2008.5+t)}} & term\ 3 \\ -4.83149 + \dfrac{102}{1 + 0.302872e^{-1.26019\,(-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
-1.42632	52	0.474899	-1.02205	2008
C Matrix		Mean of transformed function		Estimation Error
(0.000679315686)		T1=187.173422		S=0.841035

Table 19. $TQSL_1$ Variable

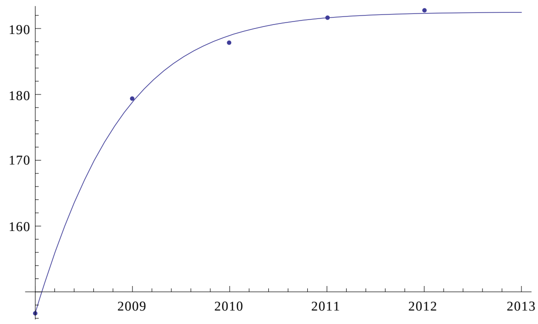


Figure 20. $TQSL_1$ as a function of time in 2008-2012 period. R^2=0.998562.

Second term,

α	β	γ	δ	μ
-5.99658	23	0.331495	-0.915049	2008.25
C Matrix		Mean of transformed function		Estimation Error
(0.005910408044)		T1=62.465461		S=0.484110

Table 20. $TQSL_2$ Variable

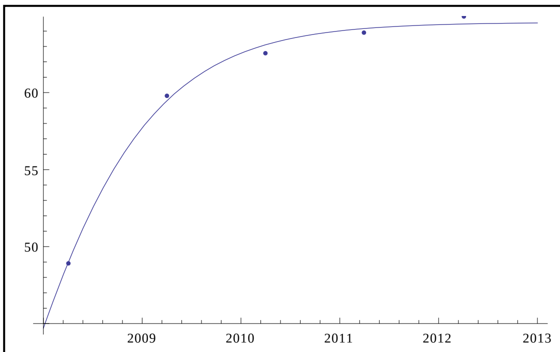


Figure 21. $TQSL_2$ as a function of time in 2008-2012 period. R^2=0.995865.

Third term,

α	β	γ	δ	μ
-3.33509	55	0.474495	-0.885117	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.000679344089)		T1=183.181246		S=0.845762

Table 21. $TQSL_3$ Variable

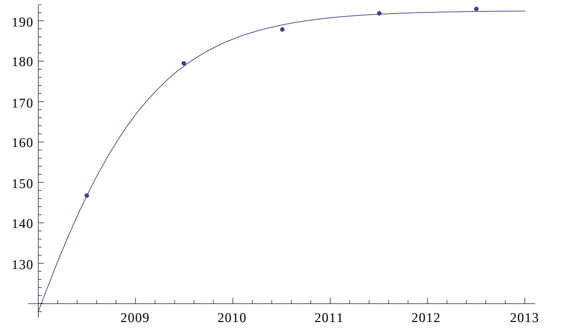


Figure 22. $TQSL_3$ as a function of time in 2008-2012 period. R^2=0.998545.

Fourth term,

α	β	γ	δ	μ
-1.70267	27	0.450443	-1.01076	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.002582147545)		T1=94.979113		S=0.901404

Table 22. $TQSL_4$ Variable

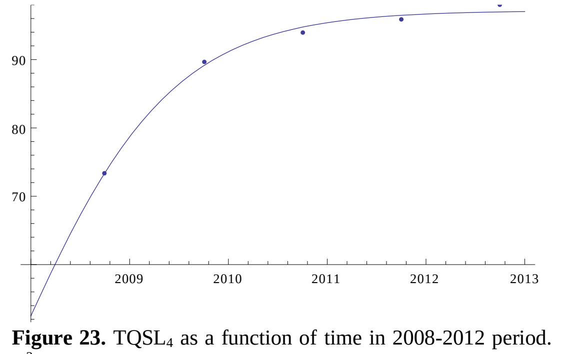


Figure 23. $TQSL_4$ as a function of time in 2008-2012 period. R^2=0.993757.

QST1 Quarterly students with quota type 1

$$qst1(t) = \begin{cases} 0.206398 + \dfrac{20}{1 + 1.04564e^{-1.3813(-2008+t)}} & term\ 1 \\ \dfrac{14}{1 + 4.72 \cdot 10^{-12} e^{26.746(-2010.25+t)}} + \dfrac{18}{1 + 11.0001e^{-0.894831(-2008.25+t)}} & term\ 2 \\ \dfrac{23}{1 + 0.243724e^{81.94(-2011.5+t)}} + \dfrac{30}{1 + 13.2678e^{-0.655784(-2008.5+t)}} & term\ 3 \\ \dfrac{14}{1 + 0.76041e^{3.92966(-2010.75+t)}} + \dfrac{18}{1 + 8.32083e^{-0.727505(-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
0.206398	20	1.04564	-1.3813	2008
C Matrix		Mean of transformed function		Estimation Error
(0.013875476790)		T1=16.793602		S=0.104748

Table 23. $QST1_1$ Variable

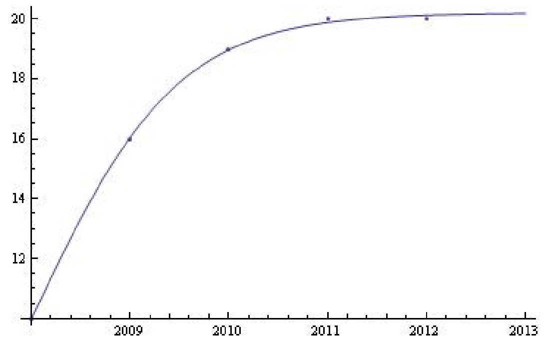


Figure 24. $QST1_1$ as a function of time in 2008-2012 period. R^2=0.999543.

Second term,

α	β_1	γ_1	δ_1	μ_1
0	14	4.76538e-12	26.746	2010.25
	β_2	γ_2	δ_2	μ_2
	18	11.0001	-0.894831	2008.25
C Matrix		Mean of transformed function		Estimation Error
$\begin{pmatrix} 0.002037998766 & -0.001193622240 \\ -0.001193622240 & 0.003567372978 \end{pmatrix}$		T1=9.343816 T2=7.034864		S=2.814901

Table 24. $QST1_2$ Variable

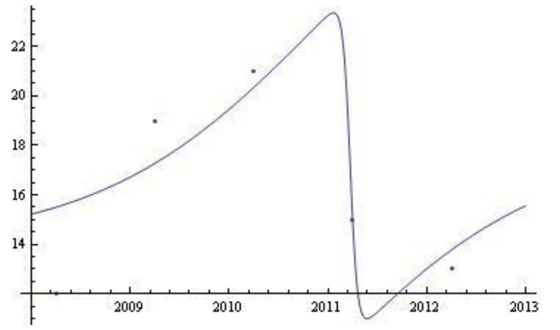


Figure 25. $QST1_2$ as a function of time in 2008-2012 period. R^2=0.795796.

Third term,

α	β_1	γ_1	δ_1	μ_1
-14.9344	15	0.0757802	39.1546	2011.5
	β_2	γ_2	δ_2	μ_2
	30	1.15316	-2.75255	2008.5
C Matrix		Mean of transformed function		Estimation Error
$\begin{pmatrix} 0.006370849146 & 0.001910306698 \\ 0.001910306698 & 0.005684729361 \end{pmatrix}$		T1=11.788674 T2=26.345729		S=0.043928

Table 25. $QST1_3$ Variable

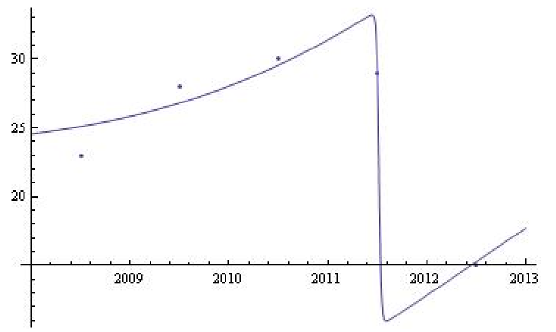


Figure 26. $QST1_3$ as a function of time in 2008-2012 period. R^2=0.963191.

Fourth term,

α	β_1	γ_1	δ_1	μ_1
0	14	0.76041	3.92966	2010.75
	β_2	γ_2	δ_2	μ_2
	18	8.32083	-0.727505	2008.75
C Matrix		Mean of transformed function		Estimation Error
$\begin{pmatrix} 0.002542641385 & -0.001112639927 \\ -0.001112639927 & 0.003892049459 \end{pmatrix}$		T1=7.220449 T2=6.661526		S=0.860572

Table 26. $QST1_4$ Variable

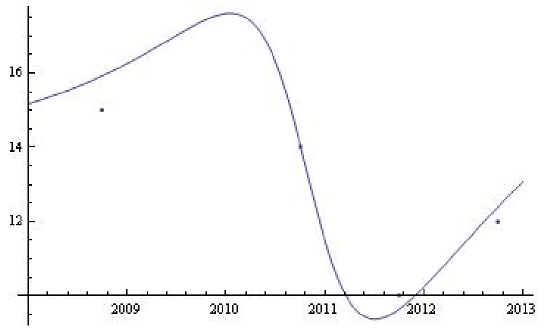


Figure 27. $QST1_4$ as a function of time in 2008-2012 period. R^2=0.959583.

QST2 Quarterly students with quota type 2

$$qst2(t) = \begin{cases} 0.888688 + \dfrac{8}{1 + 0.147305e^{-0.523292\,(-2008+t)}} & term\ 1 \\ \dfrac{12}{1 + 0.0465639e^{0.857155\,(-2008.25+t)}} & term\ 2 \\ \dfrac{18}{1 + 0.0508706e^{0.570128\,(-2008.5+t)}} & term\ 3 \\ -0.411611 + \dfrac{8}{1 + 2.14245e^{-1.29718\,(-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
0.888688	8	0.147305	-0.523292	2008
C Matrix		Mean of transformed function		Estimation Error
(1.979506539723)		T1= 7.511312		S=0.949040

Table 27. $QST2_1$ Variable

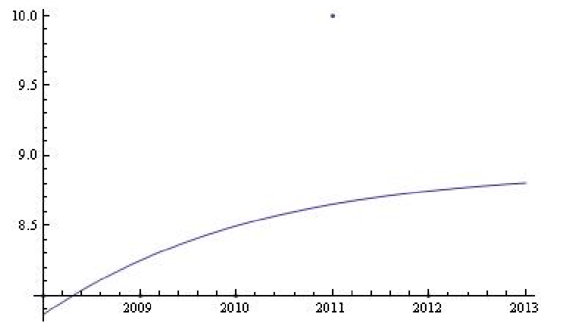


Figure 28. $QST2_1$ as a function of time in 2008-2012 period. R^2=0.155616.

Second term,

α	β	γ	δ	μ
0	12	0.0465639	0.857155	2008.25
C Matrix		Mean of transformed function		Estimation Error
(0.002385568792)		T1= 8.839519		S= 3.178865

Table 28. $QST2_2$ Variable

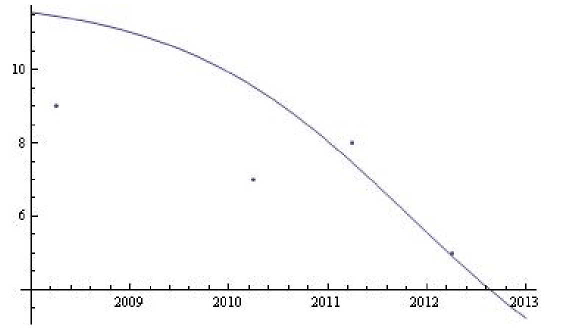


Figure 29. $QST2_2$ as a function of time in 2008-2012 period. R^2=0.466728.

Third term,

α	β	γ	δ	μ
0	18	0.0508706	0.570128	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.000870273267)		T1=15.047784		S=2.777999

Table 29. $QST2_3$ Variable

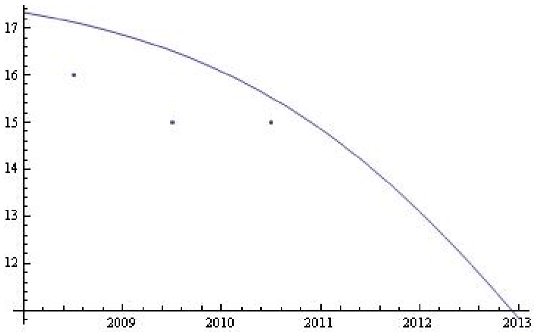


Figure 30. $QST2_3$ as a function of time in 2008-2012 period. R^2=0.344252.

Fourth term,

α	β	γ	δ	μ
-0.411611	8	2.14245	-1.29718	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.049880951786)		T1=6.011611		S=1.669814

Table 30. $QST2_4$ Variable

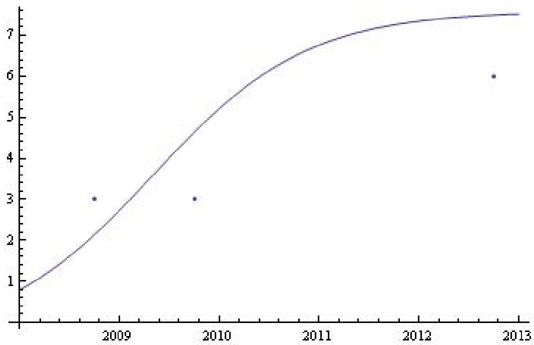


Figure 31. $QST2_4$ as a function of time in 2008-2012 period. R^2=0.668062.

QST3 Quarterly students with quota type 3

$$qst3(t) = \begin{cases} 0.0124966 + \dfrac{8}{1+\ 2.12352e^{-0.320652\,(-2008+t)}} & term\ 1 \\ \dfrac{5}{1+\ 1.59815e^{-0.608262\,(-2008.25+t)}} & term\ 2 \\ \dfrac{6}{1+\ 1.47586e^{-0.494151\,(-2008.5+t)}} & term\ 3 \\ -0.393176 + \dfrac{5}{1+\ 1.25485e^{-1.64502\,(-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
0.0124966	8	2.12352	-0.320652	2008
C Matrix		Mean of transformed function		Estimation Error
(0.258803130282)		T1= 3.787503		S= 1.924859

Table 31. $QST3_1$ Variable

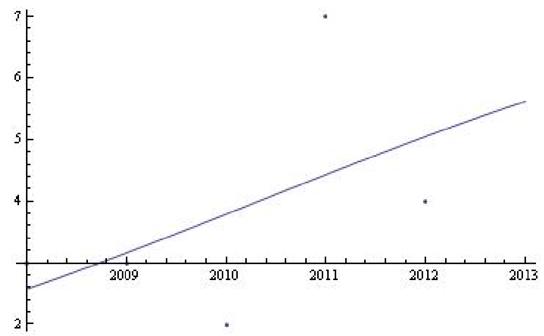


Figure 32. $QST3_1$ as a function of time in 2008-2012 period. R^2=0.24897.

Second term,

α	β	γ	δ	μ
0	5	1.59815	-0.608262	2008.25
C Matrix		Mean of transformed function		Estimation Error
(0.017421348874)		T1=3.270446		S=1.118203

Table 32. $QST3_2$ Variable

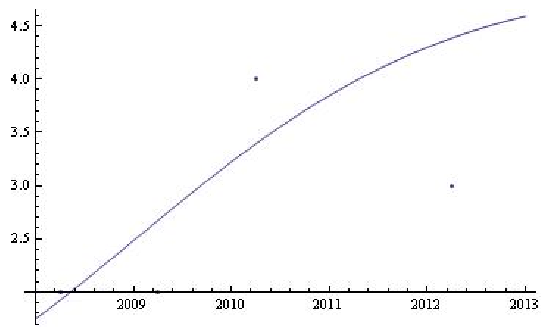


Figure 33. $QST3_2$ as a function of time in 2008-2012 period. R^2=0.452936.

Third term,

α	β	γ	δ	μ
0	6	1.47586	-0.494151	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.013184120579)		T1=3.785736		S=1.179239

Table 33. $QST3_3$ Variable

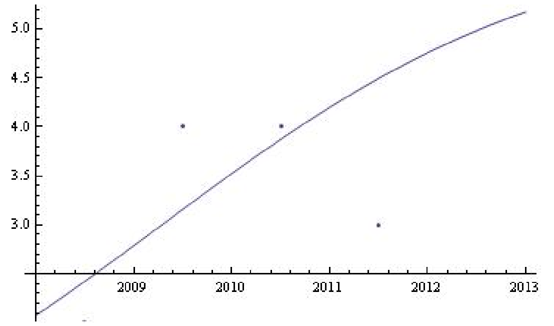


Figure 34. $QST3_3$ as a function of time in 2008-2012 period. R^2=0.526979.

Fourth term,

α	β	γ	δ	μ
-0.393176	5	1.25485	-1.64502	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.182146384354)		T1=4.193176		S= 1.371107

Table 34. $QST3_4$ Variable

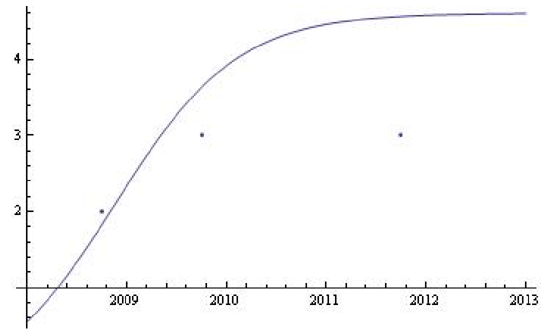


Figure 35. $QST3_4$ as a function of time in 2008-2012 period. R^2=0.477796.

QST4 Quarterly students with quota type 4

$$qst4(t) = \begin{cases} \dfrac{2.9}{1+\ 1.2773\cdot 10^{6}e^{\ -5.73568\,(-2008+t)}} & term\ 1 \\ 1.68485+\dfrac{5}{1+\ 458.881e^{\ -1.51957\,(-2008.25+t)}} & term\ 2 \\ 9.41936+\dfrac{12}{1+\ 15.8177e^{\ -0.384379\,(-2008.5+t)}} & term\ 3 \\ 7.56\cdot 10^{-7}+\dfrac{5}{1+\ 1.73789\cdot 10^{-10}e^{\ 14.9835\,(-2008.75+t)}} & term\ 4 \end{cases}$$

First term,

α	β	γ	δ	μ
0	2.9	1.2773e6	-5.73568	2008
C Matrix		Mean of transformed function		Estimation Error
(0.061805865790)		T1= 1.176670		S= 0.592191

Table 35. $QST4_1$ Variable

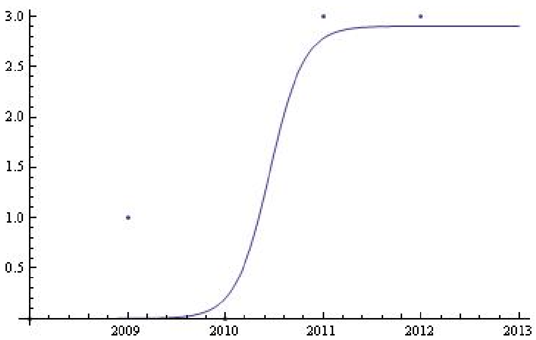


Figure 36. $QST4_1$ as a function of time in 2008-2012 period. R^2=0.910175.

Second term,

α	β	γ	δ	μ
1.68485	5	458.881	-1.51957	2008.25
C Matrix		Mean of transformed function		Estimation Error
(0.239669233569)		T1= 0.715149		S= 0.631693

Table 36. $QST4_2$ Variable

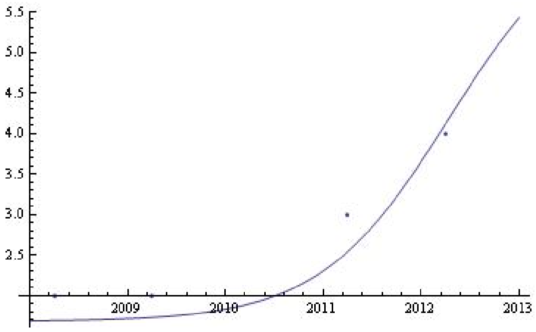


Figure 37. $QST4_2$ as a function of time in period 2008-2012 period. R^2=0.769787.

Third term,

α	β	γ	δ	μ
-0.628319	12	1.12909	-3.30855	2008.5
C Matrix		Mean of transformed function		Estimation Error
(0.031931458961)		T1= 10.628319		S= 0.936932

Table 37. $QST4_3$ Variable

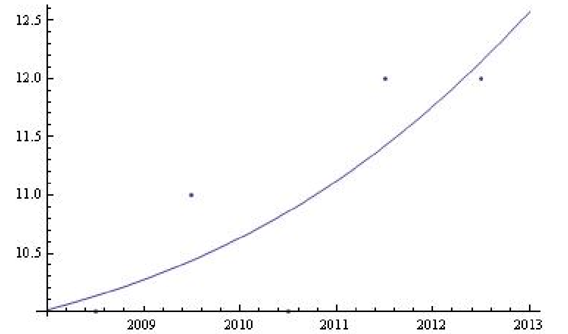


Figure 38. $QST4_3$ as a function of time in 2008-2012 period. R^2=0.643237.

Fourth term,

α	β	γ	δ	μ
7.56994e-7	2	1.73789e-10	14.9835	2008.75
C Matrix		Mean of transformed function		Estimation Error
(0.208527010191)		T1= 0.799999		S= 0.000695

Table 38. $QST4_4$ Variable

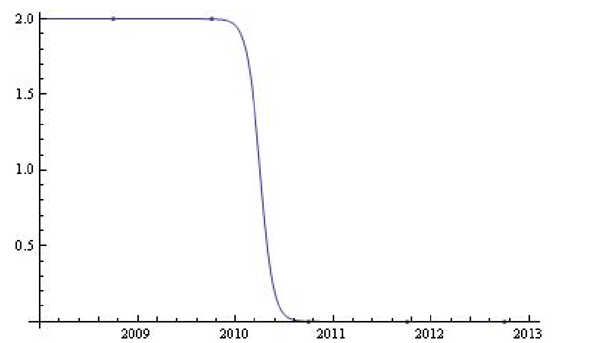


Figure 39. $QST4_4$ as a function of time in 2008-2012 period. R^2=1.

TAX1 Percentage Tax type 1 for water

$$tax1(t) = 0.0699988 + \frac{0.0100024}{1 + 1.53253 \cdot 10^{61} e^{-53.6883\,(-2008+t)}}$$

α	β	γ	δ	μ
0.0699988	0.0100024	1.53253e61	-53.6883	2008
C Matrix		Mean of transformed function		Estimation Error
(2020.204795076782)		T1=0.004501		S= 0.000004

Table 40. TAX1 Variable

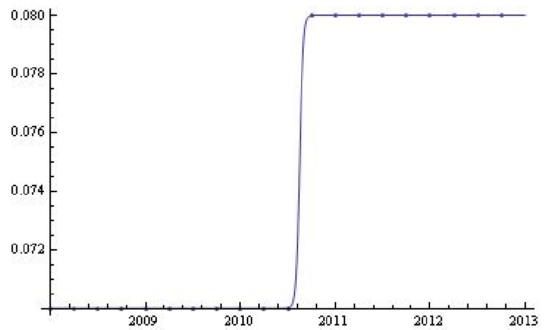


Figure 40. TAX1 as a function of time in 2008-2012 period. R^2=0.999999.

TAX2 Percentage Tax type 2 for general values

$$tax2(t) = 0.158421 + \frac{0.1}{1 + 159.501 e^{-1.05128\,(-2008+t)}}$$

α	β	γ	δ	μ
0.158421	0.1	159.501	-1.05128	2008
C Matrix		Mean of transformed function		Estimation Error
(240.993857427178)		T1=0.013579		S= 0.006230

Table 40. TAX2 Variable

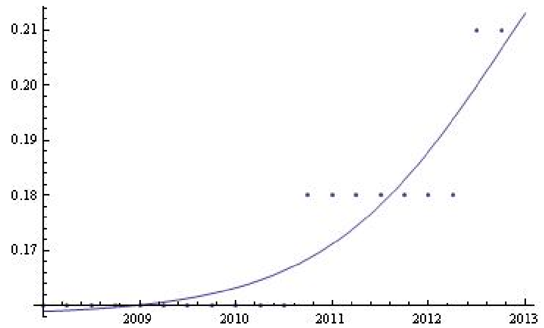


Figure 41. TAX2 as a function of time in 2008-2012 period. R^2=0.857999.

TAX3 Percentage Tax type 3 to apply to profits of an enterprise

$$tax3(t) = 0.12$$

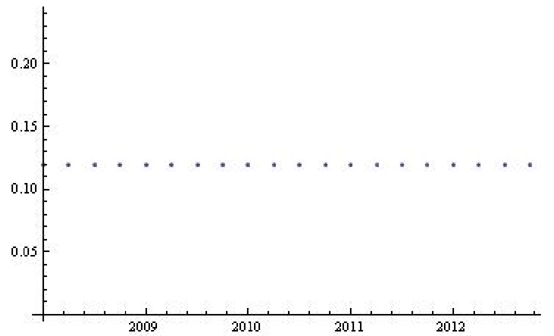


Figure 42. TAX3 as a function of time in 2008-2012 period. R^2=0.99999.

Appendix IV

Sensitivity Analysis

Independent variable x	Equations $y = mx + n$; $y = ax^2 + bx + c$	R^2	Derivative Function	Confidence interval for "m" or "$2a$"
TQSW	5455.04 -275.904 x+4.5656 x^2	0.22	-275.904+9.13119 x	(-0.671313, 18.9337)
QFCO	--	--	--	--
ACTE	--	--	--	--
OQEX	463.942 +9.40251 x-0.0088123 x^2	0.48	9.40251 -0.0176248 x	(-0.0284, -0.00686)
AINP	--	--	--	--
SSCC	--	--	--	--
BSAW	--	--	--	--
TQSL	7570.35 -110.582 x+0.453796 x^2	0.32	-110.582+0.907591 x	(0.192097, 1.62308)
QST1	-1307.78+201.596 x	0.74	201.596	(141.758, 261.434)
QST2	1029.73 +126.641 x	0.15	126.641	(-22.5973, 275.879)
QST3	--	--	--	--
QST4	1725.84 +135.429 x	0.16	135.429	(-16.2402, 287.097)
TAX1	--	--	--	--
TAX2	--	--	--	--

Table 41: Equations for the best fit of the profit after taxes "y" with the input variables. The actual data of a small business are used in the 2008-2012 period, by quarters. Dashes indicate that no direct link has been found between the considered variables.

Independent variable x	Equations $y = mx + n$; $y = ax^2 + bx + c$	R^2	Derivative Function	Confidence interval for "m" or "$2a$"
TQSW	1727.29 +20.3186 x	0.94	20.3186	(16.766, 23.8713)
QFCO	--	--	--	--
ACTE	--	--	--	--
OQEX	1724.09 +2.41122 x	0.87	2.41122	(1.99077, 2.83167)
AINP	--	--	--	--
SSCC	--	--	--	--
BSAW	--	--	--	--
TQSL	1705.36 +20.4975 x	0.88	20.4975	(17.0786, 23.9164)
QST1	33.098 +121.811 x	0.24	121.811	(26.0076,217.615)
QST2	--	--	--	--
QST3	--	--	--	--
QST4	2557.35 -157.792 x	0.75	-157.792	(-198.169, -117.414)
TAX1	--	--	--	--
TAX2	--	--	--	--

Table 42: Equations for the best fit of the profit after taxes "y" with the input variables. The actual data of a small business are used in the 2013-2025 period, by quarters. Dashes indicate that no direct link has been found between the considered variables.