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# 1 **A numerical study of the effect of nozzle diameter on Diesel** 2 **combustion ignition and flame stabilization**

## 3 **Author names**

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## 6 **Abstract**

7 The role of nozzle diameter on Diesel combustion is studied by performing CFD calculations  
8 of Spray A and Spray D from the Engine Combustion Network (ECN). These are well-  
9 characterized single-hole sprays in a quiescent environment chamber with thermodynamic  
10 conditions representative of modern Diesel engines. First, the inert spray evolution is described  
11 with the inclusion of the concept of mixing trajectories and local residence time into the  
12 analysis. Such concepts enable the quantification of the mixing rate, showing that it decreases  
13 with the increase in nozzle diameter. In a second step, the reacting spray evolution is studied  
14 focusing on the local heat release rate distribution during the auto-ignition sequence and the  
15 quasi-steady state. The capability of a well-mixed based and a flamelet based combustion model  
16 to predict Diesel combustion is also assessed. On the one hand, results show that turbulence-  
17 chemistry interaction has a profound effect on the description of the reacting spray evolution.  
18 On the other hand, the mixing rate, characterized in terms of the local residence time, drives the  
19 main changes introduced by the increase of the nozzle diameter when comparing Spray A and  
20 Spray D.

## 21 **Keywords**

22 Diesel spray, nozzle diameter, residence time, well-mixed, flamelet

## 23 **1 Introduction**

24 Fuel and thermal efficiency in internal combustion engines (ICE) relying on compression  
25 ignition (CI) are still attractive features for industries focused on diverse sectors such as  
26 automotive, road transportation and marine applications. Despite its operational advantages,  
27 pollutant emissions from CI combustion systems can potentially deteriorate local air quality  
28 and therefore human health. As a consequence, the latest advancements in CI combustion  
29 systems are driven by stricter regulations predominantly oriented to reduce permissible limits

30 for particulate matter emissions. The challenge of improving the understanding of turbulent  
31 combustion, hence pollutant formation, remains crucial for the development and enhancement  
32 of CI combustion systems.

33 In particular, the nozzle size is a geometrical parameter that brings about differences in  
34 combustion behavior between light and heavy-duty engines. From the experimental point of  
35 view, changes in nozzle orifice diameter have been observed to greatly influence the amount of  
36 soot formed in Diesel flames. In this direction, Pickett and Siebers<sup>1,2</sup> conducted experiments  
37 using a single-hole nozzle in a constant-volume vessel with pressure and temperature conditions  
38 representative of direct injection (DI) Diesel engines. Authors concluded that under the same  
39 operating condition decreasing nozzle orifice diameter increased the air-entrainment rate  
40 (relative to the amount of fuel injected) upstream of the lift-off length (LOL). As more air is  
41 entrained the average equivalence ratio at the LOL location ( $\phi_{LOL}$ ) decreases. The authors found  
42 that the decrease in  $\phi_{LOL}$  at the flame base also decreases the total amount of soot formed,  
43 proving the feasibility of mitigating soot emissions by decreasing the nozzle orifice diameter.  
44 Chengjun et al.<sup>3</sup> also observed the influence of the nozzle orifice diameter on the average value  
45 of  $\phi_{LOL}$ , hence on soot formation, in single-hole Diesel sprays in a constant-pressure vessel.  
46 The authors also found that a reduced value of  $\phi_{LOL}$  induced by the reduction of nozzle orifice  
47 diameter caused an increase in the OH zone thickness (based on OH\* chemiluminescence  
48 imaging) and a consequent reduction of the maximum soot volume fraction zone. Pastor et al.<sup>4</sup>  
49 approached the study of different nozzle diameters based on the analysis of spray dynamics and  
50 mixing field in a constant-pressure vessel. Experiments were carried-out to measure spray tip  
51 penetration, LOL and ignition delay time (ID). Under the same operating conditions,  
52 differences in results among different nozzles were explained by spray dynamics (in agreement  
53 with momentum-driven gas jets theory) and the state of the mixing field prior to the start of  
54 combustion retrieved with a 1D model.

55 It is worth mentioning that the study of reacting sprays with different nozzle orifice diameters  
56 is not limited to fundamental studies as the aforementioned cases. It can also be extended to  
57 applications in which two different nozzles are used for pilot and main injections as in the case  
58 of marine engines. In that field, Ishibashi and Tsuru<sup>5</sup> studied Diesel spray and gas injection  
59 combustion in a rapid compression machine with a 160  $\mu m$  nozzle for a pilot injection and a  
60 500  $\mu m$  for a main injection.

61 From the simulation point of view Pang et al.<sup>6</sup> conducted a numerical study on reacting sprays  
62 in a combustion vessel with nozzle orifice diameters of 100  $\mu\text{m}$ , 180  $\mu\text{m}$  and 363  $\mu\text{m}$ . Results  
63 are in line with experimental observations with shorter LOL and average leaner mixtures at this  
64 location with smaller nozzles. As fewer computational studies comparing different nozzle  
65 diameters are available, the present work aims to use computational fluid dynamics (CFD) to  
66 study the role that the nozzle orifice diameter plays on Diesel combustion. The Engine  
67 Combustion Network (ENC)<sup>7</sup> Spray A will be used as the main reference case for this work.  
68 The Spray A experiment, a single-hole nozzle injecting into a quiescent environment with  
69 thermodynamic conditions representative of CI engines, has allowed the development of an  
70 extensive database of experimental results.<sup>8-13</sup> On the simulation side, the single-hole nozzle  
71 experiment has enabled the comprehensive validation of the two-phase turbulent spray using  
72 both Lagrangian-Eulerian<sup>14-17</sup> and Eulerian<sup>18,19</sup> approaches.

73 Within the scope of the present work, different approaches to model the sub-grid flame structure  
74 will be also evaluated. Results from a detailed chemistry solver based on the well-mixed (WM)  
75 assumption and results from a model based on the flamelet assumption are compared. These  
76 two different approaches for combustion modeling allow to assess the importance of  
77 turbulence-chemistry interaction (TCI). Comparisons of such combustion models have been  
78 reported in the literature assessing the capabilities of well-mixed based models and a  
79 representative interactive linear eddy model,<sup>20</sup> transported probability density function (TPDF)  
80 based models<sup>21-23</sup> or flamelet based models.<sup>24-29</sup> From this last group of works, Lucchini et al.<sup>26</sup>  
81 assessed various assumptions for the flame structure to model a single-hole reacting spray in a  
82 constant-volume vessel. Assumptions included well-mixed and flamelet based combustion  
83 models. Capabilities of these models were evaluated for low and high-temperature and ambient  
84 density conditions in terms of global combustion parameters. From all these group of works it  
85 is consistently evidenced that combustion results heavily depend on TCI.

86 The study of TCI in this work is not limited to the analysis of global combustion parameters  
87 and is extended to the description of local phenomena during the ignition sequence and the  
88 quasi-steady state of the reacting spray. The work is organized as follows. First, the  
89 experimental target Diesel spray setups are described under the “Methodology” section along  
90 with the computational setup used to carry-out CFD calculations. The “Inert spray evolution”  
91 section comprises the validation of the computational setup for the nonreacting condition. Next,  
92 the validation and description of results for the reacting condition are included in the “Reacting  
93 spray evolution” section. Finally, concluding remarks are drawn in the “Conclusions” section.

## 94 2 Methodology

### 95 2.1 Target Diesel spray setup

96 Target conditions for the present study are based on ECN standards. Fuel injection and  
97 thermodynamic conditions studied are summarized in Table 1. Liquid n-dodecane is injected  
98 into a quiescent environment through a single-hole nozzle. For the inert condition pure  $N_2$  is  
99 present in the ambient. Meanwhile, 15% molar fraction of  $O_2$  is present (in addition to  $N_2$ ) for  
100 the reacting condition. The only varying parameter for this study is the nozzle orifice diameter.  
101 Computations are carried-out for ECN Spray A and Spray D which correspond to single-hole  
102 nozzles with diameters of  $89.4 \mu m$  (nozzle reference number 210675) and  $190.3 \mu m$  (nozzle  
103 reference number 209135), respectively.

104 The results presented in this study are validated using experimental data measured at CMT  
105 Motores Térmicos, Sandia National Laboratories or IFP Energies Nouvelles. All measurements  
106 are done under the same operating conditions in a constant-pressure vessel (CMT) or in a pre-  
107 burn constant-volume vessel (Sandia and IFPEN). The experimental data is available through  
108 the ECN.<sup>7</sup>

109 Table 1. Injection and thermodynamic conditions.

<b>Injection conditions</b>	
Fuel	n-dodecane
Nozzle diameter	$89.4 \mu m$ – Spray A
	$190.3 \mu m$ – Spray D
Injection pressure	150 MPa
Fuel temperature	363 K
<b>Thermodynamic conditions</b>	
Ambient temperature	900 K
Ambient density	$22.8 kg/m^3$
Ambient $O_2$ composition	$X_{O_2} = 0$ – nonreacting condition
	$X_{O_2} = 0.15$ – reacting condition

110

### 111 2.2 Computational setup

112 Computations were carried-out using the CFD solver CONVERGE<sup>30</sup> following the traditional  
113 Lagrangian-parcel Eulerian-fluid approach. The CFD code uses a cut-cell cartesian method for  
114 grid generation. The computational domain is a cylinder with 50 mm radius and 102 mm

115 length. The base mesh cell size is 2 *mm* for both Spray A and Spray D cases. A truncated cone-  
116 shaped fixed embedding is used near the nozzle region to improve accuracy around this critical  
117 zone. The fixed embedding adds 250  $\mu\text{m}$  and 500  $\mu\text{m}$  cells for Spray A and Spray D,  
118 respectively. In addition, adaptive mesh refinement (AMR) allows for grid resolution to be  
119 added just where is needed based on velocity, temperature and fuel mass fraction gradients. As  
120 a consequence, the minimum cell size reached due to AMR is 125  $\mu\text{m}$  for Spray A and 250  $\mu\text{m}$   
121 for Spray D cases.

122 The Eulerian fluid description is based on the Favre-averaged Navier-Stokes equations solved  
123 within a RANS framework. The standard  $\kappa - \varepsilon$  model is used with  $C_{\varepsilon 1} = 1.55$  to account for  
124 round jet correction.<sup>31,32</sup> On the other hand, the liquid-phase is described using the Lagrangian-  
125 parcel approach in conjunction with sub-models for droplet breakup, collisions, drag, and  
126 evaporation. The Kelvin-Helmholtz (KH) and the Rayleigh-Taylor (RT) models are used for  
127 the estimation of droplet breakup. Droplet collisions are accounted for by the no time counter  
128 (NTC) model. Droplet drag is predicted with a model that considers variations in the drop shape  
129 using a distortion parameter. Lastly, the droplet radius rate of change due to evaporation is  
130 estimated based on the Frossling correlation.<sup>33</sup>

131 The combustion modelling approach is based on detailed chemistry, for which a chemical  
132 mechanism with 54 species and 269 reactions<sup>34</sup> has been used. A detailed analysis of this  
133 chemical mechanism based on homogeneous reactor calculations can be found in the work by  
134 Pérez.<sup>35</sup> In terms of turbulence-chemistry interaction, two approaches have been compared. On  
135 the one hand, the well-mixed SAGE detailed chemical kinetics solver<sup>36</sup> (referred to as WM  
136 model from this point), which is available in CONVERGE. On the other hand, the unsteady  
137 flamelet progress variable (UFPV) model, which has been implemented by the authors.  
138 Regarding the WM model, the net production rate of any species  $k$  as derived from the chemical  
139 mechanism is used to solve the corresponding source term  $\dot{\omega}_k$  for governing equations at every  
140 cell in the domain, namely mass and energy conservation equations.

141 As for the UFPV model, it is based on the description of a turbulent flame as a set of strained  
142 laminar flamelets.<sup>37</sup> The concept is based on the assumption that the chemical characteristic  
143 time is small compared to the physical characteristic time (i.e. a high Damköhler number flow  
144 as in the case of Diesel-like combustion applications). In such scenario, turbulence cannot  
145 modify instantly and locally the thin layer where combustion is sustained, hence remaining

146 laminar.<sup>38</sup> Flamelets in counterflow configuration are then solved for any species  $k$  in mixture  
 147 fraction ( $Z$ ) space according to the equation:

$$\frac{\partial Y_k}{\partial t} = \frac{\chi}{2} \frac{\partial^2 Y_k}{\partial Z^2} + \dot{\omega}_k \quad (1)$$

148 Under this formulation, the chemical source term  $\dot{\omega}_k$  is obtained by solving the corresponding  
 149 reaction rates as defined by the chemical mechanism, meanwhile the strength of convective and  
 150 diffusive processes is accounted by the scalar dissipation rate ( $\chi$ ), assumed to follow a steady  
 151 profile<sup>38</sup> defined by:

$$\chi(a, Z) = \frac{a}{\pi} \exp \left[ -2(\operatorname{erfc}^{-1}(2Z))^2 \right] \quad (2)$$

152 Equation (2) can then be re-written in such a way that it becomes independent of the strain rate  
 153 ( $a$ ) by normalizing the profile using the value of the scalar dissipation rate at stoichiometric  
 154 conditions ( $\chi_{st}$ ):

$$\chi(\chi_{st}, Z) = \chi_{st} \frac{F(Z)}{F(Z_{st})} \quad (3)$$

155 The time evolution of the flamelet is stored as a so-called flamelet manifold parametrized in  
 156 terms of a progress variable ( $Y_c$ ), which describes the transition of the mixture from inert to  
 157 fully burned state as function of a linear combination of species. In this work the progress  
 158 variable is defined according to:

$$Y_c = 0.75Y_{CO} + Y_{CO_2} + Y_{H_2O} \quad (4)$$

159 At this point, the manifold generated by solving the PDE system is laminar. TCI is then  
 160 accounted using a presumed Probability Density Function (PDFs) approach for the mixture  
 161 fraction and the scalar dissipation rate assuming statistical independence between these two  
 162 variables.<sup>39</sup> For the first of these variables a beta function defined by the mean value of  $Z$  ( $\tilde{Z}$ )  
 163 and its variance ( $\tilde{Z}''^2$ ) yielding to  $P_Z(Z; \tilde{Z}, S)$ , where  $S$  is the so-called segregation factor  
 164 calculated as  $S = \tilde{Z}''^2 / (\tilde{Z}(1 - \tilde{Z}))$ . As for the scalar dissipation rate, a log-normal function  
 165 with  $\sigma = \sqrt{2}$  is used<sup>40</sup> fielding to  $P_\chi(\chi_{st}; \tilde{\chi}_{st}, \sigma)$ . Once PDFs have been defined as functions of  
 166 the independent variables  $Z$  and  $\chi_{st}$ , any average value ( $\tilde{\psi}$ ) in the manifold is defined by:

$$\tilde{\psi}(\tilde{Z}, S, \tilde{\chi}_{st}, \tilde{t}) = \int_0^\infty \int_0^Z \psi(Z, \chi_{st}, \tilde{t}) P_Z(Z; \tilde{Z}, S) P_\chi(\chi_{st}; \tilde{\chi}_{st}, \sigma) dZ d\chi_{st} \quad (5)$$

167 Considering the re-parametrization of the laminar solution in terms of the progress variable, the  
 168 average values in the manifold can be expressed as  $\tilde{\psi} = \tilde{\psi}(\tilde{Z}, S, \tilde{\chi}_{st}, \tilde{Y}_c)$ . This re-

169 parametrization is possible under the assumption that  $\tilde{Y}_c$  increases monotonically such that there  
 170 is a bijective relationship with  $\tilde{t}$ .<sup>40</sup> Equation (6) is then used for the integration of  $\chi$  (where  $J$   
 171 only depends on  $\tilde{Z}$  and  $S$ ) and relates  $\tilde{\chi}$ , recovered from the CFD calculation according to  
 172 Equation (7), and  $\tilde{\chi}_{st}$  used to query the manifold.

$$\tilde{\chi} = \left( \int_0^\infty \chi_{st} P_\chi(\chi_{st}; \tilde{\chi}_{st}, \sigma) d\chi_{st} \right) \left( \frac{1}{F(Z_{st})} \int_0^Z F(Z) P_Z(Z; \tilde{Z}, S) dZ \right) = \tilde{\chi}_{st} J(\tilde{Z}, S) \quad (6)$$

173

$$\tilde{\chi} = C_\chi \frac{\varepsilon}{\kappa} \tilde{Z}''^2 \quad (7)$$

174 It is worth mentioning that the manifold is constructed independently from the CFD calculation  
 175 by creating lookup tables where mean species mass fractions and the mean progress variable  
 176 source term can be queried as  $\tilde{Y}_k^{tab}(\tilde{Z}, S, \tilde{\chi}_{st}, \tilde{Y}_c)$  and  $\tilde{\omega}_{Y_c}^{tab}(\tilde{Z}, S, \tilde{\chi}_{st}, \tilde{Y}_c)$ , respectively. The  
 177 lookup tables are discretized with 41 points for  $\tilde{Z}$ , 17 points for  $S$ , 27 points for  $\tilde{\chi}_{st}$  and 51  
 178 points for  $\tilde{Y}_c$ .

179 Coupling of the UFPV model within the CFD framework is achieved through the chemical  
 180 source term of the species transport equation ( $\dot{\omega}_k$ ), which is calculated as:

$$\tilde{\omega}_k = \frac{\tilde{Y}_k^{tab}(\tilde{Z}, S, \tilde{\chi}_{st}, \tilde{Y}_c(t + \Delta t)) - \tilde{Y}_k^{cell}(t)}{\Delta t} \quad (8)$$

181 In Equation (8),  $\Delta t$  is the CFD time-step,  $\tilde{Y}_k^{cell}$  is the species mass fraction at the cell and  $\tilde{Y}_k^{tab}$   
 182 is the species mass fraction tabulated in the manifold in the subsequent timestep, defined in  
 183 terms of  $\tilde{Y}_c(t + \Delta t)$ , which is calculated as:

$$\tilde{Y}_c(t + \Delta t) = \tilde{Y}_c(t) + \tilde{\omega}_{Y_c}(\tilde{Z}, S, \tilde{\chi}_{st}, \tilde{Y}_c(t)) \Delta t \quad (9)$$

184 To advance in the manifold using Equation (9),  $\tilde{Y}_c(t)$  is calculated following the definition in  
 185 Equation (4) and  $\tilde{\omega}_{Y_c}$  (generic nomenclature for  $\partial \tilde{Y}_c / \partial t$ ) is retrieved from the manifold.  $\tilde{\omega}_k$  is  
 186 then used in the transport of species.

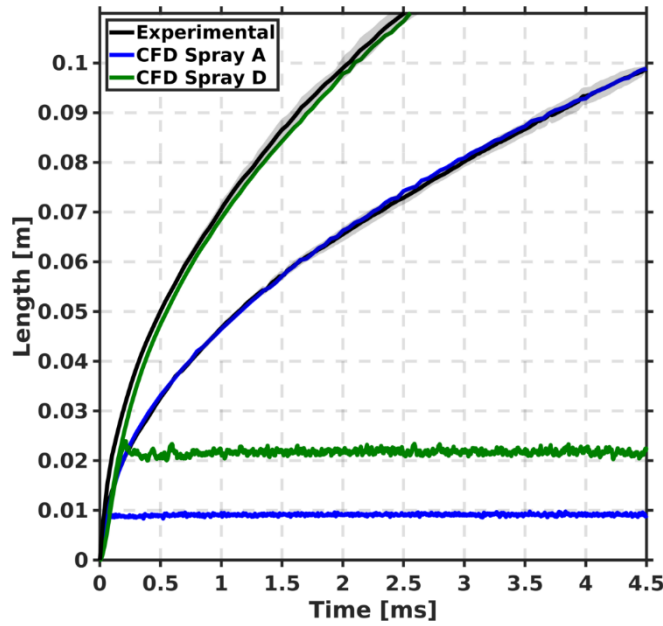
187 Finally, there's an additional aspect of the UFPV model formulation worth mentioning. In this  
 188 work a variant named UFPV-0 is introduced. For this variant, the manifold is constructed  
 189 following the same structure as for the UFPV case, but no presumed-PDF integration is taking  
 190 into account, i.e. this should correspond to a tabulated laminar flamelet model. This variant  
 191 seeks to facilitate the analysis of differences induced by the sub-grid flame structure  
 192 formulation, namely the well-mixed and the flamelet-like structure.



### 193 3 Inert spray evolution

#### 194 3.1 Validation of the computational setup

195 The validation of the computational setup is carried-out by comparing global and local  
196 quantities with experimental data available within the ECN. Following ECN guidelines, spray  
197 tip penetration is defined as the axial distance to the furthest location where the mixture fraction  
198  $Z$  reaches a value of 0.001. Figure 1 shows the experimental spray tip penetration value plotted  
199 with a solid black line with a gray shadow around the 95% confidence interval and CFD  
200 predictions for Spray A and Spray D (spray tip penetration and liquid length). The same  
201 computational setup and model settings have been used for both nozzles. An excellent  
202 agreement is observed for Spray A tip penetration, while a slight underprediction is observed  
203 for the larger nozzle. Given the fact that no change has been made in the modelling setup for  
204 both nozzles the agreement is satisfactory.

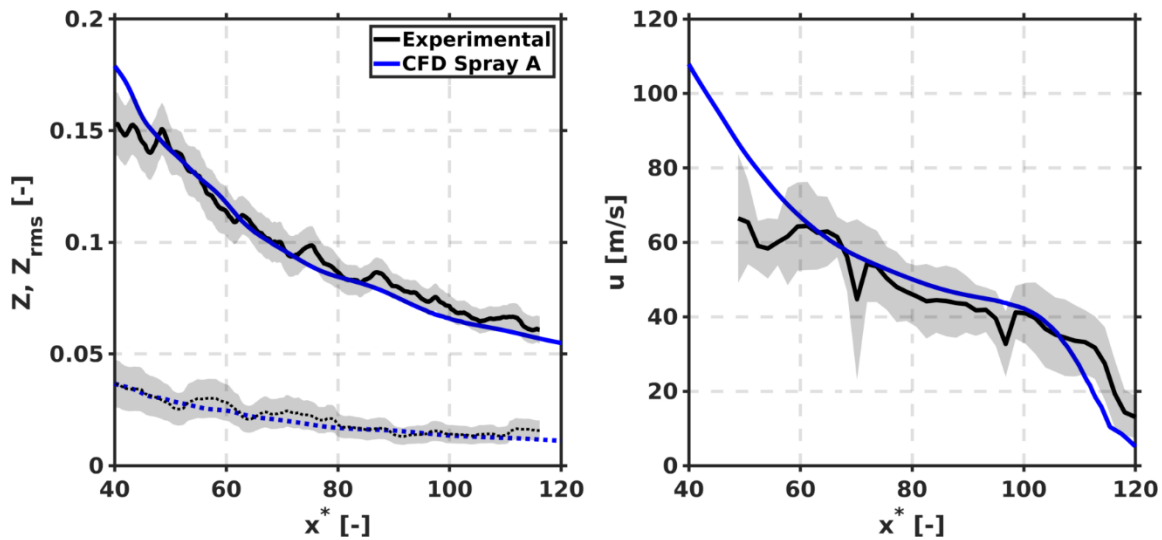


205

206 Figure 1. Spray tip penetration and liquid length for inert Spray A and Spray D.

207 In terms of local quantities, the computational setup is validated for Spray A (no experimental  
208 data available for Spray D). Mixture fraction  $Z$  and its root mean square ( $Z_{rms}$ ) and axial  
209 velocity fields are validated by comparing axial and radial profiles. Radial and axial coordinates  
210 have been normalized by the equivalent diameter ( $d_{eq}$ ) of each nozzle i.e.  $r^* = r/d_{eq}$  and  $x^* =$   
211  $x/d_{eq}$ . The equivalent diameter is defined as a function of the nozzle effective diameter ( $d_0$ )  
212 and the ratio of fuel density ( $\rho_f$ ) and air density ( $\rho_a$ ) and is calculated as  $d_{eq} = d_0\sqrt{\rho_f/\rho_a}$ .  
213 Different nozzles have been used for experimental and CFD work. As previously mentioned,  
214 nozzle 210675 (same nozzle used for the experimental measurement of spray tip penetration,

215 ignition delay and lift-off length) has been modeled for the Spray A case. Meanwhile, nozzle  
 216 210677 has been used for the experimental work to measure  $Z$  and nozzle 210678 has been  
 217 used to measure axial velocity. Nominal diameters for the three nozzles are  $89.4 \mu\text{m}$ ,  $83.7 \mu\text{m}$   
 218 and  $88.6 \mu\text{m}$  for the 210675, 210677 and 210678 reference numbers, respectively.  
 219 Normalization of spatial coordinates allows for a better comparison of experimental and CFD  
 220 results. Figure 2 and Figure 3 show axial and radial profiles for the above-mentioned local  
 221 quantities. From the left-hand panel in Figure 2 the computational setup is seen to be successful  
 222 in predicting  $Z$  (solid line) and  $Z_{rms}$  (dotted line) within the limit of the 95% confidence interval  
 223 of the experimental observation along the spray axis. These two variables are inputs for the  
 224 UFPV combustion model. Similarly, the right-hand panel in Figure 2 also shows good  
 225 agreement with experimental data in terms of axial velocity. Differences for  $x^* < 60$  are  
 226 expected due to the measurement uncertainty in that region corresponding to the limit of the  
 227 laser sheet used in the PIV experiments.<sup>10</sup>



228

229 Figure 2. Inert Spray A axial profiles for mixture fraction (solid line) and mixture fraction rms (dotted lines) at 5  
 230 ms and axial velocity at 1.5 ms.

231 Further validation is shown in terms of radial profiles for  $Z$  and  $Z_{rms}$  at two axial locations in  
 232 Figure 3 and Figure 4. At  $50 d_{eq}$  the profiles predicted by CFD for both variables (left-hand  
 233 panel) agree well with the experimental observations. As expected from the axial profiles, at  $90$   
 234  $d_{eq}$  both  $Z$  and  $Z_{rms}$  CFD results show slightly narrower profiles although in general terms the  
 235 predictions still agree well with experiments.

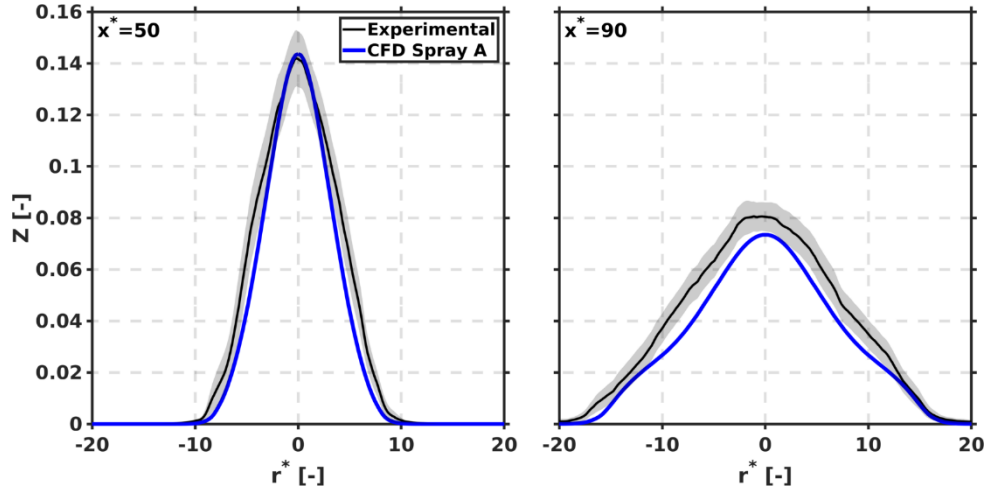


Figure 3. Mixture fraction radial profiles for inert Spray A at 5 ms.

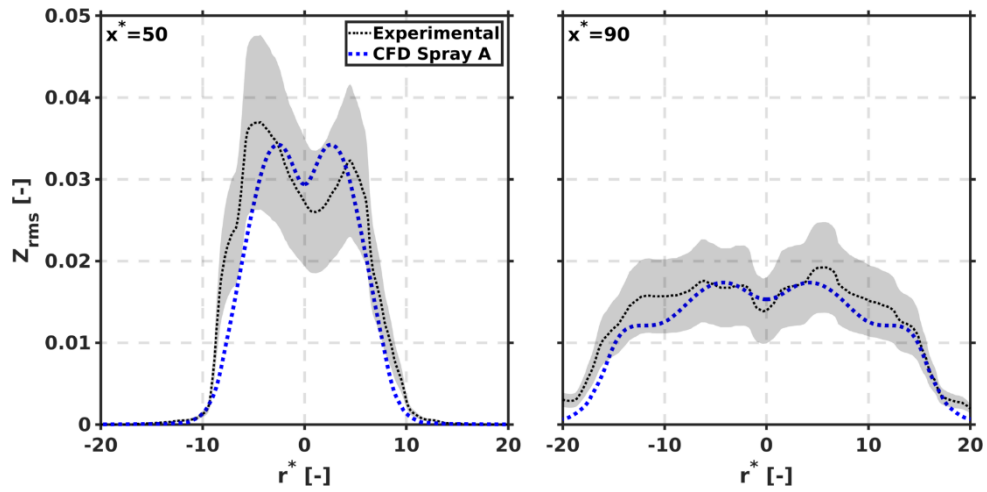


Figure 4. Mixture fraction rms radial profiles for inert Spray A at 5 ms.

### 3.2 Local residence time

To provide an additional local indicator of mixing intensity, a local residence time has been defined based on mixing trajectories (Appendix A. Mixing trajectories) as the time spent per unit of equivalence ratio ( $\phi$ ) according to:

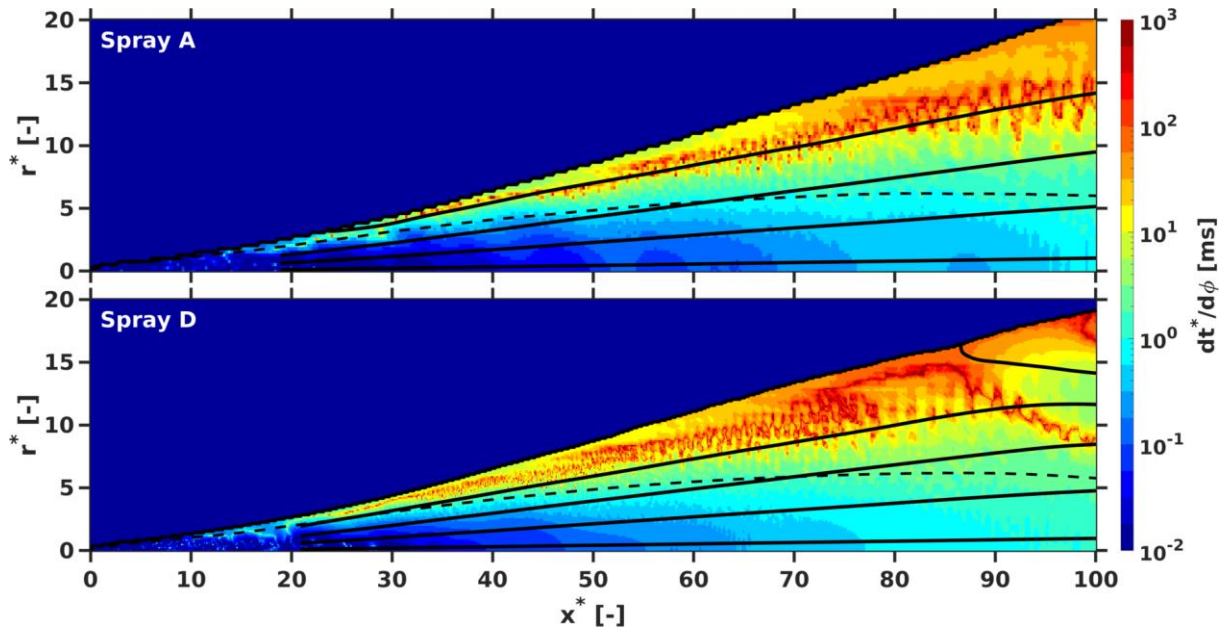
$$\frac{dt^*}{d\phi} = -\frac{dt^*}{dl} \frac{dl}{d\phi} = -\left(\sqrt{(u + u_{dif})^2 + (v + v_{dif})^2}\right)^{-1} \frac{dl}{d\phi} \quad (10)$$

The physical meaning of this parameter is the time needed to change a unit value of equivalence ratio along a mixing trajectory or, in other words, the time spent at a given  $\phi$  value. This parameter is made up of the product of two terms. First, the rate of residence time per length unit ( $dt^*/dl$ ) is solved considering convective and diffusive contributions (Equation (12) and Equation (13)). Second,  $dl/d\phi$  is obtained from the spatial gradient of  $\phi$  as projected along the direction defined by the velocity field (i.e. a mixing trajectory). To the best of the authors'

250 knowledge, this is the first time that an attempt has been made to analyze the mixing process in  
251 terms of local residence time for spray applications.

252 Figure 5 shows local residence time for both Spray A and Spray D under inert conditions. The  
253 definition of the residence time in terms of the change of equivalence ratio enables a direct  
254 comparison between both nozzle orifices. The analysis of local residence time is then made at  
255 4 *ms*, time at which the spray is already at quasi-steady state. A sample of mixing trajectories  
256 (solid lines) and the iso-contour for  $\phi = 1$  (dashed line) are also shown. For visualization  
257 purposes the color map is adjusted to logarithmic scale. It can be observed how the structure is  
258 similar for both sprays with increasing values of residence time along any trajectory when  
259 moving downstream from the orifice. Among trajectories at the same axial normalized  
260 coordinate (consequently at a similar equivalence ratio) there is also an increase of  $dt^*/d\phi$   
261 with a local maximum near the spray radius. In summary, residence time grows when moving  
262 away from the orifice, both in axial and radial direction.

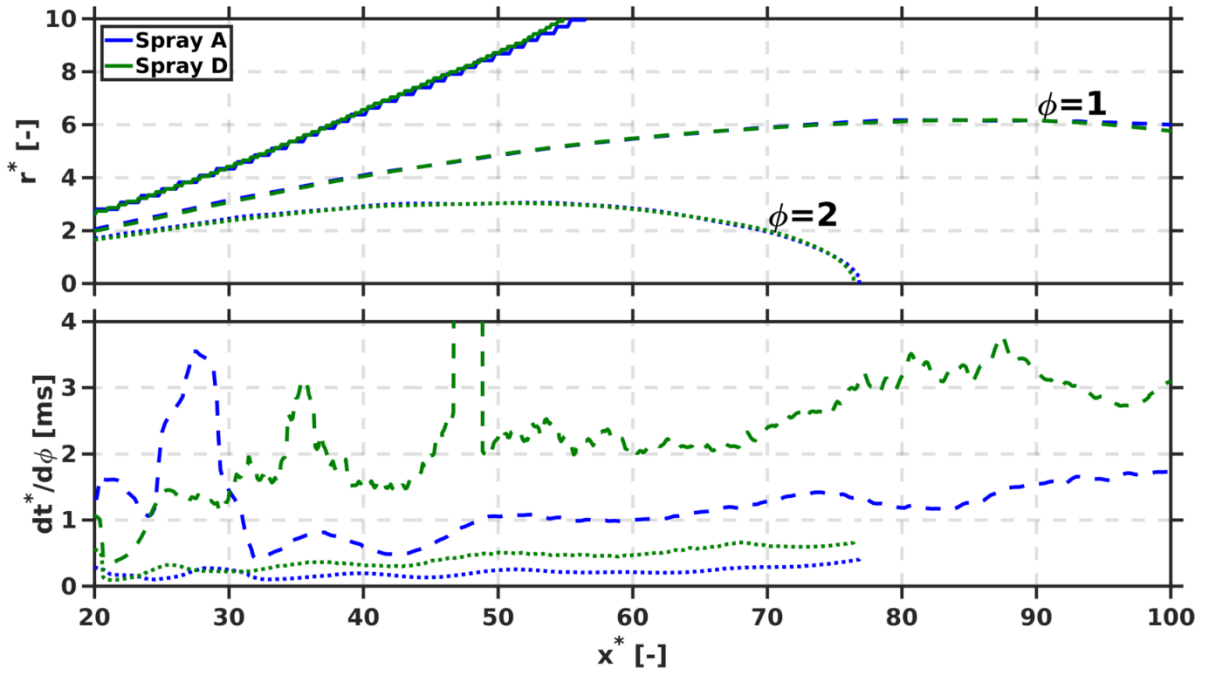
263 Due to the fact that the mixture fraction (hence equivalence ratio) is a conservative scalar, one  
264 can state that the convective plus diffusive flow of this variable remains constant between two  
265 mixing trajectories and develops with an almost constant angle compared to the axis. By  
266 integrating both the mixing trajectories and the residence time concepts, the mixing field created  
267 by the spray can be viewed as a set of mixers starting close to the nozzle, which then move  
268 away from it entraining air and spending more time on a given equivalence ratio, as farther  
269 locations are reached.



270

271 Figure 5. Local residence time for inert Spray A (top) and Spray D (bottom). Spatial coordinates have been  
 272 normalized by  $d_{eq}$ .

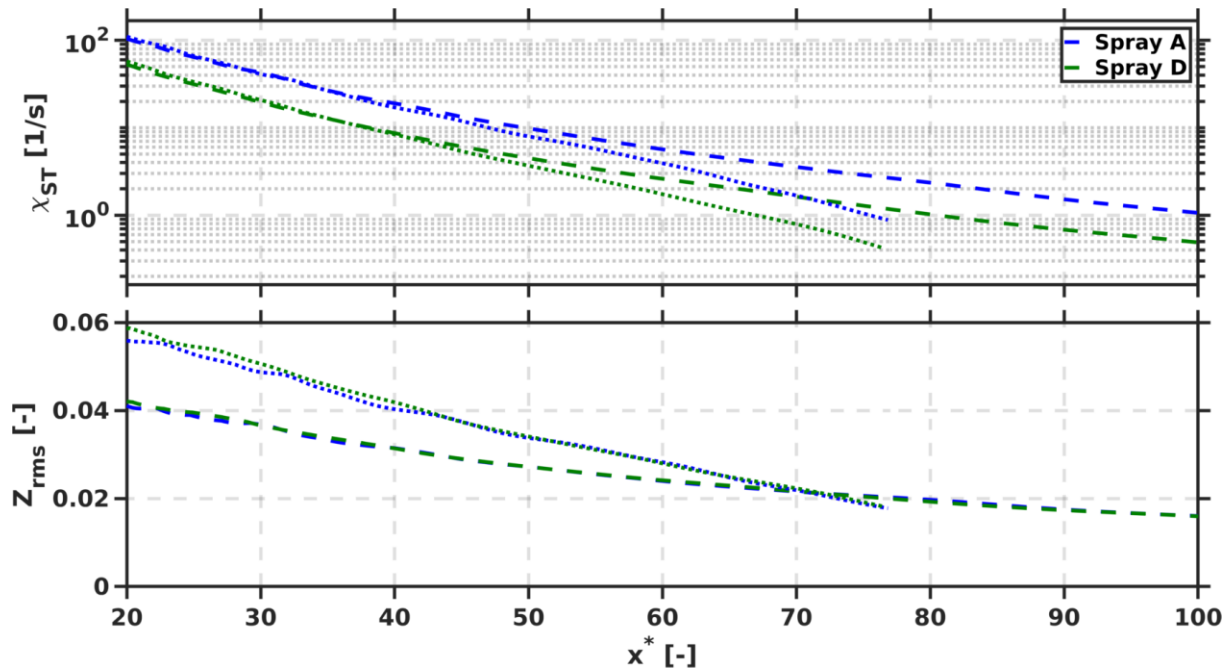
273 To enable further comparison between both nozzles, the upper part of Figure 6 shows spray  
 274 radius with a solid line and iso-contours of  $\phi = 1$  (dashed line) and  $\phi = 2$  (dotted line). It is  
 275 then confirmed that the normalization of spatial coordinates of the two different nozzles by  $d_{eq}$   
 276 results in the same value of  $\phi$  for a given value of  $(x^*, r^*)$ . Additionally, if  $dt^*/d\phi$  is studied  
 277 along the reference iso-contours of  $\phi$  (bottom part of the figure) it is then verified that this  
 278 parameter increases with axial distance, in agreement to the contours shown in Figure 5.  
 279 Furthermore, for the same value of  $\phi$ , Spray A is characterized by shorter local residence time  
 280 (thus faster mixing) compared to Spray D. Going back to the previous description of the spray  
 281 as a set of mixers defined by the mixing trajectories and leaning out while moving away from  
 282 the nozzle, the time spent on a given equivalence ratio will be always longer for the larger  
 283 nozzle by a factor around 2, i.e. approximately equal to the nozzle orifice increase. This has an  
 284 effect on combustion development, as the following sections will prove.



285

286 Figure 6. Top: Spray radius  $R$ , and iso-contours of  $\phi = 1$  (dashed line) and  $\phi = 2$  (dotted line) for Spray A and  
 287 Spray D. Bottom: Corresponding local residence time along iso-contours of  $\phi = 1$  and  $\phi = 2$ . Profiles of  
 288  $dt^*/d\phi$  have been smoothed with a moving average algorithm. Spatial coordinates have been normalized by  
 289  $d_{eq}$ .

290 To conclude the description of the inert spray evolution, Figure 7 shows the scalar dissipation  
 291 rate at stoichiometric conditions and mixture fraction rms for Spray A and Spray D, which are  
 292 input parameters for flamelet models such as the UFPV model. These variables are plotted  
 293 along the same reference iso-contours of  $\phi$  shown in the top panel of Figure 6. Results show  
 294 that higher  $\chi_{ST}$  is predicted for Spray A for a given  $\phi$ , in line with faster mixing occurring for  
 295 the smaller nozzle, which creates more important gradients. On the contrary, there are virtually  
 296 no differences among the nozzles in terms of mixture fraction rms. The implications of these  
 297 last observations will be further discussed in the description of the reactive spray in the next  
 298 sections.



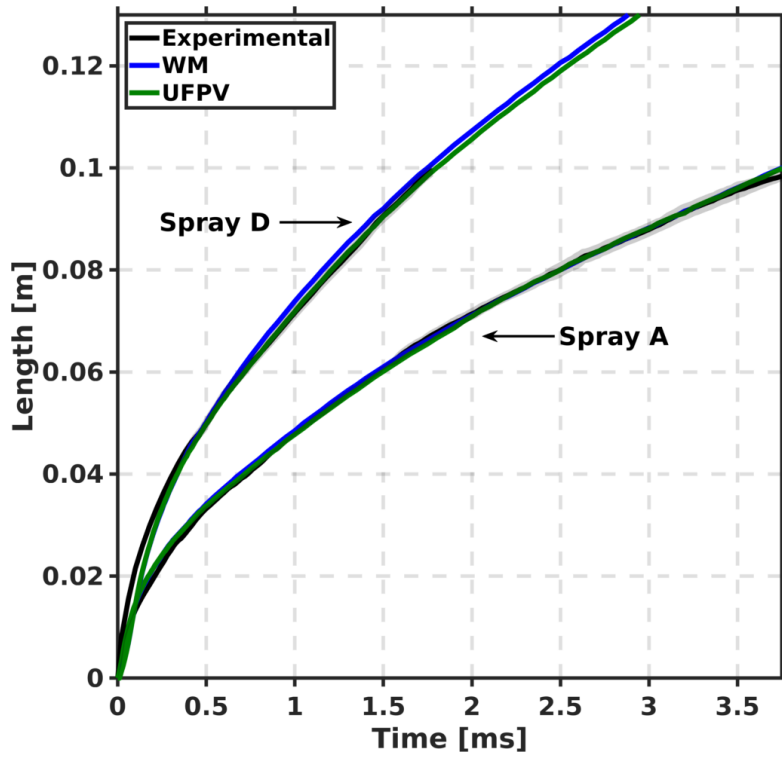
299

300 Figure 7. Scalar dissipation rate at stoichiometric conditions and mixture fraction rms along iso-contours of  $\phi =$   
 301 1 (dashed line) and  $\phi = 2$  (dotted line) for Spray A and Spray D.

## 302 4 Reacting spray evolution

### 303 4.1 Validation of the computational setup

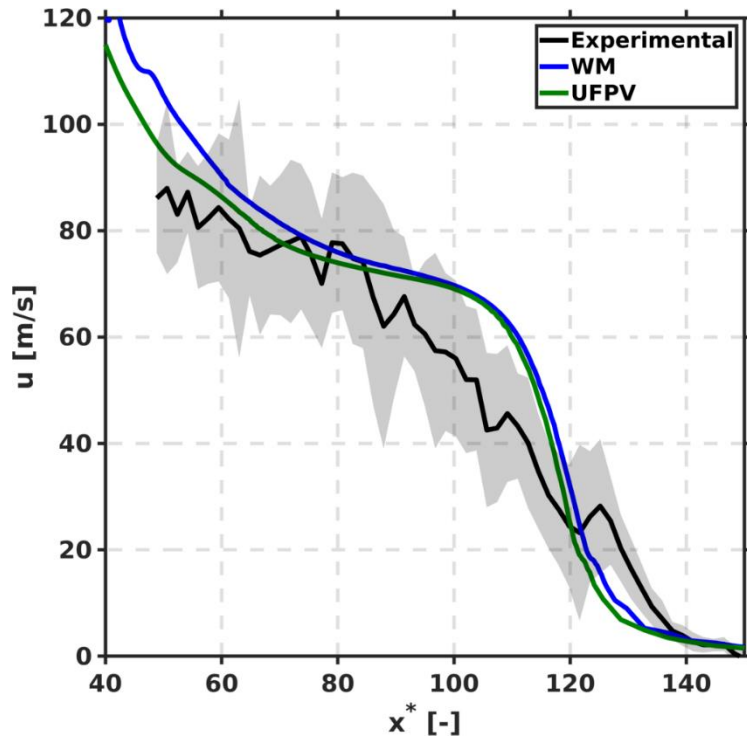
304 The computational setup validated for inert conditions is used to simulate reacting Spray A and  
 305 Spray D according to the conditions summarized in Table 1. Both the WM model and the  
 306 flamelet-based UFPV model are used. Figure 8 compares results for spray tip penetration. As  
 307 in the validation for the inert setup, here the spray tip penetration is defined as the axial distance  
 308 from the nozzle where  $Z$  reaches a value of 0.001. The experimental result is plotted with a  
 309 black solid line with gray shadow to indicate measurement uncertainty. Excellent agreement  
 310 between CFD and experimental results is observed for Spray A, meanwhile for Spray D a slight  
 311 over-prediction is observed for both combustion modelling approaches, while the agreement is  
 312 better for UFPV. Differences between both predictions are linked to the differences in ignition  
 313 delay, which triggers an acceleration of the flow.<sup>41</sup> In addition to spray tip penetration, the setup  
 314 is also validated for axial velocity with available PIV measurements for Spray A.<sup>10</sup> Figure 9  
 315 compares results for this variable along the spray axis. It is shown that both models predict  
 316 similar results which match well the experimental data.



317

318

Figure 8. Spray tip penetration for reacting Spray A and Spray D.



319

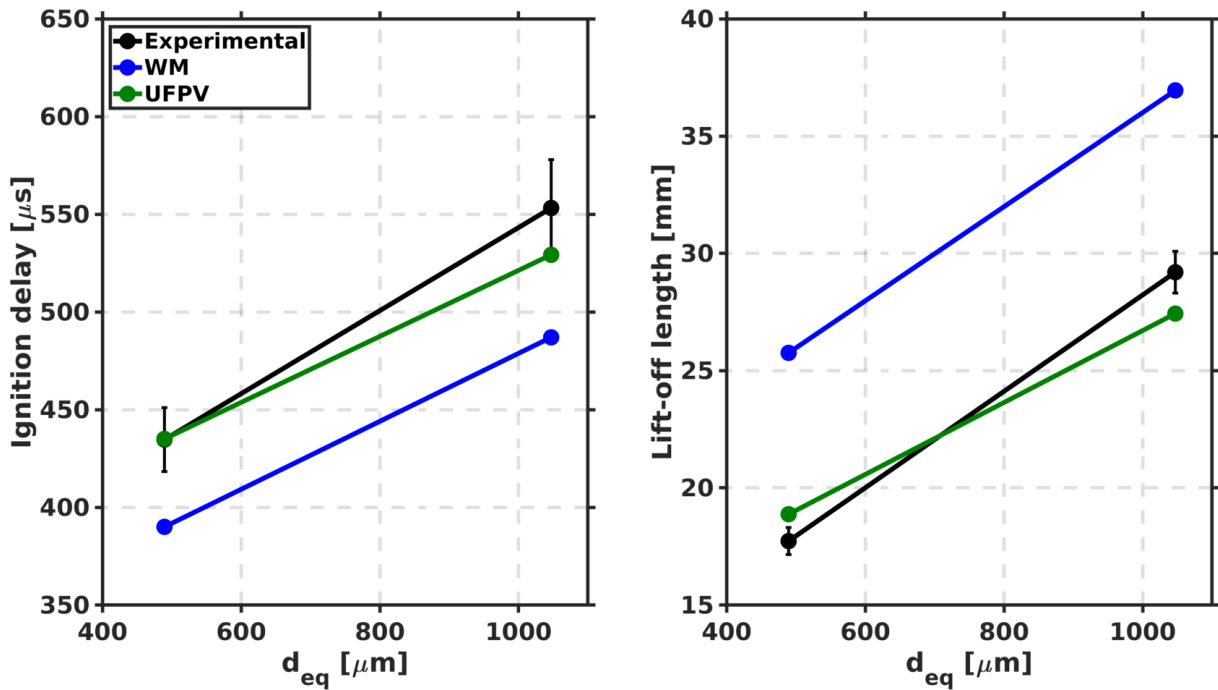
320

Figure 9. Axial velocity along spray axis for reacting Spray A at 1.5 ms.

321 After the validation of the reacting flow evolution, global combustion parameters are analyzed  
 322 here. Hence, CFD results for ID and LOL are quantitatively compared to experiments. On the  
 323 experimental side, ID and LOL are the result of analyzing schlieren and OH\*



324 chemiluminescence images, respectively. On the CFD side, ID and LOL are obtained following  
 325 ECN guidelines. ID is defined as the time from start of injection at which  $dT_{max}/dt$  is  
 326 maximum, with  $T_{max}$  being the maximum temperature in the domain. For LOL, the definition  
 327 is based on OH mass fraction. At each time after the start of combustion LOL is marked at the  
 328 closest position to the nozzle to reach 14% of the maximum OH mass fraction. Then, an average  
 329 value is obtained once LOL has stabilized.



330

331

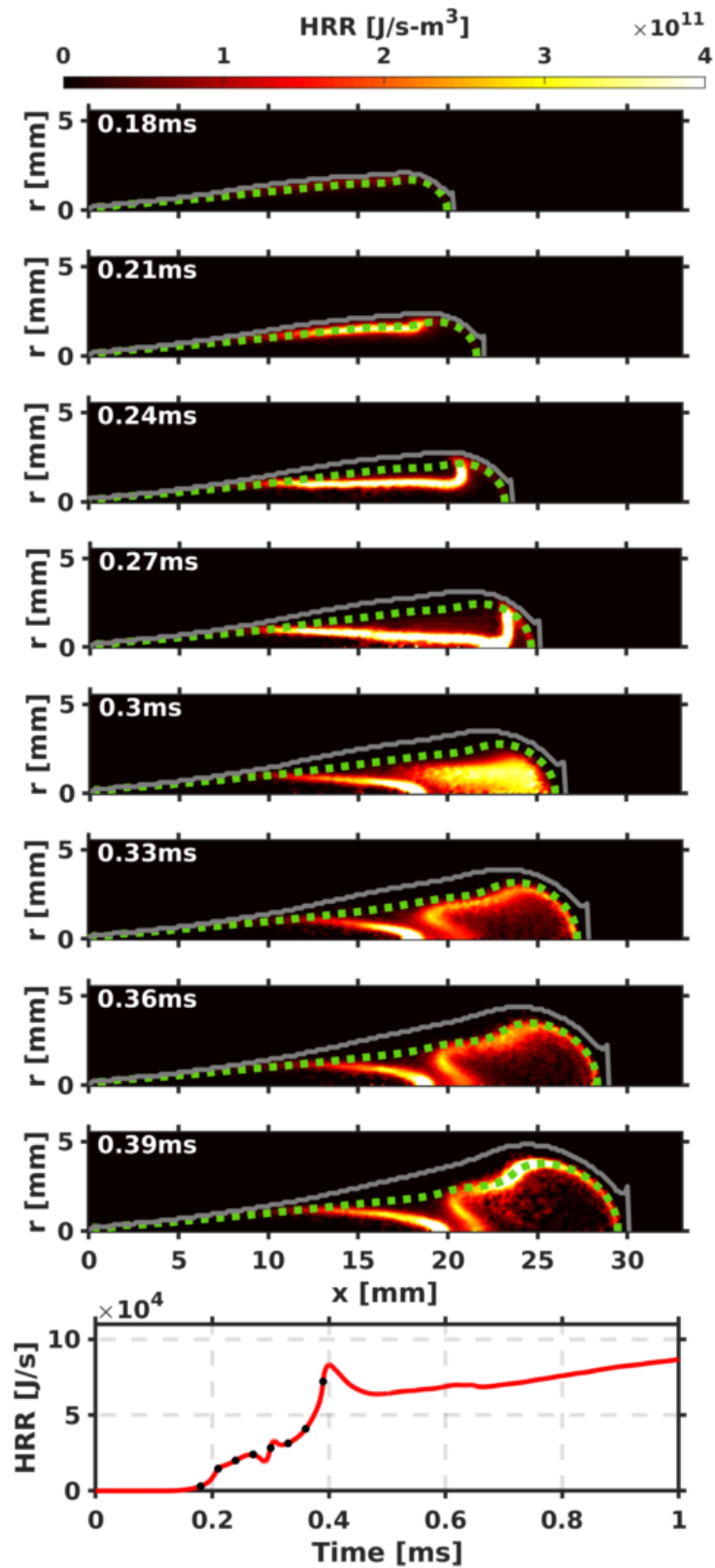
Figure 10. Ignition delay and lift-off length for Spray A and Spray D.

332 Figure 10 shows differences in predicted values compared to experimental results represented  
 333 with black dots (vertical lines on top mark the standard deviation of the measurement). A slight  
 334 underprediction of ID and an overestimation of LOL is observed when the WM model is used  
 335 for both nozzles. Differences decrease with the use of the UFPV model. In this case, ID matches  
 336 well with the experiment for Spray A with a slight underprediction for Spray D. As for LOL  
 337 maximum deviation from the experimental values is less than 7%. The detailed analysis of both  
 338 quantitative parameters will be related to the autoignition sequence and steady flame structure  
 339 in the following sections, so that observed differences among nozzles can be understood.

#### 340 4.2 Spray A auto-ignition sequence

341 In this section, a detailed analysis of the ignition sequence of Spray A is made comparing results  
 342 from the WM model and the UFPV model. The analysis makes use of the mixing trajectories  
 343 concept, which enables bridging a link between combustion development in the physical space  
 344 and in the equivalence ratio-temperature ( $\phi - T$ ) maps.

345 First, the auto-ignition sequence predicted using the WM model is presented in Figure 11. The  
346 contour of the local heat release rate (HRR) is plotted along with the spray radius. A dashed  
347 green line is plotted at the location of the iso-contour of the most reactive equivalence ratio  
348 ( $\phi_{MR}$ ). This parameter indicates the mixture composition showing the fastest ignition process  
349 when considering homogeneous reactors (HR) calculations starting from the inert adiabatic  
350 mixing condition. The HR 0D calculations done for this particular case and chemical  
351 mechanism show that the shortest ID corresponds to  $\phi_{MR} = 1.32$  and is in line with findings  
352 already published in the literature.<sup>42</sup> The last panel of the figure shows the spatially integrated  
353 HRR with black dot markers to highlight the time instants at which the local HRR was depicted.



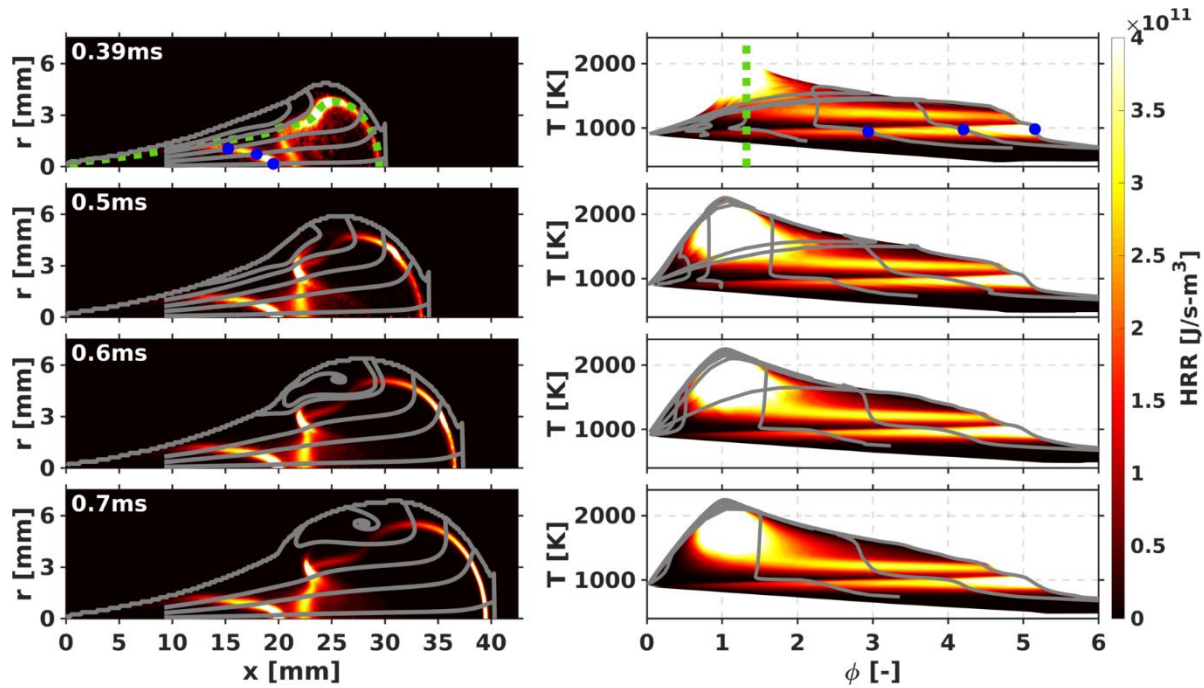
354

355 Figure 11. Time and spatially resolved local HRR for Spray A using the WM model. Dashed green line drawn at  
 356  $\phi_{MR}$ . Bottom plot shows the integrated HRR, where markers indicate the timings of the local HRR contours.

357 The sequence depicted in Figure 11 shows how the first-stage of ignition starts taking place at  
 358 around 0.21 ms near the spray radial periphery and locations close to  $\phi_{MR}$ . This observation

359 agrees well with the idea that ignition requires a certain degree of mixing of the injected fuel to  
360 reach reactive enough equivalence ratios and temperature conditions, as well as some residence  
361 time for chemistry to progress. The location of this low-temperature heat release near the spray  
362 border is consistent with longer residence time at the spray radial periphery, as described in the  
363 “Local residence time” section. Starting at 0.21 *ms* some sort of low-temperature heat release  
364 wave starts progressing towards richer mixtures (with a consistent increase in the integrated  
365 HRR in the last panel of the figure) in the spray core followed by a quasi-homogeneous state  
366 of heat release at 0.3 *ms*. Next, an abrupt decrease in heat release throughout the spray cross  
367 section is observed between 0.33 *ms* to 0.36 *ms* to then make way to the second-stage ignition  
368 occurring around the most reactive equivalence ratio iso-surface. The penultimate row in the  
369 panel shows the situation at the ID timing based on  $dT_{max}/dt$ , which occurs at 0.39 *ms*, with  
370 the start of an intense heat release zone around the most reactive equivalence ratio location.  
371 After this point, a diffusion flame is established and the integrated HRR is controlled by mixing  
372 as seen in the last row of the figure.

373 The onset time for the appearance of low-temperature heat release at around 0.21 *ms*, the  
374 consequent propagation towards the spray core followed by a quasi-homogeneous state of heat  
375 release and finally the decrease in chemical activity prior to the second-stage ignition are all  
376 features that are in line with experimental observations and supporting modelling results  
377 presented in the work by Dahms et al.<sup>11</sup> At the time of the second-stage ignition (penultimate  
378 panel in Figure 11) there are two distinctive heat release zones in the spray: a first cone-shaped  
379 structure associated with low-temperature heat release that was already present at 0.3 *ms*, and  
380 a high-temperature heat release zone near the spray radial periphery along the  $\phi_{MR}$  contour. It  
381 should also be emphasized that this high-temperature heat release zone confined around  $\phi_{MR}$   
382 is in disagreement with the above-mentioned experimental observations (supported by  
383 modelling results)<sup>11</sup> where main ignition takes place over a wide range of equivalence ratios.



384

385 Figure 12. Time resolved local HRR in spatial coordinates (panels on the left) and in  $\phi - T$  coordinates (panels  
 386 on the right) for Spray A using the WM model. Dashed green line drawn at  $\phi_{MR}$ . Blue markers to highlight the  
 387 low-temperature heat release zone.

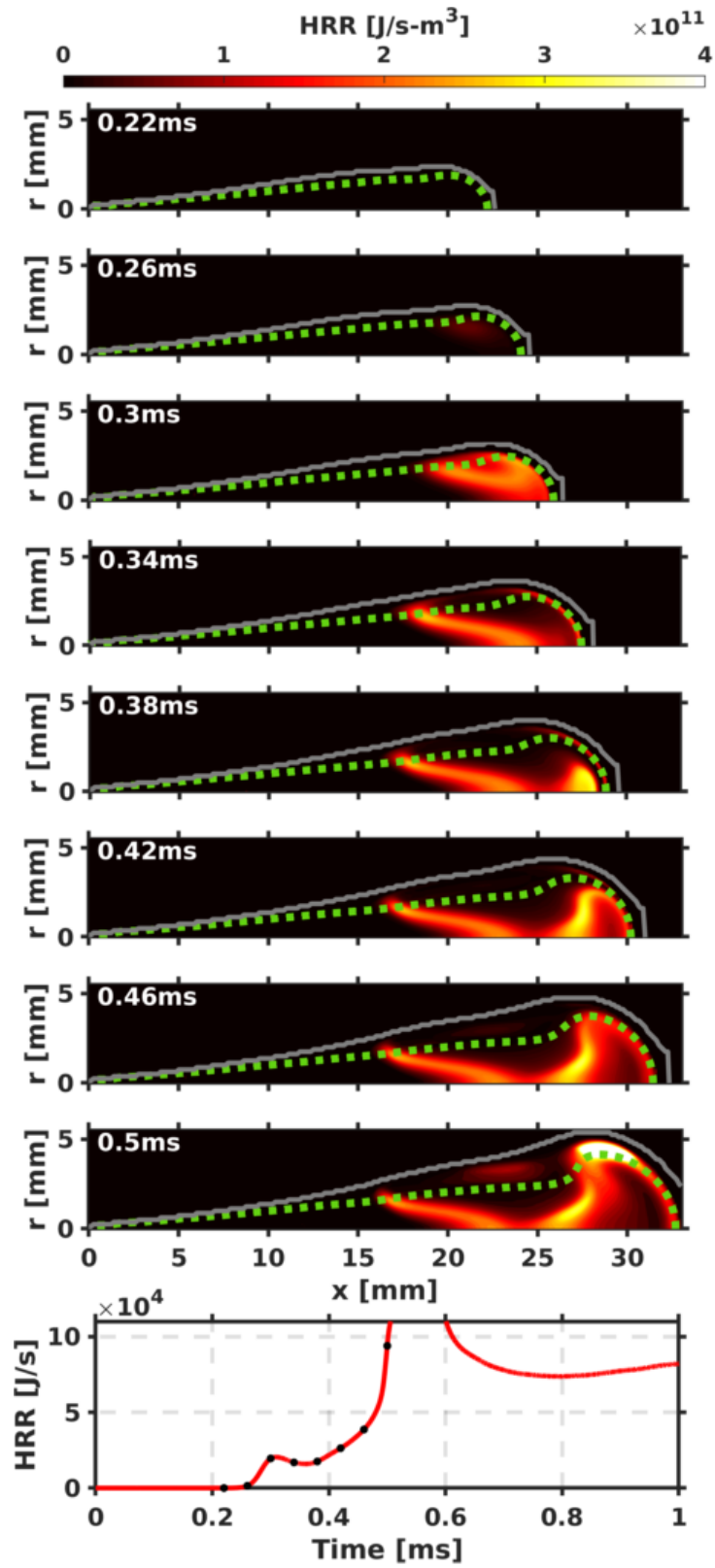
388 Finally, to analyze the post-ignition sequence Figure 12 shows on the left panels HRR contours  
 389 in spatial coordinates and on the right panels HRR contours plotted in  $\phi - T$  coordinates.  
 390 Mixing trajectories are included in both representations, with the purpose of linking spatial and  
 391  $\phi - T$  coordinates. At ID timing (top part of Figure 12) two distinctive characteristics  
 392 mentioned in the analysis of the auto-ignition sequence can be confirmed. On the one hand, the  
 393 occurrence of second ignition at  $\phi_{MR}$  is clearly observed on the  $\phi - T$  map. The intense HRR  
 394 spot observed in spatial coordinates near the spray radial periphery corresponds to the highest  
 395 temperature in the  $\phi - T$  map, which at ID time takes place at the vertical dashed line drawn at  
 396  $\phi_{MR}$ . On the other hand, low-temperature heat release is observed to occur over a wide  
 397 equivalence ratios range, when represented in  $\phi - T$  coordinates. The farthest point from the  
 398 nozzle at around  $x \cong 20 \text{ mm}$  on the first mixing trajectory (closest to the spray axis)  
 399 corresponds to the point at  $\phi \cong 5$ , the next two points at  $\phi \cong 4$  and  $\phi \cong 3$  correspond to blue  
 400 round markers on the second and third mixing trajectories, respectively. Furthermore, for all  
 401 three trajectories a second low intensity heat release front occurs just downstream of the initial  
 402 one at a slightly higher temperature (around 1200 K). This zone will later result in the LOL  
 403 stabilization region. Both low and medium temperature heat release regions remain essentially  
 404 steady for the remaining part of the reacting spray evolution, as they are located in the quasi-  
 405 steady part of the spray.

406 A noticeable feature of the autoignition sequence for the WM model is that, beyond 0.39 *ms*,  
407 the high-temperature heat release spot observed in spatial coordinates splits into two fronts.  
408 One of the fronts progresses in upstream direction, towards the position at which LOL will later  
409 stabilized. The other front moves downstream towards the spray head. The progress of this  
410 second heat release front in spatial coordinates can be linked to  $\phi - T$  space through the mixing  
411 trajectories, starting from the axis towards higher radial coordinates in the physical space, which  
412 corresponds to lines from richer to leaner values in the  $\phi - T$  space. At 0.5 *ms* the heat release  
413 front has not yet reached the two trajectories closest to the axis. Consequently, on the  $\phi - T$   
414 map these two mixing trajectories have still not reached the maximum temperature on the map  
415 (evidenced by the almost horizontal temperature between  $\phi \cong 2$  and  $\phi \cong 3$ ). At 0.6 *ms* the  
416 heat release front has just passed through the second mixing trajectory closest to the axis,  
417 causing it to reach the maximum temperature on the  $\phi - T$  map for any equivalence ratio value.  
418 Lastly, at 0.7 *ms* the heat release front has reached the spray axis and all mixing trajectories  
419 have reached the maximum temperature for any equivalence ratio value.

420 In an attempt to establish an intermediate situation that enables a better understanding of the  
421 changes when moving from WM results to UFPV results, Figure 13 shows the auto-ignition  
422 sequence for the modified version of the UFPV model, denoted as UFPV-0, where the flamelet  
423 manifold has been built without any PDFs integration (neither for mixture fraction nor for scalar  
424 dissipation rate) to be able to capture spatial details that otherwise are softened by the presumed-  
425 PDF integration as later seen in Figure 14. In this way, WM to UFPV-0 (Figure 12 vs Figure  
426 13), shows the effect of changing the sub-grid description of the flame structure (from a WM  
427 to a flamelet formulation) while UFPV-0 to UFPV (Figure 13 vs Figure 14), evaluates the  
428 influence of TCI by means of presumed-PDF integration.

429 Figure 13 shows initial heat release occurring from the spray radial periphery towards the spray  
430 axis before reaching a stabilized cone-shaped low-temperature heat release front at 0.34 *ms*.  
431 The evolution is more volumetric than for the WM model, where a well-defined low-  
432 temperature reaction front was observed in the initial stages (0.24 *ms*). Furthermore, reactions  
433 tend to be located further downstream compared to WM results already from the start, probably  
434 due to the inhibiting effect of scalar dissipation rate in locations close to the nozzle (Figure 7).  
435 The quasi-homogeneous heat release state already observed for the WM model at 0.3 *ms* seems  
436 to be occurring also for UFPV-0 at 0.38 *ms*, although spatially limited to a region closer to the  
437 spray tip and near the spray axis. After that, heat release decreases within this reaction zone,  
438 which slightly recedes upstream and towards the spray radial periphery. Then, high-temperature

439 ignition takes place (last contour plot in Figure 13). In agreement with the whole spray ignition  
440 sequence, the high-temperature heat release starts at around 27 mm, further downstream than  
441 for WM results (slightly upstream of 25 mm).

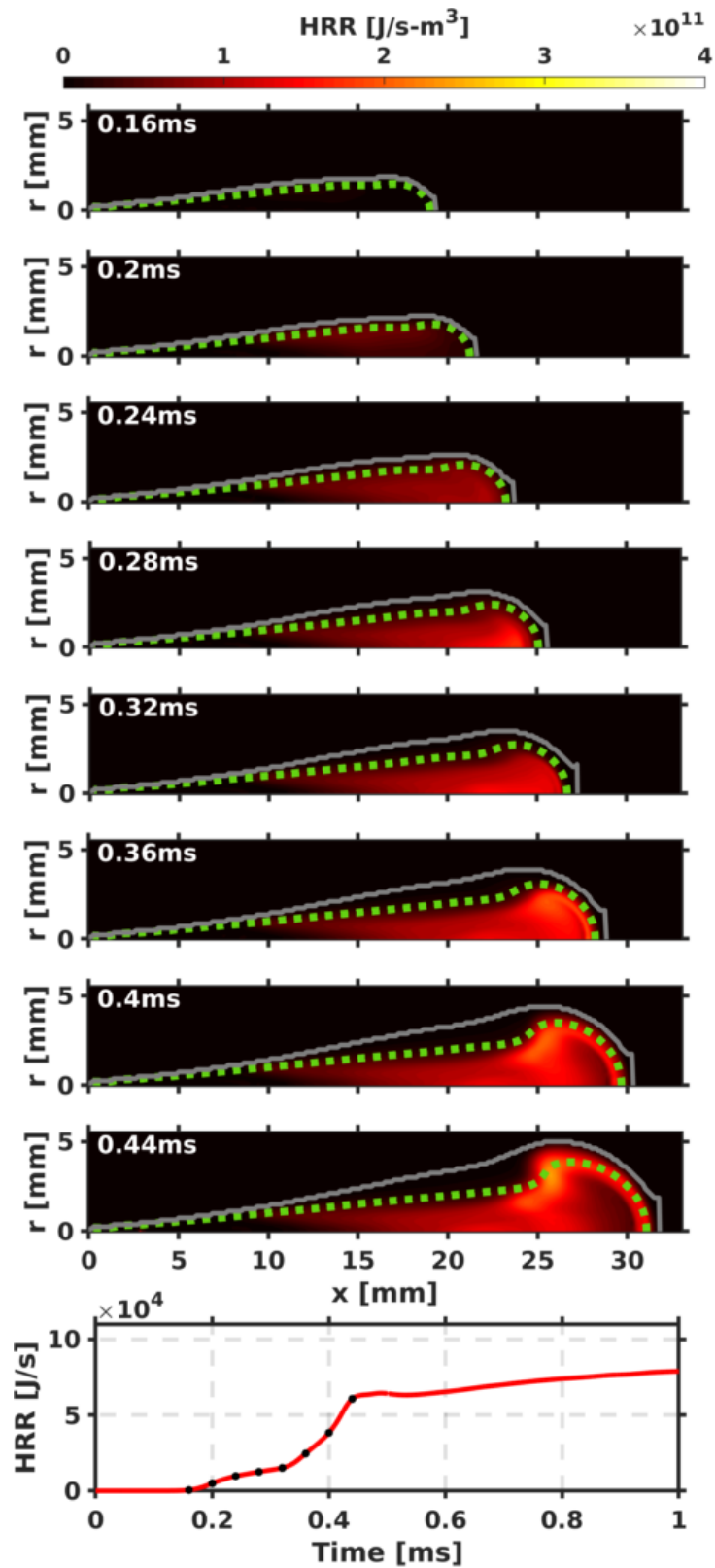


442

443 Figure 13. Time and spatially resolved local HRR for Spray A using the UFPV-0 model. Dashed green line  
444 drawn at  $\phi_{MR}$ . Bottom plot shows the integrated HRR, where markers indicate the timings of the local HRR  
445 contours.

446 The same image layout used to depict the auto-ignition sequence for WM model results has  
447 been used for UFPV model results. Consequently, Figure 14 shows HRR contours accompanied  
448 by the spray radius and the  $\phi_{MR}$  iso-contour (green dashed line). Several time instants are  
449 included up until ID time at 0.44 ms (penultimate row in Figure 14). Aside from the differences  
450 in timing, some features in ignition will be discussed to evidence the differences in ignition  
451 description.





452

453 Figure 14. Time and spatially resolved local HRR for Spray A using the UFPV model. Dashed green line drawn  
 454 at  $\phi_{MR}$ . Bottom plot shows the integrated HRR, where markers indicate the timings of the local HRR contours.

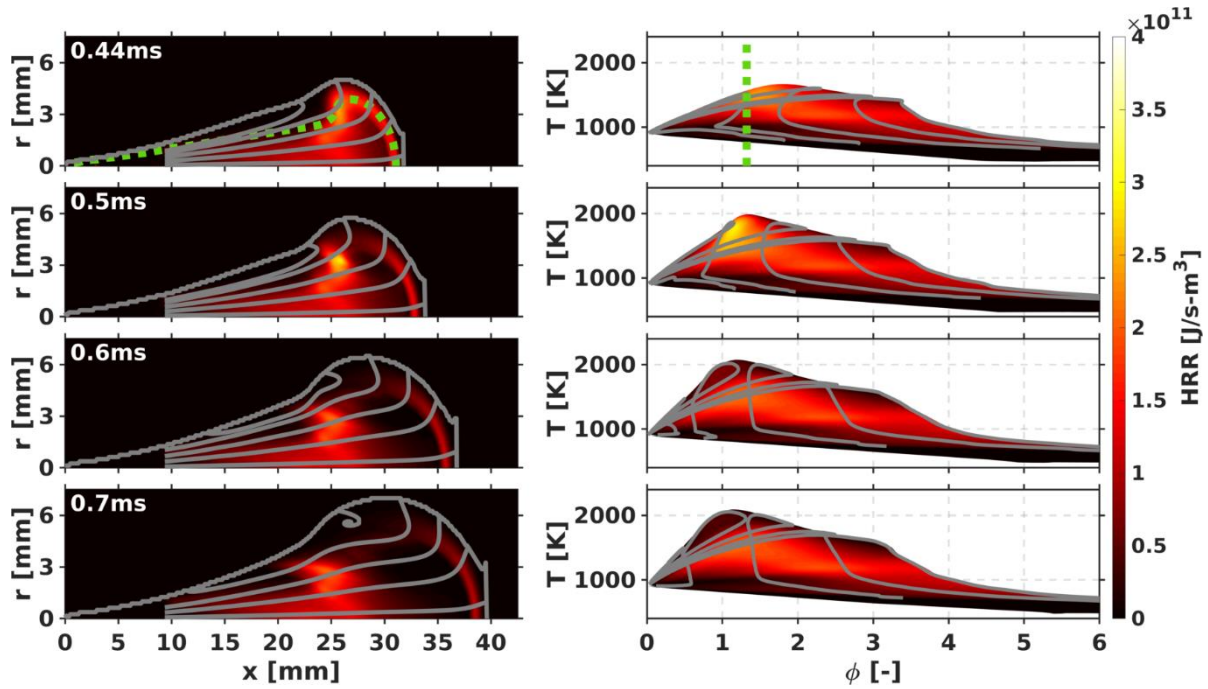
455 In a similar way as observed in WM model results, first-stage ignition starts taking place near  
 456 the spray radial periphery. Nonetheless, unlike predictions using the WM model, the UFPV

457 model predicts this first-stage ignition as taking place in a much broader area in the spray and  
458 not only around the most reactive equivalence ratio as derived from HR results. Keeping in  
459 mind that the color scale is the same, the intensity of this initial ignition is lower for the UFPV  
460 model (as also seen in the integrated HRR plot at the bottom of the figure), which will be a  
461 constant feature through the whole ignition sequence. Next, heat release progresses towards the  
462 spray core with richer mixtures and reaches a quasi-homogeneous reaction state at around 0.32  
463 *ms* close to the spray head, comparable to WM model results at 0.3 *ms*. However, the low-  
464 temperature heat release front is not confined to a concrete region in the spray as it was predicted  
465 in the latter model. Additionally, not such an abrupt decrease in chemical activity throughout  
466 the spray cross section prior to second-stage ignition is observed with the UFPV model (0.36  
467 to 0.4 *ms*) as was the case for WM (0.33 *ms*). At 0.4 *ms* an increase in HRR level is visible  
468 near the spray radial periphery close to the spray tip, where second-stage ignition eventually  
469 takes place at 0.44 *ms*. This observation is consistent with high residence time and low  $\chi_{ST}$ ,  
470 both favorable for auto-ignition.<sup>43</sup> Compared to WM results, where the main ignition was  
471 observed slightly upstream of 25 *mm* (penultimate panel on Figure 11), UFPV tends to produce  
472 a main ignition site slightly downstream of 25 *mm*. As in the WM case, the ID time coming  
473 from the analysis of  $dT_{max}/dt$  matches the timing of high-temperature HRR (last row of Figure  
474 14)

475 Observed differences between WM and UFPV results obey to two factors. First, UFPV makes  
476 use of reaction source terms based upon the flamelet formulation, compared to the  
477 homogeneous reactors in the WM model. Second, UFPV uses a presumed-PDF approach to  
478 account for turbulent fluctuations, which are neglected in the WM model. Regarding differences  
479 between UFPV-0 and UFPV (Figure 13 vs Figure 14), the effect of presumed-PDF becomes  
480 quite apparent. The main effect is the smoothing of gradients that results in a more volumetric  
481 description of the ignition event of the spray. A second important feature is the decrease in heat  
482 release seen both in the spray HRR contours and in the integrated HRR plot. Finally, while the  
483 spatial location of all such events is pretty similar, the timing becomes slightly advanced for  
484 UFPV. All these results are a consequence of the averaging of the different igniting flamelets.

485 Based on the conceptual descriptions and experimental observations reported in the literature<sup>11</sup>  
486 the UFPV model successfully captures auto-ignition key features of ECN Spray A. Low-  
487 temperature reactions are seen to be starting at the spray radial periphery and then move towards  
488 the spray axis. The quasi-homogeneous state of low-temperature heat release and the  
489 subsequent decrease in chemical reactivity before the main ignition event are also captured by

490 the model. Finally, the model predicts second-stage ignition as taking place in a broader range  
 491 of mixtures and not being confined around  $\phi_{MR}$ . Diffusion phenomena induced by the scalar  
 492 dissipation rate as well as the presumed-PDF approach allows the UFPV model to capture this  
 493 last feature that is not reproduced in a well-mixed approach. Finally, the inclusion of such sub-  
 494 grid diffusion effects by means of  $\chi_{ST}$  delays the overall temporal sequence of auto-ignition in  
 495 around 0.1 *ms* compared to the well-mixed approach, while the presumed-PDF approach  
 496 advances back the timing of events in approximately 0.05 *ms*.



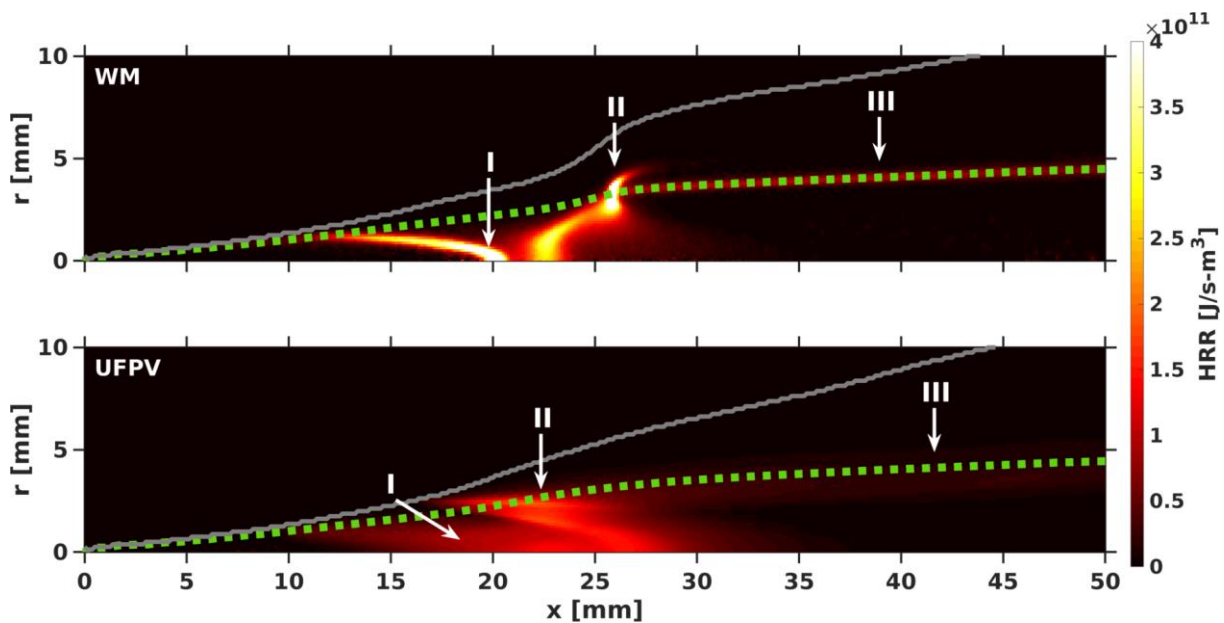
497

498 Figure 15. Time resolved local HRR in spatial coordinates (panels on the left) and in  $\phi - T$  coordinates (panels  
 499 on the right) for Spray A using the UFPV model. Dashed green line drawn at  $\phi_{MR}$ .

500 The post-ignition sequence predicted by the UFPV model is shown in Figure 15. As already  
 501 seen in the spatial representation of the HRR contour, the second-stage ignition takes place in  
 502 a broader range of equivalence ratios and not just around  $\phi_{MR}$ . Compared to WM results (Figure  
 503 12), where a defined steady low-temperature reaction zone was observed to be established  
 504 upstream (10-20 *mm*) with another intermediate reaction layer close to the axis (around 22  
 505 *mm*), and the diffusion flame stabilization occurred by the propagation of two fronts along the  
 506 stoichiometric surface, UFPV shows a much less intense low-temperature front over a wide  
 507 spray region (15-25 *mm*), with the main heat release over the whole spray cross-section at  
 508 around 20-25 *mm*. No transient front evolution can be observed around the stoichiometric  
 509 surface, as was the case for the WM model. An important feature, however, is the very different  
 510 appearance of the heat release at the LOL location, which will be analyzed in the next section.

511 4.3 Spray A quasi-steady state description

512 In spite of the almost quasi-steady appearance of the reacting spray at 0.7 ms in the previous  
513 section, the analysis of LOL for all modelling approaches is made for a later time, in which the  
514 spray head does not interact with the flame base. Figure 16 shows the contour of local HRR  
515 results for WM (top) and UFPV model (bottom). The spray radius is plotted with a gray solid  
516 line and the location of the stoichiometric equivalence ratio is plotted with a dashed green line.  
517 Both models predict heat as being release from three distinctive areas i.e. a low-temperature  
518 structure (zone I) at the flame base as a result of first-stage ignition, a diffusion flame front  
519 (zone III) around the stoichiometric equivalence ratio and an intermediate partially-premixed  
520 flame front (zone II) around the LOL location. While zones I and II are quite similar in both  
521 models, except for the fact that both fronts are narrower for the well-mixed model due to the  
522 absence of both sub-grid flamelet diffusion and presumed-PDFs integration, the location and  
523 appearance of zone II is a key difference between WM and UFPV predictions. Figure 17 shows  
524 a zoomed view of the local HRR contour around the area where LOL is stabilized including the  
525 well-mixed model, as well as the UFPV and UFPV-0 approaches. Mixing trajectories have been  
526 added (solid gray lines) to allow for the posterior link of spatial and  $\phi - T$  coordinates. Blue  
527 “x” markers are plotted at the locations where OH mass fraction reach 14% of the maximum  
528 value in the spray following ECN guidelines for the location of LOL.

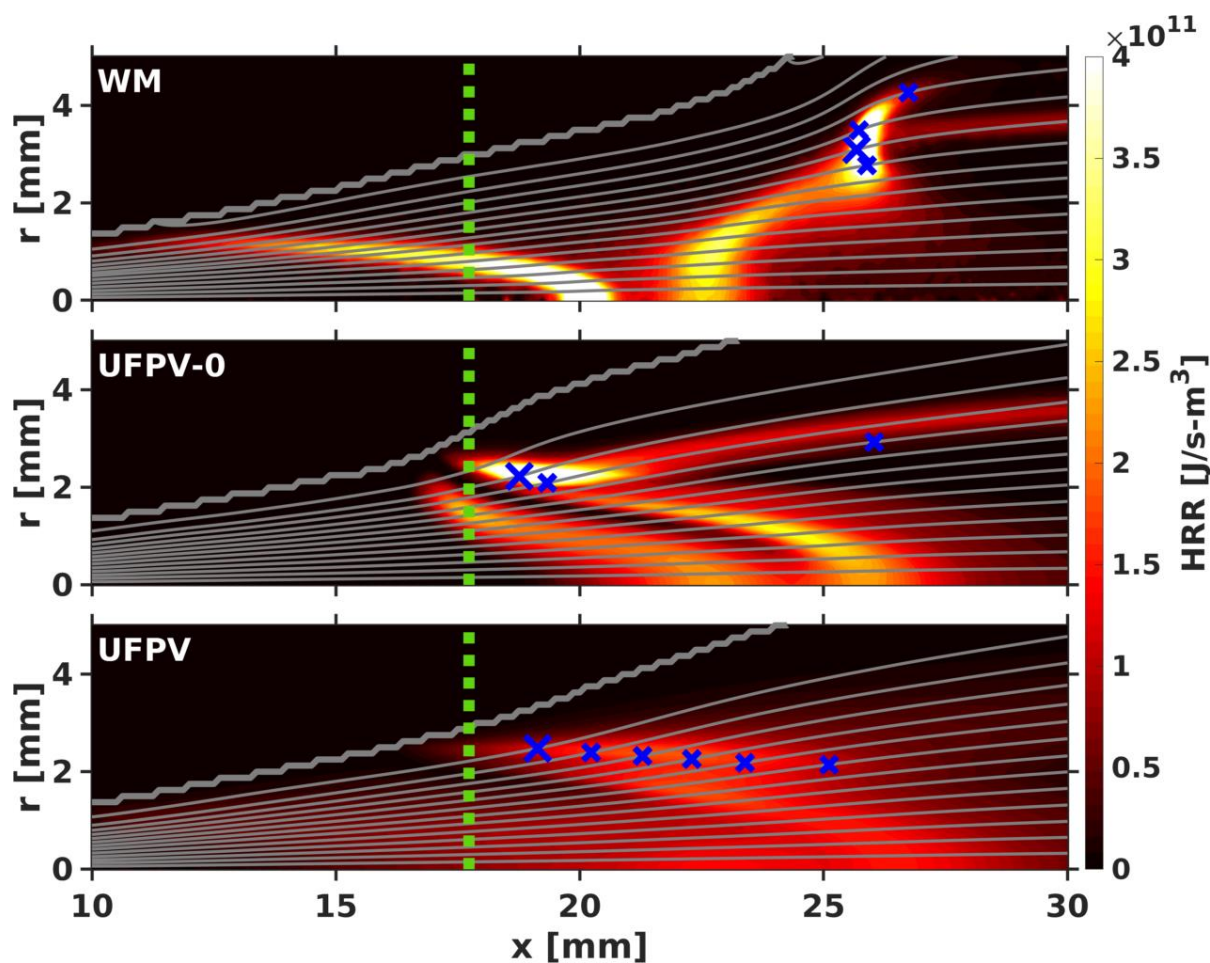


529

530 Figure 16. Local HRR contour at quasi-steady state for Spray A using WM (top) and UFPV model (bottom).  
531 Dashed green line drawn at  $\phi = 1$ .

532 From Figure 16 low and high-temperature heat release fronts (zones I and II) were shown to be  
533 spatially decoupled for the WM case. In contrast, the flame structure predicted by the UFPV

534 model around the stabilized LOL position (Figure 17 bottom) shows how zones I and II are  
535 virtually merged. Compared to the well-mixed case, the introduction of  $\chi_{ST}$  for UFPV-0 seems  
536 to contribute to the stabilization of zone I further downstream from the nozzle. This observation  
537 is consistent with a delayed temporal evolution of auto-ignition (comparing WM and UFPV-0  
538 results) as described in the “Spray A auto-ignition sequence” section since high  $\chi_{ST}$  values near  
539 the nozzle inhibit combustion. As for Zone II, sub-grid diffusion seems to contribute to the  
540 stabilization of the high-temperature front in both UFPV approaches closer to the nozzle as  
541 compared with WM results. Furthermore, the observed intermediate temperature zone close to  
542 the spray axis in the WM approach has a very similar shape in the UFPV-0 approach to the low-  
543 temperature one. The averaging role of the presumed-PDF approach is also seen, when  
544 comparing UFPV-0 and UFPV, in the sense that both low-temperature zones become eventually  
545 merged and the high-temperature heat release drops in intensity. All such features will be  
546 analyzed in  $\phi - T$  coordinates in the following.



547

548 Figure 17. Local HRR contour at quasi-steady state near LOL location for Spray A using WM (top), UFPV-0  
 549 (middle) and UFPV model (bottom). Biggest blue “x” marker for mixing trajectory closest to predicted LOL  
 550 (green dashed line).

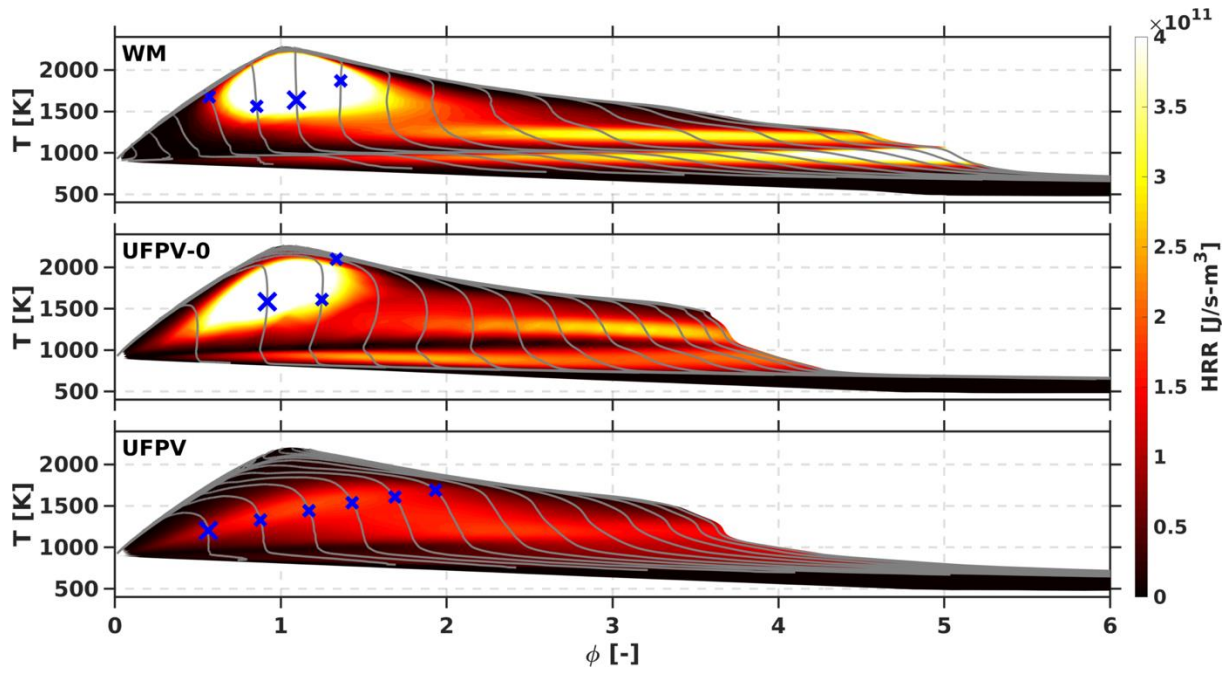
551 With the inclusion of mixing trajectories, the path followed by a “conserved gas particle” can  
 552 be depicted both in spatial and  $\phi - T$  coordinates. Analysis of WM results (Figure 18 top)  
 553 reveals how any “conserved gas particle” starts diluting almost along the inert adiabatic mixing  
 554 trajectory, with a first noticeable increase in temperature as it passes through the low-  
 555 temperature area (zone I in Figure 17 top). As already discussed, this initial flame front occurs  
 556 at a similar temperature but at a different equivalence ratio for every single trajectory. Beyond  
 557 zone I, two types of evolutions can be observed. For mixing trajectories closer to the axis, i.e.  
 558 moving through richer equivalence ratio values, the also mentioned intermediate flame front  
 559 can be observed at around 22-23 mm, and correspondingly at a temperature of around 1100 K.  
 560 After that, no heat will be released along those trajectories until reaching the stoichiometric  
 561 flame front. On the other hand, mixing trajectories reaching the stoichiometric flame front at  
 562 around 25 mm directly run into the high-temperature heat release, and the intermediate  
 563 temperature ignition is missing. This is probably due to the longer residence time associated

564 with such radially displaced trajectories, which enables reaching high-temperature ignition for  
565 similar equivalence ratio values as those closer to the axis.

566 The inclusion of UFPV-0 results (Figure 18 middle) allows to isolate the effect of  $\chi_{ST}$  in  $\phi - T$   
567 coordinates. As it was already mentioned when analyzing the effects of diffusion in spatial  
568 coordinates, there are two distinctive effects. On the one hand, when comparing the well-mixed  
569 results (no effect of  $\chi_{ST}$ ) with UFPV-0, it becomes clear that diffusion decreases reactivity for  
570 low-temperature chemistry (zone I in Figure 17) as the HRR is less intense in the zone below  
571 1000 K, which becomes evident in the  $\phi - T$  map. On the contrary, high-temperature heat  
572 release (upper part in the  $\phi - T$  centered around  $\phi = 1$ ) becomes wider in equivalence ratio  
573 and temperature ranges, especially towards the lean region. This effect speeds up the transition  
574 from low-temperature to high-temperature heat release with the consequent stabilization of  
575 LOL closer to the nozzle compared to WM results.

576 Finally, the description can also be carried-out for the UFPV model (Figure 18 bottom). In the  
577 same way as for spatial coordinates, the distinction among different flame fronts in the  $\phi - T$   
578 map is softened due to averaging. In this case, the low-temperature flame front occurs over a  
579 wide region upstream 20 mm for all trajectories, but the trends in the  $\phi - T$  map does not depart  
580 substantially from the inert adiabatic mixing line. Instead of separated reaction regions around  
581 the LOL as for the WM model, UFPV shows a single high heat release zone starting radially at  
582 around 19 mm and which reaches the spray axis at around 26 mm. All mixing trajectories flow  
583 through this zone, shown by the steep increase in temperature in the  $\phi - T$  map over a wide  
584 range of equivalence ratio values.

585 According to the previous differences in heat release zones, modelling approaches predict  
586 different locations for the LOL. On the one hand, the well-mixed approach predicts LOL to  
587 stabilize on the mixing trajectory passing through the most reactive spot in the high-temperature  
588 heat release zone close to the stoichiometric equivalence ratio. On the contrary, the UFPV  
589 model predicts LOL to stabilize at the lean high-temperature heat release zone. Furthermore,  
590 the UFPV-0 shows the underlying flame structure, with an intense stoichiometric combustion,  
591 which extends towards both the lean and rich sides, which may bring some remembrance with  
592 triple flame structures. Recent findings supported by DNS calculations under similar operating  
593 conditions show that LOL stabilization might take place at lean, stoichiometric or rich zones  
594 depending on the local flame topology,<sup>44</sup> but including triple flame effects.



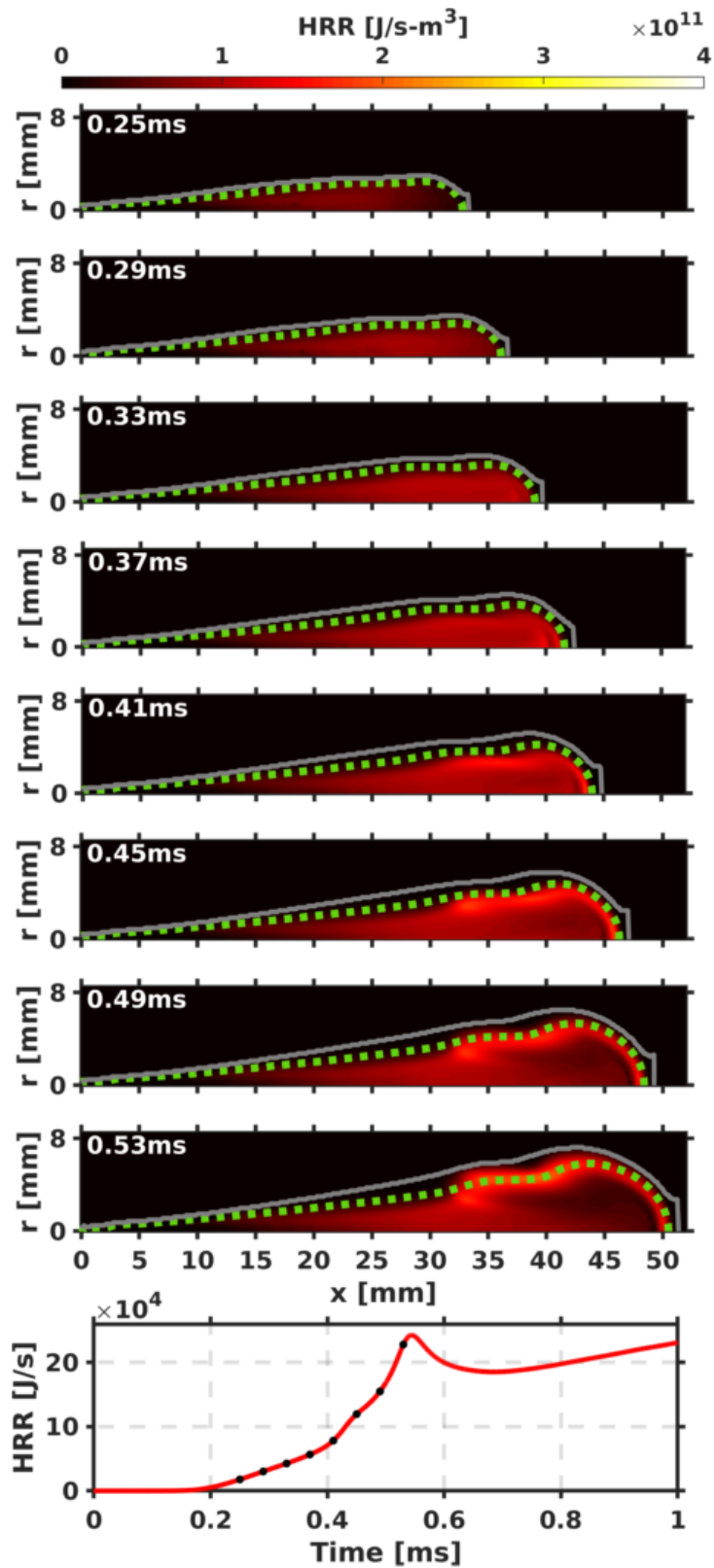
595

596 Figure 18. Local HRR contour at quasi-steady state in  $\phi - T$  space for Spray A using WM (top), UFPV-0  
 597 (middle) and UFPV model (bottom). Biggest blue “x” marker for mixing trajectory closest to predicted LOL.

598 4.4 Spray D auto-ignition sequence

599 After a detailed description of the different modelling approaches for the Spray A case, the  
 600 UFPV model is used to evaluate the influence of nozzle diameter in Diesel combustion  
 601 following similar concepts. First, the auto-ignition sequence of Spray D will be analyzed. Local  
 602 HRR contours are plotted in Figure 19 (along with the spray radius and the iso-contour of  $\phi_{MR}$ )  
 603 for several timings up to the ID timing at 0.53 ms.





604

605 Figure 19. Time and spatially resolved local HRR for Spray D using the UFPV model. Dashed green line drawn  
 606 at  $\phi_{MR}$ . Bottom plot shows the integrated HRR, where markers indicate the timings of the local HRR contours.

607 The sequence depicted in Figure 19 evidences that both nozzles (see Figure 14 for Spray A  
 608 results) share similar features on how the flame is established i.e. a cool flame originated at the

609 spray radial periphery (in a broader range of mixtures not limited to  $\phi_{MR}$ ), a quasi-  
610 homogeneous state of heat release (at 0.32 ms for Spray A and at 0.37 ms for Spray D) and  
611 finally a decrease in the HRR prior to second-stage ignition taking place at the spray radial  
612 periphery (at 0.44 ms for Spray A and at 0.53 ms for Spray D). The location of second-stage  
613 ignition for Spray A is seen to occur closer to the spray tip compared to Spray D, where this  
614 occurs essentially upstream of the spray tip front. Higher  $\chi_{ST}$  values reported above for Spray  
615 A play an important role in the spatial shift of ignition location compared to the larger nozzle.  
616 Differences in ignition location among nozzles are consistent with those reported in the  
617 literature from experimental observations by Pastor et al.<sup>4</sup>

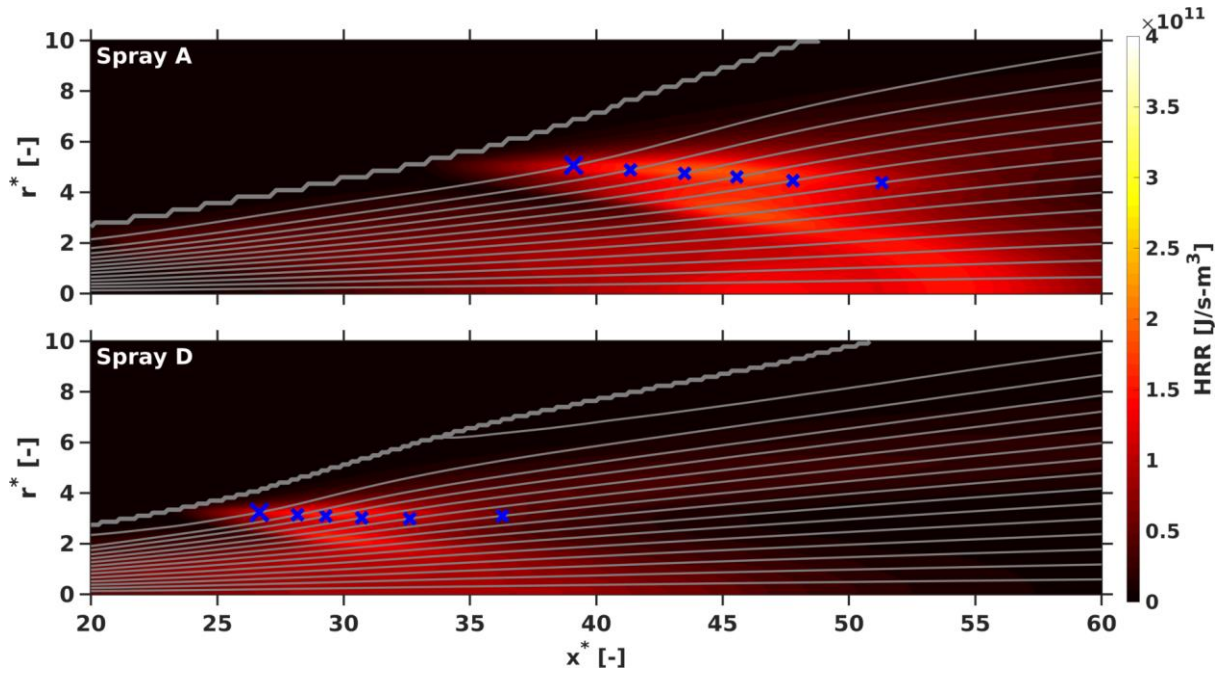
618 In agreement with previously discussed longer residence time for Spray D nozzle, a slower  
619 progression of the auto-ignition sequence is expected compared to spray A, as it takes more  
620 time to reach reactive ignitable mixtures, which might be compensated to some extent by the  
621 lower  $\chi_{ST}$  values of the larger nozzle. From simulation results, the whole ignition sequence is  
622 seen to be already delayed from the initial low-temperature stages, and eventually Spray D  
623 ignites 94  $\mu s$  later compared to Spray A. In that same direction, from experimental observations  
624 Spray D high-temperature ignition occurs 137  $\mu s$  later compared to Spray A. These differences  
625 are consistent with a slower mixing process as previously described in the “Validation of the  
626 computational setup” section. Aside from timing, the general development of the ignition  
627 sequence on Spray D occurs at richer equivalence ratio values. The longer residence time for  
628 Spray D enables the ignition of richer mixtures less favorable from the point of view of  
629 temperature and equivalence ratio. This observation will be further analyzed in the next section.

#### 630 4.5 Spray D quasi-steady state description

631 After ignition, both nozzles also share similar heat release zones at quasi-steady state. The  
632 previously observed zones I, II and III for Spray A (Figure 16 bottom) are also reproduced in  
633 Spray D, resulting in a similar flame structure, with an upstream location occurring at richer  
634 mixtures for the larger nozzle following the auto-ignition comparison. A closer look at zone II  
635 in Figure 20, where LOL is stabilized, confirms the similarity of Spray A and Spray D flame  
636 structure. The use of normalized coordinates already points at a stabilization of the flame base  
637 at a richer location in Spray D compared to Spray A.

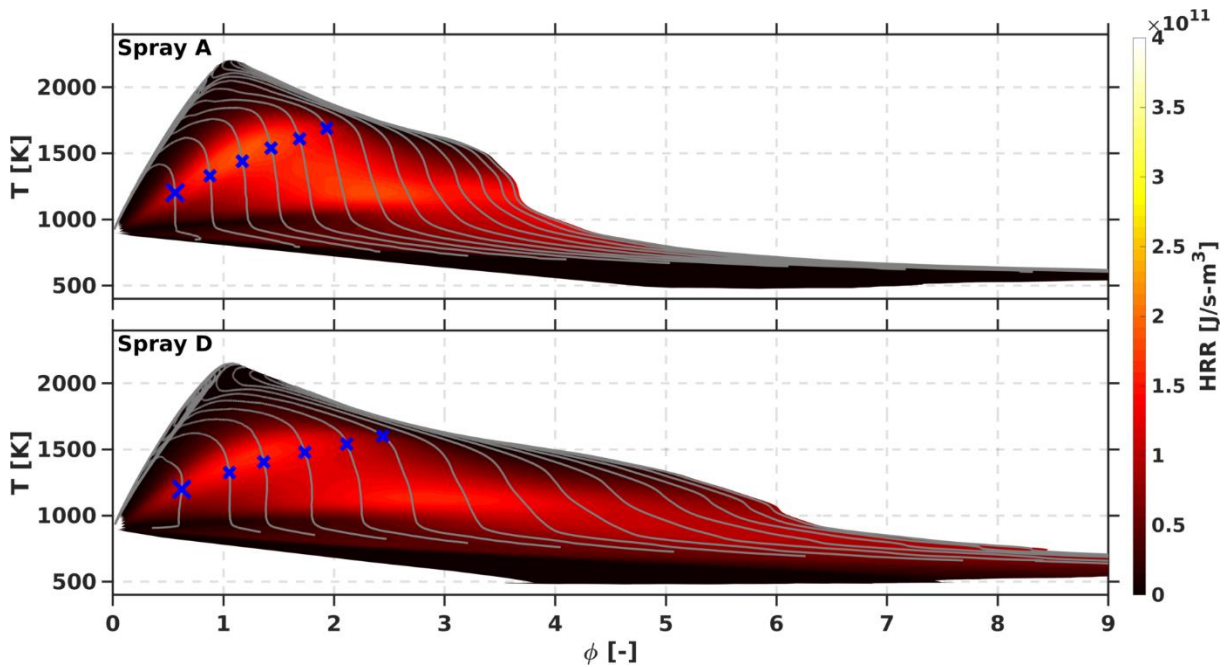
638 In Figure 20, mixing trajectories are superimposed onto the local HRR contour, and in Figure  
639 21 the corresponding  $\phi - T$  maps are shown. The positions where the mixing trajectories go  
640 above the contour of the 14% of the maximum OH mass fraction are highlighted with blue “x”

641 markers. As previously observed for Spray A the closest point to the nozzle, which defines the  
 642 LOL value in both cases, occur in the most radially displaced trajectory, i.e. at lean conditions.



643

644 Figure 20. Local HRR contour at quasi-steady state near LOL location for Spray A (top) and Spray D (bottom)  
 645 using the UFPV model. Spatial coordinates have been normalized by  $d_{eq}$ . Biggest blue “x” marker for mixing  
 646 trajectory closest to predicted LOL.



647

648 Figure 21. Local HRR contour at quasi-steady state in  $\phi$ -T space for Spray A (top) and Spray D (bottom)  
 649 using the UFPV model. Biggest blue “x” marker for mixing trajectory closest to predicted LOL.

650 Despite evident similarities between the two nozzles studied in this work, a major distinction  
 651 has already been mentioned, i.e. both ignition and lift-off length stabilization occur in more

652 fuel-rich mixtures in Spray D as compared to Spray A. This observation is sustained by a slower  
653 mixing process with consequent longer residence time for Spray D. In  $\phi - T$  coordinates Spray  
654 D richer combustion is evidenced by the presence of mixing trajectories increasing in  
655 temperature for  $\phi > 4$ , which does not happen for Spray A. Such trajectories are the ones  
656 closest to the axis, where the scalar dissipation rate is lower, which also contributes to the  
657 possibility of reaction to progress in richer equivalence ratio zones. On the other hand,  
658 trajectories linked to the LOL stabilization (blue markers in Figure 20 and Figure 21) reach the  
659 lift-off limit at similar equivalence ratio and temperature values for both nozzles. However, the  
660 different development of the mixing process in terms of spatial distribution and local residence  
661 time results in different spatial locations for this high-temperature zone between both nozzles.

## 662 **5 Conclusions**

663 The effect of nozzle orifice diameter on Diesel combustion has been studied. Spray A and Spray  
664 D from the ECN have been modeled and validated under nonreacting and reacting conditions  
665 using CFD. For the nonreacting condition, the applicability of the mixing trajectory concept has  
666 been assessed for both nozzles along with the concept of local residence time. For the reacting  
667 condition, commonly made assumptions for TCI have been evaluated for Spray A. For this  
668 purpose, auto-ignition sequence and quasi-steady state results from the WM and the UFPV  
669 models have been analyzed. Finally, a comparison of auto-ignition and quasi-steady state results  
670 of the two nozzles has been made.

671 Main conclusions from this study can be summarized in the following:

- 672 • Under the nonreacting condition, local residence time has been quantified for both  
673 nozzles. Results show that it increases at locations farther away from the nozzle both in  
674 axial and radial directions. This observation is consistent with the location at which  
675 main ignition is observed to take place. Both WM and UFPV models predict main  
676 ignition as taking place near the spray periphery. At this location in the spray two  
677 observations should be emphasized. On the one hand, a “conserved gas particle” has  
678 already diluted as it follows its mixing trajectory. On the other hand, local residence  
679 time is high enough as to let chemistry progress.
- 680 • The reduction of the nozzle diameter promotes faster mixing. Under the nonreacting  
681 condition the time spent at a given  $\phi$  is shorter in Spray A compared to Spray D. Taking  
682 into account the description of the spray as a set of trajectories where mixture fraction  
683 is progressively diluting, this means that the faster mixing for Spray A enables reaching

684 ignitable equivalence ratio values earlier. Therefore, the shorter ID time for the smaller  
685 nozzle is consistent with shorter residence time.

- 686 • For the reacting Spray A condition both WM and UFPV models predict similar global  
687 steps leading to main ignition. The main difference is related to the spatial width of the  
688 area that characterizes this event. WM results show main ignition occurring at a narrow  
689 range of mixtures centered around  $\phi_{MR}$ . On the contrary, UFPV results show how main  
690 ignition takes place on a broader range of mixtures.
- 691 • At quasi-steady state the predicted flame structure for Spray A is remarkably different  
692 among the two combustion models. The scalar dissipation rate (only accounted for in  
693 the UFPV model) seems to shift further downstream the low-temperature heat release  
694 zone compared to WM results. The downstream shift is consistent with high  $\chi_{ST}$  near  
695 the nozzle. On the high-temperature heat release zone,  $\chi_{ST}$  is observed to play an  
696 opposite role contributing to the stabilization of the LOL closer to the nozzle compared  
697 to WM results. The comparison of UFPV and the intermediate UFPV-0 model,  
698 considering the flamelet sub-grid structure but not the presumed-PDF integration,  
699 evidences that one of the reasons for the wider spatial location of reaction zones is the  
700 averaging of laminar flamelets, smoothing the gradients within the reacting zones of the  
701 spray.
- 702 • UFPV results for Spray A and Spray D show a similar ignition sequence for both  
703 nozzles. Faster mixing and higher  $\chi_{ST}$  values for Spray A cause main ignition to occur  
704 closer to the spray head compared so Spray D where main ignition occurs closer to the  
705 spray radial periphery.
- 706 • In spatial coordinates, both Spray A and Spray D share a similar flame structure at quasi-  
707 steady state. In  $\phi - T$  space Spray D is characterized by richer mixtures being able to  
708 ignite. A slower mixing process, thus longer residence time, allow richer mixtures to  
709 ignite in the larger nozzle close to the axis, with lower scalar dissipation rate also  
710 contributing to this ignition capability.

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## 717 **Appendix A. Mixing trajectories**

718 In problems of momentum transfer, fluid flow is usually analyzed in terms of streamlines, i.e.  
719 lines describing the convective flow movement. In cases where species are also transported by  
720 means of convection and diffusion, both terms have to be accounted when tracking species.  
721 Assuming azimuthal symmetry, generally occurring in single spray cases such as the one under  
722 investigation, so-called “mixing trajectories” can be obtained by integration of the equation:

$$\frac{dx}{u + u_{dif}} = \frac{dr}{v + v_{dif}} \quad (11)$$

723 In Equation (11),  $u$  and  $v$  denote the convective components of the velocity field, while  $u_{dif}$   
724 and  $v_{dif}$  allow for the consideration of the transport flow induced by the diffusion of mixture  
725 fraction. Following a RANS approach for a spray at high Reynolds number, turbulent diffusion  
726 is assumed to be much more important than the laminar one. Therefore, diffusive components<sup>45</sup>  
727 are defined in an analogue way to a Fick’s diffusion law (where the diffusion flux can be written  
728 as  $gf = \rho Z v_{dif}$ ), according to:

$$u_{dif} = -\frac{D_t}{Z} \frac{\partial Z}{\partial x} \quad (12)$$

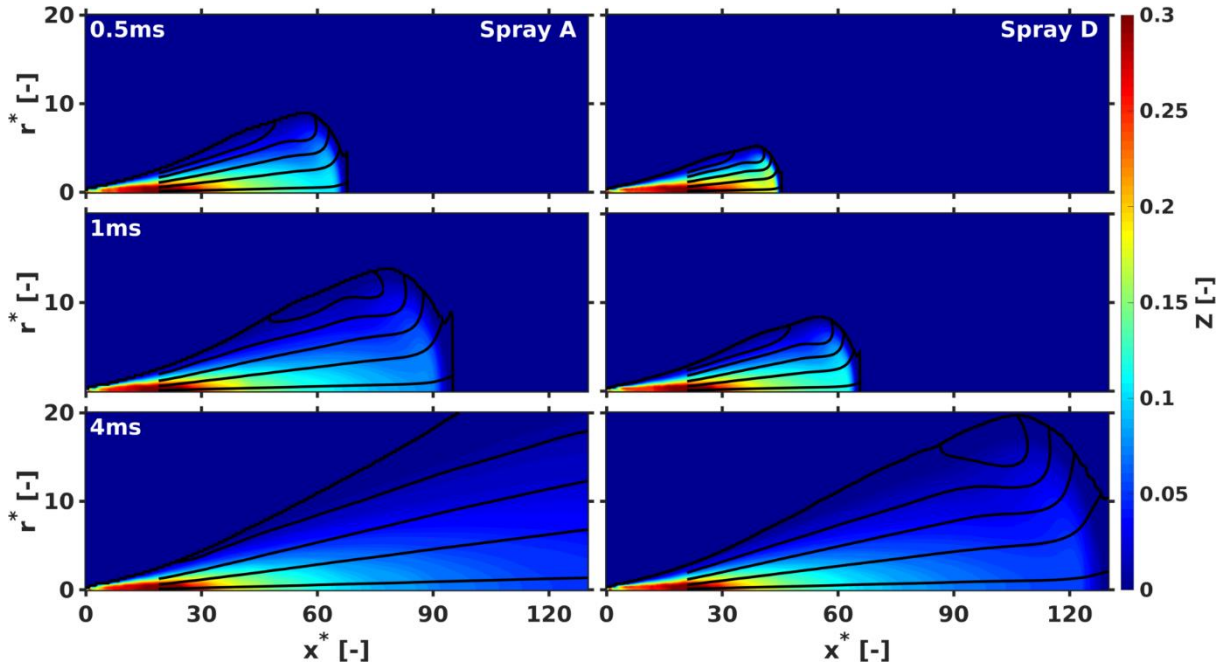
729

$$v_{dif} = -\frac{D_t}{Z} \frac{\partial Z}{\partial r} \quad (13)$$

730 where diffusivity has been assumed equal to the RANS turbulent one, calculated from turbulent  
731 viscosity via a unity Schmidt turbulent number, which has been imposed in the CFD  
732 calculations. Therefore, the diffusion coefficient in Equation (12) and Equation (13) is  
733 calculated as  $D_t = C_\mu \kappa^2 / \varepsilon$  with  $C_\mu = 0.09$ .

734 The mixing field, described by  $Z$ , is shown in Figure 22 for both Spray A and Spray D at several  
735 time instants after start of injection. The contour of the spray is delimited by the spray radius  
736 marked at the locations where  $Z$  is 1% of the value on the spray axis. Finally, mixing trajectories  
737 are also plotted on the contour plots in Figure 22. These are calculated downstream of the liquid  
738 length to avoid any effect induced by Lagrangian parcels, as the mixing trajectories are mainly  
739 an Eulerian concept.

740 Time development of mixing trajectories agrees with the general evolution of the spray, where  
 741 a transient zone progresses at the tip of the spray, behind which a quasi-steady flow is  
 742 established. In this sense, mixing trajectories are almost straight lines with a direction that  
 743 barely changes until reaching around 70-80% of the tip penetration. Transient structures can be  
 744 observed at the furthest radial locations around the tip of the spray.



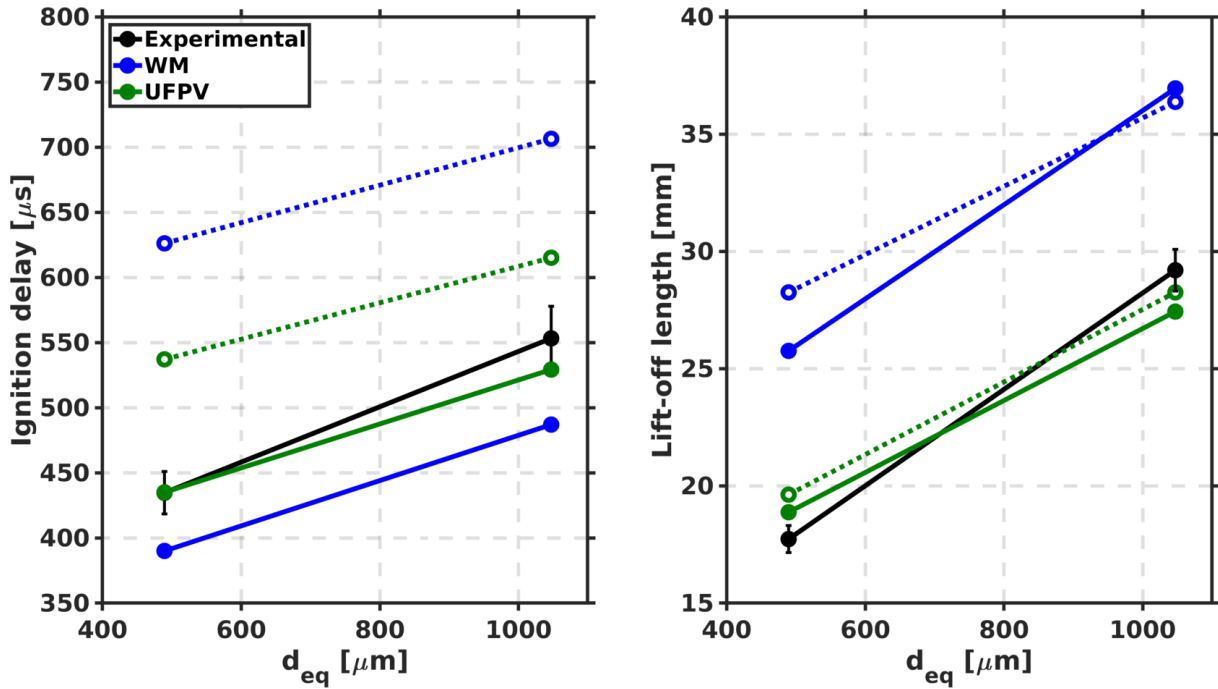
745  
 746 Figure 22. Mixture fraction field and mixing trajectories for Spray A (left) and Spray D (right).

747 It is also worth mentioning that by normalizing the axial and radial coordinates, at any given  
 748 point defined by  $(x^*, r^*)$  the same  $Z$  value for both nozzles is reached. This observation is  
 749 confirmed as trajectories for both nozzles start at  $x^* \cong 20 d_{eq}$  indicating that both nozzles have  
 750 a similar saturation  $Z$  value. This is also expected since saturation  $Z$  depends on the fuel, fuel  
 751 temperature and ambient temperature and pressure.

752 **Appendix B. Extended study of global combustion parameters**

753 As described in the “Computational setup” section the chemical mechanism by Yao et al.<sup>34</sup> has  
 754 been used in this work to describe the oxidation of n-dodecane. Figure 23 shows the predicted  
 755 values for ignition delay and lift-off length for the chemical mechanisms by Yao et al.<sup>34</sup> (as  
 756 presented in Figure 10) and Narayanaswamy et al.<sup>46</sup> This last chemical mechanism comprising  
 757 257 species and 1521 reactions is also used to describe the oxidation of n-dodecane. Despite  
 758 differences in the predicted values for ID and LOL, the trend between nozzles (i.e. longer ID  
 759 and LOL for Spray D compared to Spray A) remains the same regardless of the chemical  
 760 mechanism choice for both WM and UFPV models. This observation shows that the main

761 conclusions in this work concerning the effect of nozzle diameter on Diesel combustion hold  
 762 valid independently of the chemical mechanism used.

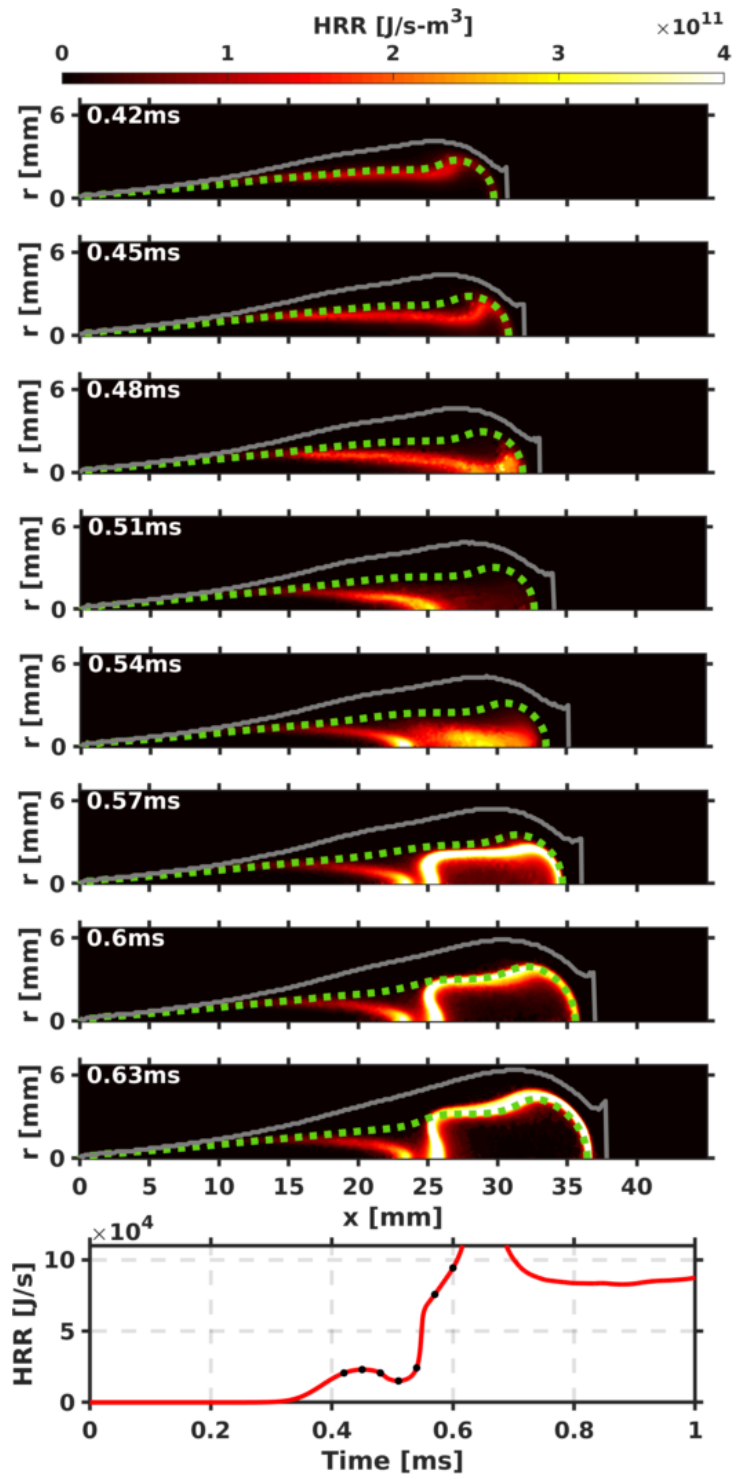


763

764 Figure 23. Ignition delay and lift-off length for Spray A and Spray D using the chemical mechanism by Yao et  
 765 al.<sup>34</sup> (solid line) and Narayanaswamy et al.<sup>46</sup> (dashed line).

766 Finally, Figure 24 and Figure 25 depict the Spray A auto-ignition sequence for the  
 767 Narayanaswamy et al.<sup>46</sup> chemical mechanism using the WM and the UFPV models,  
 768 respectively. The figures show the local HRR contour with the spray radius and the most  
 769 reactive equivalence ratio iso-contour ( $\phi_{MR} = 1.39$ ). Despite differences in the maximum HRR  
 770 level reached, both results share the same auto-ignition characteristics highlighted for the  
 771 chemical mechanism by Yao et al.<sup>34</sup> in Figure 11 for the WM model and in Figure 14 for the  
 772 UFPV model.

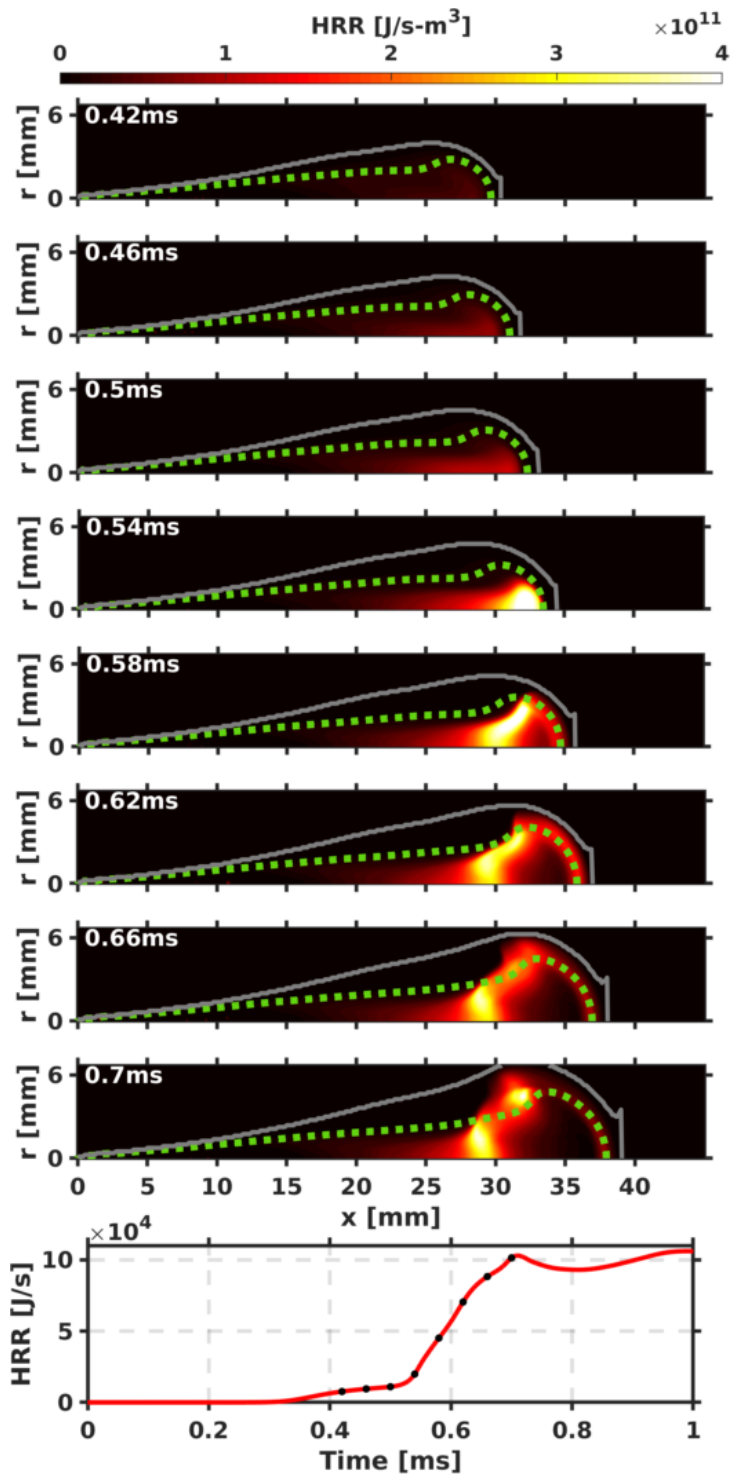




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Figure 24. Time and spatially resolved local HRR for Spray A using the WM model and the Narayanaswamy et al.<sup>46</sup> chemical mechanism. Dashed green line drawn at  $\phi_{MR}$ . Bottom plot shows the integrated HRR, where markers indicate the timings of the local HRR contours.



777

778 Figure 25. Time and spatially resolved local HRR for Spray A using the UFPV model and the Narayanaswamy  
 779 et al.<sup>46</sup> chemical mechanism. Dashed green line drawn at  $\phi_{MR}$ . Bottom plot shows the integrated HRR, where  
 780 markers indicate the timings of the local HRR contours.

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