



Parametric effects by using the strip-pair comparison method around red CIE color center

FERNANDO BRUSOLA,*  IGNACIO TORTAJADA,  ISMAEL LENGUA, BEGOÑA JORDÁ, AND GUILLERMO PERIS-FAJARNÉS

Centro de Investigación en Tecnologías Gráficas, Universitat Politècnica de València, Camino de Vera s/n 46022, Valencia, Spain

*fbrusola@upv.es

Abstract: The strip comparison method, based on the serial exploration method described by Torgerson [*Theory and Methods of Scaling*; Wiley & Sons (1958); Chap. 7], for the development of near-threshold color difference models was presented and validated with theoretical data by the authors in a previous work. In this study, we investigate parametric effects derived from the use of the strip comparison method on chromaticity-discrimination ellipses around the red CIE color center. The results obtained led to the conclusion that the strip comparison method has little effect on the parameters of the chromaticity-discrimination ellipses determined by the pair comparison method when pairs of patches in the strips are separated by a black line 0.5 mm thick or are separated by 3 mm spacing on a white background and also correlates well with the parameters reported by other authors using the pair comparison method at the threshold.

© 2020 Optical Society of America under the terms of the [OSA Open Access Publishing Agreement](#)

1. Introduction

Since the CIE committee 1.3 (colorimetry) established guidelines for the scientific community for research in the field of color-difference assessment in a coordinated manner (Robertson [1]), many studies in the field of color-difference models have been conducted [2–12] and added to previous studies developed by MacAdam, Wyszecki, and others [13–16]. More recently, after the publishing of a second set of CIE guidelines for coordinated future work on industrial color-difference evaluation (Maier [17]), Melgosa [18], chairman of the CIE committee 2.3, made a third call in 2007 to collect new datasets to complement those used to develop the CIEDE2000 [19,20] color-difference formula in order to develop new uniform color spaces to which color-difference formulas could be associated to improve their applicability in industrial environments. Since CIEDE2000, new work has been published on near-threshold color-difference assessment [21–24], and, also, to study parametric effects on color differences [25–29].

The collection of new datasets is essential for the improvement of existing color-difference models. However, if larger datasets are not currently available, it is due to the difficulty of obtaining them. The psychometric tests needed to evaluate the sensation of color difference usually require significant concentration from the observers, which often extends the duration of the tests, complicating the recruitment of volunteers. It is, therefore, interesting to develop new methods that will simplify and shorten the decision-making process during psychometric tests.

Most of the psychometric tests used, to date, to assess the sensation of color difference are based on two methods: Gray Scale (GSM) and Pair Comparison (PCM).

This study proposes the use of the Strip Comparison Method (SCM), based on the serial exploration method described by Torgerson [30], the description and validation with theoretical data of which was presented in a previous paper [31]. The method is based on printed strips containing pairs of patches arranged in rows or columns, such that the patches on one row or column of the strip are printed as close to color constant as possible, while the patches of the other row or column are printed so that, along the strip, the variation of the color difference ΔE_{ab}^* between pairs increases in approximately constant steps from a zero color difference, at one end

of the strip, to a selected maximum color difference at the other end, or as close to this as possible. Additionally, the variations of ΔL^* , Δa^* , and Δb^* on each strip should conform to the pattern of directions, in the CIELAB color space, indicated in Fig. 1. As described by Brusola et al. [31], observers are asked to indicate the number of the pair of patches in each strip from which a slight color difference is observed. Collected frequency data and measured color differences of the strip patches are then processed to obtain the parameters of the chromaticity-discrimination ellipses.

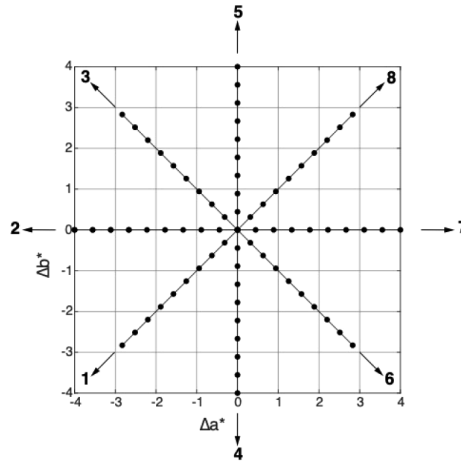


Fig. 1. Theoretical difference distribution pattern in the a^*-b^* plane for strips with 10 pairs of patches and $\Delta E_{ab}^* = 4$ maximum difference in last patch. Identification of direction vectors is in bold.

This study aims to determine the parametric effects due to the use of SCM with respect to the results that would have been obtained using PCM around the CIE red ($L^*=44$, $a^*=37$, $b^*=23$) color center.

In this paper we do not intend to validate the SCM, because it was already validated in [31] with perfect theoretical data, in the sense that when generating the frequency data simulating the observers' perception, as if they were really responding to an assumed underlying color-difference model (CIE94, in the case of the article mentioned), SCM can predict the assumed model with the necessary precision, even with the presence of certain noise level.

We have chosen red CIE color to determine the parametric effects by using SCM in this work, because, according to the results published by a good number of authors, is one of many that presents greatest discrepancies in terms of the color-discrimination ellipsoid coefficients, as shown in [29]. Obviously, other studies should be carried out, to confirm if it is possible to generalize the results of this work to other color centers.

It is necessary to take into account the laboriousness of the analysis carried out, where more than 100 observations by 8 strips and by 3 separations in the case of SCM and 100 observations by 80 pairs of patches and by 3 separations in the case of PCM have had to be made, which means more than 26,400 observations, regardless of the time needed to prepare the color samples.

2. Methods

2.1. Generation of color samples

Control strips were printed with an Epson Stylus 7000 inkjet printer on 250 g/m² Epson Premium Semigloss photo paper. Printing was performed by Adobe Photoshop from a *tif* file, with 16-bit

color depth per channel, generated directly with MATLAB routines programmed by the research team, following a procedure similar to that described by Brusola et al. [32].

Three sets of eight pairs of strips were printed, each consisting of ten pairs of patches whose color difference ΔE_{ab}^* increased from one pair to the next in approximately equal steps, up to a maximum difference of $\Delta E_{ab}^*=4$. In each of the sets, each of the eight strips should correspond to the eight theoretical directions in the chromatic plane a^*-b^* shown in Fig. 1.

Three sets of strips were generated, each corresponding to a different separation between pairs of patches on each strip. A pair of patches was printed with no separation in the first set, with a black line 0.5 mm thick separation in the second set, and with a spacing of 3 mm on a white paper background in the third set, using the same spacing pattern used by Brusola et al. [29]. Figure 2 shows the described separation pattern for vector directions #1 and #6 near the red CIE color center.

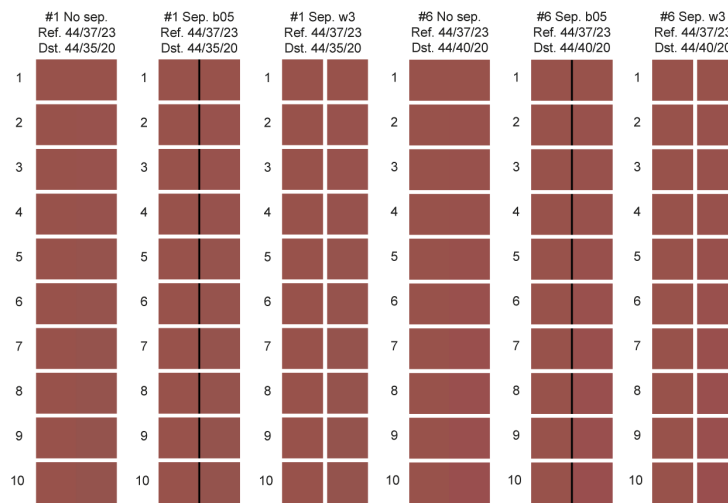


Fig. 2. Proposed strips for red CIE ($L^*=44$, $a^*=37$, $b^*=23$) color center along vector direction #1 ($\Delta L^*=0$, $\Delta a^*=-2.8$, $\Delta b^*=-2.8$) and vector direction #6 ($\Delta L^*=0$, $\Delta a^*=2.8$, $\Delta b^*=-2.8$) for three types of separation between pairs of patches: no separation (*no sep.*), black line 0.5 mm thick (*b05*) and separated 3 mm under white background (*w3*). Note: the authors cannot guarantee that the colorimetric values observed in the figure correspond to those used in the experiment due to the impossibility of controlling the color management workflows involved when readers are observing the figure displayed or printed in their own devices.

Once the strips were printed, a measurement was taken after approximately 10 minutes of drying. The measurement was made with an Xrite i1-iO automatic spectrophotometer reading system.

The effective plotter repeatability was estimated by using the parameter mean color difference from the mean (MCDM) of a set of measurements as proposed by Hunt and Pointer [33], which was obtained from the measurement of the reference patches in every strip. For all strip sets, the MCDM parameter obtained was less than $0.2 \Delta E_{ab}^*$ units. With the same set of reference patches, the following values of standard deviation from CIELAB coordinates were obtained: $\sigma_L=0.11$, $\sigma_a=0.20$, and $\sigma_b=0.30$. However, by calculating the standard deviation of Δa^* , Δb^* , and ΔL^* of color differences in the first pair of patches of all strips printed (which theoretically should be equal) the following values were obtained: $\sigma_L=0.10$, $\sigma_a=0.13$, and $\sigma_b=0.14$, which indicates that the printer is slightly more precise (precision) than exact (accuracy). The values obtained

allow us to foresee a slight deviation in the printing directions on each strip from the desired values and a good grouping of the points along the strips, as confirmed in the results section.

2.2. Visual evaluation

Each set of color strips was evaluated 10 times by 15 observers with normal color vision, in accordance with the Farnsworth-Munsell 100-tone test. The observers were asked to indicate the number of the pair of patches on every strip from which they first began to perceive a barely noticeable color difference (*jnd*, just noticeable color-difference pair). Thus, according to the above procedure, just one frequency record per observer and per strip was obtained in each evaluation.

The strips were observed in a Verivide viewing cabinet, equipped with a MASTER TL-D 90 Graphica 18W/965 lamp, with a correlated color temperature of 6500 K and a color rendering index $Ra = 98$. The display background corresponded to a neutral gray with CIELAB coordinates $L^* = 62$, $a^* = 0$, $b^* = 0$. These same conditions were reported by Brusola et al. [29] when performing the evaluation by PCM.

2.3. Statistical model and validation of results

The statistical model for processing the data is the same as that described by Brusola et al. [31]. The results obtained by SCM were compared with the results by PCM, after cutting the strips used into corresponding independent pairs. The PCM results were published in a previous work by Brusola et al. [29] to evaluate the parametric effect of sample separation on chromaticity-discrimination ellipses around the red CIE color center.

3. Results

Figures 3–5 show the detail of the adjustment of the chromaticity-discrimination ellipses by SCM for pairs of patches without separation (Fig. 3), for pair of patches separated with a black line 0.5 mm thick (Fig. 4), and for pairs of patches with a spacing of 3 mm on a white background (Fig. 5).

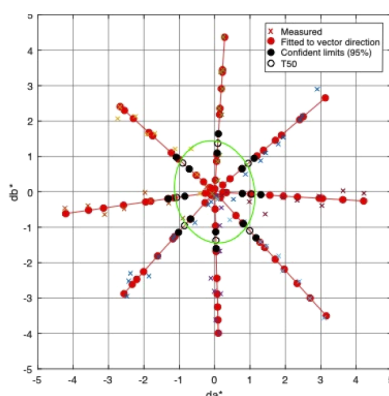


Fig. 3. Chromaticity-discrimination ellipse obtained by SCM for pairs of patches without separation.

Tables 1–3 together present the results of applying PCM with the proposed SCM, where both followed a procedure similar to the one proposed by Alman, Berns, et al. [5,7] to obtain the T50 tolerances for the theoretical direction scheme indicated in Fig. 1. In the first column of the aforementioned tables, the identifier of the theoretical vector direction is indicated according

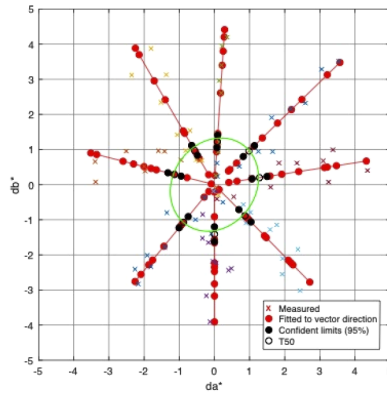


Fig. 4. Chromaticity-discrimination ellipse resulting from SCM, pairs of patches separated by a black line 0.5 mm thick.

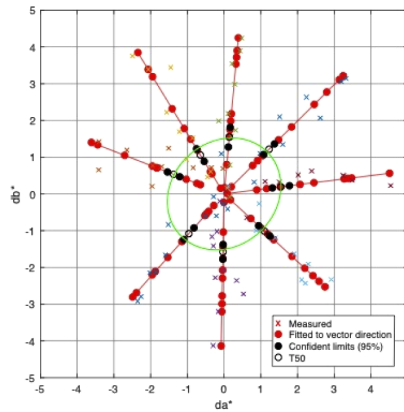


Fig. 5. Chromaticity-discrimination ellipse resulting from SCM, pairs of patches separated by 3 mm on white background

to the identifier shown in Fig. 1. The second and third columns of Tables 1–3 correspond with the unitary vector directions (da^* , db^*) obtained after linear regression passing through the origin from the actual measured differences Δa^* , Δb^* in each strip. The fourth column, R^2 , corresponds to the determination coefficient, which provides information about the percentage of the total variance explained by the indicated adjustment. The next column, $T50$, corresponds to the median of the probability distribution adjusted to the frequencies observed in each direction, which represents the threshold of perception of color differences in each direction.

The $T50$ tolerance was obtained by adjusting the frequency data to a normal cumulative probability distribution for PCM and to a normal probability density function for SCM. The LCL and UCL columns correspond to the lower and upper limits of the confidence interval of $T50$, respectively. The S column corresponds to the standard deviation associated with the corresponding psychometric curve. The last column, Standardized Residual Sum of Squares ($STRESS$) index, corresponds to the quality indicator of the adjustment obtained in each direction proposed by García et al. [34]. For the calculation of $STRESS$, the visual differences have been determined from the frequencies observed for each pair by the PCM. For SCM, the visual

Table 1. Results obtained by PCM and SCM for pairs of samples with no separation^a

V	da*	db*	R2	Method	T50	LCL	UCL	S	STRESS
1	-0.665	-0.747	98.9	PCM	1.140	1.032	1.248	0.604	14
				SCM	1.279	1.023	1.536	0.446	8
2	-0.989	-0.145	97.0	PCM	0.877	0.780	0.974	0.557	12
				SCM	1.087	0.851	1.323	0.455	21
3	-0.743	0.670	97.3	PCM	0.623	0.545	0.700	0.412	13
				SCM	1.215	0.966	1.464	0.509	7
4	0.026	-1.000	99.4	PCM	1.061	0.912	1.211	0.727	17
				SCM	1.368	1.133	1.602	0.593	10
5	0.064	0.998	99.9	PCM	0.907	0.801	1.014	0.513	14
				SCM	1.373	1.101	1.644	0.755	18
6	0.669	-0.743	99.5	PCM	1.119	0.992	1.247	0.604	14
				SCM	1.480	1.213	1.746	0.511	4
7	0.998	-0.061	97.2	PCM	0.716	0.606	0.826	0.422	16
				SCM	1.047	0.7841	1.309	0.571	15
8	0.763	0.647	97.7	PCM	1.209	1.159	1.259	0.526	6
				SCM	1.242	1.017	1.467	0.447	4

^aV: identification of vector direction according to scheme shown in Fig. 1; da* and db*: unitary vectors of the vector directions after fitting measured color difference (Δa^* , Δb^*) points on every strip to the line that passes through the origin; T50: median of the probability distribution (psychometric curve) fitted to the observed frequencies in every vector direction; S: standard deviation of the psychometric curve; STRESS: Standardized Residual Sum of Squares index proposed by García et al. [34].

Table 2. Results obtained by PCM and SCM for pairs of samples separated by a black line 0.5 mm thick^a

V	da*	db*	R2	Method	T50	LCL	UCL	S	STRESS
1	-0.633	-0.774	92.9	PCM	0.831	0.700	0.993	0.891	15
				SCM	1.384	1.177	1.591	0.744	14
2	-0.969	0.248	87.1	PCM	0.798	0.641	0.955	0.620	15
				SCM	1.172	0.978	1.365	0.539	19
3	-0.501	0.866	96.8	PCM	0.783	0.752	0.814	0.457	4
				SCM	1.128	0.965	1.291	0.533	13
4	0.001	-1.000	93.4	PCM	1.051	0.951	1.151	0.745	9
				SCM	1.414	1.202	1.625	0.628	14
5	0.066	0.998	99.9	PCM	1.000	0.940	1.054	0.192	24
				SCM	1.232	1.054	1.410	0.750	23
6	0.699	-0.715	93.4	PCM	0.733	0.681	0.784	0.624	9
				SCM	1.242	0.996	1.488	0.561	27
7	0.988	0.154	94.2	PCM	0.761	0.564	0.957	0.705	18
				SCM	1.297	1.082	1.513	0.690	3
8	0.716	0.698	97.0	PCM	1.071	0.926	1.217	0.699	16
				SCM	1.370	1.148	1.593	0.848	12

^aV: identification of vector direction according to scheme shown in Fig. 1; da* and db*: unitary vectors of the vector directions after fitting measured color difference (Δa^* , Δb^*) points on every strip to the line that passes through the origin; T50: median of the probability distribution (psychometric curve) fitted to the observed frequencies in every vector direction; S: standard deviation of the psychometric curve; STRESS: Standardized Residual Sum of Squares index by García et al. [34].

Table 3. Results obtained by PCM and SCM for pairs of samples separated by 3 mm (white background)^a

V	da*	db*	R2	Method	T50	LCL	UCL	S	STRESS
1	-0.664	-0.748	96.5	PCM	0.886	0.441	1.330	1.641	22
				SCM	1.446	1.234	1.657	0.936	13
2	-0.932	0.362	87.1	PCM	1.105	0.884	1.325	0.780	16
				SCM	1.474	1.294	1.654	0.560	17
3	-0.521	0.854	97.3	PCM	0.850	0.658	1.041	1.012	16
				SCM	1.238	1.038	1.438	0.713	19
4	-0.019	-1.000	93.9	PCM	1.204	1.087	1.320	1.086	8
				SCM	1.577	1.376	1.777	0.568	9
5	0.091	0.996	98.9	PCM	1.137	0.996	1.279	1.206	11
				SCM	1.552	1.282	1.823	0.810	22
6	0.736	-0.677	96.2	PCM	0.951	0.762	1.140	1.036	14
				SCM	1.488	1.279	1.697	0.655	11
7	0.992	0.124	95.4	PCM	1.027	0.865	1.189	1.224	12
				SCM	1.560	1.325	1.794	0.812	4
8	0.710	0.704	94.9	PCM	1.321	1.090	1.552	1.127	16
				SCM	1.728	1.522	1.933	0.913	20

^aV: identification of vector direction according to scheme shown in Fig. 1; da* and db*: unitary vectors of the vector directions after fitting measured color difference (Δa^* , Δb^*) points on every strip to the line that passes through the origin; T50: median of the probability distribution (psychometric curve) fitted to the observed frequencies in every vector direction; S: standard deviation of the psychometric curve; STRESS: Standardized Residual Sum of Squares index by García et al. [34].

differences have been determined from the accumulated frequencies calculated from the observed ones, under the assumption that an observer should perceive, in each observation, a color difference for those patches, from the one selected as *jnd* in each strip, whose ΔE_{ab}^* measurement is higher than that of the *jnd* pair.

Table 4 shows a summary of the results obtained by the two methods with respect to STRESS, T50, and S. As shown in Fig. 6(a), the average of the STRESS values obtained by the strip method is similar to that obtained by the pair method, around 14, these values having been obtained by averaging the STRESS values in each of the directions and in the three cases of separation. This indicates a very similar degree of adjustment by both methods.

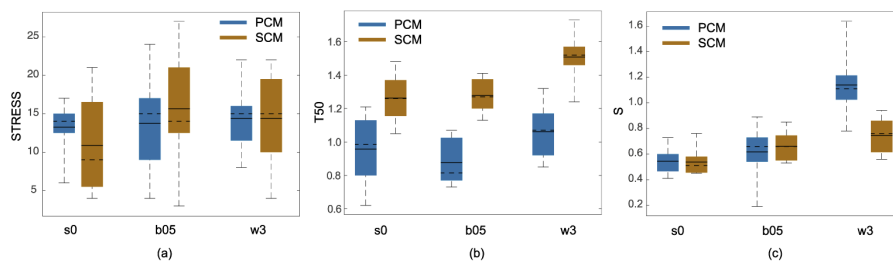


Fig. 6. Box and Whisker plot of the results shown in Table 4; continuous horizontal lines correspond to the mean values (\bar{m} , shown in Table 4) and dashed horizontal lines correspond to the median values.

Table 4. Comparison of STRESS, T50, and S values between PCM and SCM^a

V	STRESS						T50						S					
	s ₀	s ₀	b ₀₅	b ₀₅	w ₃	w ₃	s ₀	s ₀	b ₀₅	b ₀₅	w ₃	w ₃	s ₀	s ₀	b ₀₅	b ₀₅	w ₃	w ₃
	PC	SC	PC	SC	PC	SC	PC	SC	PC	SC	PC	SC	PC	SC	PC	SC	PC	SC
1	14	8	15	14	22	13	1.14	1.28	0.83	1.38	0.89	1.45	0.60	0.45	0.89	0.74	1.64	0.94
2	12	21	15	19	16	17	0.88	1.09	0.80	1.17	1.11	1.47	0.56	0.46	0.62	0.54	0.78	0.56
3	13	7	4	13	16	19	0.62	1.22	0.78	1.13	0.85	1.24	0.41	0.51	0.46	0.53	1.01	0.71
4	17	10	9	14	8	9	1.06	1.37	1.05	1.41	1.20	1.58	0.73	0.59	0.75	0.63	1.09	0.57
5	14	18	24	23	11	22	0.91	1.37	1.00	1.23	1.14	1.55	0.51	0.76	0.19	0.75	1.21	0.81
6	14	4	9	27	14	11	1.12	1.48	0.73	1.24	0.95	1.49	0.60	0.51	0.62	0.56	1.04	0.66
7	16	15	18	3	12	4	0.72	1.05	0.76	1.30	1.03	1.56	0.42	0.57	0.71	0.69	1.22	0.81
8	6	4	16	12	16	20	1.21	1.24	1.07	1.37	1.32	1.73	0.53	0.45	0.70	0.85	1.13	0.91
\bar{m}	13	11	14	16	14	14	0.96	1.26	0.88	1.28	1.06	1.51	0.55	0.54	0.62	0.66	1.14	0.75

^aV: identification of vector direction according to scheme shown in Fig. 1; PC: PCM; SC: SCM; s₀: samples with no separation; b₀₅: samples separated by a black line 0.5 mm thick; w₃: samples separated 3 mm on white background; \bar{m} : mean value; T50: median of the probability distribution (psychometric curve) fitted to the observed frequencies in every vector direction; S: standard deviation of the psychometric curve; STRESS: Standardized Residual Sum of Squares index by García et al. [34].

However, the ratio of the average of the T50 values obtained by PCM to the average of the T50 values obtained by SCM becomes 1.4 times higher. This shows a clear tendency (parametric effect) of the strip method to increase the size of the chromaticity-discrimination ellipses with respect to those obtained by the pair method in this experiment, as can be observed too in Fig. 6(b).

Conversely, the relationship between the average S values, the standard deviation of the psychometric curve, associated to some extent with the degree of confusion of the observers to decide whether they perceive the color difference or not, by SCM with respect to PCM becomes 0.84. As can be seen in Fig. 6(c) the reduction of S is only evident for samples separated 3 mm on white background. This indicates to some extent that the degree of confusion in responding to the corresponding psychometric test by the observers is lower by SCM than by PCM for the aforementioned case.

During the recording of experimental data, an approximate reduction of 50% of the assessment time was observed for the SCM with respect to the PCM, which could be explained by the fact that the pairs of patches near and below the threshold are the most time consuming, but just as by SCM the observer only has to stop and make an assessment for the set of patches near the threshold, that can take double time than the evaluation of one single pair of patches, by PCS the observer has to repeat the same laborious assessment for each of the pairs near and below the threshold, which in average can represent 25% of the pairs of patches in one strip.

Table 5 shows the parameters of the chromaticity-discrimination ellipses obtained for the different cases considered in this article. For both PCM and SCM, two fitting techniques were used, described in Table 5 as Bayesian or T50. The Bayesian method corresponds to the method described by Brusola et al. for PCM [35] and that described for SCM [36]. The fitting technique, T50, is based on the calculation procedure used by Melgosa et al. [10] from the T50 tolerance values obtained in Tables 1–4 for both PCM and SCM. Resultant chromaticity discrimination ellipses are plotted in Fig. 7. As Table 5 and Fig. 7 show, the parameters of the ellipses obtained by the Bayesian fitting technique and by T50 are very similar when used for either PCM or SCM. However, the parametric effect, as mentioned in the previous paragraph, is confirmed in the sense that the size factor (K_G) of the chromaticity-discrimination ellipses determined by SCM are larger than those determined by the pair method. However, the size factor reported by Huang et al. [22],

$K_G = 2.4$ approximately, for printed samples, and Xu et al. [21], $K_G = 3.36$ approximately, for samples displayed on a computer screen, are much more like those obtained by SCM for near red CIE color centers at the threshold.

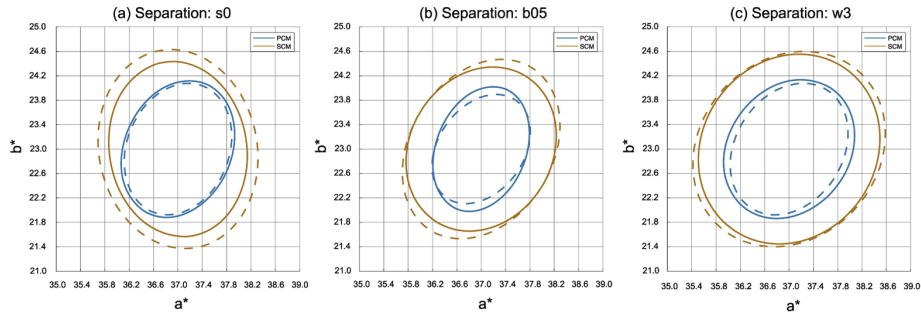


Fig. 7. Plot of the Chromaticity-discrimination ellipses shown in Table 5. Continuous lines correspond to chromaticity discrimination ellipses obtained by T50 method and dashed lines correspond to chromaticity discrimination ellipses obtained by Bayesian methods.

Table 5. Chromaticity ellipse parameters in $\Delta a^* - \Delta b^*$ plane^a

Case	Method	Fitting	g_{11}	g_{22}	g_{12}	A	B	θ	Tilt	KG	S	2σ
s0	PCM	Bayesian	1.34	0.91	-0.22	1.11	0.84	68°	36°	1.71	0.71	8°
		T50	1.19	0.84	-0.20	1.16	0.88	66°	34°	1.79	-	-
	SCM	Bayesian	0.59	0.38	0.04	1.64	1.30	101°	69°	2.59	0.37	9°
		T50	0.78	0.49	0.05	1.44	1.13	99°	67°	2.26	-	-
b05	PCM	Bayesian	1.7	1.38	-0.46	0.98	0.70	54°	22°	1.47	0.92	6°
		T50	1.69	1.02	-0.33	1.06	0.74	68°	36°	1.57	-	-
	SCM	Bayesian	0.63	0.5	-0.14	1.57	1.17	58°	26°	2.40	0.61	9°
		T50	0.67	0.56	-0.10	1.4	1.17	59°	27°	2.27	-	-
w3	PCM	Bayesian	1.14	0.92	-0.25	1.15	0.88	57°	25°	1.78	1.36	8°
		T50	0.90	0.81	-0.17	1.21	0.99	53°	21°	1.94	-	-
	SCM	Bayesian	0.41	0.40	-0.06	1.71	1.46	47°	15°	2.80	0.60	12°
		T50	0.45	0.42	-0.05	1.61	1.43	55°	23°	2.69	-	-

^a g_{11} , g_{22} , and g_{12} : metric coefficients of the ellipse; A and B : major and minor semi-axes, respectively; θ : angle of the major axis with respect to $+a^*$; $Tilt (\Delta\theta)$: angular difference between the hue angle (h_{ab}^*) of the color center and the angle of the major axis of the discrimination ellipses with respect to $+a^*$, in this order; K_G : size factor = $\sqrt{\pi AB}$; S : standard deviation of the psychometric curve; 2σ : twice the standard deviation of the posterior distribution of θ (equivalent to a credibility interval of 95% probability); s_0 : samples with no separation; b_05 : samples separated by a black line 0.5 mm thick; w_3 : samples separated 3 mm on white background.

Conversely, a certain degree of regularity of the tilt of the ellipses, $\Delta\theta$, between PCM and SCM is obtained for pairs of patches separated by a 0.5 mm black line or by a 3 mm white space, as can be seen in Figs. 7(b) and 7(c). For these cases, tilt does not differ by more than 10°, in accordance with the order of magnitude of the 95% credibility intervals (shown in column 2σ of Table 5, obtained only by Bayesian techniques). A large discrepancy of the tilt is observed between the PCM and SCM values for the case of pairs of patches with no separation, as can be seen in Fig. 7(a), where the difference can be greater than 30°.

With respect to the values of S , the standard deviation of the psychometric curve, the tendency observed in Tables 1–4 for each direction again confirms that the overall values of S obtained

by SCM are lower than those obtained by PCM, thus indicating a lower degree of confusion by SCM than PCM.

Table 6 shows the tolerance values that would minimize the classification error using different color-difference formulas, for the datasets obtained by SCM, according to the method described by Berns [37]. Classification errors are of the same order of magnitude, or less, than those reported in Table 9 by Brusola et al. [29] for PCM datasets, with an average classification error of approximately 6%.

Table 6. Optimization tolerance that minimizes number of wrong classification decisions with respect to color-difference formula, according to Berns [37] for SCM.

Formula	Dataset	SCM	
		Tol.	WD (%)
CIELAB	<i>No separation</i>	1.2	2.5
	<i>3 mm thick white line</i>	1.5	3.3
	<i>0.5 mm thick black line</i>	1.3	5.2
CIE94	<i>No separation</i>	0.6	5.7
	<i>3 mm thick white line</i>	0.8	9.4
	<i>0.5 mm thick black line</i>	0.6	7.2
CMC	<i>No separation</i>	0.7	5.8
	<i>3 mm thick white line</i>	1.0	9.5
	<i>0.5 mm thick black line</i>	0.8	7.3
CIEDE2000	<i>No separation</i>	0.6	6.3
	<i>3 mm thick white line</i>	0.8	10
	<i>0.5 mm thick black line</i>	0.7	7.4
CAM02-SCD	<i>No separation</i>	0.6	3.1
	<i>3 mm thick white line</i>	0.7	4.4
	<i>0.5 mm thick black line</i>	0.6	4.1
CAM02-UCS	<i>No separation</i>	0.7	2.9
	<i>3 mm thick white line</i>	0.9	3.4
	<i>0.5 mm thick black line</i>	0.8	3.4
	<i>Mean:</i>	0.8	5.6

As indicated by Brusola et al. [29] for the PCM datasets, the SCM datasets show that the CAM02-SCD and CAM02-UCS formulas produce the best results, thus generating a lower error rate.

There is a high degree of consistency in terms of the percentage of wrong classification decisions between the weighted color difference formulae (all except CIELAB) shown in Table 6, with the highest error in all formulae for samples separated 3 mm on white background, intermediate error for samples separated with a black line 0,5 mm thick and the lowest percentage of wrong classification decisions for samples with no separation. The discrimination tolerance is also quite consistent, corresponding the highest value for samples separated 3 mm on white background with respect to all the color difference formulas and the lowest for samples no separation. The tolerance for samples separated with a black line 0,5 mm thick is maintained in an intermediate position and is equal to the tolerance for samples without separation in only two cases: CIE94 and CAM02-SCD. In this regard, we believe that the degree of precision of the printing system used, around $\Delta E_{ab}^* = 0.2$, may be the cause of the slight fluctuations observed in Figs. 4(a), 4(b), and Table 6.

4. Conclusions

Three sets of strips were printed in the CIE red environment, each consisting of ten pairs of samples whose color difference ΔE_{ab}^* progressively increased, at approximately equal intervals, in eight different directions of the a^*-b^* color plane, from an almost zero color difference in the first pair of patches to an approximately four unit difference in the last pair of patches on each strip. Each set of strips generated corresponds to three different types of separation between the patches of the strips: no separation, with a black line of separation of 0.5 mm thick, and with a separation of 3 mm on a white background. The strips were evaluated 100 to 140 times by a panel of observers using the SCM described by Brusola et al. [31]. Once the frequency data had been collected by the strip method, the strips were cut into individual pairs and three new assessments were performed for each set by PCM, by the same panel of observers that performed the strip assessment. The results obtained enable us to conclude that the strip method has little effect on the tilt of the chromaticity-discrimination ellipses, determined by the pair method, for sets with a black line spacing of 0.5 mm or with a spacing of 3 mm on a white background. In general, the chromaticity-discrimination ellipses determined by SCM in this experiment are larger than those determined by PCM. However, the sizes obtained by SCM are more consistent with the sizes of the chromaticity-discrimination ellipses by PCM at the threshold, as reported by various authors (Huang et al. [22], Xu et al. [21]).

The results determined by SCM agree with those by PCM in terms of the behavior of the color-difference formulas evaluated, with an average classification error of 6%; the formulas of CAM02-SCD and CAM02-UCS being those that provide a smaller classification error.

In summary, the results enable us to conclude that the strip method hardly introduces parametric effects in the determination of the chromaticity-discrimination ellipses, especially when a separation line between the samples is introduced, and that SCM could be used in the future to increase the number of datasets needed to improve the color-difference formulas.

Disclosures

The authors declare no conflicts of interest

References

1. A. R. Robertson, "CIE guidelines for coordinated research on colour-difference evaluation," *Color Res. Appl.* **20**(3), 399–403 (1995).
2. R. M. Rich and F. W. Billmeyer, "Method for deriving color-difference-perceptibility ellipses for surface-color samples," *J. Opt. Soc. Am.* **65**(8), 956–959 (1975).
3. D. Strocka, A. Brockes, and W. Paffhausen, "Influence of experimental parameters on the evaluation of color-difference ellipsoids," *Color Res. Appl.* **8**(3), 169–175 (1983).
4. M. R. Luo and B. Rigg, "Chromaticity-discrimination ellipses for surface colours," *Color Res. Appl.* **11**(1), 25–42 (1986).
5. D. H. Alman, R. S. Berns, G. D. Snyder, and W. A. Larsen, "Performance testing of color-difference metrics using a color tolerance dataset," *Color Res. Appl.* **14**(3), 139–151 (1989).
6. K. Witt, "Parametric effects on surface color-difference evaluation at threshold," *Color Res. Appl.* **15**(4), 189–199 (1990).
7. R. S. Berns, D. H. Alman, L. Reniff, G. D. Snyder, and M. R. Balonon-Rosen, "Visual determination of suprathreshold color-difference tolerances using probit analysis," *Color Res. Appl.* **16**(5), 297–316 (1991).
8. T. Indow, A. R. Robertson, M. Grunau, and G. H. Fielder, "Discrimination ellipsoids of aperture and simulated surface colors by matching and paired comparison," *Color Res. Appl.* **17**(1), 6–23 (1992).
9. M. Melgosa, E. Hita, J. Romero, and L. J. Del Barco, "Color-discrimination thresholds translated from the CIE (x, y, Y) space to the CIE 1976 (L*, a*, b*)," *Color Res. Appl.* **19**(1), 10–18 (1994).
10. M. Melgosa, E. Hita, A. J. Poza, D. H. Alman, and R. S. Berns, "Suprathreshold color-difference ellipsoids for surface colors," *Color Res. Appl.* **22**(3), 148–155 (1997).
11. D. H. Kim and J. H. Nobbs, "New weighting functions for the weighted CIELAB colour difference formula," *Proc. Colour* **97**, 446–449 (1997).
12. K. Witt, "Geometric relations between scales of small colour differences," *Color Res. Appl.* **24**(2), 78–92 (1999).
13. D. L. MacAdam, "Visual sensitivities to color differences in daylight," *J. Opt. Soc. Am.* **32**(5), 247–274 (1942).

14. W. R. J. Brown and D. L. MacAdam, "Visual sensitivities to combined chromaticity and luminance differences," *J. Opt. Soc. Am.* **39**(10), 808–834 (1949).
15. W. R. Brown, W. G. Howe, J. E. Jackson, and R. M. Moris, "Multivariate normality of the color-matching process," *J. Opt. Soc. Am.* **46**(1), 46–49 (1956).
16. G. Wyszecki and G. H. Fielder, "Color difference matches," *J. Opt. Soc. Am.* **61**(11), 1501–1513 (1971).
17. T. Maier, "CIE guidelines for coordinated future work on industrial color-difference evaluation," *Color Res. Appl.* **20**(6), 399–403 (1995).
18. M. Melgosa, "Request for existing experimental datasets on color differences," *Color Res. Appl.* **32**(2), 159 (2007).
19. M. R. Luo, G. Cui, and B. Rigg, "The development of the CIE 2000 colour difference formula," *Color Res. Appl.* **26**(5), 340–350 (2001).
20. ISO/CIE 11664-6:2014, "Colorimetry – Part 6: CIEDE2000 Colour-difference formula."
21. H. Xu and H. Yaguchi, "Visual evaluation at scale of threshold to suprathreshold color difference," *Color Res. Appl.* **30**(3), 198–208 (2005).
22. M. Huang, H. Liu, G. Cui, M. R. Luo, and M. Melgosa, "Evaluation of threshold color differences using printed samples," *J. Opt. Soc. Am. A* **29**(6), 883–891 (2012).
23. S. Wen, "A color difference metric based on the chromaticity discrimination ellipses," *Opt. Express* **20**(24), 26441–26447 (2012).
24. M. Huang, H. Liu, G. Cui, and M. R. Luo, "Testing uniform color spaces and colour-difference formulae using printed samples," *Color Res. Appl.* **37**(5), 326–335 (2012).
25. CIE Publication 101, "Parametric effects in colour-difference evaluation," CIE Central Bureau, Vienna (1993).
26. S. Guan and M. R. Luo, "Investigation of parametric effects using small colour differences," *Color Res. Appl.* **24**(5), 331–343 (1999).
27. J. H. Xin, C. C. Lam, and M. R. Luo, "Investigation of parametric effects using medium colour-difference pairs," *Color Res. Appl.* **26**(5), 376–383 (2001).
28. G. Cui, M. R. Luo, B. Rigg, and W. Li, "Colour-difference evaluation using CRT colours. Part II: parametric effects," *Color Res. Appl.* **26**(5), 403–412 (2001).
29. F. Brusola, I. Tortajada, B. Jordá, and M. Melgosa, "Parametric effects on the evaluation of threshold chromaticity differences using red printed samples," *J. Opt. Soc. Am. A* **36**(4), 510–517 (2019).
30. W. S. Torgerson, *Theory and Methods of Scaling*, (Wiley & Sons, 1958), Chap. 7.
31. F. Brusola, I. Tortajada, I. Lengua, B. Jordá, and G. Peris, *Centro de Investigación en Tecnologías Gráficas, Universitat Politècnica de València, Camino de Vera s/n 46022 València, Spain*, are preparing a manuscript to be called "Strip-pair comparison method for developing threshold color-difference models: Theoretical model validation."
32. F. Brusola, I. Tortajada, B. Jordá, I. Lengua, and L. Dunai, "Método para la reproducción precisa de muestras de color impresas a partir de los datos de caracterización necesarios para la generación de perfiles de color ICC," *Proc. XI Congreso Nacional de Color*, Ourense, Spain, 109–112 (2016).
33. R. W. G. Hunt and M. R. Pointer, *Measuring color* (John Wiley & Sons, 2011), Ch. 9.
34. P. A. García, R. Huertas, M. Melgosa, and G. Cui, "Measurement of the relationship between perceived and computed color differences," *J. Opt. Soc. Am. A* **24**(7), 1823–1829 (2007).
35. F. Brusola, I. Tortajada, I. Lengua, B. Jordá, and G. Peris, "Bayesian approach to color-difference models based on threshold and constant-stimuli methods," *Opt. Express* **23**(12), 15290–15309 (2015).
36. F. Brusola, I. Tortajada, I. Lengua, B. Jordá, and G. Peris, *Centro de Investigación en Tecnologías Gráficas, Universitat Politècnica de València, Camino de Vera s/n 46022 València, Spain*, are preparing a manuscript to be called "Strip-pair comparison method for developing threshold color-difference models: Bayesian approach."
37. R. S. Berns, "Deriving Instrumental Tolerances from Pass-Fail and Colorimetric Data," *Color Res. Appl.* **21**(6), 459–472 (1996).