Invariant energy in short-term personality dynamics

Antonio Caselles ♮[1], Salvador Amigó ♯, and Joan C. Micó ♭,

(♮) IASCYS member, Departament de Matemàtica Aplicada,
Universitat de València, (retired),
(♯) Departament de Personalitat, Avaluació i Tractaments Psicològics,
Universitat de València,
(♭) Institut Universitari de Matemàtica Multidisciplinar,
Universitat Politècnica de València.

1 Introduction

This work presents a way to find a time-invariant energy associated to the short-term personality dynamics as a consequence of an arbitrary stimulus. First of all, the short-term personality dynamics is presented as a second order differential equation [1], derived from the original first order integro-differential equation [2] called as the response model. In the following, the referred second order formulation can be presented from the perspective of the classical mechanics, through the Lagrangian and the Hamiltonian functions [1]. However, although the Hamiltonian is a true energy (a sum of kinetic and potential energies), it depends explicitly on time. Consequently it is not an invariant energy [3]. Besides, both the second order differential equation and the Hamiltonian are mathematically similar (in Physics) to the mathematical approach to a particle with variable mass subjected to a retrieving force with a time-dependent retrieving constant. This problem is known in the specialized literature as the harmonic oscillator with time-dependent mass and frequency. Fortunately, this problem can be transformed into a formulation that provides the well-known Ermakov-Lewis invariant [4], which can be reinterpreted as an energy invariant [5]. Thus, starting from our second order differential equation that describes the short-time personality dynamics, and following one of the methods presented in [4], an Ermakov-Lewis invariant is found, which can be interpreted as a personality energy-invariant.

2 The response model and its Lagrangian-Hamiltonian approach

The response model [2] is given by the following integro-differential equation:

$$\left.\begin{array}{c}\frac{dq(t)}{dt} = a\cdot(b-q(t)) + \delta\cdot s(t)\cdot q(t) - \sigma\cdot\int_{t_0}^{t} e^{\frac{x-t}{\tau}}\cdot s(x)\cdot q(x)dx \\ q(t_0) = q_0\end{array}\right\} \tag{1}$$

[1] e-mail: antonio.caselles@uv.es

The function $s(t)$ represents an arbitrary stimulus and $q(t)$ the General Factor of Personality (GFP). For more details about the meaning of Eq. (1) as well as its parts and parameters see [2]. Taking the time derivative in Eq. (1) and making the subsequent substitutions, the second order differential equation, and its initial conditions arise:

$$\left.\begin{array}{c} \ddot{q}(t)+\gamma(t)\cdot\dot{q}(t)+v(t)\cdot q(t)=\frac{a\cdot b}{\tau} \\ q\left(t_0\right)=q_0 \\ \dot{q}\left(t_0\right)=a\cdot\left(b-q_0\right)+\delta\cdot s_0\cdot q_0 \end{array}\right\} \tag{2}$$

In Eq. (2):

$$v(t)=\left(\frac{a}{\tau}+\sigma\cdot s(t)-\frac{\delta}{\tau}\cdot s(t)-\delta\cdot\dot{s}(t)\right) \tag{3}$$

$$\gamma(t)=\frac{d}{dt}\left(\ln\left(u(t)\right)\right)=\frac{\dot{u}(t)}{u(t)} \tag{4}$$

$$u(t)=u_0 e^{\left(a+\frac{1}{\tau}\right)\left(t-t_0\right)-\delta\int_{t_0}^{t}s(x)dx} \tag{5}$$

Eq. (2) is an equivalent version of Eq. (1), and u_0 is an undetermined constant by the moment. In addition, s_0 is the stimulus' value in the initial time $t=t_0$. Note that this version is equivalent to that of a harmonic oscillator with time-dependent mass $u(t)$ and time-dependent frequency $v(t)$ subjected to a constant force $a\cdot\frac{b}{t}$. The difference with respect to the physical problem is that, here, the frequency $v(t)$ can take an arbitrary sign during its evolution, while in physics it is always positive (due to it, $v(t)$ always appears as $\omega^2(t)$). Eq. (2) is the version of the response model to be used from now onward. The Lagrangian, (L), the momentum (p) and the Hamiltonian (H) corresponding to Eq. (2) are [1]:

$$L(t,q,\dot{q})=\frac{1}{2}u(t)\cdot\dot{q}^2-\frac{1}{2}u(t)\cdot v(t)\cdot q^2+\frac{a\cdot b}{\tau}u(t)\cdot q \tag{6}$$

$$p=\frac{\partial L}{\partial q}=u(t)\cdot\dot{q} \tag{7}$$

$$H(t,p,q)=\frac{\partial L}{\partial\dot{q}}-L(t,q,\dot{q})=\frac{1}{2}\frac{p^2}{u(t)}+\frac{1}{2}u(t)\cdot q^2-\frac{a\cdot b}{\tau}u(t)\cdot q \tag{8}$$

Note that Eq. (8) represents actually energy in its physical sense because it can be rewritten as:

$$H(t,q,p)=T(t,p)+V(t,q) \tag{9}$$

where $T(t,p)=\frac{1}{2}\frac{p^2}{u(t)}$ is the kinetic energy and $V(t,q)=\frac{1}{2}u(t)\cdot v(t)\cdot q^2-\frac{a\cdot b}{\tau}u(t)\cdot q$ is the potential energy. However, it is not an invariant energy due to it is explicitly time-dependent. Would it be possible to get an invariant energy with the suitable changes? This is the goal of the following section.

3 Getting the invariant energy

In [4], Ray and Reid provide several methods to get invariants related to Eqs. (2) and (8); these are known as Ermakov-Lewis invariants (note that a collection of invariants can be got). Here we follow what we think is the most intuitive Ray and Reid's, which works directly on Eq. (2) [4]. First of all, the change $Q(t) = \sqrt{u(t)} \cdot q(t)$ provides the equation:

$$\ddot{Q}(t) + \Omega(t) \cdot Q(t) = \frac{a \cdot b}{\tau} \sqrt{u(t)} \tag{10}$$

where:

$$\Omega(t) = v(t) - \left(\frac{1}{2} \frac{\dot{u}(t)}{u(t)} - \frac{1}{2} \frac{\dot{u}^2(t)}{u^2(t)} \right) \tag{11}$$

That is, the mentioned change reduces Eq. (2) to Eq. (10), which is the equation of a harmonic oscillator with frequency $\Omega(t)$ subjected to a retrieving force $\frac{a \cdot b}{\tau}\sqrt{u(t)}$. Two new consecutive changes are needed now: the first on Eq. (10), on the dependent variable $x(t) = \frac{Q(t)}{C(t)} + A(t)$, and the second on the independent variable $T = \int_{t_0}^{t} \frac{dr}{C^2(r)}$, where $C(t)$ and $A(t)$ are undetermined auxiliary functions by the moment. These changes provide:

$$\begin{aligned} &\ddot{x}(T) + C^3(t)\Big(\ddot{C}(t) + \Omega(t) \cdot C(t)\Big) \cdot x(T) + \\ &+ C^3(t)\Big(- \ddot{C}(t) \cdot A(t) - 2\dot{C}(t) \cdot \dot{A}(t) - C(t) \cdot \ddot{A}(t) - \Omega(t) \cdot C(t) \cdot A(t) - \tfrac{a \cdot b}{\tau} \sqrt{u(t)} \Big) = 0 \end{aligned} \tag{12}$$

where $\ddot{x}(T) = \frac{d^2 x(T)}{dT^2}$. In order that Eq. (12) becomes an equation with constant parameters, we force the following to hold:

$$\ddot{C}(t) + \Omega(t) \cdot C(t) = \frac{k}{C^3(t)} \tag{13}$$

$$\ddot{A}(t) + 2\frac{\dot{C}(t)}{C(t)} \dot{A}(t) + k \frac{A(t)}{C^4(t)} + \frac{a \cdot b}{\tau} \frac{\sqrt{u(t)}}{C(t)} \tag{14}$$

where k is an undetermined constant. Then Eq. (12) becomes:

$$\ddot{x}(T) + k \cdot x(T) = 0 \tag{15}$$

The Lagrangian, momentum and Hamiltonian corresponding to Eq. (15) are:

$$L_x(t, x, \dot{x}) = \frac{1}{2} \tag{16}$$

$$p_x = \frac{\partial L}{\partial \dot{x}} = \dot{x} \tag{17}$$

$$H_x(t, x, p_x) = \frac{\partial L}{\partial x} \dot{x} - L_x(t, x, \dot{x}) = \frac{p_x^2}{2} + \frac{k}{2} x^2 \tag{18}$$

Note that the Hamiltonian H_x is explicitly time-independent, therefore it is invariant. In fact, by undoing the changes proposed before, we obtain the corresponding Ermakov-Lewis invariant [4] that, such as Padmanabhan emphasises in [5], it has been demonstrated that is an invariant energy:

$$\begin{aligned} E = \frac{1}{2}\Bigg(\sqrt{u(t)}\cdot C(t)\cdot \dot{q} + C^2(t)\cdot \dot{A}(t) + \Big(\frac{1}{2}C(t)\frac{\dot{u}(t)}{\sqrt{u(t)}} - \sqrt{u(t)}\cdot \dot{C}(t)\Big)q\Bigg)^2 + \\ +\frac{k}{2}\Big(\frac{\sqrt{u(t)}}{C(t)}q + A(t)\Big)^2 \end{aligned} \tag{19}$$

Note that in the invariant energy E given by Eq. (19), $C(t)$ and $A(t)$ must hold Eqs. (13) and (14). In addition, Eq. (15) can be solved analytically in function of $A(t)$ and $C(t)$ auxiliary variables by undoing the changes proposed before, and consequently:

$$\begin{aligned} q(t) &= \frac{C(t)}{\sqrt{u(t)}}\Big(- A(t) + k_1 \int_{t_0}^{t} \frac{dr}{C^2(r)} + k_2\Big) \quad if\ k = 0 \\ q(t) &= \frac{C(t)}{\sqrt{u(t)}}\Bigg(- A(t) + k_1\cdot exp\Big(\sqrt{k}\int_{t_0}^{t} \frac{dr}{C^2(r)} + k_2\cdot exp\Big(- \sqrt{k}\int_{t_0}^{t} \frac{dr}{C^2(r)}\Big)\Bigg) \quad if\ k > 0 \\ q(t) &= \frac{C(t)}{\sqrt{u(t)}}\Bigg(- A(t) + k_1\cdot sin\Big(\sqrt{-k}\int_{t_0}^{t} \frac{dr}{C^2(r)} + k_2\cdot cos\Big(\sqrt{-k}\int_{t_0}^{t} \frac{dr}{C^2(r)}\Big)\Bigg) \quad if\ k < 0 \end{aligned} \tag{20}$$

4 Application case: a dose of methylphenidate

The application case consists in one subject that consumed 10 mg of methylphenidate, and its GFP was measured every 7.5 minutes during 180 minutes (3 hours), with the 5 adjectives scale, GFP-FAS [6], in the interval [0, 50]. The initial condition was also measured before consumption, with value q_0, which is considered as initial condition for Eq. (20). The stimulus equation is given by:

$$s(t) = s_0 e^{-\beta\cdot t} + \begin{cases} \frac{\alpha\cdot M}{\beta-\alpha}(e^{-\alpha\cdot t} - e^{-\beta\cdot t}) : \alpha \neq \beta \\ \alpha\cdot M\cdot t\cdot e^{-\alpha\cdot t} : \alpha = \beta \end{cases} \tag{21}$$

Note that taking into account Eq. (21), calculating Eqs. (3), (5) and (11) (necessary to compute Eqs. (19) and (20) through Eqs. (13) and (14)), become very complex, due to the nonlinear character of Eqs. (13) and (14). Thus, the version that considers $k = 0$ in both Eqs. (19) and (20) is chosen for simplicity. In addition, the invariant energy, Eq. (19), for $k = 0$ can be split into three terms:

$$\begin{aligned} E = T_e + V_e + R_e = \frac{1}{2}\Big(\sqrt{u(t)}\cdot C(t)\dot{q} + C^2(t)\cdot \dot{A}(t)\Big)^2 + \\ +\frac{1}{2}\Bigg(\Big(\frac{1}{2}C(t)\frac{\dot{u}(t)}{\sqrt{u(t)}} - \sqrt{u(t)}\cdot \dot{C}(t)\Big)q\Bigg)^2 + \\ +\Big(\sqrt{u(t)}\cdot C(t)\cdot \dot{q} + C^2(t)\cdot \dot{A}(t)\Big)\Bigg(\Big(\frac{1}{2}C(t)\frac{\dot{u}(t)}{\sqrt{u(t)}} - \sqrt{u(t)}\cdot \dot{C}(t)\Big)q\Bigg) \end{aligned} \tag{22}$$

where T_e is a kinetic energy, V_e a potential energy and R_e a lost energy. This splitting is based on the fact that $A(t)$ has the same dimensions as q, $C(t)$ is non dimensional, and that $A(t)$

and q variables and its derivatives play these roles in the different kinds of energies observed in similar problems in Physics. Once the initial values of $A(t)$, $A_0 = q_0$, $\dot{A}_0 = \dot{q}_0$, have been determined and taking $k_1 = 1$ and $k_2 = 1$, the initial values of $C(t)$ become $C_0 = -1.053$, $\dot{C}_0 = -0.066$. In addition, due to the term $(a + \frac{1}{\tau})(t - t_0)$, in the $u(t)$ function of Eq. (5), can be divergent, depending on the a or τ values, the constant u_0 is taken as $u_0 = e^{-(a+\frac{1}{\tau})(T-t_0)}$, where T is the border time of the experiment (180 min.). Then the calibration of Eq. (20) can be seen in Fig. 1.while the three energies split in Eq. (22) can be seen in Fig. 2 and the total energy E in Fig. 3. Observe the invariance of this last energy.

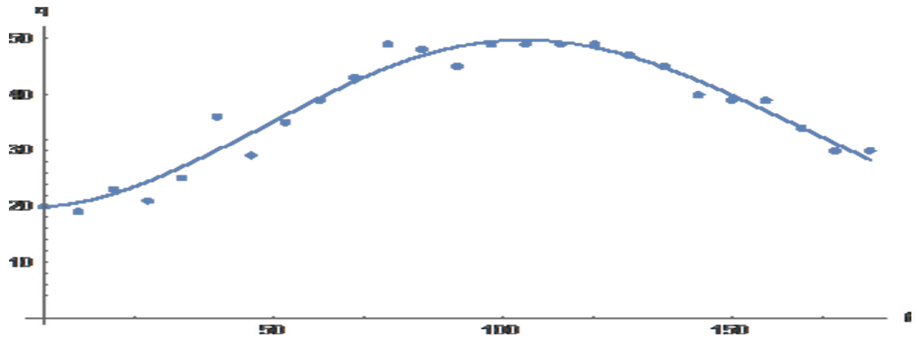

Figure 1: Experimental values (dots) and the theoretical values (curve) of the GFP (q) versus time.

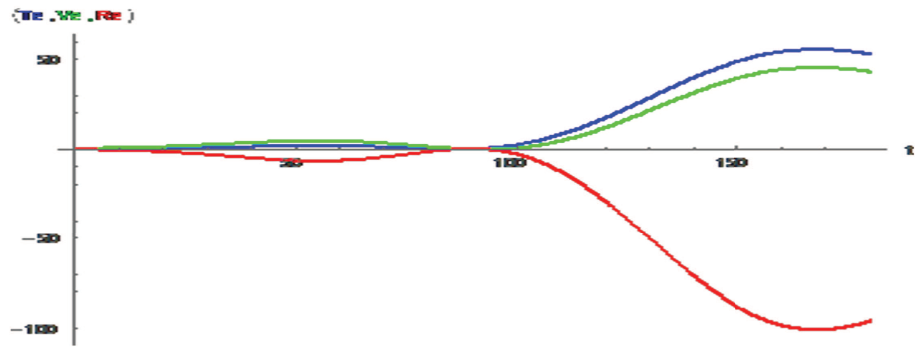

Figure 2: Kinetic energy T_e (upper curve), potential energy V_e (intermediate curve) and lost energy R_e (lower curve) versus time.

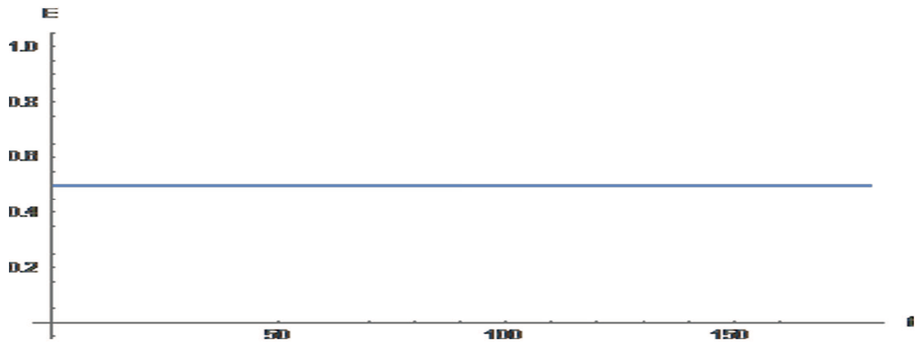

Figure 3: Invariant energy E versus time.

5 Conclusions and future work

We want to emphasize that an invariant energy has been found in the context of the short-term personality dynamics. However, it has been found at the price to compute the dynamics of the auxiliary variables $A(t)$ and $C(t)$ through the nonlinear Eqs. (13) and (14). Even in the case here considered, $k = 0$, the calculation complexity is present. However, this case provides the splitting of the invariant energy into three interesting terms: kinetic, potential and lost energies. Nevertheless, we are conscious that this theoretical progress in personality theory must find its own applications. Firstly, consider that having an invariant in the context of any theory is highly important because it permits to characterize or classify non-well identified problems in its corresponding theory, for instance, the classification of orbits in the two-bodies problem. Thus, a future work should be to try to classify personality typologies by the found invariant energy, for instance, those related to health, even, to try to change a disordered personality by another stable one through planned stimuli, such as the self-regulation therapies suggest [7]. On the other hand, When the short-term personality dynamics, given by Eq. (1), tries to be applied to two or more consecutive stimuli, the model calibrations become useless due to the arising of habituation phenomena. However, due to the invariant energy is always conserved, the habituation phenomena could be predicted and thus, this invariant could be used to predict the long-term personality dynamics, i.e., when two or more consecutive stimuli influence on an individual [8].

References

[1] Micó, J.C. Amigó, S. Caselles, A. Hamiltonian approach to human personality dynamics: an experiment with methylphenidate. *Proceedings of the Modelling for Engineering and Human Behaviour*, 209-212, Universitat Politècnica de València, 2018.

[2] Micó, J.C. Amigó, S. Caselles, A. Advances in the General Factor of Personality Dynamics. *Revista Internacional de Sistemas*, 22: 34-38, 2018.

[3] Landau, L. D., Lifshitz, E. M. *Mecánica (Vol. 1 del Curso de Física Teórica)*. Barcelona, Ed. Reverté, 2002.

[4] Ray, J. R., Reid, J. L. Invariants for forced time-dependent oscillators and generalizations. *Physical Review A*, 26: 1042-1047, 1982.

[5] Padmanabhan, T. Demystifying the constancy of the Ermakov-Lewis invariant for a time dependent oscillator. *arXiv-1712.07328v1 [physics.class-ph]*, 1-5, 2017.

[6] Amigó, S., Micó, J. C., Caselles, A. Five adjectives to explain the whole personality: a brief scale of personality. *Revista Internacional de Sistemas*, 16: 41–43, 2009.

[7] Amigó, S., Caselles, A., Micó, J. C. The self-regulation therapy to reproduce drug effects: a suggestion technique to change personality and the DRD3 gene expression. *International Journal of Clinical and Experimental Hypnosis*, 61: 282–304, 2013.

[8] Caselles, A., Micó, J. C. Amigó, S. Cocaine addiction and personality: A mathematical model. *British Journal of Mathematical and Statistical Psychology*, 63: 449–480, 2010.