Numerical modeling of double-negative acoustic metamaterials: Should losses be included?

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Abstract
Viscous and thermal losses of acoustic waves are usually neglected or accounted for as boundary impedance. It is known, however, that acoustic losses become relevant in devices with some dimension in the millimeter range or below. On the other hand, the new class of structures called acoustic metamaterials can be affected by acoustic losses, but in this case the extension of these effects is less known. Acoustic metamaterials are intricate periodic structures where, at frequencies low enough (corresponding to wavelengths much larger than the structure period), elementary units interact producing interesting unusual effects.

In this paper advanced modeling tools based on the Boundary Element Method (BEM) and the Finite Element Method (FEM) are used to study the effect of losses in an acoustic metamaterial scaled to different sizes. The conclusions are expected to give insight on the practical limitations when using acoustic metamaterials.

Keywords: acoustic metamaterials, numerical methods, visco-thermal losses.

PACS no. 43.20.Hq, 43.20.Rz

1 Introduction

Viscous and thermal losses are present in any acoustic wave and show as attenuation over large distances or as a boundary effect. Usually the lossless wave equation is employed to describe the sound field, and loss is assigned to attenuation terms or surface impedances.[1]

However, when the dimensions of the acoustic domain are of a similar scale as the viscous and thermal boundary layers, neglecting losses can lead to unrealistic results. These dimensions are in the millimeter/micrometer range for audible frequencies. Well-known examples are acoustic couplers, acoustic transducers, or hearing aids.[2,3]

The periodic arrangement of specially designed structural units known as acoustic metamaterial can show unusual properties at low frequencies which can be employed to make feasible theoretical proposals like acoustic cloaking.[4,5] Acoustic metamaterials, where the acoustic wave propagates in a fluid rather than a solid, have recently joined the list of structures where acoustic fluid losses might be relevant, with the added difficulty of their necessarily intricate internal structure. The dimensions of these materials are usually designed larger than the boundary layers but much smaller than the wavelength, and therefore it is not very clear in what circumstances and to what extent viscous and thermal losses affect their very special properties.

On the other hand, new numerical modeling tools have been developed in the last 10-15 years, which can cope with viscous and thermal losses, and employ either the Finite Element Method (FEM) or the Boundary Element Method (BEM).[6,7,8] They are however much more computationally costly than their corresponding lossless implementations, and therefore are often replaced by lossless implementations when evaluating metamaterials.

The aim of this paper is evaluating how losses can affect an acoustic metamaterial at different length scales. To do this, a particular acoustic metamaterial in the literature has been chosen.[9] The metamaterial is scaled up and the effect of losses in its special properties studied.

Section 2 gives an overview of the numerical methods with losses used in the calculation. Section 3 briefly describes the acoustic metamaterial test case, taken from [9]. The scaling method and its results are shown in section 4. Section 5 is dedicated to conclusions and future work.

2 Numerical models with losses

Full numerical implementations of viscous and thermal losses are based on the linearized Navier-Stokes equations:[1]

$$
\begin{aligned}
&\frac{\partial \rho}{\partial t} + \rho_o \nabla \vec{v} = 0 \qquad \rho_o T_o \frac{\partial s}{\partial t} = \lambda \Delta T \\
&\rho_o \frac{\partial \vec{v}}{\partial t} = -\nabla p + \left(\eta + \frac{4}{3}\mu\right) \nabla(\nabla \vec{v}) - \mu \nabla \times (\nabla \times \vec{v}) \\
&s = \frac{C_p}{T_o}\left(T - \frac{\gamma - 1}{\beta\gamma} p\right) \qquad \rho = \frac{\gamma}{c^2}(p - \beta T)
\end{aligned}
\tag{1a-1e}
$$

Equations (1a) to (1e) represent respectively conservation of mass, conservation of energy, conservation of momentum and thermodynamic conditions. The symbols are: ρ, density, ρ_0, static density, $\vec{v}$, particle velocity, T, temperature, T_0, static temperature, s, entropy, λ, thermal conductivity, p, sound pressure, η, bulk viscosity, μ, coefficient of viscosity, C_p, specific heat at constant pressure, γ, ratio of specific heats and β, rate of increase of pressure with temperature at constant volume. In addition to linear variations, no flow, homogeneous fluid and dimensions of the setup and wavelength larger than the molecular mean free path (~10^{-7} m) are assumed.

2.1 Finite Element Method

The direct implementation of the linearized Navier-Stokes equations (1a-1e) in the Finite Element Method has been proposed and developed by several authors.[5,9] The implementation was made originally in the open FEM package Elmer and it has been included later in the Comsol FEM package, which has been employed in this paper in its version 5.2.[10]

This is a computationally intensive implementation, mainly because the thin viscous and thermal boundary layers need to be properly represented. For this reason, a sparing but sufficient meshing is important, requiring an understanding of every case and how much detail is required.

FEM with losses is the most extended implementation. It was included early in a commercial package, and its implementation has been progressively refined and optimized. A growing number of examples including losses using FEM can be found in the literature and in the Comsol webpage.

2.2 Boundary Element Method

The BEM implementation with losses is based on the Kirchhoff derivation of the Navier-Stokes equations (1a-1e):[1,11]

$$\begin{aligned} \left(\Delta + k_a^2\right) p_a &= 0 \\ \left(\Delta + k_h^2\right) p_h &= 0 \\ \left(\Delta + k_v^2\right) \vec{v}_v &= \vec{0} \quad with \quad \nabla \cdot \vec{v}_v = 0 \end{aligned} \tag{2a-2c}$$

Eqs. (2a-2c) represent, the so-called *acoustic (a)*, *thermal (h)* and *viscous (v)* modes, which can be dealt with independently in the acoustic domain. They are however linked through the temperature and velocity boundary conditions. The pressure is separated into two components $p = p_a + p_h$, while the velocity is separated into three components $\vec{v} = \vec{v}_a + \vec{v}_h + \vec{v}_v$. The viscous velocity or rotational velocity does not have a corresponding pressure. Correspondingly, the wavenumbers k_a, k_h and k_v are based on the lossless wavenumber k and physical properties of the fluid, such as the viscosity, bulk viscosity and thermal conductivity coefficients, air density, and specific heats. Eq. (2a) is a wave equation, while Eqs. (2b-2c) are diffusion equations, given the large imaginary part of k_h and k_v. Eq. (2c) is a vector equation and therefore can be split into three components, giving a total of five unknowns.

The implementation in BEM is made by discretizing equations (2a-2c) one by one and combining them into a single matrix equation using the boundary conditions. The matrix equation is solved for the acoustic pressure p_a and subsequently other variables are obtained on the boundary. From the boundary values, any domain field point can be calculated. [7,8]

3 Description of the acoustic metamaterial

The results shown in this paper are based on the acoustic metamaterial presented in reference [9], where details can be found. In short, the authors of [9] proposed a metamaterial that is formed by elementary units as the one shown in Fig. 1. The units are arranged in a periodic lattice, and the sound waves propagate in between two planes separated by the distance *h*, where the units are embedded. Such a metamaterial shows extraordinary properties within a certain frequency band, where double negative behaviour is predicted; i.e., where both the dynamical bulk modulus and mass density have negative values. In addition, at frequencies where the density is near zero, interesting behaviours like tunnelling of acoustic waves through narrow channels, restoring of a plane wave after scattering, and perfect transmission through bends and corners were also theoretically characterized.

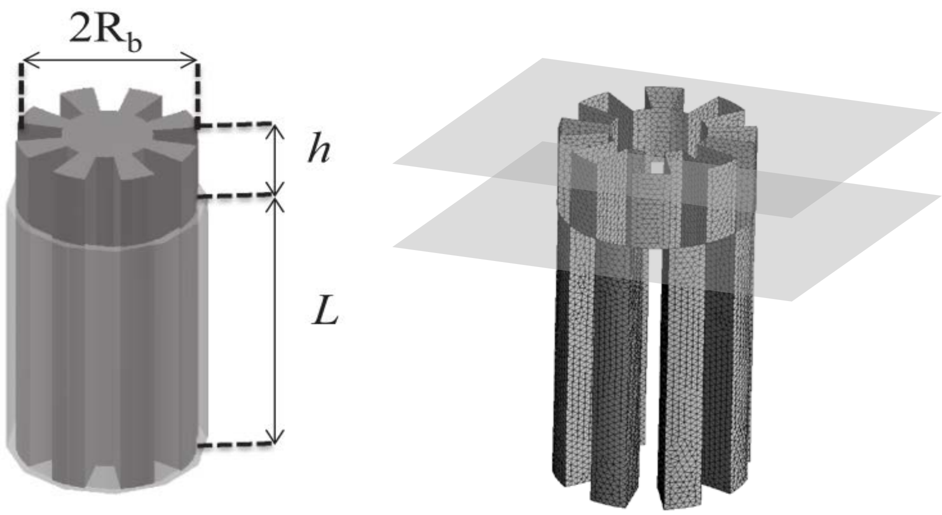

Figure 1 – A single unit of the example metamaterial from reference [9]. Left: schematic view with some of its defining parameters (radius R_b, height of the space between planes h, and depth of the wells, L). Right: BEM mesh version of the same unit, where only the domain boundary is meshed. The positions of the planes where the unit is embedded are shown as transparent rectangles.

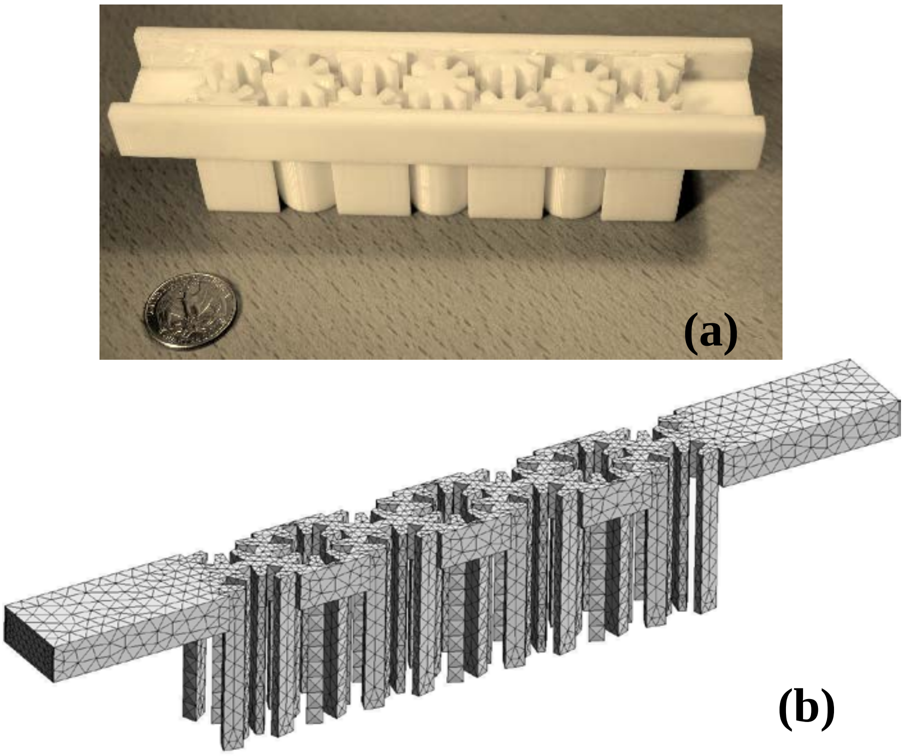

Figure 2 – a) Sample of acoustic metamaterial measured in Ref. [9]; b) corresponding BEM mesh.

The results in Ref. [9] are based on an analytical model of the material and on lossless FEM simulations, both predicting the double-negative behavior. The paper also includes measurements made on a 3D printed sample, as shown in Fig. 2a. Unfortunately, the expected double-negative behavior were not supported by experimental data.

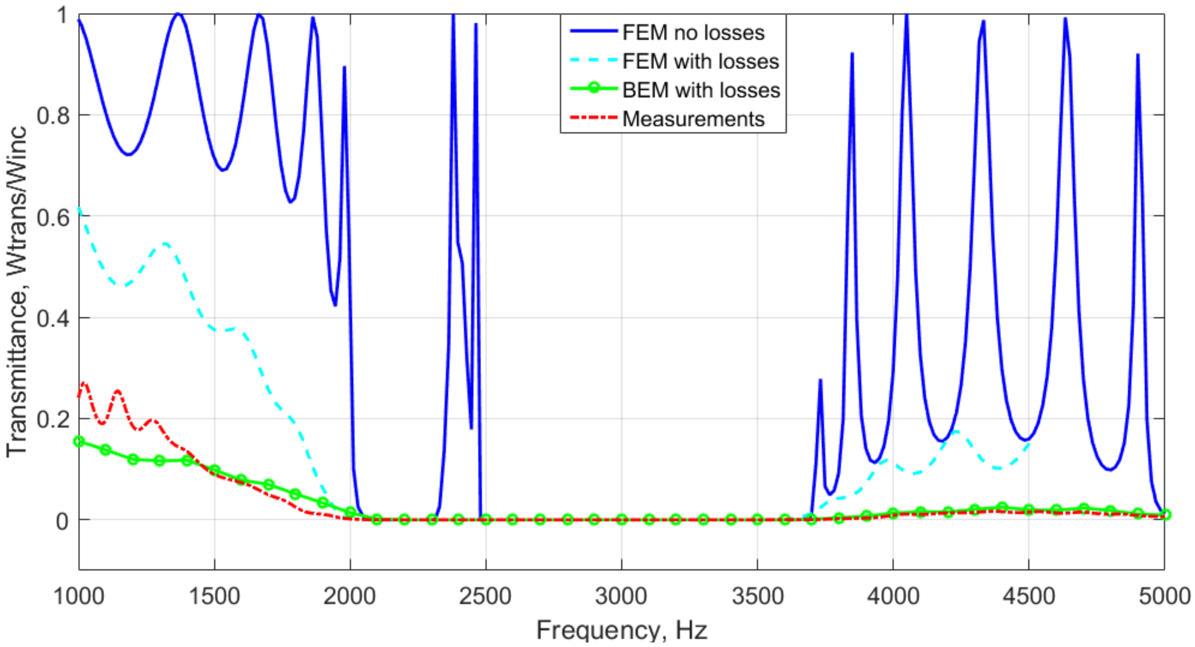

Figure 3 – Transmittance results for the metamaterial sample (sample A in Ref. [9]). Solid blue curve, lossless FEM result; dashed light blue curve, FEM with losses; dash-dotted red curve, measurements (from [9]); green-marker curve, BEM with losses. Double-negative frequency is ~2400 Hz.

In order to test the performance of the material, the transmittance (insertion loss) is simulated using the numerical methods with losses described in the previous section, and compared with the results in [9]. The transmittance is defined as the ratio of the acoustic power after an incident plane wave has traversed the material, with respect to the incident power. It should vary between 0 (no transmission) to 1 (full transmission). The transmittance results are shown in Fig. 3.

The double-negative behavior is predicted by the lossless FEM model at around 2400-2500 Hz. Lossless BEM results (not shown), also predict this behavior at a slightly higher frequency. However, measurements and models with viscous and thermal losses lack the transmittance peak at this frequency: the extraordinary behavior seems to be hindered by losses. This result was already deduced in [9] from the measurements, and it is confirmed by the numerical models.

4 Scaling of the metamaterial and results

The metamaterial described in the previous section has a length (from the input to the output) of 127,5 mm. In this section models of scaled-up versions of this setup are used. The lossless description of the metamaterial should give, and it actually does for the lossless models, the same transmittance results at a corresponding scaled-down frequency. Viscous and thermal losses, however, do not vary with scale in the same way.[1] The thickness of the viscous and thermal boundary layers is inversely proportional to $\sqrt{f}$. Moreover, viscous losses strongly depend on the incidence angle. The scaled models are computationally demanding, but no more than the original setup. The physical dimensions are made larger, but the calculation frequencies are divided by the same factors. This means the same mesh can be reused. The transmittance is calculated with the same piston velocity excitation in all cases, which gives a growing incident power, since the section of the sample grows as the square of the scale factor.

Fig. 4 shows the transmittance at the double-negative frequency for different scale factors. The frequency at which each point is calculated is the double-negative frequency (2600 Hz in the BEM model), divided by the scale factor. The lossless calculation gives a fairly good transmission as expected, but in the models with losses the transmission improves very slowly as the scale grows. Even for the larger scale, where the metamaterial is about 2,5 m long, the transmittance is below 10^{-2}. The FEM model shows a similar tendency, despite its evaluation of an overall lower transmittance. Curves also show that the improvement of the transmittance is less significant as the scale grows, stabilizing at a rather low value.

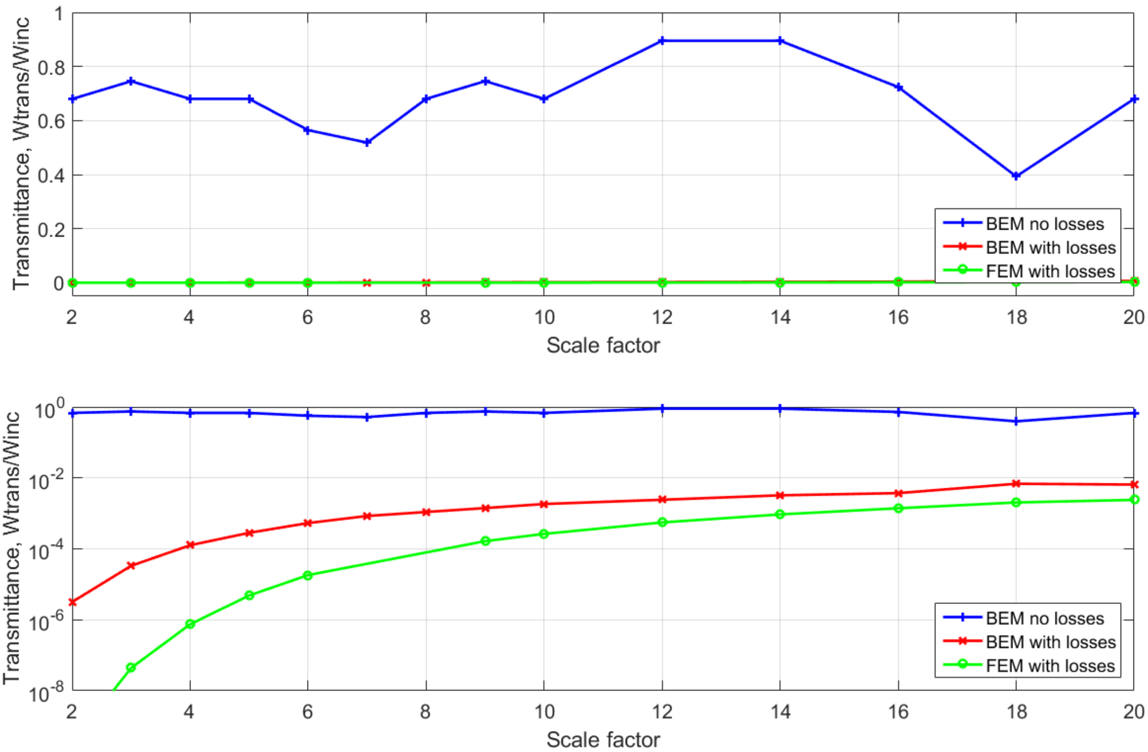

Figure 4 – Transmittance at the scaled double-negative frequency as a function of the scaling factor. The lossless calculation (blue '+') shows high transmission, while BEM (red 'x') and FEM (green 'o') with losses predict a quite poor transmission. Large versions of the metamaterial hardly improve. The lower figure is equivalent to the upper one, but using a logarithmic ordinate axis.

5 Conclusions and future work

The results in section 4 seem to contradict the general belief assuming that a sufficiently large acoustic metamaterial will not be affected by viscous and thermal losses. On the contrary, results indicate that the example metamaterial cannot be made large enough so that losses become negligible. Even for large versions of the metamaterial, most of the incident power never reaches the output of the material.

This conclusion is the same whether the material is modeled using FEM or BEM with losses. These methods are based on the same physical background but have very different implementations. No measurements have been done on other than the original setup.

In the future, more work can be done in order to support the main conclusion and understand better the effect of losses on acoustic metamaterials. Besides performing verification measurements, other metamaterial setups may be tried. A detailed analysis of the simulation results can also reveal more details about what is the contribution of the different parts of the setup to the overall losses, and how the metamaterial design can minimize them.

Acknowledgements

J. Sanchez-Dehesa acknowledges the support by the Spanish Ministerio de Economía y Competitividad, and the European Union Fondo Europeo de Desarrollo Regional (FEDER) through Project No. TEC2014-53088-C3-1-R.

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