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Topological Krasner hyperrings with special emphasis on isomorphism theorems

Manoranjan Singha and Kousik Das

Department of Mathematics, University of North Bengal, Darjeeling-734013, India
(manoranjan.math@nbu.ac.in, das.kousik1991@nbu.ac.in)

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Abstract

Krasner hyperring is studied in topological flavor. It is seen that Krasner hyperring endowed with topology, when the topology is compatible with the hyperoperations in some sense, fruits new results comprising algebraic as well as topological properties such as topological isomorphism theorems.

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1. Introduction and relevant literature

The theory of hyperalgebra which is extended in this article gets birth in the year 1934 but it gets acquaintance during the last two decades and so far it is wide in various branches of Mathematics including Physics and Chemistry: geometry [25, 26], graph theory [5, 27], codes [31, 10], cryptography [4], probability [20], automata [19], artificial intelligence [17], lattice theory [14, 15], chemistry [1, 12], physics [8, 21], and all credits for these go to the hyperoperations. It is investigated that how the difference between hyperoperation and binary operation affects on the theory of topological Krasner hyperring, especially on the topological isomorphisms. Hyperring, introduced by Krasner [16] is one of the most general structures so far in the literature that satisfies the ring-like axioms. Later, many mathematicians, like Ameri [3, 2], Massouros

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[18], Spartalis [29], Davvaz [7], Stratigopoulos [30], Kemprasit [24] extended this field of study. In literature, a topological ring is a combination of two structures, namely a topological space and a ring. These two structures are connected in such a way that one affects another. In this paper, we generalize this concept as topological Krasner hyperring, supported by illustrative examples. We also present an example that makes difference between the classical and the new concept. In the later part, we use the notion of complete parts to study isomorphism theorems on hyperrings.

Let's begin with some basic definitions and results which will be used as ready references in the sequel. On a nonempty set $\mathcal{H}$, a *hyperoperation* is a function $+ : \mathcal{H} \times \mathcal{H} \to \mathcal{P}^*(\mathcal{H})$, where $\mathcal{P}^*(\mathcal{H})$ is the collection of nonempty subsets of $\mathcal{H}$. For nonempty subsets A, B of $\mathcal{H}$ and $x \in \mathcal{H}$, consider

$$A + B = \bigcup_{a \in A,\, b \in B} a + b,\ x + A = \{x\} + A \text{ and } A + x = A + \{x\}.$$

A *Krasner hyperring* is an algebraic structure $(\mathcal{H}, +, \cdot)$ satisfying the following axioms:

(1) $(\mathcal{H}, +)$ is a canonical hypergroup, i.e., $+$ is a hyperoperation on $\mathcal{H}$ such that
  - (a) for every $x, y, z \in \mathcal{H}$, $x + (y + z) = (x + y) + z$,
  - (b) for every $x, y \in \mathcal{H}$, $x + y = y + x$,
  - (c) there exists $0 \in \mathcal{H}$ such that $0 + x = \{x\}$ for every $x \in \mathcal{H}$,
  - (d) for every $x \in \mathcal{H}$, there exists a unique $x' \in \mathcal{H}$ such that $0 \in x + x'$, (write $-x$ instead of such x'),
  - (e) $z \in x + y$ implies $y \in -x + z$ and $x \in z - y$;

(2) $(\mathcal{H}, \cdot)$ is a semigroup having zero as a bilaterally absorbing element, i.e., $0 \cdot x = x \cdot 0 = 0$ for all $x \in \mathcal{H}$.

(3) The multiplication, '$\cdot$' is distributive with respect to the hyperoperation $+$.

Throughout this context, *hyperring* stands for *Krasner hyperring*. The following elementary facts are the consequences of the above axioms:
$-(-x) = x$, for any nonempty subset X of $\mathcal{H}$, $-X = \{-x : x \in X\}$ and $-(x+y) = -x-y$. Also, for all $a, b, c, d \in \mathcal{H}$, $(a+b)\cdot(c+d) \subseteq a\cdot c+b\cdot c+a\cdot d+b\cdot d$.

A nonempty subset $\mathcal{K}$ of the hyperring $\mathcal{H}$ is said to be a *subhyperring* of $\mathcal{H}$ if $(\mathcal{K}, +, \cdot)$ is itself a hyperring. The subhyperring $\mathcal{K}$ is a *hyperideal* of $\mathcal{H}$ if $h \cdot k \in \mathcal{K}$ and $k \cdot h \in \mathcal{K}$ for all $h \in \mathcal{H}$ and $k \in \mathcal{K}$. The subhyperring $\mathcal{K}$ is said to be *normal* in $\mathcal{H}$ if and only if $h + \mathcal{K} - h \subseteq \mathcal{K}$ for all $h \in \mathcal{H}$. For a normal hyperideal $\mathcal{K}$ of a hyperring $\mathcal{H}$ the following results hold:

(1) $x + \mathcal{K} = \mathcal{K} + x$ for all $x \in \mathcal{H}$,
(2) $(x + \mathcal{K}) + (y + \mathcal{K}) = x + y + \mathcal{K}$ for all $x, y \in \mathcal{H}$,
(3) if $x, y \in \mathcal{H}$, $x + y + \mathcal{K} = z + \mathcal{K}$ for all $z \in x + y$,
(4) $x + \mathcal{K} = y + \mathcal{K}$ for all $y \in x + \mathcal{K}$.

Let $\mathcal{K}_1, \mathcal{K}_2$ be two hyperideals of a hyperring $\mathcal{H}$ such that $\mathcal{K}_2$ is normal in $\mathcal{H}$. Then,

(1) $\mathcal{K}_1 \cap \mathcal{K}_2$ is a normal hyperideal of $\mathcal{K}_1$,

(2) $\mathcal{K}_2$ is a normal hyperideal of $\mathcal{K}_1 + \mathcal{K}_2$.

For a normal hyperideal $\mathcal{K}$ of a hyperring $\mathcal{H}$, define an equivalence relation $\mathcal{K}^*$ as follows:

$$x \equiv y (mod\, \mathcal{K}) \text{ if and only if } (x - y) \cap \mathcal{K} \neq \phi.$$

Then, for all $x \in \mathcal{H}$, $\mathcal{K}^*(x) = x + \mathcal{K}$. The collection $[\mathcal{H} : \mathcal{K}^*] = \{\mathcal{K}^*(x) : x \in \mathcal{H}\}$ of all equivalence classes forms a hyperring together with the hyperoperations $\oplus$ and multiplication $\odot$ defined as follows:

$$\begin{aligned} \mathcal{K}^*(x) \oplus \mathcal{K}^*(y) &= \{\mathcal{K}^*(z) : z \in \mathcal{K}^*(x) + \mathcal{K}^*(y)\}, \\ \mathcal{K}^*(x) \odot \mathcal{K}^*(y) &= \mathcal{K}^*(x \cdot y). \end{aligned}$$

A *homomorphism* from a hyperring $(\mathcal{H}, +, \cdot)$ into another hyperring $(\mathcal{H}', +', \cdot')$ is a map $f : \mathcal{H} \to \mathcal{H}'$ such that $f(x+y) \subseteq f(x) +' f(y)$ and $f(x \cdot y) = f(x) \cdot' f(y)$, for all $x, y \in \mathcal{H}$. A homomorphism f from $(\mathcal{H}, +, \cdot)$ into $(\mathcal{H}', +', \cdot')$ is said to be a *good homomorphism* if $f(x + y) = f(x) +' f(y)$, for all $x, y \in \mathcal{H}$. An onto homomorphism is called *epimorphism*. An *isomorphism* from $(\mathcal{H}, +, \cdot)$ onto $(\mathcal{H}', +', \cdot')$ is a bijective good homomorphism and if such map exists, then write $\mathcal{H} \cong \mathcal{H}'$. If f is an isomorphism from $\mathcal{H}$ onto $\mathcal{H}'$, then f^{-1} is an isomorphism from $\mathcal{H}'$ onto $\mathcal{H}$. For a homomorphism $f : \mathcal{H} \to \mathcal{H}'$, the kernel of the homomorphism is defined as $\ker f = \{x \in \mathcal{H} : f(x) = 0_{\mathcal{H}'}\}$. It is seen (Example 1.2 [24]) that the kernel of a homomorphism may be empty, but, if it is nonempty (i.e., $\ker f \neq \phi$), then the following results ([24]) hold:

(1) $f(0_{\mathcal{H}}) = 0_{\mathcal{H}'}$;
(2) $f(-x) = -f(x)$, for all $x \in \mathcal{H}$;
(3) $\ker f$ is a hyperideal of $\mathcal{H}$;
(4) If f is injective, then $\ker f = \{0_{\mathcal{H}}\}$;
(5) If f is a good homomorphism and $\ker f = \{0_{\mathcal{H}}\}$, then f is injective;
(6) If f is a good homomorphism, $f(\mathcal{H})$ is a subhyperring of $\mathcal{H}'$.

Note that, if f is onto, then $f(x) = 0_{\mathcal{H}'}$ for some $x \in \mathcal{H}$, i.e., $\ker f \neq \phi$.

A nonempty subset C of a hyperring $\mathcal{H}$ is said to be a complete part of $\mathcal{H}$ if for any nonzero natural number n and for all $x_1, x_2, \cdots, x_n$ of $\mathcal{H}$, the following implication holds:

$$C \cap \sum_{i=1}^{n} x_i \neq \phi \Rightarrow \sum_{i=1}^{n} x_i \subseteq C.$$

Let A and B be two nonempty subsets of the hyperring $\mathcal{H}$ such that A is a complete part of $\mathcal{H}$ and $x \in \mathcal{H}$. Then,

(1) $-x + x + A = x - x + A = A$;
(2) $-A$ is a complete part of $\mathcal{H}$;
(3) $x + A$ and $A + x$ are complete parts;
(4) $B \subseteq -x + A$ if and only if $x + B \subseteq A$.

For more details about hyperring we refer to [16, 9, 24, 7].

2. Topological Hyperring and Isomorphism Theorems

To define topological hyperring, the codomain of the hyperoperation is to be topologized, but there is no straightforward way to obtain such topology. So, let's consider the following.

Lemma 2.1 ([13])**.** *For a topological space $(\mathcal{H},\tau)$, the family $\mathcal{B}$ consisting of the sets $S_V=\{U\in\mathcal{P}^*(\mathcal{H}):U\subseteq V\}$, where $V\in\tau$ is a base for a topology τ^* on $\mathcal{P}^*(\mathcal{H})$.*

Now, we are in a situation to define topological hyperring.

Definition 2.2. Let $(\mathcal{H},+,\cdot)$ be a Krasner hyperring endowed with some topology τ. Then, $\mathcal{H}$ is said to be a *topological Krasner hyperring*, denoted by $(\mathcal{H},+,\cdot,\tau)$, if with respect to the product topology on $\mathcal{H}\times\mathcal{H}$ and the topology τ^* on $\mathcal{P}^*(\mathcal{H})$, the following maps

(TH1) $(x,y)\mapsto x+y$ from $\mathcal{H}\times\mathcal{H}$ to $\mathcal{P}^*(\mathcal{H})$;
(TH2) $x\mapsto -x$ from $\mathcal{H}$ to $\mathcal{H}$;
(TH3) $(x,y)\mapsto x\cdot y$ from $\mathcal{H}\times\mathcal{H}$ to $\mathcal{H}$;

are continuous. For the ease of writing throughout this context, let's write *topological hyperring* instead of *topological Krasner hyperring*.

Every topological ring is a topological hyperring. Here, we consider some other examples.

Example 2.3. Consider the hyperring $(R,+,\cdot)$, where $R=\{0,1,2\}$, the hyperoperation $+$ and the binary operation '$\cdot$' are defined as follows

$+$	0	1	2
0	{0}	{1}	{2}
1	{1}	{0,2}	{1}
2	{2}	{1}	{0}

$\cdot$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	0

Let R be endowed with the topology $\tau=\{\phi,\{1\},\{0,2\},R\}$. Then, $(R,+,\cdot,\tau)$ is a topological hyperring.

Example 2.4. Consider the unit interval $[0,1]$ as a subspace of $\mathbb{R}$ with standard topology. For $x,y\in[0,1]$, let $+$ be the hyperoperation defined as follows

$$x+y=\begin{cases}\{\max\{x,y\}\}, & \text{if} \quad x\neq y;\\ [0,x], & \text{if} \quad x=y.\end{cases}$$

Then $([0,1],+,\cdot)$ is a topological hyperring, where '$\cdot$' is the usual multiplication on $\mathbb{R}$.

Remark 2.5. Unlike in topological rings, some results may fail to hold in the new setting. For, in the above Example 2.4, $\frac{1}{2}\in[0,1]$ and $[0,\frac{1}{2})$ is open in $[0,1]$, but $\frac{1}{2}\oplus[0,\frac{1}{2})=\{\frac{1}{2}\}$, which is not open in $[0,1]$.

Lemma 2.6. *In a topological hyperring $(\mathcal{H},+,\cdot,\tau)$, the following results hold.*

(1) *For $a\in\mathcal{H}$, the map $T_a(x)=a+x$ from $\mathcal{H}$ to $\mathcal{P}^*(\mathcal{H})$ is continuous.*

(2) *Let U be open and a complete part of $\mathcal{H}$. Then, for $a \in \mathcal{H}$, $a+U$ is an open subset of $\mathcal{H}$.*
Moreover, if A is any subset of $\mathcal{H}$, then $A+U$ is an open subset of $\mathcal{H}$.
(3) *The map $I(x) = -x$ from $\mathcal{H}$ to $\mathcal{H}$ is a homeomorphism.*

Proof. (1) Being a restriction of the map $+ : \mathcal{H}\times\mathcal{H} \to \mathcal{P}^*(\mathcal{H})$, T_a is continuous for any $a \in \mathcal{H}$.
(2) Consider the basic open set S_U of $\mathcal{P}^*(\mathcal{H})$ and $a \in \mathcal{H}$. Then,

$$\begin{aligned} T_{-a}^{-1}(S_U) &= \{x \in \mathcal{H} : T_{-a}(x) \in S_U\} \\ &= \{x \in \mathcal{H} : -a + x \subseteq U\} \end{aligned}$$

Suppose $\mathfrak{S} = \{x \in \mathcal{H} : -a + x \subseteq U\}$. Take $y \in \mathfrak{S}$, then $-a + y \subseteq U$. So, $y \in 0 + y \subseteq (a + (-a)) + y \subseteq a + (-a + y) \subseteq a + U$. Again, if $z \in a + U$, then $-a + z \subseteq -a + a + U = U$, which implies $z \in \mathfrak{S}$. Hence, $T_{-a}^{-1}(S_U) = a + U$, which is open in $\mathcal{H}$.
For $A \subseteq \mathcal{H}$, $A + U = \bigcup\limits_{a\in A}(a + U)$, which is also open for being arbitrary union of open sets.
(3) The inverse of an element in the canonical hypergroup $(\mathcal{H}, +)$ is unique. So, the map I on $\mathcal{H}$ is a homeomorphism. □

Remark 2.7. The open subsets in the above Example 2.3 are complete parts.

Theorem 2.8. *In a topological hyperring $(\mathcal{H}, +, \cdot, \tau)$, the following straightforward results easily hold.*

(1) *For a neighborhood V of zero, there exist a neighborhood U of zero and a neighborhood W of x, where $x \in \mathcal{H}$ such that $U + U \subseteq V$ and $U \cdot W \subseteq V$.*
(2) *If V is any neighborhood of zero, then $-V$ is also a neighborhood of zero.*
(3) *Every neighborhood U of zero contains a symmetric neighborhood of zero (i.e., $U \cap (-U)$).*
(4) *If U is a neighborhood of zero and $n > 1$, then there exists a symmetric neighborhood V of zero such that $\underbrace{V + V + \cdots + V}_{n \text{ terms}} \subseteq U$.*

Proof. The proofs are straightforward. □

Any subhyperring of a topological hyperring is also a topological hyperring when considering the relative topology on it, such subhyperrings are called *topological subhyperrings*.

Let $\mathcal{I}$ be a hyperideal of a hyperring $(\mathcal{H}, +, \cdot)$ and $\mathcal{H}/\mathcal{I} = \{x + \mathcal{I} : x \in \mathcal{H}\}$. Then, $(\mathcal{H}/\mathcal{I}, \oplus, \odot)$ is a hyperring, called *quotient hyperring* of $\mathcal{H}$ by $\mathcal{I}$, where $(x + \mathcal{I}) \oplus (y + \mathcal{I}) = \{z + \mathcal{I} : z \in x + y\}$ and $(x + \mathcal{I}) \odot (y + \mathcal{I}) = (x \cdot y) + \mathcal{I}$ for $x, y \in \mathcal{H}$.

Let $\mathcal{I}$ be a normal hyperideal of a topological hyperring $(\mathcal{H}, +, \cdot, \tau)$ and $\Phi_{\mathcal{I}}$ be the canonical map of $\mathcal{H}$ onto $\mathcal{H}/\mathcal{I}$, defined by $\Phi_{\mathcal{I}}(x) = x + \mathcal{I}$ for $x \in \mathcal{H}$. Let's topologize $\mathcal{H}/\mathcal{I}$ by declaring the map $\Phi_{\mathcal{I}}$ to be quotient, i.e., a subset $\mathcal{A}$

of $\mathcal{H}/\mathcal{I}$ is open in $\mathcal{H}/\mathcal{I}$ if and only if $\Phi_{\mathcal{I}}^{-1}(\mathcal{A})$ is open in $\mathcal{H}$. This topology is called the *quotient topology* on $\mathcal{H}/\mathcal{I}$ and denoted by τ_Φ.

Theorem 2.9. *Let $\mathcal{I}$ be a normal hyperideal of a topological hyperring $(\mathcal{H}, +, \cdot, \tau)$ such that the members of τ are complete parts and $\Phi_{\mathcal{I}}$ be the above mentioned map, then the following results hold.*

1. *$\Phi_{\mathcal{I}}$ is continuous, open good epimorphism.*
2. *$(\mathcal{H}/\mathcal{I}, \oplus, \odot, \tau_\Phi)$ is a topological hyperring.*
3. *If $\mathcal{V}$ is a fundamental system of neighborhoods of zero (i.e., 0) in $\mathcal{H}$, then $\{\Phi_{\mathcal{I}}(V) : V \in \mathcal{V}\}$ is a fundamental system of neighborhoods of zero (i.e., $0 + \mathcal{I} = \mathcal{I}$) for the quotient topology τ_Φ of $\mathcal{H}/\mathcal{I}$.*

Proof. (1) For $x, y \in \mathcal{H}$, $\Phi_{\mathcal{I}}(x + y) = \{z + \mathcal{I} : z \in x + y\} = (x + \mathcal{I}) \oplus (y + \mathcal{I}) = \Phi_{\mathcal{I}}(x) \oplus \Phi_{\mathcal{I}}(y)$. $\Phi_{\mathcal{I}}$ is continuous as per the definition of the quotient topology. Let O be an open subset of $\mathcal{H}$. Claim that $\Phi_{\mathcal{I}}^{-1}(\Phi_{\mathcal{I}}(O)) = O + \mathcal{I}$. To prove $O + \mathcal{I} \subseteq \Phi_{\mathcal{I}}^{-1}(\Phi_{\mathcal{I}}(O))$, take $y \in O + \mathcal{I}$. Then, $y \in p + \mathcal{I}$ for some $p \in O$, which implies $y + \mathcal{I} = p + \mathcal{I}$. Thus, $y \in \Phi_{\mathcal{I}}^{-1}(\Phi_{\mathcal{I}}(O))$. Now, take $x \in \Phi_{\mathcal{I}}^{-1}(\Phi_{\mathcal{I}}(O))$, then $x + \mathcal{I} \in \Phi_{\mathcal{I}}(O)$, which implies $x + \mathcal{I} = r + \mathcal{I}$ for some $r \in O$. Thus, $x \in r + \mathcal{I} \subseteq O + \mathcal{I}$. Hence, $\Phi_{\mathcal{I}}$ is an open map (by (2) of Lemma 2.6).
(2) $\Phi_{\mathcal{I}} \times \Phi_{\mathcal{I}}$ is the map from $\mathcal{H} \times \mathcal{H}$ to $\mathcal{H}/\mathcal{I} \times \mathcal{H}/\mathcal{I}$ defined by $(\Phi_{\mathcal{I}} \times \Phi_{\mathcal{I}})(x, y) = (\Phi_{\mathcal{I}}(x), \Phi_{\mathcal{I}}(y))$ for all $(x, y) \in \mathcal{H} \times \mathcal{H}$. As $\Phi_{\mathcal{I}}$ is a continuous open surjection, so is $\Phi_{\mathcal{I}} \times \Phi_{\mathcal{I}}$. Then, $\oplus \circ (\Phi_{\mathcal{I}} \times \Phi_{\mathcal{I}}) = \Phi_{\mathcal{I}} \circ f$ and $\odot \circ (\Phi_{\mathcal{I}} \times \Phi_{\mathcal{I}}) = \Phi_{\mathcal{I}} \circ f$, where $f = +$ and $\cdot$ respectively. So, the continuity of f implies both $\oplus$ and $\odot$ are continuous (by Theorem 5.3 of [33, p. 33]). Let $\mathfrak{I} : \mathcal{H}/\mathcal{I} \to \mathcal{H}/\mathcal{I}$ be defined by $\mathfrak{I}(x + \mathcal{I}) = (-x) + \mathcal{I}$ for $x \in \mathcal{H}$. Also, $\mathfrak{I} \circ \Phi_{\mathcal{I}} = \Phi_{\mathcal{I}} \circ -$ is continuous, as $-$ is continuous; hence $\mathfrak{I}$ is continuous (by Theorem 5.3 of [33, p. 33]).
(3) For every neighborhood $\mathfrak{U}$ of zero in $\mathcal{H}/\mathcal{I}$, $\Phi_{\mathcal{I}}^{-1}(\mathfrak{U})$ is a neighborhood of zero in $\mathcal{H}$, so there exists $V \in \mathcal{V}$ such that $V \subseteq \Phi_{\mathcal{I}}^{-1}(\mathfrak{U})$. Then, $\Phi_{\mathcal{I}}(V) \subseteq \Phi_{\mathcal{I}}(\Phi_{\mathcal{I}}^{-1}(\mathfrak{U})) = \mathfrak{U}$. ☐

Remark 2.10. It is clear from the above Theorem 2.9 that if $\mathfrak{A}$ is some open subset of $\mathcal{H}/\mathcal{I}$, then there exists open subset $\mathcal{A}$ of $\mathcal{H}$ such that $\mathfrak{A} = \mathcal{A}/\mathcal{I}$.

Theorem 2.11. *Let $\mathcal{B}$ be a subhyperring and $\mathcal{I}$ be a normal hyperideal of a topological hyperring $(\mathcal{H}, +, \cdot, \tau)$ such that $\mathcal{I} \subseteq \mathcal{B}$. If the members of τ are complete parts, then the quotient topology of $\mathcal{B}/\mathcal{I}$ is identical with the topology induced on the subhyperring $\mathcal{B}/\mathcal{I}$ of $\mathcal{H}/\mathcal{I}$ by the quotient topology of $\mathcal{H}/\mathcal{I}$.*

Proof. Let $\Phi_{\mathcal{B},\mathcal{I}} : \mathcal{B} \to \mathcal{B}/\mathcal{I}$ and $\Phi_{\mathcal{H},\mathcal{I}} : \mathcal{H} \to \mathcal{H}/\mathcal{I}$ be the canonical surjections. Let $\mathfrak{O}$ be open for the quotient topology of $\mathcal{B}/\mathcal{I}$. Then, $\Phi_{\mathcal{B},\mathcal{I}}^{-1}(\mathfrak{O})$ is open in $\mathcal{B}$ and $\Phi_{\mathcal{B},\mathcal{I}}^{-1}(\mathfrak{O}) = \mathcal{B} \cap \mathcal{Q}$ for some open subset $\mathcal{Q}$ of $\mathcal{H}$. Claim that $\mathfrak{O} = \mathcal{B}/\mathcal{I} \cap \Phi_{\mathcal{H},\mathcal{I}}(\mathcal{Q})$. For, clearly $\mathfrak{O} \subseteq \mathcal{B}/\mathcal{I} \cap \Phi_{\mathcal{H},\mathcal{I}}(\mathcal{Q})$. For the converse, take $\alpha \in \mathcal{B}/\mathcal{I} \cap \Phi_{\mathcal{H},\mathcal{I}}(\mathcal{Q})$. Then, $\alpha = b + \mathcal{I}$ for some $b \in \mathcal{B}$ and $\alpha = q + \mathcal{I}$ for some $q \in \mathcal{Q}$, which implies $q \in b + \mathcal{I} \subseteq \mathcal{B} + \mathcal{I} = \mathcal{B}$ as $\mathcal{I} \subseteq \mathcal{B}$. Consequently, $q \in \mathcal{B} \cap \mathcal{Q} = \Phi_{\mathcal{B},\mathcal{I}}^{-1}(\mathfrak{O})$, so $\alpha = q + \mathcal{I} \in \mathfrak{O}$.

Now suppose $\mathfrak{R}$ be open in $\mathcal{B}/\mathcal{I}$ for the topology on $\mathcal{B}/\mathcal{I}$ induced by the quotient topology on $\mathcal{H}/\mathcal{I}$. Then, $\mathfrak{R} = \mathcal{B}/\mathcal{I} \cap \mathfrak{S}$ for some open subset $\mathfrak{S}$ of

$\mathcal{H}/\mathcal{I}$. So, $\Phi^{-1}_{\mathcal{B},\mathcal{I}}(\mathfrak{R}) = \mathcal{B} \cap \Phi^{-1}_{\mathcal{H},\mathcal{I}}(\mathfrak{S})$, which is open in $\mathcal{B}$. Hence, $\mathfrak{R}$ is open for the quotient topology of $\mathcal{B}/\mathcal{I}$. □

Corollary 2.12. *Let $\mathcal{B}$ be a subhyperring and $\mathcal{I}$ be a normal hyperideal of a topological hyperring $(\mathcal{H}, +, \cdot, \tau)$. If the members of τ are complete parts, then the quotient topology on $(\mathcal{B}+\mathcal{I})/\mathcal{I}$ is identical with the topology induced by the quotient topology of $\mathcal{H}/\mathcal{I}$.*

Let's define topological isomorphism and prove some topological isomorphism theorems.

Definition 2.13. A homomorphism f between two topological hyperrings satisfying $\ker f \neq \phi$ is said to be a *topological homomorphism* if it is a continuous open mapping. If f is a good, one to one and onto topological homomorphism, then it is called a *topological isomorphism* and in this case the hyperrings are *topologically isomorphic.*

Example 2.14. Consider the topological hyperring $([0, 1], +, \cdot, \tau_u)$ as in Example 2.4, where τ_u is the subspace topology induced from $\mathbb{R}$ with standard topology. $[0, 1)$ being a normal hyperideal of $[0, 1]$, the quotient hyperring $[0, 1]/[0, 1)$ is a topological hyperring with respect to the quotient topology induced by canonical projection $\Phi : [0, 1] \to [0, 1]/[0, 1)$. Now, consider $X = \{0, 1\}$ together with the hyperoperation $\oplus$ and the binary operation $\odot$ defined as follows:

$\oplus$	0	1
0	$\{0\}$	$\{1\}$
1	$\{1\}$	$\{0,1\}$

$\odot$	0	1
0	0	0
1	0	1

Then, $(X, \oplus, \odot, \tau')$ is a topological hyperring, where $\tau' = \{\phi, \{0\}, X\}$. If we define $\psi : [0, 1]/[0, 1) \to \{0, 1\}$ by $\psi(x + [0, 1)) = [x] =$ the greatest integer less than or equal to x, then, ψ is a topological isomorphism.

Example 2.15. For a hyperring $(R, +, \cdot)$ and for some positive integer n, the collection $M_n(R)$ of all $n \times n$ matrices over R forms a Krasner hyperring with respect to the hyperaddition $\oplus$ and multiplication $\odot$ defined as, for $A = (a_{ij}), B = (b_{ij}) \in M_n(R)$, $A \oplus B = \{C \in M_n(R) : C = (c_{ij}), c_{ij} \in a_{ij} + b_{ij}\}$ and $A \odot B = (a_{ij} \cdot b_{ij})$. Now, replace $(R, +, \cdot)$ by the topological hyperring $([0, 1], +, \cdot, \tau_u)$ of Example 2.4, where τ_u is the subspace topology induced from $\mathbb{R}$ with standard topology. If we topologize $M_n([0, 1])$ by identifying it with $[0, 1]^{n^2}$, then, $M_n([0, 1])$ is a topological Krasner hyperring. In a similar manner, we can also obtain the topological Krasner hyperring $M_n(M_n([0, 1]))$.

Now, let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$ be an element of $M_4([0, 1])$ and consider the map $f : M_4([0, 1]) \to M_2(M_2([0, 1]))$ defined by

$$f(A) = \begin{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} & \begin{pmatrix} a_{13} & a_{14} \\ a_{23} & a_{24} \end{pmatrix} \\ \begin{pmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix} & \begin{pmatrix} a_{33} & a_{34} \\ a_{43} & a_{44} \end{pmatrix} \end{pmatrix}.$$

Then, f is a topological isomorphism.

Theorem 2.16. *Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two topological hyperrings such that the open subsets of $\mathcal{H}_1$ are complete parts. Let f be an open, continuous and good topological homomorphism from $\mathcal{H}_1$ onto $\mathcal{H}_2$ such that $\ker f$ is normal in $\mathcal{H}_1$. Then, $\mathcal{H}_1/\ker f$ and $\mathcal{H}_2$ are topologically isomorphic.*

Proof. Clearly, $\psi : \mathcal{H}_1/\ker f \to \mathcal{H}_2$ defined by $\psi(x + \ker f) = f(x)$, for all $x \in \mathcal{H}_1$ is the required isomorphism[24, 7]. So, it is only required to prove that ψ is open and continuous. Consider an open subset $\mathfrak{U}$ of $\mathcal{H}_1/\ker f$. Then, by *Remark* 2.10 there exists an open subset U of $\mathcal{H}_1$ such that $\mathfrak{U} = U/\ker f$. So, $\psi(\mathfrak{U}) = \psi(U/\ker f) = f(U)$, which is open in $\mathcal{H}_2$. To prove ψ continuous, consider an open subset V of $\mathcal{H}_2$. Then,

$$\begin{aligned} \psi^{-1}(V) &= \{x + \ker f : \psi(x + \ker f) \in V\} \\ &= \{x + \ker f : f(x) \in V\} \\ &= f^{-1}(V)/\ker f, \end{aligned}$$

which is open in $\mathcal{H}_1/\ker f$ as $f^{-1}(V)$ is open in $\mathcal{H}_1$. □

Theorem 2.17. *Let f be a homomorphism from a topological hyperring $(\mathcal{H}, +, \cdot, \tau)$ into a topological hyperring $(\mathcal{H}', +', \cdot', \tau')$ such that $\ker f (\neq \phi)$ is normal in $\mathcal{H}$ and members of τ are complete parts. Let $\mathcal{J}$ be a normal hyperideal of $\mathcal{H}$ such that $\mathcal{J} \subseteq \ker f$. Then, the homomorphism g from $\mathcal{H}/\mathcal{J}$ to $\mathcal{H}'$ satisfying $g \circ \Phi_{\mathcal{J}} = f$ is continuous (open, a topological homomorphism) if and only if f is. In particular, if $\mathcal{J} = \ker f$, g is a topological isomorphism (good monomorphism) if and only if f is a topological good epimorphism (good homomorphism).*

Proof. Continuity and openness are consequences of Theorem 5.3 of [33, p. 33] and Theorem 2.9. To prove $\ker g \neq \phi$, take $x \in \ker f$. Then, $f(x) = 0_{\mathcal{H}'}$, which implies $g(x + \mathcal{J}) = 0_{\mathcal{H}'}$. So, $x + \mathcal{J} \in \ker g$.

For the second part, suppose $\mathcal{J} = \ker f$. Let f be a topological good homomorphism. Then, for $x, y \in \mathcal{H}$,

$$\begin{aligned} g((x + \mathcal{J}) \oplus (y + \mathcal{J})) &= g(\{z + \mathcal{J} : z \in x + y\}) \\ &= \{(g \circ \Phi_{\mathcal{J}})(z) : z \in x + y\} \\ &= f(x + y) = f(x) +' f(y) \\ &= (g \circ \Phi_{\mathcal{J}})(x) +' (g \circ \Phi_{\mathcal{J}})(y) \\ &= g(x + \mathcal{J}) +' g(y + \mathcal{J}) \end{aligned}$$

and $g((x + \mathcal{J}) \odot (y + \mathcal{J})) = g((x \cdot y) + \mathcal{J}) = (g \circ \Phi_{\mathcal{J}})(x \cdot y) = f(x \cdot y) = f(x) \cdot' f(y) = (g \circ \Phi_{\mathcal{J}})(x) \cdot' (g \circ \Phi_{\mathcal{J}})(y) = g(x + \mathcal{J}) \cdot' g(y + \mathcal{J})$.
Now, $g(0_{\mathcal{H}} + \mathcal{J}) = (g \circ \Phi_{\mathcal{J}})(0_{\mathcal{H}}) = f(0_{\mathcal{H}}) = 0_{\mathcal{H}'}$. Therefore, $0_{\mathcal{H}} + \mathcal{J} \in \ker g$.

Let $x \in \mathcal{H}$ be such that $g(x+\mathcal{J}) = 0_{\mathcal{H}'}$. Then, $f(x) = 0_{\mathcal{H}'}$ and hence $x \in \ker f$, which implies $x + \ker f = 0_{\mathcal{H}} + \ker f$. Clearly, g is surjective if and only if f is.

For the converse, suppose g is a topological good monomorphism. Then, for $x, y \in \mathcal{H}$,

$$\begin{aligned} f(x+y) &= (g \circ \Phi_{\mathcal{J}})(x+y) \\ &= g(\Phi_{\mathcal{J}}(x) \oplus \Phi_{\mathcal{J}}(y)) \\ &= g((x+\mathcal{J}) \oplus (y+\mathcal{J})) \\ &= g(x+\mathcal{J}) +' g(y+\mathcal{J}) \\ &= (g \circ \Phi_{\mathcal{J}})(x) +' (g \circ \Phi_{\mathcal{J}})(y) \\ &= f(x) +' f(y). \end{aligned}$$

and $f(x \cdot y) = (g \circ \Phi_{\mathcal{J}})(x \cdot y) = g((x \cdot y) + \mathcal{J}) = g((x+\mathcal{J}) \odot (y+\mathcal{J})) = g(x+\mathcal{J}) \cdot' g(y+\mathcal{J}) = (g \circ \Phi_{\mathcal{J}})(x) \cdot' (g \circ \Phi_{\mathcal{J}})(y) = f(x) \cdot' f(y)$. □

Corollary 2.18. *Let $\mathcal{I}, \mathcal{J}$ be normal hyperideals of a topological hyperring $\mathcal{H}$ such that $\mathcal{J} \subseteq \mathcal{I}$ and the open subsets of $\mathcal{H}$ are complete parts. Then, the following results hold:*

(1) *the map $f : \mathcal{H}/\mathcal{J} \to \mathcal{H}/\mathcal{I}$ defined by $f(x+\mathcal{J}) = x + \mathcal{I}$ is a topological good epimorphism.*
(2) *$(\mathcal{H}/\mathcal{J})/(\mathcal{I}/\mathcal{J})$ and $\mathcal{H}/\mathcal{I}$ are topologically isomorphic.*

Proof. (1) Here, $f \circ \Phi_{\mathcal{J}} = \Phi_{\mathcal{I}}$ is an open, continuous good epimorphism from $\mathcal{H} \to \mathcal{H}/\mathcal{I}$ by Theorem 2.9. Then, by Theorem 2.17, f is a topological good epimorphism.
(2) As f is surjective, so, $\ker f \neq \phi$ and

$$\begin{aligned} \ker f &= \{x+\mathcal{J} : f(x+\mathcal{J}) = 0 + \mathcal{I}\} \\ &= \{x+\mathcal{J} : f \circ \Phi_{\mathcal{J}}(x) = \mathcal{I}\} \\ &= \{x+\mathcal{J} : \Phi_{\mathcal{I}}(x) = \mathcal{I}\} \\ &= \{x+\mathcal{J} : x+\mathcal{I} = \mathcal{I}\} \\ &= \{x+\mathcal{J} : x \in \mathcal{I}\} \\ &= \mathcal{I}/\mathcal{J}. \end{aligned}$$

Then, by Theorem 2.17, $g : (\mathcal{H}/\mathcal{J})/(\mathcal{I}/\mathcal{J}) \to \mathcal{H}/\mathcal{I}$ satisfying $g \circ \Phi_{\mathcal{I}/\mathcal{J}} = f$ is the required topological isomorphism. □

Corollary 2.19. *Let $\mathcal{I}, \mathcal{J}$ be hyperideals of a topological hyperring $\mathcal{H}$ such that*

- *$\mathcal{J}$ is normal and $\mathcal{I}$ is open in $\mathcal{H}$;*
- *open subsets of $\mathcal{H}$ are complete parts.*

Then, $\mathcal{I}/(\mathcal{I} \cap \mathcal{J})$ is topologically isomorphic to $(\mathcal{I}+\mathcal{J})/\mathcal{J}$.

Proof. Here, $f : \mathcal{I} \to (\mathcal{I}+\mathcal{J})/\mathcal{J}$ defined by $f(x) = x+\mathcal{J}$, for all $x \in \mathcal{I}$ is an epimorphism such that

$$\begin{aligned} \ker f &= \{x \in \mathcal{I} : f(x) = 0+\mathcal{J}\} \\ &= \{x \in \mathcal{I} : x+\mathcal{J} = \mathcal{J}\} \\ &= \{x \in \mathcal{I} : x \in \mathcal{J}\} \\ &= \mathcal{I} \cap \mathcal{J} \neq \phi. \end{aligned}$$

As $\mathcal{I}$ is open in $\mathcal{H}$, by Theorem 22.1 [22, p. 140] f is a quotient map and hence, f is a topological good epimorphism (by Theorem 2.9). So, by Theorem 2.17, $g : \mathcal{I}/(\mathcal{I}\cap\mathcal{J}) \to (\mathcal{I}+\mathcal{J})/\mathcal{J}$ satisfying $g \circ \Phi_{\mathcal{I}\cap\mathcal{J}} = f$ is the required topological isomorphism. □

Theorem 2.20. *Let $\mathcal{K}$ be a complete part dense subhyperring of a topological hyperring $\mathcal{H}$, and let $\mathcal{I}$ be a closed normal hyperideal of $\mathcal{K}$, $\overline{\mathcal{I}}$ its closure in $\mathcal{H}$. If the open subsets of $\mathcal{H}$ are complete parts, then $g(x+\mathcal{I}) = x+\overline{\mathcal{I}}$ is a topological isomorphism from $\mathcal{K}/\mathcal{I}$ to the dense subhyperring $(\mathcal{K}+\overline{\mathcal{I}})/\overline{\mathcal{I}}$ of $\mathcal{H}/\overline{\mathcal{I}}$.*

Proof. As the open subsets of $\mathcal{H}$ are complete parts, Lemma 3.2 ([11]), continuity of multiplication map and Lemma 3.9 ([28]) imply $\overline{\mathcal{I}}$ is a normal hyperideal of $\mathcal{H}$. Here, the kernel of the restriction $\Phi_{\overline{\mathcal{I}}}|_{\mathcal{K}} : \mathcal{K} \to (\mathcal{K}+\overline{\mathcal{I}})/\overline{\mathcal{I}}$ is

$$\begin{aligned} \ker \Phi_{\overline{\mathcal{I}}}|_{\mathcal{K}} &= \{x \in \mathcal{K} : \Phi_{\overline{\mathcal{I}}}(x) = \overline{\mathcal{I}}\} \\ &= \{x \in \mathcal{K} : x+\overline{\mathcal{I}} = \overline{\mathcal{I}}\} \\ &= \mathcal{K} \cap \overline{\mathcal{I}} = \mathcal{I} \neq \phi \end{aligned}$$

and g satisfies $g \circ \Phi_{\mathcal{I}} = \Phi_{\overline{\mathcal{I}}}|_{\mathcal{K}}$. $\Phi_{\overline{\mathcal{I}}}|_{\mathcal{K}}$ is a continuous good epimorphism, so, by Theorem 2.17, g is a continuous isomorphism from $\mathcal{K}/\mathcal{I}$ to $(\mathcal{K}+\overline{\mathcal{I}})/\overline{\mathcal{I}}$. As $\mathcal{K}$ is dense in $\mathcal{H}$ and $\Phi_{\overline{\mathcal{I}}}$ is continuous, $\Phi_{\overline{\mathcal{I}}}(\mathcal{K}) = (\mathcal{K}+\overline{\mathcal{I}})/\overline{\mathcal{I}}$ is dense in $\mathcal{H}/\overline{\mathcal{I}}$.

Consider an open subset $\mathfrak{O}$ of $\mathcal{K}/\mathcal{I}$ and let $\Phi_{\mathcal{I}}^{-1}(\mathfrak{O}) = P$. Then, *Remark* 2.10 and (1) of Theorem 2.9 imply $g(\mathfrak{O}) = \Phi_{\overline{\mathcal{I}}}(P)$ and $P+\mathcal{I} = P$. P being open in $\mathcal{K}$, there exists an open subset U of $\mathcal{H}$ such that $P = U \cap \mathcal{K}$. Claim that $(U+\overline{\mathcal{I}}) \cap \mathcal{K} = P$. To prove $(U+\overline{\mathcal{I}}) \cap \mathcal{K} \subseteq P$, let $k \in \mathcal{K}$ such that $k \in (U+\overline{\mathcal{I}})$. Then, there exist $u \in U$ and $h \in \overline{\mathcal{I}}$ such that $k \in u+h$. U being a neighborhood of u, there exists a symmetric neighborhood V of zero such that $u+V \subseteq U$. Then, $(h+V) \cap \mathcal{I} \neq \phi$, so there exists $v \in V$ such that $(h+v) \cap \mathcal{I} \neq \phi$. As $\mathcal{I}$ is a closed subset of $\mathcal{H}$, $\mathcal{I}$ is also a complete part of $\mathcal{H}$ and hence $(h+v) \subseteq \mathcal{I}$. Consequently, $u+h \subseteq (u-v)+(v+h) \subseteq (u+V)+\mathcal{I} \subseteq U+\mathcal{I}$. As $\mathcal{K}$ is a complete part, $u+h \subseteq \mathcal{K}$ and hence, by Proposition 2.3 ([28]) $u+h \subseteq (U+\mathcal{I}) \cap \mathcal{K} = (U+\mathcal{I}) \cap (\mathcal{K}+\mathcal{I}) = (U \cap \mathcal{K})+\mathcal{I} = P+\mathcal{I} = P$. The reverse inclusion also follows from Proposition 2.3 ([28]). Therefore, $g(\mathfrak{O}) = \Phi_{\overline{\mathcal{I}}}(P) = \Phi_{\overline{\mathcal{I}}}(U) \cap ((\mathcal{K}+\overline{\mathcal{I}})/\overline{\mathcal{I}})$, an open subset of $(\mathcal{K}+\overline{\mathcal{I}})/\overline{\mathcal{I}}$, for if $x+\overline{\mathcal{I}} \in \Phi_{\overline{\mathcal{I}}}(U)$ where $x \in \mathcal{K}$, then $x \in \Phi_{\overline{\mathcal{I}}}^{-1}(\Phi_{\overline{\mathcal{I}}}(U)) \cap \mathcal{K} = (U+\overline{\mathcal{I}}) \cap \mathcal{K} = P$, which implies $x+\overline{\mathcal{I}} \in \Phi_{\overline{\mathcal{I}}}(P) = g(\mathfrak{O})$. □

Conclusion. As in algebra, distinguishing and classifying topological hyperrings is of great importance in the theory of topological Krasner hyperring too.

This article provides basic tools for this task too. In short, the essence of this article is to show identities in difference between binary and hyper operations. It is hoped that our investigations in the present article will throw much light towards extension of the theory of hyperalgebra and pave the way for further research in this direction.

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