A Model-based Damage Identification using Guided Ultrasonic Wave Propagation in Fiber Metal Laminates

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Abstract: *Fiber metal laminates (FML) are lightweight hybrid structural materials that combine the ductile properties of metal with high specific stiffness of fiber reinforced plastics. These advantages led to a dramatic increase in such materials for aeronautical structures over the last few years. One of the most common and vulnerable defects in FML is impact-related delamination, often invisible to the human eye. Guided ultrasonic waves (GUW) show high potential for monitoring structural integrity and damage detection in thin-walled structures by using the physical phenomena of wave propagation interacting with the defects. The focus of this research project is on describing an inverse solution for the detection and characterization of defect in FML. Model-based damage analysis utilizes an accurate finite element model (FEM) of GUW interaction with the damage. The FEM is developed by the project partners from mechanics at Helmut-Schmidt-University in Hamburg, Germany, and will be treated as a black-box for further analysis. A Bayesian approach (Markov chain Monte Carlo) is employed to characterize the damage and quantify its uncertainties. This inference problem in a stochastic framework requires a very large number of forward solves. Therefore, a profound investigation is carried out on different reduced-order modeling (ROM) methods in order to apply a suitable technique that significantly improves the computational efficiency. The proposed method is well illustrated on a simpler case study for the damage detection, localization and characterization using 2D elastic wave equation. The damage in this case is modeled as a reduction in the wave propagation velocity. The inference problem utilizes a parameterized projection-based ROM coupled with a surrogate model instead of the underlying high-dimensional model.*

1 INTRODUCTION

Fiber reinforced plastics (FRPs), due to their very high strength to weight ratio, are often the favorite choice of material for engineers in building lightweight structures. Although FRPs possess high specific stiffness, they exhibit a weak bearing behaviour and impact resistance. In order to overcome these disadvantages of FRPs, fiber metal laminates (FMLs) are developed in the late 20^{th} century. FMLs have the ability to demonstrate elastic-plastic behavior, as a corollary, a part of the energy introduced by impacts is absorbed by plastic deformations of the metal layers impeding its failure. The most commonly used FML is glass laminate aluminium reinforced epoxy (GLARE), which has excellent fatigue strength, high specific strength and low weight. However, due to its complex structure with different materials, its application is very challenging in terms of its production as well as the damage detection. Guided ultrasonic waves (GUW) have an immense potential in ensuring integrity of the structure and have been extensively used over the last decade. It has been shown that the propagation behavior of GUW changes when interacting with a damage.[1,2]

Numerical studies like finite element methods (FEM) play a crucial role for a well founded analysis of wave propagation and to assess the suitability of the GUW for damage detection. Furthermore, based on these numerical models, the requirements for sensors and actuators

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can be derived with regard to their sensitivity through the solution of an inverse problem. Often high-dimensional FEM analysis will be very expensive which restricts us to use them directly for an inverse problem analysis. To alleviate this burden, projection-based model order reduction techniques are commonly used. There exits two approaches towards solving an inverse problem: the method of maximum likelihood estimation (MLE) and Bayesian estimation. The former results into the best single point estimation of the parameter while the latter models the parameter as a random variable and produces a probability density function (PDF) associated with it. The fact that the likelihood function is often extremely complicated with several local maxima, inhibits the use of MLE approach. Therefore, the inverse optimization problem is reformulated to a stochastic inference problem.[3]

Based on the current status of this research project, we consider a two-dimensional elastic hyperbolic wave equation as a test case, upon which a parameterized reduced order model is developed and Bayesian inference is applied to estimate the damage parameters. The remainder of this paper is organized as follows. Section 2 and section 3 describes the numerical model and the model order reduction approach used in this project respectively. Bayesian stochastic framework for damage identification is described in section 4. Section 5 discusses the results of parametric model reduction and damage characterization. Finally, conclusion and future works are given in section 6.

2 NUMERICAL MODEL

As the FEM model for wave propagation in FML is currently being developed by the project partners from mechanics group at Helmut-Schmidt-University in Hamburg, several potential inverse problem algorithms for damage characterization are simultaneously analyzed at Technical University Braunschweig. This led to the use of a simpler model, a two dimensional elastic hyperbolic wave equation, instead of the FEM model itself.

A 2D plate of isotropic and heterogeneous medium with multiple damages (two damages) is considered and the wave propagation is modeled by the equation:

$$\ddot{u} - c^2 \Delta u = f. \tag{1}$$

Here, $u(\mu, t)$ is displacement of the plate, Δ is the Laplacian in $\mathbb{R}^2$, $c(x, y)$ describes the wave velocity at any given point (x, y) on the plate, and $f(x, y, t)$ is the excitation function. The system is parameterized by $\mu \in \mathbb{R}^{3d}$, where d represents the number of damages and the factor 3 accounts for the number of parameters x, y, c for each of the damages.

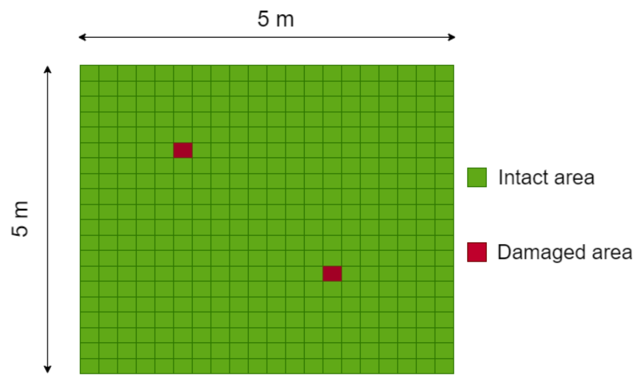


Figure 1: Distribution of wave propagation velocity in the plate

The plate has a side of 5 m and the damage was modeled as a change in the wave propagation velocity. Approximating the spatial derivatives using central difference operators, the considered hyperbolic wave equation can be written as follows:

$$\ddot{u} - Au = f \tag{2}$$

with $A(\mu) = C(x,y)\Delta$. Here, $A(\mu) \in \mathbb{R}^{N\times N}$ is a parameterized symmetric positive definite matrix, $C(x,y) \in \mathbb{R}^{N\times N}$ is a matrix with squared wave velocities at any given x-y coordinate, and $u(\mu,t)$, $f(x,y,t) \in \mathbb{R}^N$ at any given instant of time $t \in [0,T]$. The plate is discretized with an element size of 0.02 m in both x and y directions. The wave propagation velocity in the intact area is assumed to be 0.5 ms^{-1}. Figure 1 represents the distribution of wave propagation velocity in the plate. The green intact area of the plate has the highest velocity (0.5 ms^{-1}) whereas the brown regions represent the damages with relatively lower propagation velocity. Based on Courant-Friedrichs-Lewy condition[4], the time step for numerical integration of the system is evaluated as 0.02 s in order to avoid the convergence issues.

3 PARAMETRIC MODEL ORDER REDUCTION

The numerical simulation of large-scale engineering problems requires a huge computational effort. To overcome this computational cost, projection-based model reduction techniques are often employed to reduce the model without a considerable loss of accuracy. The order reduction is accomplished by projecting the full order solution to the reduced order space using an orthogonal projection matrix $\Phi \in \mathbb{R}^{N\times n}$ such that,

$$u \approx u_h = \Phi\alpha \qquad\qquad \ddot{u} \approx \ddot{u}_h = \Phi\ddot{\alpha}. \tag{3}$$

where, u_h is the approximation of displacement u. Inserting (3) into (2) and projecting it onto the lower dimensional space leads to the reduced order problem,

$$\begin{aligned} \Phi\ddot{\alpha} - A\Phi\alpha &= f \\ \Phi^T\Phi\ddot{\alpha} - \Phi^T A\Phi\alpha &= \Phi^T f \\ \ddot{\alpha} - A_r\alpha &= f_r \end{aligned} \tag{4}$$

where, $\alpha(\mu,t) \in \mathbb{R}^n$, $A_r(\mu) \in \mathbb{R}^{n\times n}$ and $f_r(x,y,t) \in \mathbb{R}^n$ at any given instant of time t. The projection matrix Φ can be obtained by proper orthogonal decomposition (POD) of adaptively extracted features of the system. The displacements of the system that are numerically evaluated at m discrete time steps are saved in an observation matrix called snapshot matrix $U \in \mathbb{R}^{N\times m}$

$$U = \begin{bmatrix} | & | & & | \\ u(t_1) & u(t_2) & \dots & u(t_m) \\ | & | & & | \end{bmatrix}. \tag{5}$$

The snapshot matrix is then split into its basis and coefficients using singular value decomposition, $U = P\Sigma V^T$. Here, $\Sigma \in \mathbb{R}^{m\times m}$ is a diagonal matrix containing singular values σ_j, $P \in \mathbb{R}^{N\times m}$ is a left singular matrix with proper orthogonal modes (POMs) and $V \in \mathbb{R}^{m\times m}$ is a right singular matrix. The projection error incurred for considering upto σ_k singular values can be measured as

$$E = \frac{\sum_{j=k+1}^{m} \sigma_j^2}{\sum_{j=1}^{m} \sigma_j^2} \tag{6}$$

see Kerschen and Golinval, 2002[5]. Using (6), the required level of accuracy to capture the energy of the system can be chosen and subsequently, the number of POMs that enriches the projection matrix can also be decided

$$\Phi = [p_1, p_2, ..., p_n]. \tag{7}$$

As the governing equation depends on several parameters like x-y coordinates of the central position of the damage(s) and wave propagation velocity c in the damaged area(s), a parametric model order reduction (PMOR) is targeted. Due to its affine parameter dependency of the wave equation, PMOR involves an offline training phase, where the projection matrix Φ is built. This ensures that the projection matrix need not vary with the model parameters during the inverse problem analysis (online phase). After an intensive literature review, it was found that there was only one previous work that studied PMOR for hyperbolic wave equation using classical POD-Greedy approach[6]. However, in this project, an adaptive POD-Greedy procedure with kriging based on the work of Paul-Dubois-Taine[7] is applied to accomplish the PMOR through an optimized exploration strategy. This includes construction of a surrogate model for evaluating the reduced model error estimates, finding the largest error estimate, solving the full order model for the corresponding parameter sample with largest error estimate and subsequently updating the reduced-order model (ROM). The error estimate[6,8,9] at time t used in this procedure is as follows:

$$e_h(\mu) = \|u - u_h\| \leq \sqrt{\left(\frac{\gamma}{\beta}\left\|e_{h,0}\right\|^2 + \frac{1}{\beta}\left\|\dot{e}_{h,0}\right\|^2\right)} + \frac{1}{\sqrt{\beta}}\int_0^t \|r(s)\|\, ds \tag{8}$$

where, $e_{h,0}$ and $\dot{e}_{h,0}$ are the error estimate and its derivative at $t = 0$ respectively. β and γ are the coercivity and continuity constants of $A(\mu)$ and the residual is given by r with $s \in [0, T]$. After each greedy iteration, more error estimates are available to build the surrogate model. As the greedy algorithm proceeds, it eventually makes the error model more accurate and thereby finds a more optimal reduced space. It is essential to ensure that Φ remains orthogonal in this procedure. The offline phase can be terminated whenever the largest error estimate in an iteration is less than the specified threshold error value. Once the projection matrix Φ which is enriched with the required number of POMs is obtained, the solution can be evaluated using (3).

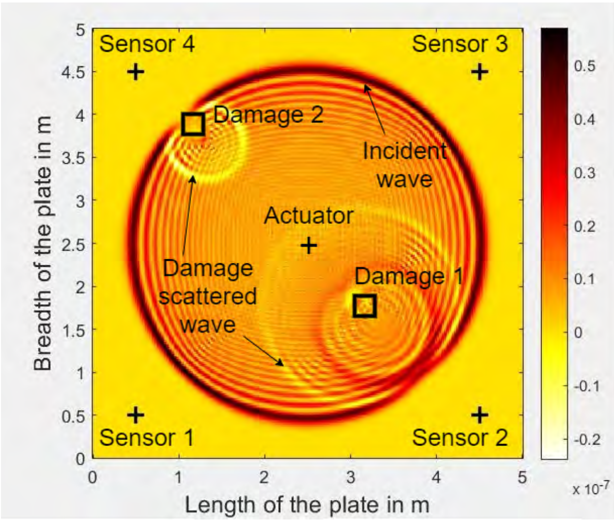
4 BAYESIAN INFERENCE FOR DAMAGE CHARACTERIZATION

Given the shape and size of the damage, the parameters $\mu = \{x, y, c\}$ for a damage are estimated using the Bayesian stochastic framework. The parameter vector μ is represented by a prior probability distribution $P(\mu|I)$ conditioned upon the prior knowledge I on the parameters. The posterior PDF $P(\mu|D, I)$ given data D and prior information is given by the Bayes' formula:

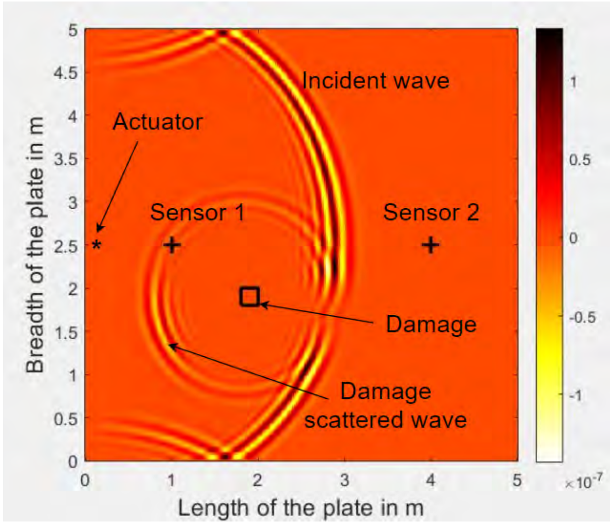
$$P(\mu|D, I) = \frac{P(D|\mu, I)P(\mu|I)}{P(D|I)} \tag{9}$$

where, $P(D|\mu, I)$ is the likelihood function that describes how likely are the candidate parameters to produce the given measurement data. The denominator $P(D|I)$ is called as marginal likelihood or evidence which ensures the integration of the posterior PDF results to 1. Unlike the deterministic approach that yields point estimates of the damage parameters, Bayesian inference method aims to describe posterior distribution for a given set of measurement data D. This allows the researcher to quantify the uncertainties associated with those parameters. The L_2 norm of the residual between the measurements and model output recorded at each sensor is considered to identify the damage. This quantity implicitly signifies the time-of-flight information. In this test case problem, four sensors are located at 4 corners of the plate with an actuator in the center that establishes a pitch-catch configuration to characterize the damage (see Figure 2(a)). The presence of model and measurement errors are described together by the variable ε. For convenience, ε is assumed to be an independent Gaussian variable with its mean at zero and standard deviation of σ_ε, $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon)$. This uncertainty is added to the model output to generate synthetic data D which is used to carry out this inference problem.

The evaluation of posterior distribution is often analytically intractable and hence one tend to draw samples numerically from the posterior. A more commonly used procedure is Markov chain Monte Carlo (MCMC) method, which results into a dependent sequence of samples from a stationary distribution, asymptotically equal to that of the target distribution. Of several existing MCMC variants, we describe Metropolis-Hastings (MH) algorithm[10] and the same is used in this work.



(a) Multiple damages



(b) Single damage

Figure 2: A snapshot of wave propagation and damage scattering

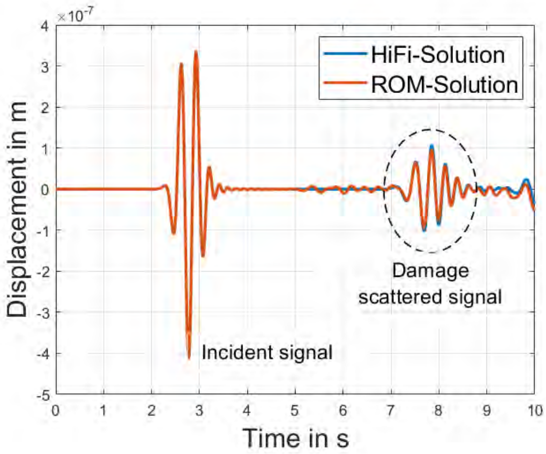
An arbitrary sample from the prior distribution μ_i is picked, then the algorithm produces a proposal candidate sample μ_i^* using a stochastic model $P(\mu_i^*|\mu_i)$ which denotes the probability of attaining μ_i^* conditioned upon the current sample. Both the samples μ_i^*, μ_i are then used to evaluate the ratio r:

$$r = \frac{P(D|\mu_i^*, I) \times P(\mu_i^*|I) \times P(\mu_i|\mu_i^*)}{P(D|\mu_i, I) \times P(\mu_i|I) \times P(\mu_i^*|\mu_i)} \tag{10}$$

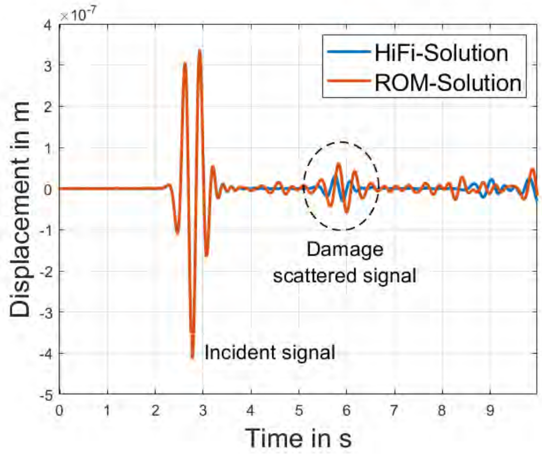
which is nothing but the ratio of their posteriors multiplied by the ratio of the candidate generating stochastic models. The current sample μ_i is updated to the proposal candidate sample μ_i^* if the ratio $r > z$, where z is a random value between 0 and 1. This acceptance-rejection sampling is iteratively carried out for a large specified number of samples, N_T, which ensures that the resulting Markov chain is stationary. Often, when starting from an arbitrary sample, there exist an initial phase of non-stationary period n_B while building the chain. This period is called 'burn-in' period and the samples until n_B have to be discarded to represent the final posterior distribution.

5 RESULTS

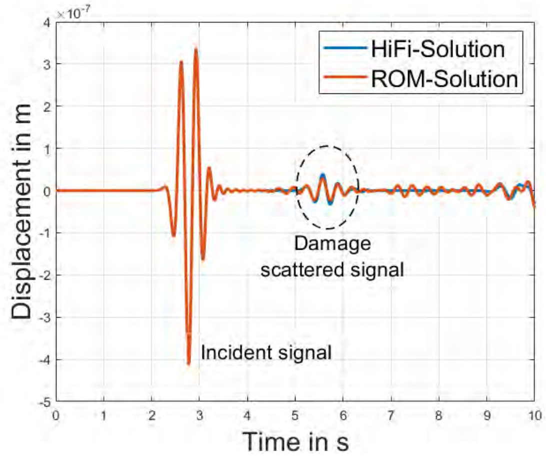
For convenience, model reduction is carried out on a slightly different setup with 2 sensors and one damage as shown in figure 2(b). The PMOR is trained in the parametric domain, $\mathcal{P} = \{x \times y \times c \mid [0.5, 4.5] \times [0.5, 4.5] \times [0.05, 0.45]\}$. The application of adaptive POD-Greedy algorithm as described in section 3 produced 800 global modes that could very well capture dynamics of the system for any sample μ from $\mathcal{P}$. Figure 3 depicts the reconstruction of wave signal measured at sensor 1, as shown in figure 2(b), for four randomly selected parameter samples in $\mathcal{P}$. On a 4-core Intel(R) Core(TM) i7-10510U CPU @ 1.80 GHz processor with 16 GB RAM, the evaluation of high-fidelity (HiFi) 2D elastic wave equation took 1.73 s while the reconstruction using global modes took 1.92 s. Based on the computation time, the first instinct questions the purpose of model reduction. But the actual computational efficiency of PMOR can be realized when applied to a much sophisticated higher-dimensional problem, for example, the FEM-model of composite structures which involves the evaluation of individual element shape functions. However, the application of adaptive POD-Greedy PMOR on hyperbolic wave equation is very well demonstrated through this test case.



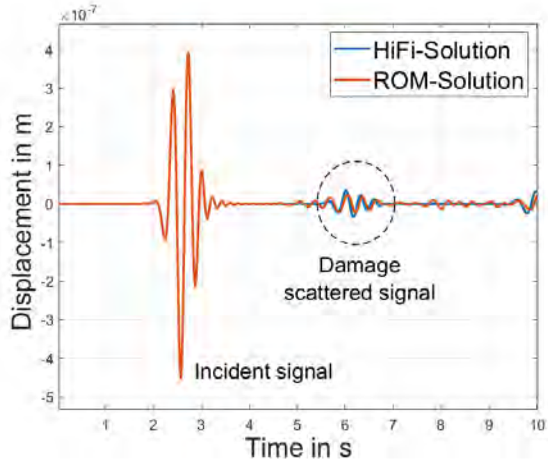
(a) $\mu = [2.2, 2.2, 0.05]$



(b) $\mu = [1.4, 3.0, 0.05]$



(c) $\mu = [1.68, 3.0, 0.3]$



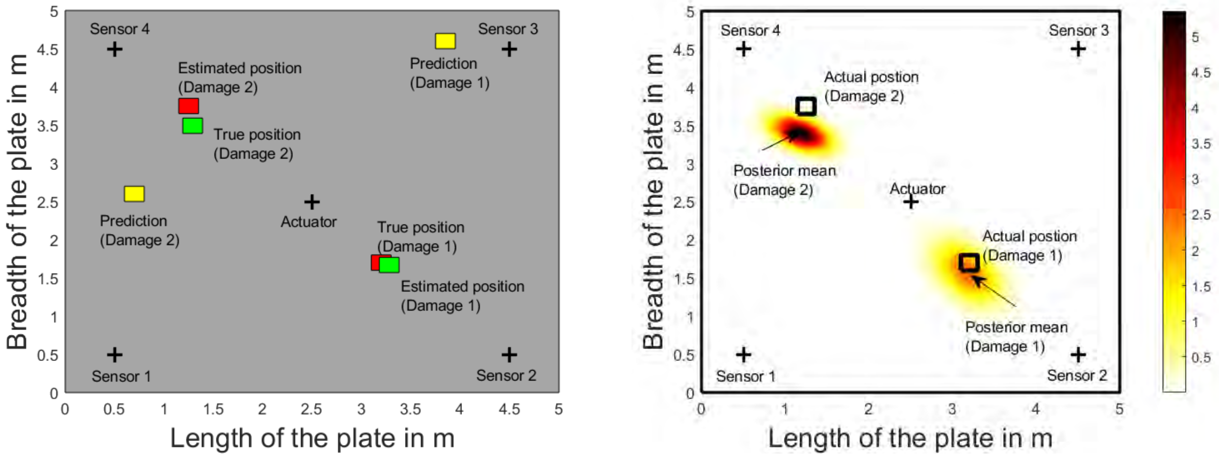
(d) $\mu = [1.68, 2.6, 0.15]$

Figure 3: Comparison of reduced-order solution with high-fidelity solution

Bayesian inference for damage characterization was informed by the reduced-order model instead of the high-fidelity model. In order to embed multiple damages, the configuration shown in figure 2(a) is used to estimate the damage parameters and quantify their uncertainties. The MCMC approach described in section 3 is performed to localize and characterize the damages. The measurement data is obtained by adding a zero mean Gaussian-type errors to the model output. The data used for Bayesian inference is generated as follows:

$$D = M(\mu, t) + \varepsilon \tag{11}$$

where, $M(\mu, t)$ is the noise-free model output and ε is the normally distributed measurement error of 5%. The damage localization parameters, i.e., the x-y coordinates are uniformly distributed in $[0.5, 4.5]$ m and the localized wave propagation velocity in the damaged areas are also uniformly distributed in $[0.05, 0.45]$ ms^{-1}. By MCMC-MH algorithm, 35000 samples from the posterior distribution are drawn. Figure 4(a) illustrates the point estimate and figure 4(b) shows the joint posterior PDF of the x-y coordinates of the center location of the damages in 2D view. The posterior PDF is not normalized with the evidence. The identified center locations of damage 1 and damage 2 are 0.21 m and 0.11 m respectively away from their actual locations accounting for relative errors of 4.2% and 2.2% with respect to the minimum sensor spacing. Similarly, the propagation velocities in damage 1 and damage 2 are estimated to have relative errors of 3.3% and 6.08% respectively. These small quantities of errors in parameter estimation describes the effectiveness of Bayesian inference approach.



(a) Deterministic estimate **(b)** Stochastic estimate

Figure 4: Illustration of the localization of the damages with center locations at $(3.2, 1.7)$ and $(1.25, 3.75)$ using the deterministic and stochastic approaches

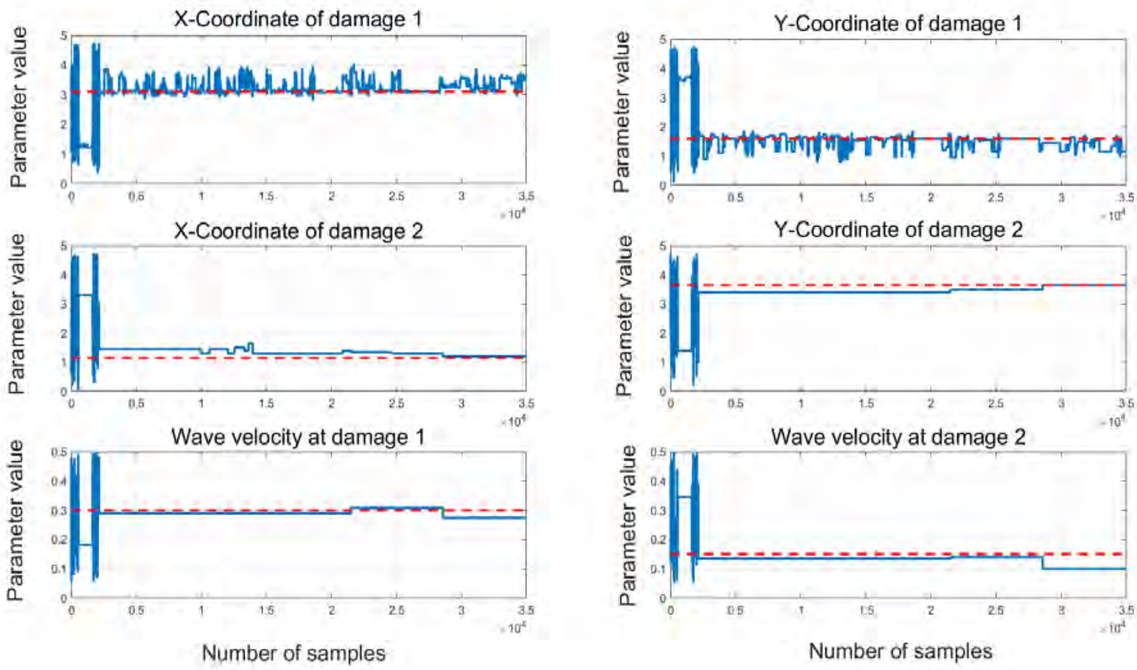


Figure 5: Trace plot of the estimated damage parameters using Bayesian approach with their true values indicated in red dashed lines.

Trace plots and histograms for the damage parameters corresponding to 5% measurement error are shown in figure 5 and 6 respectively. Trace plots show the Markov chains for each parameter while the histograms represent their marginal posterior distributions. The true values are indicated in red dashed lines in each of these plots. The histograms indicate that all the parameters appear normally distributed with some skewness around their true values.

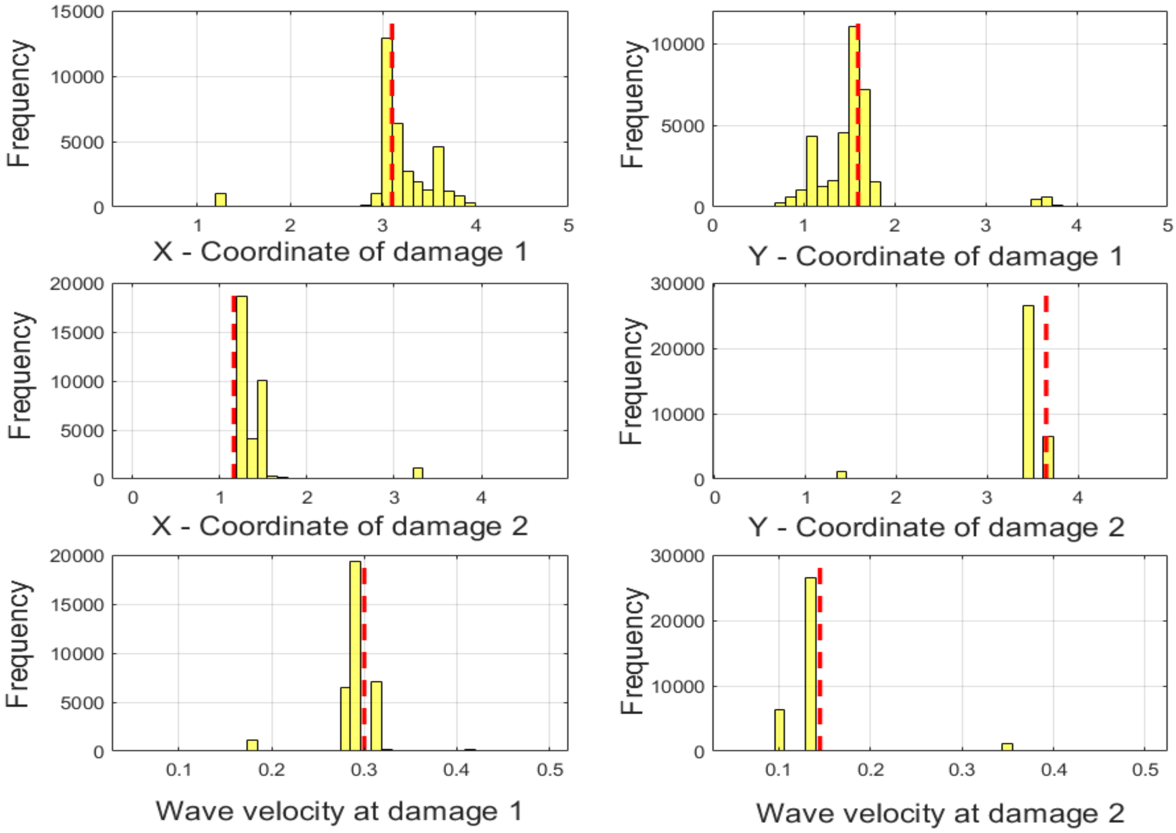


Figure 6: Histograms of the estimated damage parameters using Bayesian approach

Table 1: CoVs of damage parameters for 5% measurement error

Parameters	Damage 1	Damage 2
x	0.144	0.122
y	0.324	0.101
c	0.318	0.374

The uncertainties associated with parameters are analyzed using coefficient of variation (CoV)[11]. CoV is defined as the ratio of the standard deviation to the mean of the distribution. The CoVs for the estimated damage parameters are listed in table 1. The values of CoVs increase as the standard deviation of the error model in (11) increases. This is illustrated in the table 2 containing the CoVs for 7% and 10% error. Therefore, it is crucial to recognize the fact that the estimation uncertainties are positively correlated with the errors, i.e., uncertainties magnifies with the rise in modeling and measurement errors.

Table 2: CoVs of damage parameters for 7% and 10% measurement errors

	7 %		10 %	
Parameters	Damage 1	Damage 2	Damage 1	Damage 2
x	0.287	0.274	0.422	0.421
y	0.382	0.219	0.451	0.407
c	0.412	0.433	0.487	0.574

6 CONCLUSIONS AND FUTURE WORK

This work implemented considerable amount of the future work associated with this research project. An investigation of the applicability of parametric model-order reduction and Bayesian framework for damage identification is presented here. The effectiveness of the proposed approaches for model-order reduction and damage identification is validated by a numerical experiment on a two-dimensional elastic wave equation. The numerical study showed that not only the damages are localized but also the defects are well characterized, i.e., the wave velocities in the damaged areas are also estimated in this case study. The described adaptive POD-Greedy procedure with kriging produced a global projection matrix over the entire parametric domain which is used to evaluate the reduced-order solution. Subsequently, the Bayesian approach for inferring the damage parameters employed the reduced-order model rather than the high-fidelity model. Unlike pinpointing the estimate of parameters through the classical deterministic method, the Bayesian inference produced a distribution for the damage parameters. These distributions are not only used to identify the damage parameters with certain confidence levels but also to quantify their associated uncertainties.

Future work concerns the application of the presented methods on a finite element model of guided wave propagation in fiber metal laminate structures for the damage identification. Obviously, the anisotropic nature of the material could possibly impose challenges which need to be addressed. The number of parameters involved in the constitutive modeling of composite materials is usually large and hence a prior sensitivity analysis should be performed in order to ignore the less influential parameters in damage characterization. Also the embedded sensors and actuators could potentially act as defects, hence novel techniques are required to solve this artefact. It is ultimately aimed to detect, localize and characterize the class and degree of the damage in FML structures. This study can also be further extended to estimate the in-plane residual strength of the structure corresponding to the estimated parameters using a suitable artificial neural network model.

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