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An orbital model for the Parker Solar Probe mission: Classical vs relativistic effects

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Abstract

The Parker Solar Probe is a spacecraft designed to study the Sun's corona from inside. It is providing unprecedented detailed information on the density and composition of the Sun's atmosphere as well as the electromagnetic fields, plasma and solar wind. On the other hand, this probe is to achieve record speeds in the International Celestial Reference Frame (ICRF) never obtained before in any previous mission. It is expected that in the last perihelion of 2025 it would move at 0.064% of the speed of light with respect to the barycenter of the Solar System. By this time it will approach only 9.86 solar radii to the center of the Sun. These orbital conditions make the Parker's Solar Probe also an interesting experiment concerning the validity of General Relativity (GR). The combination of a high velocity and a relatively intense gravitational field increases the values of the post-Newtonian terms governing the orbital corrections by GR. In this paper, we consider an orbital model for the Parker Probe trajectory, including the important effect of radiation pressure, to calculate the relativistic corrections. From this model, we compare the magnitude of the corrections in order to evaluate the possibility of obtaining a test of GR from spacecraft missions orbiting close to the Sun.

Keywords: Parker Solar Probe; Orbital models; Tests of general relativity; Post-Newtonian approximation

1. Introduction

Robotic spacecraft and space probes have been tremendously important in the last sixty years as the principal way of conducting exploratory investigations of the Solar System. NASA's has conducted the majority of these missions starting with the Pioneer 5 launched on March, 11th, 1960 and which was aimed at the study of the interplanetary magnetic fields between the orbits of the Earth and Venus. Later on, the "grand tour" project was designed for the successive exploration of Jupiter, Saturn, Uranus and Neptune in the seventies and eighties of the past century by the Pioneer 10 and 11 as well as the Voyager 1 and 2 missions (Butrica, 1998). These missions made extensive use of the flyby manoeuvers as conceived by G. Flandro and M. Minovitch (Flandro, 1966). These manoeuver exploits the energy interchange between the spacecraft and the Sun-planet system to gain (or reduce) the spacecraft kinetic energy in the Solar System Barycentric Frame.

As early as 1958, at the beginning of the Space Age, the Fields and Particles Group of the National Academy of Sciences proposed a mission to pass inside the orbit of Mercury to study the particles and fields in the vicinity of the Sun. The Parker Solar Mission is the realization of this concept after sixty years of the original proposal. Despite the long wait, the interest of a mission for the close investigation of the Sun is still intact in the scientific community,

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especially among specialists in Solar Physics but, as we will discuss in this paper, also celestial mechanics and Relativity.

After some redesign of several missions proposals, the Parker Solar Probe was finally launched on August, 12th, 2018 on board a Delta IV Heavy rocket. It reached the first perihelion on November, 6th of the same year, following a previous flyby of Venus on October, 3th. The mission was planned to perform a total of seven flybys of Venus to obtain a progressive shortening of the orbital period of the spacecraft in its orbit around the Sun and also a closer perihelion to the center of the star. The probe is expected to be operational until the end of 2025 reaching its perihelion 26th on December, 12th of that year. At its closest approach, it will be located at only 6.9 million kms from the center of the Sun (or 9.86 solar radii) and it would move at 192 km/s becoming the fastest probe ever as well as the one that has approached closest to the Sun. Apart from the Venus flybys, the trajectory is also modified and fine-tuned employing trajectory correction maneuvers carried out at specified times. The propulsion is provided by 12 4.4-N blowdown monoprop hydrazine thrusters.

The most essential component of this spacecraft is the hexagonal solar shield that protects all the scientific instrumentation from the intense solar radiation received during the approximation to the perihelion. This solar shield is designed with reinforced carbon–carbon composite also used for protection against friction heat in reentry vehicles. As any deviation from the correct attitude might be fatal for the spacecraft, automatic reorientation is achieved using reaction wheels. This solar shield has a diameter of 2.3 m and its modelling is important for the dynamics of this spacecraft because of the influence of radiation pressure in its trajectory.

The mission is named after astrophysicist E. N. Parker who in 1958 proposed the existence of the Solar wind (Parker, 1958). Its main objective is the investigation of this stream of plasma particles moving outwards from the Sun but it is also aimed at studying in depth the Solar corona through the corresponding instrumentation: (i) flux-gate and search-coil magnetometers and plasma sensors are used to analyze the electric and magnetic fields as well as the plasma density and electron temperature. (ii) Energetic particles instrumentation is used to analyze the most energetic electrons, protons and heavy ions (iii) wide-field telescopes have the objective of obtaining images of the corona and heliosphere (iv) Electrostatic analyzers and Faraday cups will obtain number counts of electrons and protons as well as their velocity, density and temperature. For further details about the mission and early results see Guo et al. (2021).

Despite it has not been considered as a principle objective in the mission proposal, we could also add to the list the study of the spacecraft trajectory to understand the contributions of the many perturbations acting upon it. In particular, the radiation pressure and the corrections provided by General Relativity. Utilizing the Deep Space Network the position of spacecraft in the International Celestial Reference Frame is tracked based on the Delta-Differential One-Way Ranging (Delta-DOR) technique (Border et al., 2015). The difference in arrival times of radio signals emitted by these spacecraft can be used to pinpoint their positions in space with great accuracy. The Parker Solar Probe uses the X-band radar in the frequency range of 8.0-12.0 GHz. Assuming sufficient signal intensity and minimal interference (a signal-to-noise ratio of 18 dB Hz or greater) the accuracy in the location of the spacecraft can reach maximum errors of 0.04 m. However, as the Parker Solar Probe mission is aimed at the Sun we must not forget that the Solar corona is a source of *X*-rays and, consequently, may reduce the signal-to-noise ratio in the *X*-band and the accuracy of the radar. On the other hand, we will see in this paper that the relativistic effects can manifest as a difference in the spacecraft ephemeris as large as several kilometers. As this is $\simeq 10^5$ times larger than the maximum accuracy of the tracking precision, this opens the possibility of using the Parker Solar Probe's mission to obtain new constraints on the Post-Newtonian parameters if other perturbations (especially the effect of the radiation pressure) are taking into account with enough accuracy to be determined in this work.

There is a long tradition, starting in the sixties of the past century, to test General Relativity by using spacecraft and radar technologies. In 1964, I. Shapiro calculated that the round-trip travel time of a radar signal aimed at a spacecraft or celestial body would be increased by a certain amount that depends upon the mass of a nearby object (Shapiro, 1964). In particular, for echoes bouncing from the surface of Mercury or Venus, when these planets are on the other side of the Sun and aligned with the Earth, this echo delay could be as large as 200 μs due to the effect of the Sun. This was successfully tested in 1968 by using the Haystack radar at the MIT Lincoln Laboratory (Shapiro et al., 1968). Shapiro echo delays are now, routinely, considered in the interpretation of telemetry observations. There is also a source of errors in this effect arising from the uncertainty in the masses of the Sun and the planets as well as the parameters β and γ of the post-Newtonian approximation. In this paper, we will also consider the contribution of the Shapiro effect for the processing the telemetry data to constrain both β and γ from the spacecraft trajectory positions in the ICRF (International Celestial Reference Frame).

Another example is the Gravity Probe B experiment in which a satellite carried four London moment gyroscopes. Alignment of these gyroscopes was achieved with a reference telescope and, after some corrections involving classical electromagnetic effects, the predictions of General Relativity for the geodetic and frame-dragging precessions in the direction of the spin of the gyroscopes were checked with an error no larger than a 20% (Everitt et al., 2011). More recently, the BepiColombo mission to Mercury has, among other goals, the measurement of the parameters β

and γ of the Post-Newtonian formalism in General Relativity (Milani et al., 2002; Schettino and Tommei, 2016).

An even more recent example: the satellites GSAT-0201 and GSAT-0202 of the European GNSS constellation were launched in 2014 and they ended up, unintentionally, in eccentric orbits. Herrmann et al. (Herrmann et al., 2018) then showed that they can be used to test the gravitational redshift and relativistic Doppler effects with an accuracy four orders of magnitude larger than those obtained with Gravity Probe A back in the seventies of the past century (Vessot and Levine, 1979).

More generally, high accuracy tracking of spacecraft also allows for a continuous test of General Relativity in many missions, even if the design team have not considered this as an initial objective. In this paper, we evaluate the contribution of General Relativity to the trajectory of the Parker Solar Probe close to the Sun to show that it implies a difference in the tracked position larger enough to be detected and analyzed. We compare with the magnitude of other perturbations, including the effect of planets and satellites in the Solar System as well as the Solar Radiation pressure exerted upon the shield and the solar panels of the spacecraft. To gauge the contributions of relativistic effects vs the classical perturbations is an important enterprise in celestial mechanics and spacecraft dynamics which is still the object of intense research. Recently, Phillip et al. (Philipp et al., 2018) have used the XHPS integrator to simulate both the post-Newtonian relativistic effects and the contribution of other factors such as solar radiation pressure (SRP), Earth albedo, atmospheric drag and thermal radiation pressure to the orbit of some artificial satellites around the Earth such as GRACE and those in the GNSS constellation. These authors found that for Earth satellites SRP is, usually, the dominant effect with thermal radiation pressure and Earth albedo being of the same order of magnitude as the relativistic effects. In this work, we show, similarly, that SRP is a key contribution to the Parker Solar Probe acceleration in comparison with other classical or relativistic effects.

The paper is organized as follows: In Section 2 we discuss the orbital model and the perturbations corresponding to all the planets and other celestial bodies in the Solar System, the effect of the radiation pressure and the contribution of General Relativity. In Section 3 we discuss the magnitude of the different perturbations and their influence in the spacecraft trajectory as well as the key signature of General Relativity in the orbital model. Finally, in Section 4 we discuss the interest to fundamental physics of the careful analysis of the Doppler and tracking data of spacecraft missions, specially those with extreme orbits never studied before as the Parker Solar Probe or the new Solar Orbiter (Müller et al., 2020).

2. The orbital model

To model the trajectory of any celestial body or spacecraft, we must choose an adequate coordinate system. In interplanetary missions it is now standard to use the International Celestial Reference Frame (ICRF). The ICRF is a quasi-inertial frame of reference centered at the barycenter of the Solar system. Their axes are measured from the positions or extragalactic sources (such as quasars) using very long baseline interferometry (Giorgini, 2019). The X axis of this frame points towards the vernal equinox, the Y axis is perpendicular to it and lies in the same plane that the equator of the Earth, with the Z axis pointing towards the North Pole.

All modern planetary ephemerides are obtained by using the Einstein-Infeld-Hoffmann equations of motion, i. e., the equations of the post-Newtonian formalism at first order. In this formalism, the complete equation of motion for the Solar System Barycenter is also calculated (Fienga et al., 2008).

The equations of motion for the spacecraft can be written as follows:

$$\dot{\mathbf{v}} = \mu_{\text{Sun}} \frac{\mathbf{r}_{\text{Sun}} - \mathbf{r}}{|\mathbf{r}_{\text{Sun}} - \mathbf{r}|^3} + \mathbf{P}, \tag{1}$$

where μ_{Sun} is the mass constant of the Sun, i.e., the product of the gravitational constant times the mass of the Sun, $\mathbf{r}$ and $\mathbf{r}_{\text{Sun}}$ are the coordinate position vectors of the spacecraft and the center of the Sun with respect to the origin of the Solar System Barycenter Frame and $\mathbf{P}$ contains all perturbation forces acting upon the spacecraft. The acceleration of the spacecraft is denoted as $\dot{\mathbf{v}}$, i. e., we denote derivatives with respect to coordinate time by using the dot convention.

There are three main sources of perturbation to be taken into account in our model:

$$\mathbf{P} = \mathbf{P}_{\text{planets}} + \mathbf{P}_{\text{solarpressure}} + \mathbf{P}_{\text{RG}}, \tag{2}$$

each of them are described below:

1. In the first place we must consider the gravitational interactions of the spacecraft with all the planets and other bodies in the Solar System. In particular, the closer and larger ones (the planets from Mercury to Saturn). These contribute to the spacecraft acceleration in the ICRF with terms of the same form as those of the Sun:

$$\mathbf{P}_{\text{planets}} = \sum_i \mu_i \frac{\mathbf{r}_i - \mathbf{r}}{|\mathbf{r}_i - \mathbf{r}|^3}, \tag{3}$$

   where the sum extends over all celestial bodies in the Solar System, $i = \text{Mercury}, \text{Venus}, \text{TheEarth}, \ldots$.
2. Another major source of perturbations in missions to the Sun is the radiation pressure. This force has even been considered as a method of spacecraft propulsion (Wright, 1992) and its effects in spacecraft trajectory and orientation have been studied and applied in mission design since the beginnings of the space age (Georgevic, 1973). Here, we assume a simple model in which, by attitude control, the spacecraft shield is always perpendicular to the incident light rays. Under

the hypothesis of perfect reflection the force (per unit area of the shield) acting upon the spacecraft by solar pressure would be:

$$\mathbf{P}_{\text{solarpressure}} = \frac{S_0 R_0^2}{m\,c} \frac{\mathbf{r} - \mathbf{r}_{\text{Sun}}}{|\,\mathbf{r} - \mathbf{r}_{\text{Sun}}|^3}, \tag{4}$$

where $S_0 = 1367\ \text{W/m}^2$ is the intensity of Solar radiation at 1 AU from the Sun, $R_0 = 1$ AU, c is the speed of light in vacuum, m is the mass of the spacecraft, and we consider that the radiation pressure force points outwards from the center of the Sun.

3. The contribution of General Relativity considered as a perturbation of the classical Newtonian gravitational force. In Celestial Mechanics the effect of General Relativity is usually computed in the context of the Einstein-Infeld-Hoffman (EIH) formalism (Will, 2016) which is equivalent to the linearized field equations and the post-Newtonian equations of motion (Brumberg, 2007). For our problem these equations can be written as follows:

$$\begin{aligned}\mathbf{P}_{\text{RG}} = {} & \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B\mathbf{n}_{BA}}{r_{AB}^2}\left[v_A^2 + 2v_B^2 - 4(\mathbf{v}_A\cdot\mathbf{v}_B) - \frac{3}{2}(\mathbf{n}_{AB}\cdot\mathbf{v}_B)^2\right.\\ & \left. -4\sum_{C\neq A}\frac{Gm_C}{r_{AC}} - \sum_{C\neq B}\frac{Gm_C}{r_{BC}} + \frac{1}{2}((\mathbf{x}_B - \mathbf{x}_A)\cdot\mathbf{a}_B)\right]\\ & + \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B}{r_{AB}^2}[\mathbf{n}_{AB}\cdot(4\mathbf{v}_A - 3\mathbf{v}_B)](\mathbf{v}_A - \mathbf{v}_B)\\ & + \frac{7}{2c^2}\sum_{B\neq A}\frac{Gm_B\mathbf{a}_B}{r_{AB}} + \mathcal{O}(c^{-4}),\end{aligned} \tag{5}$$

where A denotes the Parker Solar Probe and B, C, other bodies in the Solar System. Here, $\mathbf{a}_A$ is the relativistic contribution to the acceleration in the ICRF of the celestial bodies considered, r_{AB} is the distance between the Parker Solar Probe and the body B, r_{BC} is the distance between the celestial bodies B and $C, \mathbf{n}_{BA}$ is the unit vector in the direction from the Parker Solar Probe to the body $B, \mathbf{v}_A$ is the velocity vector of the probe (v_A being its modulus), c is the speed of light, G is the gravitational constant and $\mathbf{m}_B$ the mass of the body B. The EIH equations were originally derived by Einstein and his collaborators Einstein et al. (1938). The idea is that we can build a Lagrangian for the motions of a system of particles in General Relativity valid up to order $\mathcal{O}(c^{-4})$ because the emission of gravitational radiation is an effect that modifies the particles trajectories at $\mathcal{O}(c^{-5})$. The derivation of this Lagrangian was deduced by Landau and Lifshitz and the corresponding Euler–Lagrange equations are those given in Eq. (5). Usually, the expression in Eq. (5) is written with the Newtonian term included but we have separated these terms in Eqs. (1) and (3).

For a thorough discussion on the topic of the post-Newtonian celestial dynamics we suggest to the reader to check out the references by C. Will Will (2006, 2016). The objective of experimental General Relativity in connection with the orbital dynamics in the Solar System is, fundamentally, to obtain increasingly more accurate values of the parameters β and γ in the parametrized post-Newtonian formalism. If General Relativity is correct we will have $\beta = \gamma = 1$. Here, γ is a measure of the space curvature produced by a unit test mass and can be constrained in the Shapiro effect or with light deflection by large masses. The parameter β can be understood as a measure of the nonlinearity in the superposition law of gravity for g_{00} and it is determined with the perihelion shift of Mercury, for example. From experiments with the delay of radio waves of the Cassini spacecraft the bound $|\,\gamma - 1\,| < 2.3\times10^{-5}$ was obtained by Bertotti and Tortora in 2003 Bertotti et al. (2003). The best bound for $|\,\beta - 1\,|$ is 8×10^{-5} as obtained from observations of the perihelion shift of Mercury with the Messenger mission Verma et al. (2014). It has been suggested that with the ESA's mission BepiColombo radio-science experiment an improvement of one order of magnitude in the possible deviations of these parameters from unity could be achieved Milani et al. (2002). However, this analysis is still to be performed.

Celakoska and Trenčevski Celakoska and Trenčevski (2009) also derived the EIH equations for the extended metric of the parametrized post-Newtonian formalism in terms of the parameters β and γ. These equations can be used to analyze the trajectories of planets or spacecraft in a variety of modified gravity theories:

$$\begin{aligned}\mathbf{P}_{\text{PPN}} = {} & \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B\mathbf{n}_{BA}}{r_{AB}^2}\left[\gamma v_A^2 + (1+\gamma)v_B^2\right.\\ & - 2(1+\gamma)(\mathbf{v}_A\cdot\mathbf{v}_B) - \frac{3}{2}(\mathbf{n}_{AB}\cdot\mathbf{v}_B)^2 - 2(\beta+\gamma)\sum_{C\neq A}\frac{Gm_C}{r_{AC}}\\ & \left. -(2\beta - 1)\sum_{C\neq B}\frac{Gm_C}{r_{BC}} + \frac{1}{2}((\mathbf{x}_B - \mathbf{x}_A)\cdot\mathbf{a}_B)\right]\\ & + \frac{1}{c^2}\sum_{B\neq A}\frac{Gm_B}{r_{AB}^2}[\mathbf{n}_{AB}\cdot((2+2\gamma)\mathbf{v}_A - (1+2\gamma)\mathbf{v}_B)](\mathbf{v}_A - \mathbf{v}_B)\\ & + \frac{3+4\gamma}{2c^2}\sum_{B\neq A}\frac{Gm_B\mathbf{a}_B}{r_{AB}} + \mathcal{O}(c^{-4}).\end{aligned} \tag{6}$$

Notice that Eq. (6) reduces to Eq. (5) for $\beta = \gamma = 1$.

There are other effects that could be taken into account in a detailed model as they are also the source of perturbations. Nevertheless, these other effects have a negligible impact on the trajectory in comparison with the ones cited above and can be neglected in the model. Charged particles emitted by the Sun, i.e., the solar wind, contributes with a ram pressure over spacecraft. This is estimated by the following expression:

$$P = m_p n V^2, \tag{7}$$

where m_p is the proton mass, n the plasma density and V the speed of the solar wind (Gary, 1995). At a distance of 1 AU this expression predicts a pressure in the range of a few nPa (10^{-9} N/m^2). We can compare this value with the standard pressure of 9.08 μPa (10^{-6} N/m^2) for the radiation pressure exerted on a perfectly reflective surface located at 1 AU from the Sun and oriented normally to the light rays. This means that the forces arising from the solar wind are, typically, three orders of magnitude smaller than the ones arising from radiation pressure.

A perturbation factor more difficult to estimate is the magnetic force arising from the movement of the spacecraft in the magnetic field of the Sun as it collects a charge from the surrounding plasma. In the case of a satellite of the Earth, Lämmerzahl et al. (Lämmerzahl et al., 2006) assumed a charge $Q < 10^{-7}$ C, a typical velocity of $v = 30$ km/s and a magnetic field of 0.2 G. From these parameters a magnetic Lorentz acceleration (for a typical spacecraft mass of 1000 kg) of 10^{-11} km/s^2 is obtained. As we will see in the next section this is up to three orders of magnitude smaller than the contribution of the radiation pressure and one order of magnitude below the effect of the relativistic corrections. The average magnetic field of the Sun at the photosphere is around several gauss (Babcock, 1963) but it can raise to thousands of gauss in the sunspots regions. On the other hand, the closer perihelion of the Parker Solar Probe would be at 9.86 solar radii from the center of the Sun and the expected magnetic field would be of only a fraction of a gauss. The experiment FIELDS, which is a main objective of the mission, would provide direct data about the values of the magnetic field in the solar corona. The spacecraft charge would be more difficult to determine as it depends on the geometry and electrostatic properties of the spacecraft. Ergun et al. (Ergun et al., 2010) have shown that, under three conditions that are present in missions close to the Sun the spacecraft acquires a negative potential as a consequence of the formation of an electrostatic barrier. These conditions are: (i) Photoelectron density exceeding the ambient plasma density (ii) Debye length much smaller than the spacecraft size (iii) Thermal electron energy much larger than the escape energy for photoelectrons. Modelling of charging for the Parker Solar Probe is, consequently, a complex problem and it would require more quality data on the corona environment. On the other hand, the velocity of the spacecraft close to the perihelion would reach $v \simeq 300$ km/s which implies a factor of 10 in the magnetic force in comparison with artificial satellites of the Earth. Considering all these factors a conservative estimate $10^{-11} - 10^{-12}$ km/s^2 can be given for the maximum magnetic force per unit mass acting upon the probe.

A spacecraft can also carry a magnetic moment that generates a force as it moves through the gradient of a magnetic field. The general estimate of 2 Am^2 has been given as an upper bound to this magnetic moment and this gives rise to a force $F = 4 \times 10^{-11}$ N in the gradient of the magnetic field of the Earth (Lämmerzahl et al., 2008). Gradients of the magnetic field in the Solar corona is still to be measured but assuming the force would be similar we obtain a perturbation acceleration of the order of 10^{-16} km/s^2 that it is even smaller than the magnetic Lorentz force.

In the next section we will calculate these perturbations and their contributions to the orbital model to verify that the relativistic corrections are within the range of experimental analysis in the Parker Solar Probe mission.

3. Numerical results

We will now apply the orbital model discussed in the previous section to the orbit of the Parker Solar Probe in the approach to the perihelion number 13. This perihelion is planned to be reached by September 2022. We have chosen this section of the orbit because it would correspond to a maximum velocity larger than 160 km/s. In this manoeuvre the spacecraft would reach a distance smaller than 10.5 million kms to the center of the Sun. This perihelion would be attained after the fifth flyby of Venus in which the spacecraft is also actively controlled and a burning of the hydrazine thrusters takes place that modifies the trajectory. Anyway, we will start the integration long after this burning so the probe is then moving under the action of natural forces alone.

The orbital model outlined in the previous section was implemented using the Mathematica 10 software Inc (2016). Particular care was taken to keep the accuracy goal of the numerical integrators of the ordinary differential equations up to 40 digits. This was attained by setting the precision goal of every variable and algorithm accordingly. Mathematica numerical integrators are based upon standard variable time-step methods for differential equations which are well-suited for celestial mechanics applications.

The ephemeris of the planets and satellites used in this model were obtained from the Horizons web interface from the Jet Propulsion Laboratory Giorgini (2019). The mass constants of the Sun, the planets and satellites considered in our model are also listed below in Table 1.

Table 1

Celestial body	Mass constant (km^3/s^2)
Sun	132712440041.939380
Mercury	22031.78
Venus	324858.592
Earth	398600.435436
Moon	4902.800066
Mars	42828.375214
Jupiter	126686531.9
Saturn	37931206.159
Uranus	5793951.322
Neptune	6835099.5
Ganymede	9887.834
Callisto	7179.289
Titan	8978. 14

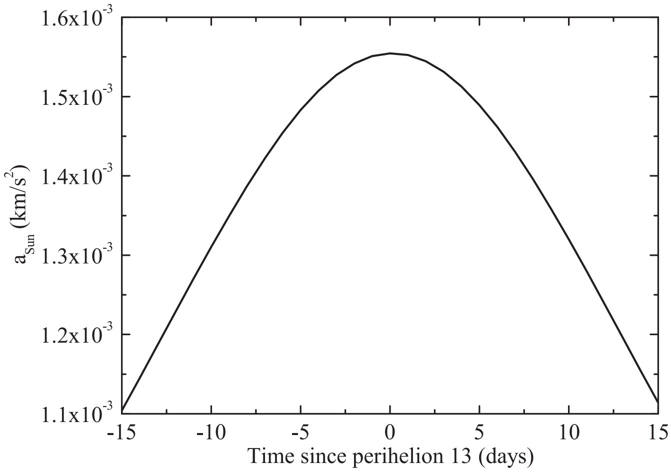

Fig. 1. Acceleration imparted by the Sun's gravitational attraction on the Parker Solar Probe in the period of fifteen days before and after perihelion 13.

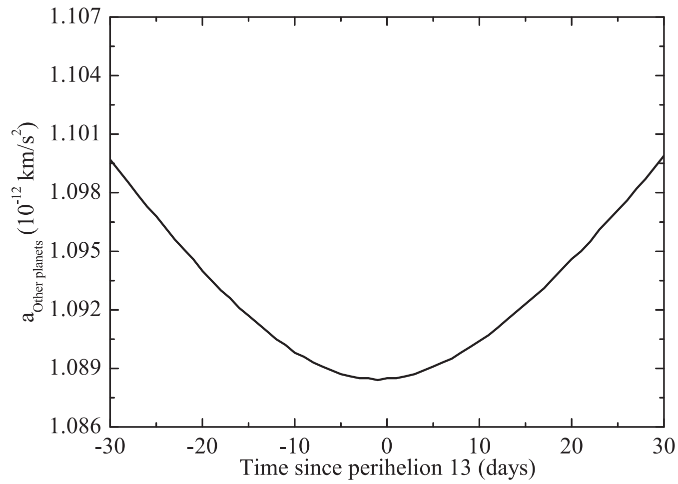

Fig. 3. The same as Fig. 2 but for the rest of the planets and the largest asteroids and satellites in the Solar System.

We will know evaluate the relative importance of the main gravitational attraction by the Sun and the set of perturbing effects, including radiation pressure acting upon the shield and relativistic corrections. In Fig. 1 we show the modulus of the acceleration imparted by the gravitational attraction of the Sun during the perihelion approach. We observe that its magnitude is of the order of a few meters per second squared. This value would be used as a reference to compare with the perturbing acceleration corresponding to other minor effects. As a first step, we have obtained the perturbing acceleration as a consequence of the gravitational interaction of the spacecraft with the planets, satellites and major asteroids in the Solar system. In Figs. 2 and 3 we have plotted the acceleration's modulus for the perturbing effect exerted by the planets and major asteroids and satellites in the Solar System. In Fig. 2 we include only the effect of the planets from Mercury to Saturn (in comparison with the EIH relativistic contribution to the classical Newtonian acceleration) and Fig. 3 corresponds to the rest of the planets and minor bodies. These effects imply a perturbing acceleration seven orders of magnitude smaller than the main term of the model corresponding to the gravitational attraction by the Sun. If we refer to the planets from Uranus to Neptune, Pluto, the satellites and the most massive asteroids the total perturbation effect is ten orders of magnitude below that of the Sun.

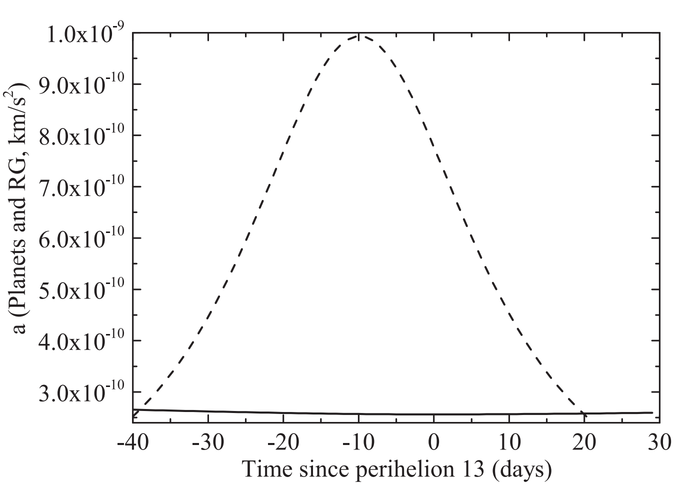

Fig. 2. Modulus of the acceleration imparted on the Parker Solar Probe by the planets from Mercury to Saturn on its approach to perihelion 13 vs time in days (solid line). The relativistic contribution is also shown (dashed line).

To model the radiation pressure exerted upon the solar shield and the solar panels we have taken into account the parameters listed in Table 2. To estimate the area of the solar shield we have considered a diameter of 2.3 m. As these parameters might not be exact some residual uncertainty in the model may persist but the accuracy would be enough to elucidate the possibility of detecting the effects of General Relativity. The intensity of the radiation pressure also varies slightly with the solar cycle Smith and Gottlieb (1974) and to study these variations is, precisely, one of the objectives of the Paker Solar Probe mission Guo et al. (2021).

To analyze the radiation pressure exerted upon the spacecraft we have used a simple box-wing model without shadowing effects because it is assumed that, on the close approximation to the Sun, both the shield and the Solar panels are facing, perfectly, towards the Sun.

In Fig. 4 we show the result of our analysis for the solar radiation pressure quantified in terms of the modulus of the

Table 2

Parameter	Value	Units
Solar shield reflection coefficient	1.8	-
Solar panels reflection coefficient	1.38	-
Solar shield's area	4	m^2
Solar panel's area	1.6	m^2
Parker Solar Probe's mass	655	kg
Intensity of Solar radiation at 1 AU	1367	W/m^2

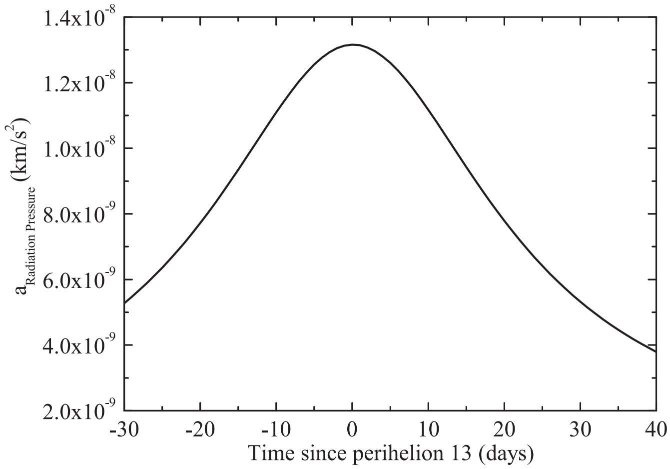

Fig. 4. Force per unit mass (in modulus) corresponding to the radiation pressure acting upon the Parker Solar Probe during perihelion 13.

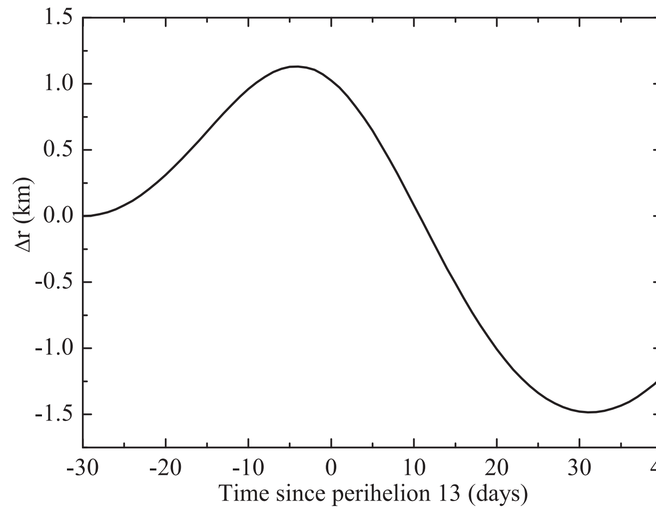

Fig. 5. Difference of the absolute distance to the center of the Solar system barycenter frame between the orbital model in this paper and the one of NASA as given by the Horizons online system (Giorgini, 2019).

perturbing acceleration. We notice that this is the most important perturbation source for the analysis of the trajectory of the Parker Solar Probe. Therefore, a careful model would be required in order not to mask the estimation of the contribution of General Relativity for the ephemeris of the spacecraft. This model should use a finite element method to analyze the radiation reflection and absorption. Another important effect arises from the quadrupolar field of the Sun quantified in terms of the J_2 coefficient takes the approximate value $J_2 \simeq 2.2 \times 10^{-7}$. The order of magnitude of the accelerations imparted by the zonal harmonic terms is $g_0 J_2 (R/r)^2$, where g_0 is Sun's gravity at the point of observation, R is Sun's radius and r is the distance of the spacecraft to the center of the Sun. As this distance is always larger than 9 solar radii for perihelion 13 we have that the J_2 force per unit mass is around 4×10^{-12} km/s^2 that may cause a deviation of several kilometers in the period of roughly one week corresponding to the perihelion passage. In any case, this effect is below the relativistic corrections. In the next subsection we will analyze the impact of this effect on the spacecraft trajectory.

Finally, we show the effect of the corrections provided by General Relativity as quantified by the Einstein-Infeld-Hoffmann equations of motion in Eq. (5).

As shown in Fig. 2 where have plotted the modulus of the extra acceleration imparted to the Parker Solar Probe by these relativistic corrections as a function of time referenced to the perihelion passage.

We observe that the order of magnitude of this effect interpreted as an anomalous extra force (per unit mass) is around 10^{-10} km/s^2. The displacement corresponding to this acceleration acting continuously for a period of 10 days is, roughly, 37 km and, consequently, well within the accuracy of telemetry for this mission. Therefore, in principle, it would be possible to detect the effect of GR on the trajectory but another question is to constrain the post-Newtonian parameters with reasonable accuracy. In the next section, we will analyze this question to obtain bounds on the necessary accuracy of the radiation pressure modelling if the GR effects are intended to be measured in this kind of Solar mission.

As a comparison, we show in Fig. 5 the discrepancy of our model with the one obtain from the Horizon's ephemeris system (Giorgini, 2019). This difference, apart from numerical errors, can be attributed to the fact that we have mismodelled the radiation pressure because that would require a full-scale geometrical model of the spacecraft.

3.1. Error bounds and post-Newtonian parameters

In this section, we will analyze the possibility of obtaining error bars for the post-Newtonian parameters by using the trajectory of the Parker Solar Probe mission. This would also apply to other spacecraft missions to the vicinity of the Sun such as the ESA Solar Orbiter Müller et al. (2020). We will use the spacecraft distance to the Sun, R, as the value to compare the expected contribution of different effects, classical and relativistic, to the telemetry measurements.

Notice also that ΔR is the contribution to the distance to the center of the Sun of the perturbation effects we are considering, separately, in this section. Therefore, ΔR can be equal to zero at some points of the trajectory.

In Fig. 6 we show the contribution to ΔR (perturbation in the distance to the Sun) of the EIH non-Newtonian acceleration as given in Eq. (5). The initial time is $t = -27.4023$ hours before perihelion 13. We see that ΔR grows to several kilometers in 80 h and, therefore, this effect can be measured, in principle, with X–band telemetry. The question is to what extent this would allow to set up constraints on the post-Newtonian parameters β and γ. In Fig. 6 we also compare with the error induced

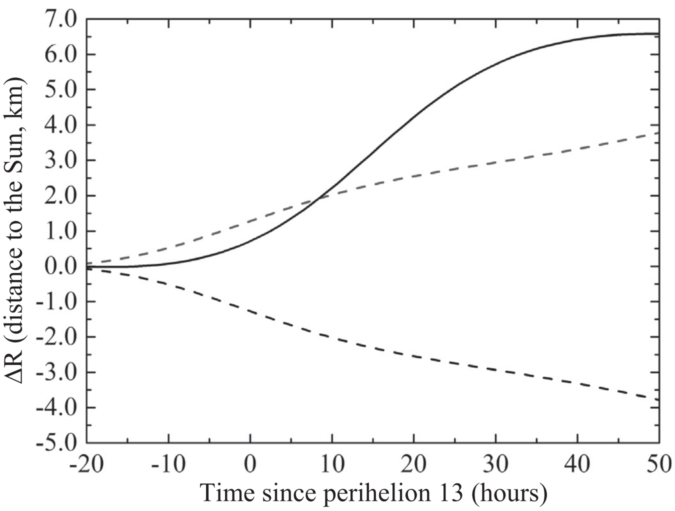

Fig. 6. Contribution to ΔR (perturbation of the distance to the Sun) for the relativistic EIH terms (solid line) and a ±5% error in the solar shield geometry (upper line corresponds to a solar shield 5% larger and the lower line to a shield a 5% smaller.

by assuming a ±5% uncertainty in the geometry of the solar shield or in the radiation pressure model in general. It is important to notice that a crude estimate of the spacecraft geometry, orientation or radiation flux has a crucial impact on the orbital model.

In fact, this is detrimental for any accurate measurement of relativistic effects whit any spacecraft mission to the Sun. We have not enough information about the spacecraft design to develop a precise thermal model. Anyway, we conclude that a model with a 10^{-6} accuracy (both in the geometrical model and the radiation flux parameters) would be necessary to unveil the relativistic effects with enough accuracy to set bounds on the β and γ parameters as discussed below.

The uncertainty in the masses of the Sun and the planets are of minor importance. As an example of this kind of error source, we have considered a ±1 change in the last significant digit figure of Jupiter's mass constant as tabulated in Table 1, i. e., a change of ± 0.1 km^3/s^2. Fig. 7.

These changes only give rise to errors of the order of magnitude of a fraction of a centimeter so they do not suppose a significant contribution in comparison to the relativistic effects or the perturbations on the PPN parameters as shown below. Of much greater importance is the effect of the Sun's quadrupole and the, relatively, large uncertainty that we still have on the J_2 parameter Mecheri et al. (2004). We have computed the contribution to ΔR of the Sun's quadrupolar field for a standard value of $J_2 = 2.2 \times 10^{-7}$ in Fig. 8.

Although the contribution of the quadrupolar field is only of a few meters it competes with that of the perturbations in $\mid \beta - 1 \mid$ and $\mid \gamma - 1 \mid$ of the order of 10^{-2}. Finally, in Fig. 9 we show the effect, according to the extended EIH equations of motion in Eq. (6), of a variation of $\beta - 1$ of $\pm 10^{-2}$ (assuming $\gamma = 1$) and of $\gamma - 1$ of the same order (assuming $\beta = 1$).

We see that in the flyby manoeuvre the X-band telemetry would be sufficiently accurate to constraint $\mid \beta - 1 \mid$ and $\mid \gamma - 1 \mid$ with this precision. However, a determination of further significant figures of J_2 is equally important to be able to say anything significant about the PPN parameters as well as an accurate model of the spacecraft geometry and solar radiation pressure. Moreover, the effect of a change in β can be distinguished from a change in γ because in the latter case we obtain a crossing in the perturbation of the distance to the Sun from a negative value before perihelion to a positive value afterwards.

Another effect to be taken into account is the Lense-Thirring frame dragging Iorio et al. (2011). This effect

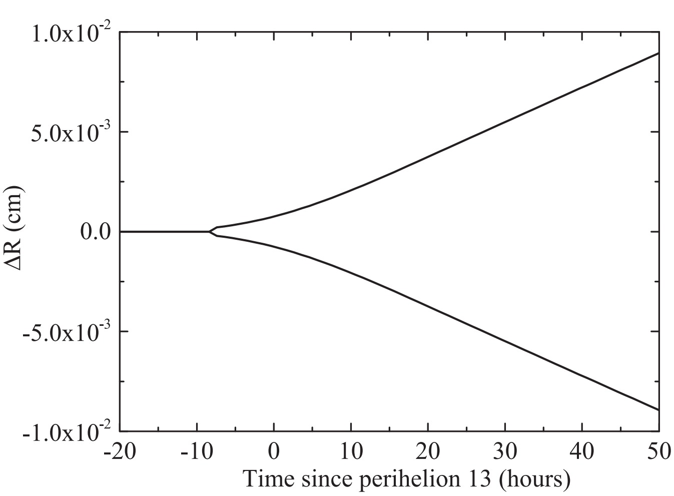

Fig. 7. Contribution to ΔR (perturbation of the distance to the Sun in centimeters) as a consequence of an increase of 0.1 km^3/s^2 in the mass constant of Jupiter (upper line) or a decrease of the same magnitude (lower line).

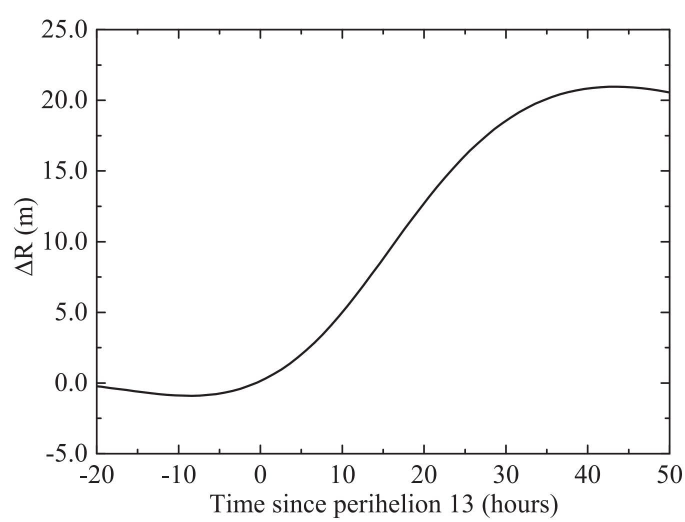

Fig. 8. Contribution to ΔR (perturbation of the distance to the Sun in meters) as a consequence of the quadrupolar field of the Sun for $J_2 = 2.2 \times 10^{-7}$.

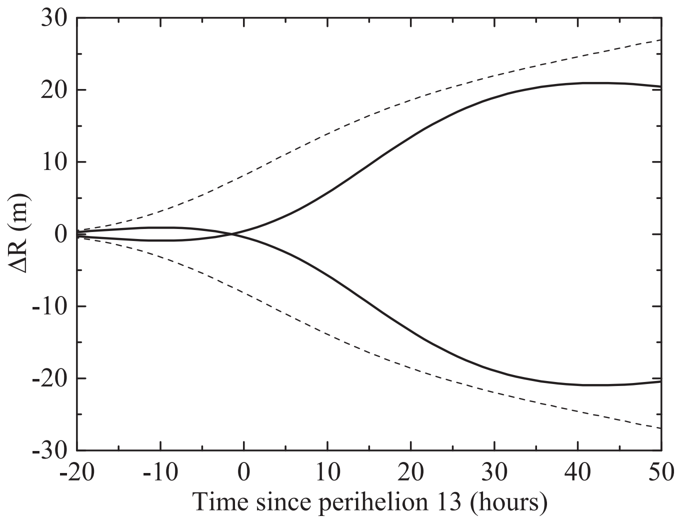

Fig. 9. Contribution to ΔR (perturbation of the distance to the Sun in meters) of the Einstein-Infeld-Hoffmann non-Newtonian acceleration of the Parker Solar Probe with $\beta = 1.01, \gamma = 1$ (upper dashed line), $\beta = 0.99$, $\gamma = 1$ (lower dashed line), $\beta = 1, \gamma = 1.01$ (upper solid line after the perihelion passage) and $\beta = 1, \gamma = 0.99$ (lower solid line after the perihelion passage).

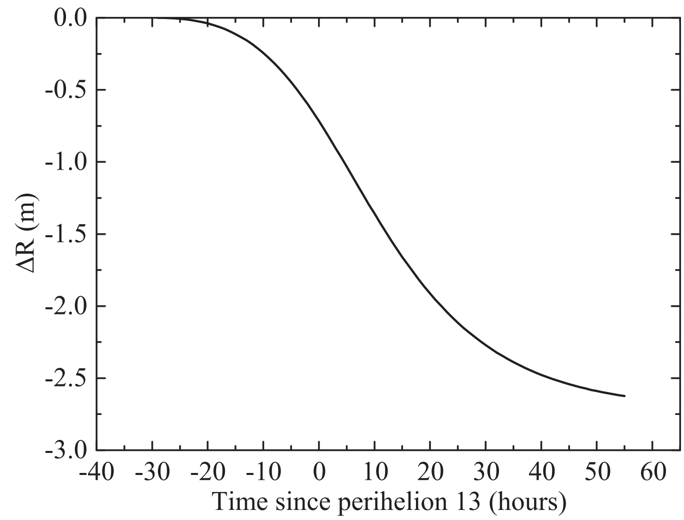

Fig. 10. Contribution to ΔR (perturbation of the distance to the Sun in meters) of the Lense-Thirring acceleration of the Parker Solar Probe.

can be understood as the interaction with a gravitomagnetic field of the form:

$$\vec{B}_g(\vec{r}) = -\frac{G}{cr^3}\left[\vec{S} - 3\left(\vec{S}\cdot\hat{r}\right)\hat{r}\right], \tag{8}$$

where **r** is the vector radius from the source to the location of the spacecraft and **S** is the angular momentum of the source. This gravitomagnetic field corresponds to an extra acceleration of the spacecraft given by expression similar to the Lorentz force:

$$\vec{A}_{\mathrm{GM}} = -2\left(\frac{\vec{v}}{c}\right) \times \vec{B}_g. \tag{9}$$

If we assume that the Sun is a sphere of uniform density the angular momentum would be given by:

$$\vec{S} = (2/5)MR^2\omega\vec{n}, \tag{10}$$

where M is the mass of the Sun, $M = 1.989 \times 10^{30}$ kg, $R = 696340$ km is the radius and $\omega = 2\pi/T$ is the angular velocity (assuming the rotation period of $T = 27$ days). The inclination and longitude of the ascending node defines the orientation of the Sun's axis in the ICRF and it is approximately given by Carrington's elements Acedo (2014):

$$\begin{aligned} \iota_c &= 7.25^\circ \\ \Omega_c &= 73.67^\circ + 0.013958^\circ(t - 1850), \end{aligned} \tag{11}$$

where t is the year of observation. The contribution to ΔR of the Lense-Thirring effect for the perihelion 13 of the Parker Solar Probe is shown in Fig. 10. It is important to notice that this relativistic effect would have a smaller contribution than the perturbation in the values of the PPN parameters β and γ described in Fig. 9. Another source of error comes from the uncertainty in the Sun's mass constant. This could be more important than the corresponding uncertainty for any planet as a consequence of the proximity of the spacecraft to the Sun and the large mass. On the other hand, the mass constant of the Sun is better known than that of any other body in the Solar system. Assuming an uncertainty of $\pm 10^{-6}$ km^3/sec^2 we obtain a discrepancy in the position of the spacecraft of the order of a few centimeters. Not enough to significantly overlap the relativistic effects for perturbed values of β and γ in the range of interest. The results are shown in Fig. 11.

Finally, we have assumed than in the analysis of the spacecraft trajectory the effect of the Shapiro echo delay Shapiro (1964, 1968) has be taken into account. This corre-

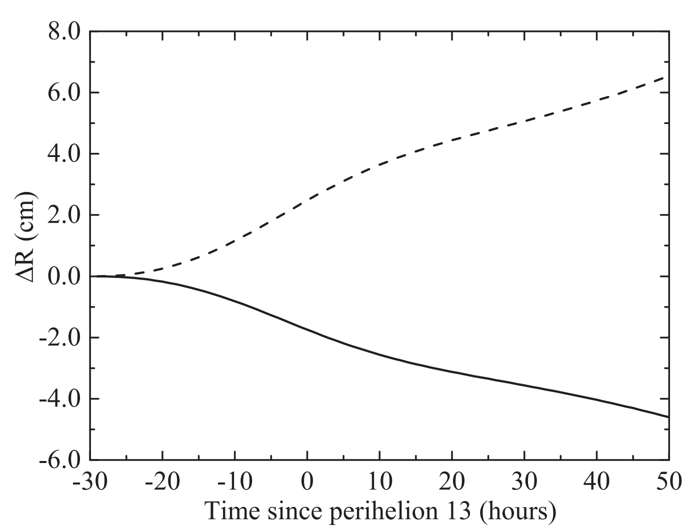

Fig. 11. Contribution to ΔR (perturbation of the distance to the Sun in centimeters) as a consequence of an increase of 10^{-6} km^3/s^2 in the mass constant of the Sun (upper line) or a decrease of the same magnitude (lower line).

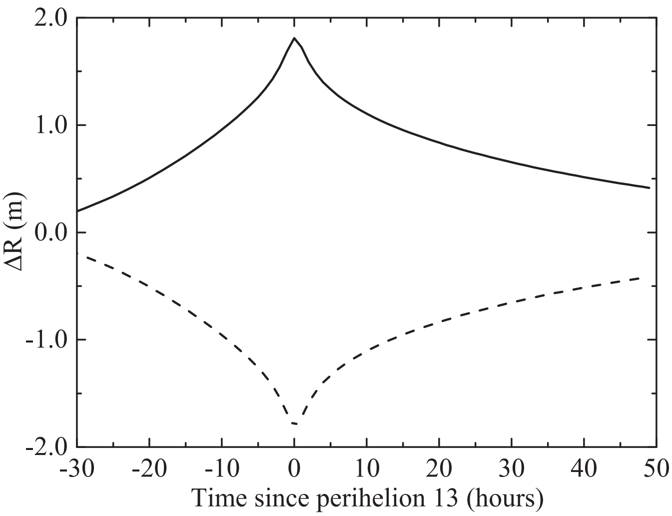

Fig. 12. Contribution to ΔR (perturbation of the distance to the Sun in meters) of the apparent position of the Parker Solar Probe inferred from the radar signals as a consequence of an extra Shapiro effect for $\Delta\gamma = 0.0001$ (upper solid line) and $\Delta\gamma = -0.0001$ (lower solid line).

sponds to a delay in the reception of radar signals coming from the spacecraft in the form:

$$\Delta t = -(1+\gamma)\frac{R_s}{2c}\ln(1 - \mathbf{R}\cdot\mathbf{x}), \tag{12}$$

where R_s is the Schwarzschild radius of the Sun in our case, $\mathbf{R}$ is the unit vector from the Earth to the Sun and $\mathbf{x}$ is the unit vector from the Earth to the spacecraft. This amounts to an apparent error in the position of the spacecraft due to the uncertainty in the value of γ of the order of magnitude of 1 meter (for $\Delta\gamma = 10^{-4}$ that should be fitted simultaneously with the relativistic effects on the real spacecraft trajectory shown in Fig. 9. As an example, in Fig. 12 we show the effect on the Shapiro effect on terms of the error in the determination of the distance of the spacecraft to the Sun for a value of $\Delta\gamma = \pm 10^{-4}$.

Summarizing, to obtain information about the PPN parameters β, γ and J_2 we will need a good model of the spacecraft geometry and radiation pressure accurate to one part in one million (both geometrically and in the radiation pressure) unless new modified gravity effects were found near the Sun and this is unlikely considering the previous constraints obtained in the Messenger Verma et al. (2014) and Cassini Bertotti et al. (2003) missions.

4. Conclusions and remarks

A century after the proposal of General Relativity the testing of its consequences is still an ongoing enterprise (Will, 2006). Being a nonlinear theory of the gravitational field it is also difficult to make predictions. In this theory solving the field equations has required, in certain cases, a vast amount of computational power. This has been the case, for example, for the merging of black holes even on the most simple geometrical configurations (Anninos et al., 1995).

But even for the corrections of Newtonian dynamics in the Solar System we are still at the beginning of an era of experimental General Relativity. Recently, the Messenger mission, whose objective was to study the geology, magnetic field and chemical composition of Mercury, played also a key role in the improvement of the measurement of the parameters β and γ in the parametrized post-newtonian approximation (PPN) by using the orbital data (Genova et al., 2016).

The Deep Space Network is the international array of spacecraft communication facilities located at Goldstone (U.S.A.), Madrid (Spain) and Canberra (Australia) that has allowed for an implementation of the Delta-DOR interplanetary radio tracking technique with increasing accuracy for more than fifty years (Border et al., 2015). The detection of spacecraft signals at these, widely separated, ground stations on Earth allows for the precise measurement of the difference in the time that these signals are received. This information can be used to pinpoint the spacecraft location on the Solar System Barycenter frame with a precision that could not be attained without the participation of the array of antennas. In the case of the Parker Solar Probe which uses the X-Band for communication an accuracy in the scale of a fraction of a meter is possible, in principle, for the determination of the spacecraft coordinates at any instant.

The study of the trajectory of the Parker Solar Probe is, consequently, an opportunity to analyze the validity of the Post-Newtonian equations of motion in an orbit with a low perihelion in which a record velocity of a 0.064% of the speed of light would be achieved during its closest approach to the Sun. To disclose the effect of General Relativity from the classical perturbations, a careful analysis of the contributions to the orbital model should be made. In particular, more emphasis on the development of a solar radiation pressure model should be made because this is the dominant perturbation term in the vicinity of the Sun. Our theoretical analysis shows that radiation pressure should be modelled with a precision of 1 part in 10^6 to obtain significant information about the PPN parameters: β, γ and J_2. This model should include both the geometry and the Sun's radiation flux with the same accuracy of 1 part in 10^6. In connection with this objective, it is important to notice that the Sun's radiation flux is usually given with an accuracy of only 10^{-3} (Kopp and Lean, 2011). Therefore, more precise modelling of the Sun's radiation flux and its variations would be necessary to obtain information about the PPN parameters in spacecraft missions to the Sun.

The highly eccentric orbits of the Parker Solar Probe (with eccentricity around 0.88) also made this mission specially suited to study the possibility of a flyby anomaly similar to the, still unsolved, flyby anomalies found in some spacecraft flybys of the Earth (Acedo, 2017). These anomalous orbital energy changes have been attributed to a non-standard gravitation effect related to the mass and angular momentum of the celestial object around which the spacecraft performs its flyby (Anderson et al., 2008).

Another recent issue in Astronomy that has motivated a widespread interest in the study of close flybys of the Sun is the orbital behaviour of the so-called Oumuamua interstellar object that crossed the inner Solar System in 2017. The anomalous acceleration detected in this object has been hotly debated and there is still no consensus about its origin (Seligman et al., 2019). The question about the possible cometary nature of this object and the interpretation of the anomalous acceleration as originating from cometary outgassing is now disfavoured by modelling and observations (Rafikov, 2018). Therefore, a careful analysis of the trajectory of a spacecraft in a similar orbit could help in the discussion about the unexplained Oumuamua's orbit close to the Sun.

Further work along these lines is to be published in the future.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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