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Sensitivity analysis of control networks in terms of minimal detectable displacements

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ABSTRACT

Sensitivity analysis is one aspect of the calculation process in displacement measurement. For this purpose, sensitivity measures are used in the form of minimal detectable displacements (MDD), derived from the definition of minimal detectable bias (MDB). Analyses were performed based on disturbing the parameter vector with the MDD value (calculated from principal component analysis). Analyses were considered for displacements of the levelling network based on the global vector ($\mathbf{MDD}_{\text{network}}$). The calculations were conducted using the least squares adjustment with pseudo-random observations. The mean success rate (MSR) was used to perform the detection analyses. The effectiveness of the global test agrees with the assumed power of the test. Local tests for a single point showed 48.6 % displacements of more than one point. It should be considered using another local test for the case of displacements of multiple points. This study concludes that in addition to the accuracy of the designed network points, the network configuration is also important in sensitivity analyses. The research shows that sensitivity analyses should be considered at the design stage of control networks, *i.e.* to determine at what level a given network is able to detect displacement.

I. INTRODUCTION

To (better) design the optimum control network, it is important to obtain a priori information about the magnitudes and rates of displacement (or deformation). In the literature on geodetic network design (*e.g.* Alizadeh-Khameneh, 2015), accuracy and reliability criteria for one observation epoch are used. In control networks (apart from accuracy and reliability analyses), the aim is to design a network that is sensitive to the displacements (or coordinate differences) between two or more measurement epochs (Niemeier, 1982; Niemeier *et al.*, 1982). Even-Tzur in works (2010; 2002) also indicates that monitoring networks should be analysed according to three criteria: accuracy, reliability and sensitivity.

The network's sensitivity is a criterion to be taken into account for the optimisation of control networks (Kuang, 1991) as it gives information on what level of displacement (or deformation) will be detected. Sensitivity measures, namely Minimal Detectable Displacement (MDD), are calculated to determine this level. The concept of MDD is based on the definition of Minimal Detectable Bias (MDB) pioneered in the work of (Baarda, 1968; 1967). It can be noted that as the concept of reliability, the measure of internal reliability is the internal reliability index $\{\mathbf{R}\}_{\text{ii}}$ and the minimal detectable bias in observation (MDB). The transfer of considerations into displacement studies was initiated in the 1970s and 1980s, mainly by German authors, *e.g.*, Pelzer (1972) and Niemeier (1982; 1981). Thus, Prof. Pelzer can be considered the precursor of displacement sensitivity analyses in control networks (Pelzer, 1972).

Although nearly fifty years have passed, the subject of network sensitivity to displacement detection is still commonly addressed in research. The main issues found in the literature on the subject include:

- Sensitivity of the control network to displacements (or deformations).
- Definition of the minimal detectable displacement.
- Selection of statistical test parameters for displacement assessment
- Examples of the use of sensitivity analyses for applied solutions.

Considering the subject of network sensitivity analysis, Heck in his work (1986), gives the quantities that are required to carry it out. These should therefore be listed:

- Network configuration (coefficient matrix $\mathbf{A}$).
- Measurement accuracy (weighting matrix $\mathbf{P}$, a priori variance factor σ_d^2).
- The distribution of test statistics to be used in the displacement (deformation) analysis for H_0 and H_a (non-centrality parameters λ_0 or probabilities α_0 and β_0).

Knowing that all this information is available before the measurements are made, sensitivity analysis can be performed during the planning stage. If the sensitivity of the designed network is not suitable for the intended use, the parameters used must be modified so that the final requirements are fulfilled. A new proposal is extending the application area of the MDD. The research focuses on the probabilistic aspect of the combination of significance and detectability (Prószyński and Łapiński, 2021).

II. MATHEMATICAL BASICS

Modelling multidimensional data acquired as part of the measurements performed requires a specific mathematical model of the object, showing the relationships between real quantities in the adopted reference system. A common object known to the surveyor is a geodetic network, particularly a displacement control network, which consists of geometric elements defined on a set of points of a given network.

The well-known Gauss-Markov model is assumed to adjust the measured values (Eq. 1):

$$\mathbf{A}\,\mathbf{x} = \mathbf{l} + \mathbf{v} \quad , \quad \mathbf{C_l} \tag{1}$$

where $\mathbf{A}\ (n \times u)$ = design matrix
$\mathbf{x}\ (u \times 1)$ = vector of parameters
$\mathbf{l}\ (n \times 1)$ = vector of observation
$\mathbf{v}\ (n \times 1)$ = vector of random errors in $\mathbf{l}$ (with opposite sign $\mathbf{v} = -\boldsymbol{\varepsilon}$)
$\mathbf{C_l}\ (n \times n)$ = covariance matrix for $\mathbf{l}$ (positive definite)
n = number of observations
u = number of parameters

After standardising (*i.e.* reducing to unit weight) the model from Equation 1, the final model in the stochastic approach will take the form (Eq. 2):

$$\mathbf{A}_\text{s}\,\mathbf{x} = \mathbf{L} + \mathbf{v}_\text{s} \quad , \quad \mathbf{C_l} \equiv \mathbf{I} \tag{2}$$

where $\mathbf{A}_\text{s} = \mathbf{D}\,\mathbf{A}, \mathbf{L} = \mathbf{D}\,\mathbf{l}, \mathbf{v}_\text{s} = \mathbf{D}\,\mathbf{v}$
$\mathbf{D}$ = standardisation matrix such that $\mathbf{D}^\text{T}\mathbf{D} = \mathbf{C}_\mathbf{l}^{-1}$

The control networks for displacement analysis are local networks without connections to the known reference system, so there is the network defect, which is determined by the fact that the matrix of coefficients in the system of observation Equations 2 is of non-full rank (Eq. 3):

$$rank(\mathbf{A}_\text{s}) = u - d_s \tag{3}$$

where d_s = the number of network defects

The coefficient matrix $\mathbf{A}_\text{s}$ in the system of normal equations is a singular matrix therefore $det(\mathbf{A}_\text{s}^\text{T}\mathbf{A}_\text{s}) = 0$, and it is impossible to compute the inverse of such a matrix (the case with an infinite number of system solutions). To obtain a solution, the generalised inverse of the matrix $\mathbf{A}_\text{s}^\text{T}\mathbf{A}_\text{s}$ can be used (Perelmuter, 1979), where a special case is the use of Moore-Penrose pseudoinverse. Then the determined parameter vector is of the least Euclidean norm and the covariance matrix of the unknowns (parameters) $\mathbf{C}_{\hat{\mathbf{x}}}$ has the smallest trace value (Equations 4 and 5) under the condition $\hat{\mathbf{v}}_\text{s}{}^\text{T}\hat{\mathbf{v}}_\text{s} = min$ as a postulate of the least squares method.

$$\hat{\mathbf{x}}^\text{T}\hat{\mathbf{x}} = min \tag{4}$$

$$Tr\ (\mathbf{C}_{\hat{\mathbf{x}}}) = min \tag{5}$$

The parameter vector is determined from the formula (Eq. 6):

$$\hat{\mathbf{x}} = \mathbf{Q}_{\hat{\mathbf{x}}}\,(\mathbf{A}_\text{s}^\text{T}\mathbf{L}) = (\mathbf{A}_\text{s}^\text{T}\mathbf{A}_\text{s})^+(\mathbf{A}_\text{s}^\text{T}\mathbf{L}) \tag{6}$$

where $\mathbf{Q}_{\hat{\mathbf{x}}}$ = cofactor matrix of the parameter vector
$(\dots)^+$ = Moore-Penrose pseudoinverse

Based on Equation 6, the displacement vector $\mathbf{d}$ is obtained (Eq. 7):

$$\mathbf{d} = \hat{\mathbf{x}}_2 - \hat{\mathbf{x}}_1 \tag{7}$$

where $\hat{\mathbf{x}}_1$ = vector of parameters in the initial position
$\hat{\mathbf{x}}_2$ = vector of parameters in the present position

while the cofactor matrix of displacement vector $\mathbf{d}$ (Eq. 8):

$$\mathbf{Q_d} = \mathbf{Q}_{\hat{\mathbf{x}}_1} + \mathbf{Q}_{\hat{\mathbf{x}}_2} \tag{8}$$

The global test procedure, as with many geodetic tasks, involves separating two hypotheses: the null (H_0) and the alternative (H_a) (Eqs. 9 and 10):

$$H_0\colon\ E(\mathbf{d}) = \mathbf{0} \tag{9}$$

$$H_a\colon\ E(\mathbf{d}) \neq \mathbf{0} = \mathbf{d}_\text{A} \tag{10}$$

where $\mathbf{d}_\text{A}\ (u \times 1)$ = the displacement vector

If the null hypothesis H_0 is rejected, the alternative hypothesis H_a is accepted.

The test statistic takes the form (Eq. 11):

$$T = \frac{\mathbf{d}^\text{T}\mathbf{Q}_\mathbf{d}^+\,\mathbf{d}}{\sigma_\text{d}^2} \sim \chi^2(h) \tag{11}$$

where σ_d^2 = the a priori variance factor $\sigma_d^2 = 1$
h = rank of the cofactor matrix in the vector of parameters

Checking the global test condition consists in comparing the corresponding test statistic being the value of T from Equation 11 and comparing it with the threshold value $\kappa = \chi^2_{h,1-\alpha_0}$.

Possible cases for checking the condition in Equation 11:

- When $T < \kappa$, then the null hypothesis H_0 is satisfied with probability $1 - \alpha_0$ (α_0 being the significance level). In this case, there are no displacements in the given network, and thus the resulting vector $\mathbf{d}$ is due only to random errors between the two measurement periods.
- When $T \geq \kappa$, then the alternative hypothesis H_a is accepted. In this case, there are displacements in the given network, and we cannot identify a point (or points) that has moved. It is only possible to say that at least one point has moved.

The single point test statistic T_i will take the form (Eq. 12):

$$T_i = \frac{\mathbf{d}_i^T \mathbf{Q}_i^{-1} \mathbf{d}_i}{\sigma_d^2} \sim \chi^2(h_i) \tag{12}$$

where $\mathbf{d}_i(h_i \times 1)$ = the displacement vector of the i-th point,
$\mathbf{Q}_i(h_i \times h_i)$ = the cofactor matrix of the i-th point,
h_i = rank of the matrix of the point in the network.

The test statistic is compared with the threshold value $\kappa_i = \chi^2_{h_i,1-\alpha_0}$:

- $T_i < \kappa_i$ then the point under test is not displaced, with probability $1 - \alpha_0$.
- $T_i \geq \kappa_i$ then the point under test shall be regarded as displaced.

The terms global test and local test are used for this paper regarding the parameter vector. The former is when the detectability of a displacement is tested on the whole network. In turn, analyses of detectability in terms of performed statistical tests for individual points in this paper will be called a local test of a single point.

Based on equations 10 and 11, the value of the non-centrality parameter λ of the distribution calculated from the pre-set or well-defined displacement vector $\mathbf{d}_A$ (Eq. 13):

$$\lambda = \frac{\mathbf{d}_A^T \mathbf{Q}_\mathbf{d}^+ \mathbf{d}_A}{\sigma_d^2} \tag{13}$$

The value of the non-centrality parameter calculated from Equation 13 is compared with the lower limit of the value of the non-centrality parameter obtained from Baarda's nomograms (note that only for the χ^2 distribution) (Baarda, 1968) or using a computational algorithm (Aydin and Demirel, 2005). The threshold value of the non-centrality parameter λ_0 is determined by the significance level α_0 (type I error probabilities) and the power of the test $1 - \beta_0$ (β_0 - type II error probabilities).

Then, if the inequality (Eq. 14):

$$\lambda \geq \lambda_0 \tag{14}$$

where $\mathbf{d}_A$ is detectable.

Then it can be concluded that the given network is sensitive to expected displacements. If the inequality is not true, the network must be redesigned to be capable of detecting expected displacement values.

Based on the inequality (Equation 14) and the knowledge about the magnitude of the displacement of the test object (vector $\mathbf{g}$), it is possible to make an evaluation to determine the detectable displacement.

Then the alternative hypothesis takes the form (Eq. 15):

$$H_a\colon\ \mathbf{d_A} = c\,\mathbf{g} \tag{15}$$

where c = scalar factor.

The displacement vector $\mathbf{d_A}$ is detectable when (Eq. 16):

$$\lambda = \frac{(c\,\mathbf{g})^T\, \mathbf{Q}_\mathbf{d}^+\, (c\,\mathbf{g})}{\sigma_d^2} > \lambda_0 \tag{16}$$

Based on Equation 16, the value of the scalar factor c can be calculated (Eq. 17):

$$c \geq \sigma_d \sqrt{\frac{\lambda_0}{\mathbf{g}^T \mathbf{Q}_\mathbf{d}^+\, \mathbf{g}}} \tag{17}$$

As a result, it is possible to determine the minimal detectable displacement vector $\mathbf{MDD}$ in the assumed model of the network (Eq. 18):

$$\mathbf{MDD} = c_{min} \cdot \mathbf{g} \tag{18}$$

The Minimal Detectable Displacement vector, a measure of network sensitivity, was obtained.

The papers (Aydin, 2011; Niemeier, 1982) take into account that in some cases, the directional vector $\mathbf{g}$ is not known. Then, in order to calculate the minimal detectable displacement, the eigenvector $\mathbf{g}_{\min}$ belonging to the minimum eigenvalue λ_{min} calculated from the decomposition of the cofactor matrix $\mathbf{Q}_\mathbf{d}^+$ should be used. This is the direction with the maximum deformability of the network (Niemeier, 1982). The formula takes the form (Eq. 19):

$$c_{min} \geq \sigma_d \sqrt{\frac{\lambda_0}{\mathbf{g}_{\min}^T \mathbf{Q}_\mathbf{d}^+\, \mathbf{g}_{\min}}} \tag{19}$$

Finally, the minimal detectable displacement vector for the whole network $\mathbf{MDD}_{newtork}$ in the eigenvector direction $\mathbf{g}_{min}$ is obtained for a given power test $1-\beta_0$ and significance level α_0(Eq. 20):

$$\mathbf{MDD}_{network} = c_{min} \cdot \mathbf{g}_{min} \tag{20}$$

Using the Moore-Penrose pseudoinverse (Eq. 6), in this solution, the cofactor matrix $\mathbf{Q_d}$ (Eq. 8) and displacement vector $\mathbf{d}$ (Eq. 7) are related to the centre of gravity of the net. This solution gives the lowest minimal detectable displacement value (Łapiński, 2019).

III. Empirical tests

For a computational example of sensitivity analysis, let us assume a levelling network consisting of eight points numbered P1 from P8. These are eight unknowns in the adjustment process. The defect of the network (d_s) is an equal one. The geometry of the vertical network is shown in Figure 1.

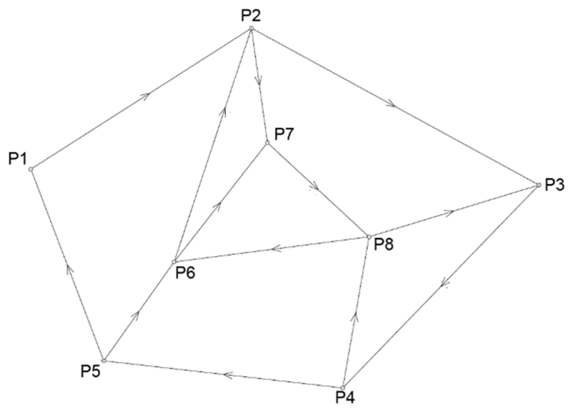

Figure 1. The geometry of the levelling network.

The observations consist of 13 differences in elevation between network points from the two measurement epochs - Table 1.

Table 1. Observations in levelling network

No.	Observation code		Number of stations in one measurement epoch
1	P1	P2	5
2	P2	P3	6
3	P3	P4	5
4	P4	P5	4
5	P5	P1	4
6	P5	P6	1
7	P6	P7	2
8	P7	P8	3
9	P8	P6	3
10	P4	P8	3
11	P8	P3	3
12	P6	P2	2
13	P2	P7	1

The numerical study considered the case of uncorrelated observations. The calculation of displacements with the method of observation differences was applied, assuming the same number of stations in levelling lines for both measurement epochs. The standard deviations for the differences $\sigma_i = \sigma_0\sqrt{\mathrm{n}}$, where $\sigma_0 = 1\,mm$, n – number of stations. It was assumed that the observations did not have gross errors or were correctly eliminated by diagnostic tests.

The study assumed a sample number of 50 000 using Monte-Carlo Simulations (Koch, 2018) (with such a sample is the compatibility of results between simulations). But at the same time, imposed a condition on the pseudo-random vector (corresponding to standardised random errors), *i.e.*, the typical true error is between 0.85 and 1.15. A self-developed algorithm in MATLAB was implemented to perform the sensitivity analyses and the presentation of the results.

In the first step, the value of minimal detectable displacement for the network was calculated based on Equation 20, assuming $\chi^2(h)$ distribution, $\alpha_0 = 0.05$, $1-\beta_0 = 0.80$. The values of $\mathbf{MDD}_{\mathrm{network}}$ are presented in Table 2.

Table 2. Summary of $\mathbf{MDD}_{\mathrm{network}}$ values

$\boldsymbol{MDD}_{network}$ [mm]	6.61 0.70 -3.87 -3.02 1.21 0.45 0.13 -2.21
$\|\boldsymbol{MDD}_{network}\|$ [mm]	8.65

A displacement sensitivity analysis was then performed by distorting the parameter vector with the value of the minimal detectable displacement. The known parameter vector is then converted into a vector of observations (with a fixed matrix $\mathbf{A}_s$), then the pseudo-random part of the observations is added. An adjustment by the least squares method is performed based on the calculated new observation vector. The result of each adjustment is a displacement vector, and the value of the non-centrality parameter is calculated from Equation 13, which is then compared with its threshold value (Eq. 14).

The results from the analyses performed are presented using the mean success rate (MSR) proposed in the work (Hekimoglu and Koch, 1999). According to the case study, the experiments are appropriately tested in the simulation procedure. After applying the appropriate test procedure, those that exceeded the threshold value (so-called successful results) are counted for each experiment. The MSR is calculated as follows (Eq. 21):

$$MSR = \frac{number\ of\ successful\ results}{number\ of\ experiments} \tag{21}$$

The results for each analysis are related to the case of displacement detection, *i.e.* information based on a statistical test. Three different graphs were produced to present the results. The first one shows the effectiveness of individual statistical tests, *i.e.* global test, local test, both tests at the same time and in the case of lack of information about a possible displacement. The effectiveness of the static test means that a displacement of at least one point in the network was detected. The calculated efficiencies for each statistical test are shown in Figure 2.

Based on Figure 2, the highest efficiency, *i.e.* with a value of 80.8 %, was obtained for the local test. It means that the displacement of at least one point in the network was detected. The effectiveness of the global test is 79.5 %, where 75.9 % refers to the displacement detection effectiveness for both tests simultaneously. It can be said that the tests have similar effectiveness, with the exception of 15.6 % of all tests where displacement was not detected. It should be added that an 80 % probability power of the tests was assumed.

The second graph shows more detailed results for the test performed for each point in the network. Efficiencies are presented with information on the number of points in the network considered to be displaced. These results are shown in Figure 3.

Based on Figure 3, the highest efficiency, *i.e.* with a value of 32.2 %, is characterised by the single point detection case. Although, for two points of the network, the efficiency of detecting displaced points is slightly lower, reaching a value of 30.9 %. It can be seen from Figure 3 that the effectiveness decreases as the number of detected points increases. In this analysis, a maximum of 5 network points were detected. Although for the case of five points detected, the effectiveness is very low, equal 0.5 %. The sum of all values is 80.8 %, which is the total effectiveness of the local test. It is worth reminding that there are 8 points in the network.

More detailed results, as they relate to the effectiveness of detecting individual point numbers in the network, are presented in the third type of graph, as illustrated in Figure 4.

Based on Figure 4, the highest efficiency of 70.5 % was obtained for point P1. In the second order, the highest efficiency was obtained for point P3. The successive points are P4, P8, P5, P2, P6, and P7. Figure 4 (100 % cumulative columnar) for each network point should be interpreted as follows, *e.g.* for point P1 in 70.5 % of all trials, the local test detected displacement. In comparison, the displacements were not detected for 29.5 % of all experiments.

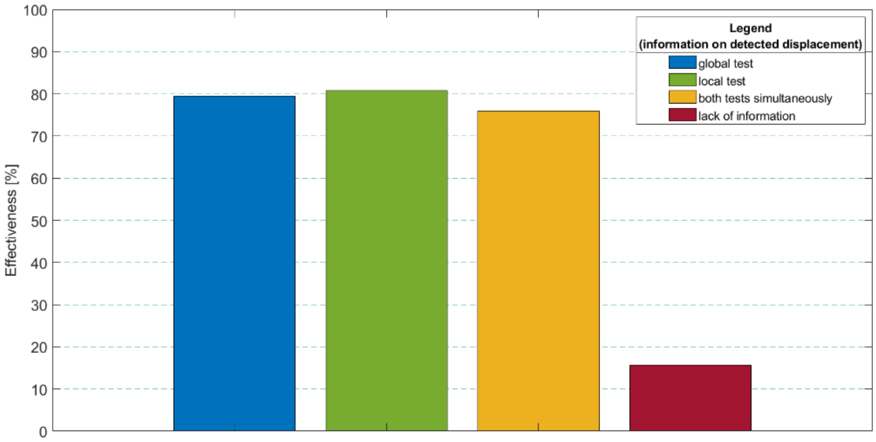

Figure 2. Effectiveness diagram for different statistical tests.

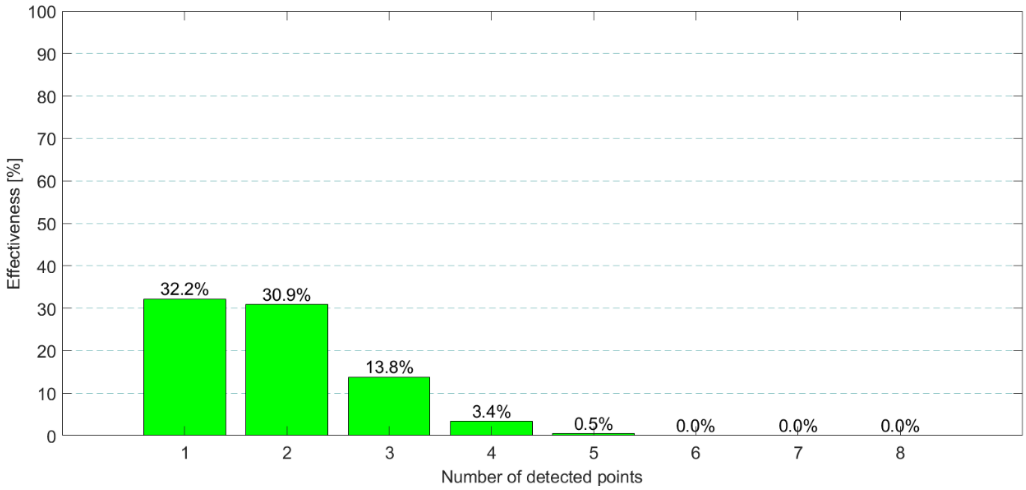

Figure 3. Determination of the effectiveness of local tests with information on the number of points considered to be displaced.

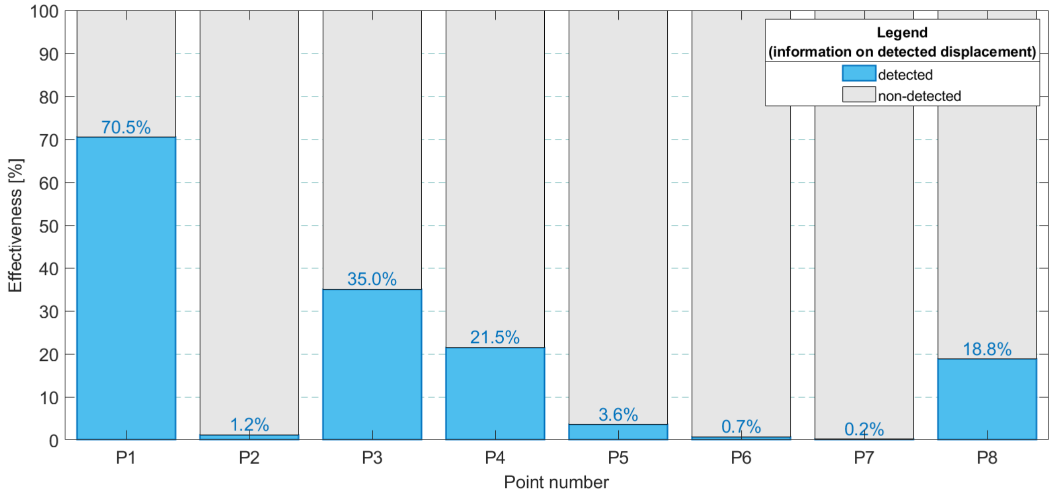

Figure 4. Summary of the detection effectiveness of each network points.

IV. Conclusions

This work on geodetic displacement measurements deals with sensitivity analyses. The disturbance analyses with the value of the minimal detectable displacement (constant vector) in the direction of the maximum deformability of the network were carried out. The work aimed to check the efficiency of displacement detection with a large simulated test sample.

The results are shown in Figures 2-4. Concerning Figure 4, to indicate why there is such an order of detection efficiency of individual points, reference should be made to the cofactor matrix $\mathbf{Q_d}$. This matrix also calculates the accuracy characteristics, so the relationship between the value of the minimal detectable displacement for the whole network and the accuracy characteristics can be assumed. Table 3 shows the values of the mean errors of point position of all network. Thus, it confirms the obtained order of the resulting effectiveness. Only for point P7, a low efficiency was obtained, despite having a higher accuracy of 0.16 mm than point P6 and 0.03 mm than point P2. However, it should be noted that this relates to a (very low) efficiency of less than 1.2 %. Point P1 has the lowest point accuracy and, according to work (Kutterer, 1998), can be the weakest area of the network.

Table 3. Mean error of point position

Point number	Mean error of point position [mm]
P1	2.0
P2	1.1
P3	1.6
P4	1.5
P5	1.2
P6	0.9
P7	1.1
P8	1.2

By distorting the parameter vector (displacements) with the value of $\mathbf{MDD}_{network}$ the effectiveness of the global test agrees with the assumed power of the test. Local tests for a single point showed that there are displacements of more than one point in 48.6 %. It should be considered using another local test for the case of displacement of multiple points.

When designing a network, the geometry of the network is important, as is the accuracy of the determination of the network points (mean error). Therefore, sensitivity analysis is used as one of the criteria for network optimisation to meet the assumed requirements for the control network. Depending on the purpose of the geodetic network, different criteria can be selected for implementation in the optimisation procedure. The definition of $\mathbf{MDD}_{\mathrm{network}}$ can be used to compare the magnitude of minimal detectable displacements for different control networks design options.

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