New Approaches to Parameter Estimation with Statistical Censorship by means of the CEV Algorithm: Characterization of its properties for high-performance normal processes

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Abstract – Parameter estimation for characterize a population using algorithms is in constant development and perfection. Recent years show that data-based decision making is complex when there is uncertainty generated by statistical censorship. The purpose of this paper is to evaluate the effect of statistical censorship on the normal distribution (common in many processes). Estimation parameter properties will be characterized with the Conditional Expected Value (CEV) algorithm, using different censorship percentages and sample sizes. Estimation properties will be focused on the monitoring and decision making in industrial processes with presence of censorship.

Key words: *Statistical Censorship; Data Analysis; Decision Making, Algorithm Optimization.*

1 Introduction

When measured parameters in a data set have a certain uncertainty of not representing the studied population and the real parameters are on the other side of the detection frontier, statistical censorship is present. Thus, statistical censorship in a data set will lead to wrong decision making.

Various authors have dedicated efforts in the study and applications of algorithms and methods for parameter estimation in censored data sets. The following stand out: Application of maximum likelihood models on data with high statistical censorship and kurtosis (Salinas and Martinez 2013). Data simulation models applied to soil science (Sedda, et al. 2012). Time failure data analysis, censored by clinical trial intervals, demographic studies, medical and public health tracing (Zhu, Tong and Sun 2008). Undetected polluting particle concentration in the environment (Krall, Simpsom and Peng 2015). And, evaluation of censored data on average life span of products with bayesian methods (Shen, et al. 2017).

Data censorship is usually mistaken with statistical truncation. Censorship data are often eliminated in order to create a truncated data set, before calculating its parameters (Krall, Simpsom and Peng 2015). Censorship speaks of incomplete information, truncation talking about missing data (this is the main difference). In truncated scenarios outlying data are never registered, by mistake or because there is no evidence of their existence.

Statistic censorship is common in industrial and medical application, usually as a result of limited resources. For example, when a material breaking strength is tested, the precision of the testing machine might influence the results (Steiner and Mackay 2000). Another major concern occurs when the median value falls below the system detection limits, causing a great loss in measurement information (Mason and Keating 2011). This type of censorship data might fall to the right or the left of the data distribution.

In a normal distribution with high statistical censorship, asymmetry and kurtosis significantly affect any estimate over the average or the standard deviation. Therefore, if statistical behaviours are unknown, it is impossible to monitor a process with the Shewhart control graphics. Figure 1 shows the normal density distribution (PDF) and its censorship version ($Tc \sim N(\mu,\sigma)$). Differences in height and distribution of the data are observed, which indicates there is a significant change in the position and dispersion of data.

Figure 1 (Normal density distribution evolution, C = -2 to 1, left censorship)

Even if the level of statistical censorship in a data set is known, and the data comes from a standard normal distribution, the different shapes data adopts due to censorship make it almost impossible to adjust the information. Some Statistical and mathematical software as SPSS, Statgraphics, RStudio, Matlab, allow to test the statistical behaviour of data using statistics as the Anderson–Darling, Kolmogorov–Smirnov y Chi-squared. However, the high kurtosis and skewness, and natural randomness of data in a censoring distribution make assigning an exact distribution of the error. This is why developing a data estimation model ignoring such complexities is unfavorable for decision-making.

Others authors have studied parameter estimation for specific processes, as diffuse regression algorithms, variance analysis for predicting energy consumption (Azadeh, Seraj and Saberi 2011). Maximum likelihood estimation of tipping points in high performance processes with linear disturbances (Akhavan Niaki and Khedmati 2013). Capacity analysis of processes with more than one correlated quality variable, exhibiting no normal characteristics (Abbasi, Taghi and Niaki 2010). Useful remainder life time algorithms for reducing machine maintenance (Hong-Zhong, et al. 2015). Parameter estimation using an inverse censorship Weibull distribution with intervals (Singh and Tripathi 2018). And censorship data analysis in biological science (Ulín Montejo 2017).

Decision-making in industrial processes control has been historically made in the Shewhart control graphics. These standardized graphic have well known properties especially when process parameters are under control. For example, in a process with $\mu= 0$ and $\sigma = 1$ under control, the average sampling vector from a control output must behave as T~N($\hat{X}$,S). Likewise the sampling deviations vector must behave as T~N($\hat{S}$,S). Therefore if observations are censored ((left or right from the distribution) its parameters $\hat{X}$ and $\hat{S}$ will change significantly and its statistical shape is uncertain.

For example, for $\mu = 0$ and $\sigma= 1$ **(N(0,1))**, assuming the process is under control with a left censorship level of C = 0.55. The percentage of censorship can been defined by a random variable *T*, normally distributed with mean, μ and standard deviation, σ, censored to the left of **C** by:

Equation 1

$$Z_c = \frac{C - \mu}{\sigma}$$

Where *Zc* is the standard value of the censorship in ***C***, and $\phi(Z_C)$ the cumulative distribution function (CDF) from standardized normal model at the point ***C***. Therefore the percentage of censorship for a censored normal to the left can be described as follows:

Equation 2

$$\Phi\left(Z_c\right) = \int_{-\infty}^{Z_c} \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(Z_C\right)^2}{2}} \right) dZ$$

Equation 3

$$Pc = \Phi\left(\frac{0.55-0}{1}\right) = 70.88\%$$

Consequently, the censorship proportion for a PDF censored to the right can be described as:

Equation 4

$$\Phi\left(Z_c\right)=\int_{Z_c}^{\infty}\left(\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{\left(Z_c\right)^2}{2}}\right)dZ$$

When the proportion censorship is unknown, the possible censorship proportion must be calculate based on hypothetical under control parameters.

This study has as main objective to identify the properties a censorship random variable may have, whose uncensored origin accommodates to a standard normal distribution. Knowing properties such as, sensitivity to parameter change when the variable is under statistical censorship will enhance the concept of ARL (Average run length). This will increase the detection sensitivity in future industrial process failures and reduce production control costs. Finally, the parametric statistical models are used to estimate data described in Steiner and Mackay, based on the substitution of each censoring observation by a conditional expected value (CEV) (Steiner and Mackay 2000), initially described by (Lawless, J.F. 1982)

2 Methodology

2.1 Statistical forms similar to the censored PDF

This investigation evaluates the statistical behaviors of the averages vector and the deviation vector for three different dispersion levels; N(0;0,7), N(0;1), N(0;1,6). Those statistical behaviors are also evaluated from the output control from censored processes, for four different sample sizes; N = 5, 10, 15 and 20. The purpose of this is to demonstrate the complexity in adjusting censorship information to different levels in any particular distribution. Information will be presented in tables that contain the statistics probability that best fits to the goodness of fit test of Anderson-Darling, Kolmogorov–Smirnov and Chi-squared for each data simulation.

2.2 Calculation of CEV Weights for Censored Data

2.2.1 CEV weights for PDF censorship left

The CEV algorithm is based on replacing the censorship information with a conditional expected value, denoted as "*Wc weight*". This value is based on the sampled average and standard deviation calculation for each of the sub groups in the matrix generated by a process of monitoring $\bar{X}$ and S.

The weight *Wc* for normal observations censorship by the left is obtained as follows:

Equation 5

$$Wc=E\left(T\middle|T\leq C\right)=\mu-\sigma\left(\frac{\phi\left(Zc\right)}{\Phi\left(Zc\right)}\right)$$

Where the terms *Ø(Zc)/ϕ(Zc)* can be denoted in the random function *V(Zc)*, with a relationship between the probability density function (PDF) and the cumulative distribution function (CDF) (Lawless, J.F. 1982):

Equation 6

$$V\left(Zc\right)=\left(\frac{\phi\left(Zc\right)}{\Phi\left(Zc\right)}\right)=\left(\frac{\phi\left(\frac{C-\mu}{\sigma}\right)}{\Phi\left(\frac{C-\mu}{\sigma}\right)}\right)$$

Therefore, the PDF or *Ø(Zc)*, in the fix censorship point C is:

Equation 7

$$\phi(Zc) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\left(\frac{C-\mu}{\sigma}\right)^2}{2}}$$

Where C ϵ R; μ ϵ R; σ > 0

Therefore, the new data used for estimating the new parameters are denoted as:

Equation 8

$$w_i = \begin{cases} t, If\ t > C \\ W_{c,} If\ t \leq C \end{cases}$$

2.2.2 CEV weight for PDF censored right

The weight *Wc* for normal observations censorship by the right is obtained as follows:

Equation 9

$$Wc = E\left(T \middle| T \geq C\right) = \mu + \sigma\left(\frac{\phi(Zc)}{\Phi(Zc)}\right)$$

The new data used for estimating new parameters are denoted as:

Equation 10

$$w_i = \begin{cases} t, If\ t < C \\ W_{c,} If\ t \geq C \end{cases}$$

2.3 Maximum likelihood process

The estimation algorithm is feed by the Equation 2 or Equation 4 (According to the censorship side), and the Equation 5 or Equation **9** (According to the censorship side).

The media and standard deviation are calculated by:

Equation 11

$$\hat{\mu}_i = \sum_{i=1}^{n} \frac{w_i}{n}$$

Equation 12

$$\hat{\sigma}_i = \sqrt{\frac{\sum_{i=1}^{n}(w_i - \hat{\mu}_i)^2}{r + (n-r)\lambda(Z_C)}}$$

Where *r* is the number of non-censorship observations, *n* is the totally of the data, *i* is equal to the number of iterations. Described the Equation 12 the parameter λ(Zc) for left-censored data is defined as:

Equation 13

$$\lambda\left(Z_C\right)=\frac{\phi\left(Z_c\right)}{\Phi\left(Z_c\right)}\left[\frac{\phi\left(Z_c\right)}{\Phi\left(Z_c\right)}+z\right]$$

And $\lambda(Zc)$ for right-censored data is described as:

Equation 14

$$\lambda\left(Z_C\right)=\frac{\phi\left(Z_c\right)}{\Phi\left(Z_c\right)}\left[\frac{\phi\left(Z_c\right)}{\Phi\left(Z_c\right)}-z\right]$$

$\lambda(Zc)$ Is between zero and one. When the value is near one the censorship percentage is small and when the censorship is high the value is close to zero. To find the maximum likelihood estimate, the algorithm is applied iteratively until the estimates converge.

Figure 2 (CEV maximum likelihood process)

Note: The functions *Wc, Wi,* $\phi(Zc)$ and the parameter $\lambda(Zc)$ should be estimated depending if its left-censored or right censored.

2.4 Simulations and estimation analysis

In this phase the precision level of the algorithm prediction is evaluated in 14-censored point through the distribution (N(0;0,7) - N(0;1) - N(0;1,6)), in order to evaluate the behaviour of the estimations. This behaviour is important because monitoring a random normal variable in a trusted way is needed even in the presence of censored data, knowing the strength and weakness of the CEV algorithm. 560,000 runs were performed for the CEV algorithm (exposed in section 2.3), through simulations in Matlab for each case of data dispersion. A total of 1,680,000 simulations were carried out.

2.5 Analysis of the results

The final considerations and properties of the algorithm are detailed. Abnormalities are identified and viewpoints are evaluated for use in monitoring high-throughput processes. Finally, unusual sources of variability present in the estimates are evaluated.

3 Results

3.1 Similar statistical shapes to the censorship PDF.

Through data fit testing, it was found that the PDF with left censorship for the averages vector Tc~N($\hat{X}$,S) and the standard deviations vector Tc~N($\hat{S}$,S), data can be modeled with probability distributions that simulate positive asymmetries. On right censorship the data can be modeled with probability distributions that simulate negative asymmetries in the averages vector Tc~N($\hat{X}$,S). Properties in the deviations vector Tc~N($\hat{S}$,S) with censorship to the right keeps the same properties as the left censorship, for example a total sampling censorship, will result in sampling standard deviation equal to 0. All these will be met, as long certain censorship percentages are not exceeded; these percentages are also affected by the sampling size.

3.1.1 Left Zc statistical distributions

With low dispersions N(0;0.7) and low censorship proportions, there are significant variations in the goodness and fit statistics, this is due to normal censorship sensibility, causing anomalous observations making it impossible to adjust the distribution. When it is possible to adjust the distribution, several tests have shown similar behaviors to the normal and logistic probability distributions. During the simulations it was found that the censorship around 20%, the goodness and fit tests also show similarities with the Weibull distribution.

Additionally for the Vector Tc~N($\hat{X}$,S), censorship data between 66% and 52% tend to adjust as a Gamma distribution; this is due to particular asymmetries form in this censorship level. When the censorship proportions is between 72% and 56% is similar statistical fit to the LogNormal and Gamma distributions. When the censorship is between 16% and 8% there is a similar significant fit to the PDF. This last result is particular important, because in some cases there is not a significant difference between moderate censorship normal and an uncensored normal, there are however anomalous behaviors that will generate observations outside limits from a normal control graphic.

Tables Table 1 and Table 2, show the censorship percentages from which most of the censorship data can be adjusted. It can be observed that increasing the sample size in the process matrix increases the censorship percentage in which data can be adjusted.

Table 1 Percentages of censorship from which PDF with left censoring is adjusted

	N 5		**N 10**		**N 15**		**N 20**	
Normal	Tc~N(X,S)	Tc~N(S,S)	Tc~N(X,S)	Tc~N(S,S)	Tc~N(X,S)	Tc~N(S,S)	Tc~N(X,S)	Tc~N(S,S)
N(0;0.7)	36,0%	44,3%	64,0%	71,6%	71,6%	78,4%	71,6%	84,1%
N(0;1)	59,9%	46,0%	65,5%	70,9%	70,9%	75,8%	65,5%	80,2%
N(0;1.6)	52,5%	52,5%	70,2%	70,2%	73,4%	73,4%	73,4%	73,4%

3.1.2 Right Zc statistical distributions

In this case, censorships by the right has tendency to adjust to Gamma 3P, Logistic, Weibull and Beta probability distributions. This happens mainly when the combination of certain parameters in this distributions achieve simulating data with asymmetric shapes on the left. Statistical results show a wide range in the distribution adjustments, generating certain uncertainty on which statistical forms adjust better to censorship data. The results show that adjustment PDF with statistical censoring depends of minimal variations in the data set, causing large statistical changes. With a large square error, the uncertainty in the decision making of the process increases.

Table 2 Percentages of censorship from which PDF with right censoring is adjusted

	N 5		**N 10**		**N 15**		**N 20**	
Normal	Tc~N(X,S)	Tc~N(S,S)	Tc~N(X,S)	Tc~N(S,S)	Tc~N(X,S)	Tc~N(S,S)	Tc~N(X,S)	Tc~N(S,S)
N(0;0.7)	36,0%	44,3%	64,0%	71,6%	71,6%	78,4%	71,6%	84,1%
N(0;1)	46.0%	46,0%	65,5%	70,9%	70,9%	75,8%	65,5%	80,2%
N(0;1.6)	43.8%	52,5%	63.5%	70,2%	73,4%	73,4%	73,4%	73,4%

In this study complex probabilities distributions as Beta 4 parameters were evaluated. This distribution has a lower quadratic error when the variability is low and the distribution of data is very skewed, allowing to have better skewness and kurtosis estimations (Beckman and Tiet jen 2007). In this case Tc~Beta(α,β,a,c), appeared in the adjustment tests 60% of the time. Finally, even do some probability distributions are predominant to the right or left of the censorship, the best indicator in each case is the asymmetry and kurtosis estimator, since all depends on the amount of skewness in the average or deviation vectors.

3.2 Maximum likelihood process evolution and parameter convergence

Evaluating the estimation algorithm for three normal dispersion levels, it can been seen that the CEV algorithm tends to underestimate the real average value with left censorship and overestimate the real average value with right censorship, when censorship levels are below 50%. This is found in all the evaluated cases, since all the weights applied on the data matrix result in the process parameters being biased when the censorship is low. Figures Figure 3 and Figure 4 show the maximum likelihood process evolution for the average and standard deviation estimation under a censorship of 92.34% and 7.66% respectively. The process converges rapidly, according to the censorship level and accuracy desire.

Figure 3 (Likelihood process N(0;0.7) N20, Pc = 92.34%)

Figure 4 (Likelihood process N(0;0,7) N20, Pc = 7.66%)

The number of iterations required to converge was significantly higher for N (0;0.7) than for N(0;1.6) when the levels of censorship are high, see Figure 5 and Figure 6. For high censorship levels the normal distribution shows a lower censorship level at point C = 1 for the left side, and C = -1 for the right case when its variability is higher. However, the variability of the distribution affects the precision of the estimates in each of the iterations when the censoring is less than 50%.

Figure 5 (Likelihood process N(0;1.6) N20, Pc = 73.40%)

Figure 6 (Likelihood process N(0;1.6) N20, Pc = 26.60%)

3.3 Precision level in the CEV algorithm prediction.

Studying the precision level in the algorithm prediction for fourteen censorship points through each distribution (N(0;0.7) - N(0;1) - N(0;1.6)) it was found that for high censorship levels, the calculated weights for the algorithm allow making a better adjustment for the real distribution parameter. The better estimations (closer to 0) are between 63.95% and 36.05%. See figures Figure 7 and Figure 8.

Figure 7 Estimation of the mean with left censorship

Figure 8 Estimation of the mean with right censorship

Figure Figure 9 and Figure 10 show the behavior of the CEV algorithm standard deviation estimations for different censorship levels. For high censorship levels, the weights calculated by the algorithm allow a better parameter estimation for the distribution. The closer estimations to the real standard deviations for each case are between 63.95% and 36.05%.

Figure 9 Estimation of the standard deviation with left censorship

Figure 10 Estimation of the standard deviation with right censorship

Since the algorithm estimates the parameters based on the control process matrix, there are no significant differences in the estimations between different sample sizes. However, there are between different dispersion levels.

3.4 Variations in estimates

The analysis of variation of the simulated estimates is done using a multifactorial ANOVA, which contains the residuals of the squared estimates; the factors sample size (N) and Level of censorship (C) at two levels. Multifactorial ANOVAs were also computed to test if there is no significant difference between right and left censorship estimation variation. Figure 11 has a plot of LSD for two levels of censoring and for four sample sizes. It also contains the P values and a plot showing the interaction between the studied factors.

Figure 11 ANOVA – Variation of estimates $\hat{\boldsymbol{\mu}}$. Left and right censorship. Sample size N and level of censorship

In the simulations, the variation of $\hat{\mu}$ has no significant differences between the left CEV case and the right CEV case. However, significant differences were found in the variations of $\hat{\mu}$ between low and high censoring in the data. The reasons for these differences are correlated with the variation of the vector of sample averages, which at high censoring contains little dispersion in its sample observations, generating that at high censoring the CEV algorithm varied less in its estimate $\hat{\mu}$; even if it makes it little powerful estimating the true value μ.

By increasing the sample size, the CEV algorithm shows fewer differences in variations $\hat{\mu}$ when the censorship is high, While at low censorship the CEV algorithm shows more significant changes in the variations $\hat{\mu}$ as the sample size increases. Finally, the interaction of the variations $\hat{\mu}$ by sample size and level of censoring becomes more significant as the dispersion of the censored PDF increases.

Contrary to the variations of the estimates $\hat{\mu}$, the variations of the estimates $\hat{\sigma}$ are bigger at higher censorship levels. The reason is due to the properties of the censored standard deviations vector, this vector contains sampling observations equal to zero (result of a full censorship sample). When the sample (random reasons) contains a single not censorship observation, causes a significant variation in the censored standard deviation vector. See Figure 12

Figure 12 ANOVA – Variation of estimates $\hat{\boldsymbol{\sigma}}$. Left and right censorship. Sample size N and level of censorship.

3.5 Application example: Parameter estimation

A semiconductor fabrication example will be used to explain the definitions and results obtained. Semiconductor fabrication is an important step for photo-lithography, in this process a light sensitive material is applied to the silicium sheet, the circuit pattern is exposed to the resistance using high intensity UV light and the remaining material is removed by plasma engraving or humid chemical. It is quite typical to follow the baking process to increase adhesion and etching resistance (Mongomery 2009). If there is a problem in the lower detection limit in the measurement system, and suppose that the true value μ is known to be below this limit C = 1.5628, whose average should be around 1.52 microns with a standard deviation of 0.13 ; then it is the typical case of left censorship.

For applying the CEV algorithm and estimating the real μ y σ parameters, 50 samples of size 5 were taken in a control process. Table 3 shows the first ten samples.

Table 3 Flow width measurements (microns) for baking process.

	Sample				
n	**1**	**2**	**3**	**4**	**5**
1	**1,5628**	**1,5628**	1,6744	**1,5628**	1,6914
2	**1,5628**	**1,5628**	1,6075	**1,5628**	1,6109
3	**1,5628**	**1,5628**	**1,5628**	**1,5628**	1,5674
4	**1,5628**	1,6352	**1,5628**	**1,5628**	**1,5628**
5	**1,5628**	**1,5628**	**1,5628**	**1,5628**	1,6441
6	1,5955	**1,5628**	**1,5628**	**1,5628**	**1,5628**
7	1,6274	**1,5628**	1,8366	**1,5628**	**1,5628**
8	**1,5628**	**1,5628**	1,6637	1,6067	**1,5628**
9	**1,5628**	1,7277	**1,5628**	**1,5628**	**1,5628**
10	**1,5628**	1,6697	**1,5628**	**1,5628**	**1,5628**

Once the process in Figure 2 is applied (CEV Estimation Process), the following results are obtained:

- Initial censored average μ_c = 1.5930
- Initial censored standard deviation σ_c = 0.0575
- Estimated average under control $\hat{\mu}$ = 1.5351
- Estimated standard deviation under control $\hat{\sigma}$ = 0.1014
- Estimated censorship ratio, Pc = 61.67%
- Iterations number = 68
- Wc = 1.4723

4 Considerations and Conclusions

A random variable may not always have relatively small parameters. Whenever possible, it is pertinent to evaluate the dispersion of the data using the expression of Equation 15, this in order to evaluate the variability characteristic of the data with respect to the cases raised in the present investigation, identifying the properties to obtain with the CEV algorithm.

Equation 15

$$Z_{ci} = \frac{(Xic - \mu_c)}{\sigma_c}$$

Where *Xic* is the original censored value, μ_c is the original average of the censored data, and σ_c is the censored standard deviation.

It can be observed in 3.1, that when there is censorship calculating probabilities for a random variable is problematic since its statistic forms are uncertain. Therefore the maximum likelihood process is proposed for variables who's behaviour is normalized and the possible assignable causes of variability are under control.

The CEV algorithm needs a large number of data in order to perform iterations, that’s why the matrix size generated in a sampling process affects the CEV maximum likelihood processes.

In section 3.5, the estimates, although better than the original parameters, have a degree of uncertainty. What the proposed scientific method of estimation allows is not to leave the decision to estimate the mean and standard deviation parameters to intuition. Knowing the algorithm estimation properties under the censorship with a normal distribution, will allow developing a more trustable control graphic, reducing associated costs in processes with statistical censorship.

Implementation of parameter estimation algorithms in processes with controlled censorship will open a door to reducing sampling costs in industrial processes. Research is now focused on demonstrating the effectiveness of a process control chart adapted to the CEV algorithm.

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