Antenna Element Design Using Characteristic Modes Analysis

Jacob J. Adams, Simone Genovesi, Binbin Yang, Eva Antonino-Daviu

***Abstract*—This paper provides a comprehensive review of recent applications of characteristic mode analysis (CMA) to innovative antenna element designs, including multi-port antennas, circularly polarized, wideband antennas, reconfigurable antennas, and dielectric resonator antennas. Emphasis is made on the interpretation of the characteristic modes for those unfamiliar with the method and on the physical insights gained from characteristic eigenvalues and eigenvectors of an antenna. In addition, we review CMA-based design strategies and specific design examples that highlight the application of CMA to various types of antennas. Ultimately, this paper seeks to demonstrate the value of CMA-based design insights for antenna engineering and to look towards promising new research directions for CMA and antenna research.**

***Index Terms*—Characteristic modes, shape synthesis, MIMO, wideband antennas, dielectric resonator antennas, reconfigurable antennas, circular polarization.**

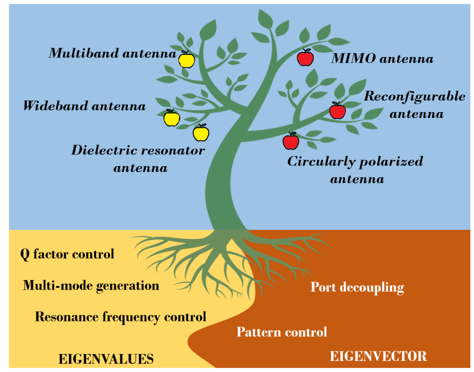


Fig. 1: Illustration of the primary relationships between antenna applications and characteristic mode properties.

I. Introduction

The Theory of Characteristic Modes was initially formulated for scattering problems [1], but it was quickly realized to have applications in antenna and radiation problems as well [2], [3]. The application of characteristic mode analysis (CMA) to antenna input parameter calculation was first investigated in [3], which laid the foundation for many later studies on CMA-based antenna design. Newman arguably conducted the first platform-based antenna analysis using CMA in [4], and antenna shaping or shape synthesis was studied as early as 1982 [5]. Around the same time, the increasing power of desktop computers and the growing availability of computational electromagnetics codes led to a period of rapid new research into novel antenna designs. However, only a handful of CMA studies were published during this time, until a review of the theory in this very publication [6] brought CMA once more into the spotlight, reviving interest within an antenna community in search of new methods to address a growing number of challenging problems.

Research over the past decade has shown that CMA offers important physical insight into antenna behavior from the modal perspective and can be a powerful tool for antenna modeling and design. Broadly speaking, CMA offers two unique features for antenna design: (i) CMA decomposes the antenna response into constitutive parts (characteristic modes), making it easier to study and control than the total response. (ii) CMA gives valuable information about the properties of the antenna structure irrespective of the feed, which allows for more fundamental analysis of the structure, unimpeded by a specific feed choice. However, CMA should be viewed as just one powerful tool in the antenna engineer's repertoire rather than a panacea. Proper interpretation of CMA results relies on the engineer's understanding of the method and their ability to apply this knowledge appropriately to their specific problem. This paper first describes an interpretation of the characteristic modes from the point of view of antenna element design, and then reviews ways in which CMA has been used for novel antenna element designs.

II. Interpretation of Eigenvalues λ_n and Eigenvectors $\boldsymbol{J}_n$ for Antenna Design

The interpretation of the eigenvalues and eigenvectors is central to the application of CMA to antenna design. The eigenvalues contain information about the net magnitude and type (electric or magnetic) of energy stored by the mode (relative to its radiated power) while the eigenvectors represent currents and thus pertain to the radiation properties as well as potential excitation schemes. Figure 1 illustrates several of the key antenna properties that can be manipulated through proper selection of the modal eigenvalues and eigenvectors, ultimately leading to applications across a wide range of antenna problems.

As described in [PAPER2] of this special issue, currents on an antenna are a weighted sum of the characteristic modal currents, and thus many other antenna properties, from the field patterns to the terminal currents, are a linear combination of the characteristic modes. Thus, the input impedance at a set of physical or conceptual terminals on the device can be expressed as a parallel combination of modal "circuits", as illustrated in Figure 2, wherein energy dissipated in each circuit is in fact radiated by that characteristic mode. This picture provides the antenna engineer a framework to conceptualize the underlying physics of their device. While a weighted sum

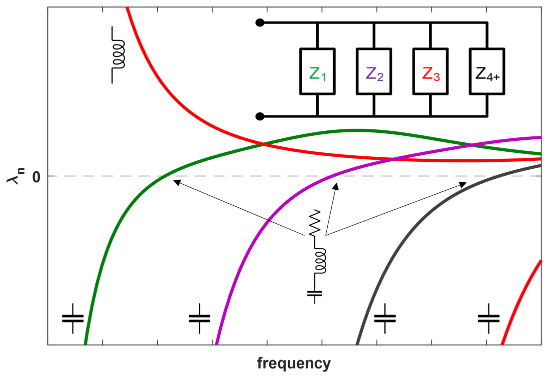


Fig. 2: An example characteristic eigenvalue spectrum showing capactive, inductive and resonant regions. Inset: illustration of the conceptual antenna input impedance as parallel modal impedances.

of all modes must be calculated in principle, in practice, CMA has been most frequently and effectively applied to the design of resonant antennas where only a few modes are well-excited and the sum can be truncated when considering radiated energy. Thus, a useful model is to consider the properties of modal "circuits" of a few critical modes in detail while higher order modes are lumped into a single reactance that produces negligible radiation.

Modes can generally be considered as "capacitive" modes with eigenvalues that begin at $-\infty$ in the low frequency limit or "inductive" modes that tend toward $+\infty$ in the low frequency limit and then approach resonance as frequency rises. In most cases, the eigenvalues of capacitive modes approach zero and resonate while inductive modes are sometimes referred to as "non-resonant" modes due to the tendency of their eigenvalues to remain above zero. Figure 2 illustrates common eigenvalue behavior, but these categories are not absolute and characteristic modes do not always fit neatly into these patterns. Still, this model provides a useful conceptual framework for antenna behavior.

Commonly, eigenvalues cross from capacitive to inductive at a single resonant frequency, and the modal "circuit" of each resonant mode can be approximated in this vicinity as a series resonator or a high pass series ladder network, akin to the spherical modes [7], [8]. Thus, the antenna response is modeled as a parallel combination of several series resonators, each with a well-defined and calculable resonant frequency and Q factor that can be used for design prior to selection of a feed. Such mode-based circuit models have also proven useful for a number of antenna applications, including rapid impedance and mutual coupling analysis for multi-port antennas [9], [10].

III. Antenna Design Using CMA

Characteristic mode analysis has been applied to numerous antenna problems, with interest exploding over the past decade. Even classical antenna designs such as the U-slot patch have revealed compelling new physical and design approaches using CMA [11], [12]. In this section, we highlight recent work on antenna elements and design methods that rely on CMA. Two other papers in this special issue [PAPER4], [PAPER5] discuss applications to platform-mode antennas and array antennas.

A. Multi-port Antennas

The inherent properties of the characteristic modes make CMA a natural choice for modeling and designing multiport antennas, and the method has drawn particular interest for multi-input, multi-output (MIMO) applications. For example, the characteristic modes have orthogonal radiation patterns, which is a key property sought after in MIMO antenna design. Thus, the characteristic modes represent a compelling basis for developing new MIMO antenna designs, as a design that can excite the characteristic modes would be guaranteed to have negligible coupling. Of course, it is impossible to excite a single characteristic mode in practice, but modal current distributions can give insight into feed configurations that produce low mutual coupling.

In developing the proper characteristic modes for a multiport antenna, the critical features to study are (i) the eigenvalues, which should be near resonance ($|\lambda| < 1$ suggested in [13]) for at least as many modes as are required for the design, and (ii) the current modes on the antenna, which give an indication as to how the feed should be constructed and how each mode will radiate. When designing an antenna with multiple characteristic modes, one can either assume that the modes are already present on a large, fixed conducting platform or that the antenna element itself should be redesigned to support multiple modes. This paper primarily deals with the latter situation and the design of individual multi-port elements, while the former approach is reviewed in [PAPER4].

The selection and design of a radiating element often begins with a canonical resonant antenna, such as a loop or microstrip antenna. These structures can then be analyzed using CMA and modified heuristically [13]–[18]. For example, in [15], the modes of a wide loop antenna were analyzed and found to support four modes with low eigenvalues, although they resonate at different frequencies. By selectively placing capacitive gaps at positions where some modes have current nulls, their eigenvalues and thus, resonant frequencies, can be controlled somewhat independently and are moved near each other for MIMO operation.

The desired multi-mode response can also be obtained through numerical optimization or shape synthesis [19]–[23]. Since CMA allows access to the antenna responses without specifying a feed, shape synthesis can be conducted independent of excitation, with many important parameters such as resonance frequency [6], radiation efficiency [24], quality factor [25], [26], and radiation pattern [27] of each individual mode directly available from the modal solutions. Because any possible port response is a superposition of these modal responses, such mode-based synthesis approach avoids imposing unnecessary constraints arising from a feed and allows a more thorough exploration of the design space. For example, in [20], [21], a feed-independent shape synthesis technique for multi-port antennas is developed, where modal properties (resonant frequency, Q factor) of several modes are included in the cost function for shape synthesis. This CMA-assisted shape-first,

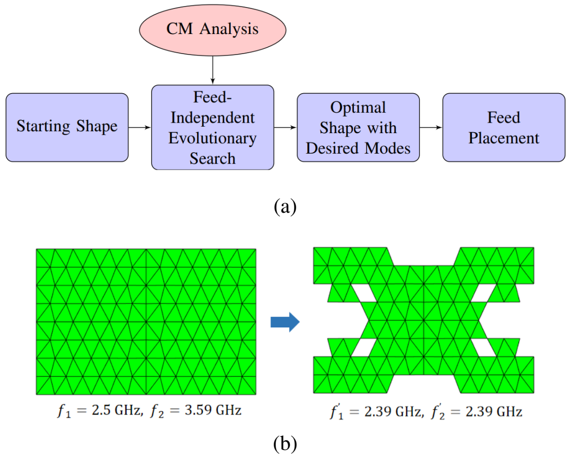


Fig. 3: (a) Design flow for CMA-assisted feed-independent antenna synthesis; (b) an example shape synthesis for a 2-port microstrip MIMO antenna at 2.4 GHz [21].

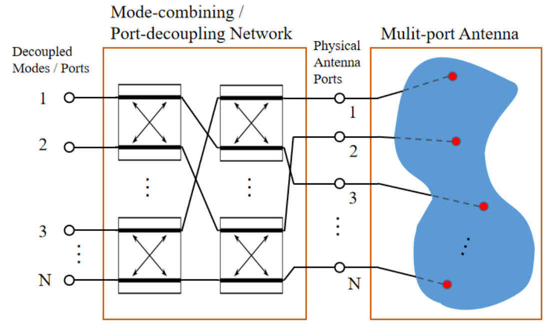


Fig. 4: A mode-decoupling network concept for a multi-port antenna. While difficult to isolate many physical antenna ports on a complex aperture, CMA gives design insight into decoupling networks that isolate modes through linear combinations of physical port currents.

feed-next design methodology is illustrated in Figure 3 for a 2-port MIMO antenna system at 2.4 GHz [21].

Element feeding is another essential step in the design of a MIMO antenna. In general, a single characteristic mode can only be excited in isolation by a feed network distributed over the entire antenna. Of course, this is impossible in practice, and thus, antennas fed by local excitations will inevitably produce coupling to the other modes with low eigenvalues at the design frequency and as a result, ports will experience mutual coupling. To mitigate coupling, feeds can be arranged in such a way that modal excitation coefficients are small for all modes except for the intended one. Recalling that the modal excitation coefficient is related to the inner product of the modal current distribution and the exciting electric field [2], the feed design must somehow apply each excitation in a location or direction with which only the intended mode aligns. This approach becomes challenging as the number of required ports and modes increases as a single feed will often excite multiple characteristic modes.

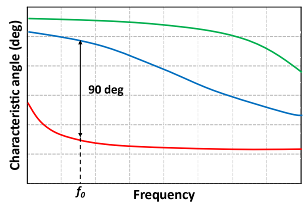
Thus, an alternative technique is to combine signals from multiple physical feed points on the antenna to create decoupled input ports elsewhere in the circuit, as illustrated in Figure 4. In [28], the authors apply the concept of “characteristic port modes” [29] in which the antenna structure stays fixed and a mode decoupling network is used to extract combinations of characteristic modes that are orthogonal at the observation terminals. The mode decoupling network maps physical antenna ports that excite several characteristic modes to MIMO terminal ports by combining their currents with varying magnitude and phase. CMA can give insight into how to select excitation points to take advantage of the natural symmetry of the modes so that simple combiner networks such as hybrid couplers can be used. For example, [13] demonstrated a four port MIMO antenna that extracts four-port modes from eight physical excitation points on each antenna element using a combination of power dividers and branchline couplers. The symmetry of the modal currents was essential to the development of the decoupling network in this design. In another case, it is shown that if the modal currents are slowly varying in the band of interest, a simple analytical decoupling network consisting of a transformer array can be designed using CMA [30], irrespective of the symmetry of the antenna or feeds.

The multi-port design approaches outlined here employing characteristic modes and characteristic port modes can also be applied to a range of problems beyond MIMO antenna design, including diversity antennas [14], [31], [32] and space-modulated arrays [33].

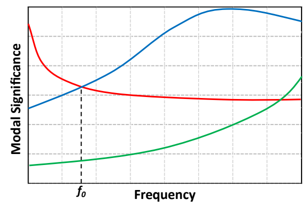
B. Circular Polarization

The intrinsic orthogonality of characteristic mode currents and radiated fields can be exploited to obtain a structure radiating a circularly polarized wave. Two conditions have to be simultaneously satisfied for providing the desired circular polarization (CP). First, two modes with orthogonal linear polarization must exist with the necessary 90 degrees phase difference between their characteristic angles (CAs), and second, both modes must exhibit the same modal significance (MS) at the working frequency, f_0. The condition on the CAs is represented in Figure 5(a) where two characteristic modes differ by 90° at frequency f_0 and the desired MS behavior is shown in Figure 5(b). If these two conditions are strictly satisfied then the correspondent axial ratio (AR) is equal to 0 dB as shown in Figure 5(c) at frequency f_0 [6] but begins to rise as the MS and CA begin to deviate from this ideal.

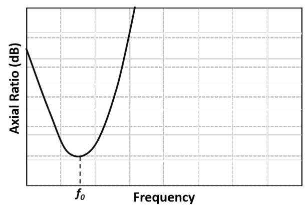
The described guidelines can be also exploited to produce more than two characteristic modes with the proper amplitude-phase condition in order to widen the CP frequency bandwidth. Several antennas are shown in Figure 5(d) that began with known designs and were enhanced using through CMA of the modal currents and eigenvalues. For example, in [34] an E-shaped microstrip patch antenna is tailored to provide two pairs of modes that are able to radiate circularly polarized fields. The physical insight provided by the CMA also allows control of the level of cross-polar radiated field. A further example of bandwidth broadening is reported in [35] where three pairs of characteristic modes are properly excited. A slightly different approach has been adopted in [36] where a pair of characteristic modes has been excited by using a set of



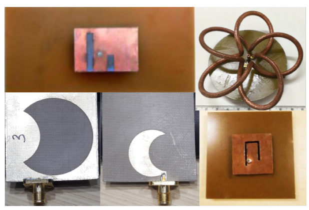
(a)



(b)

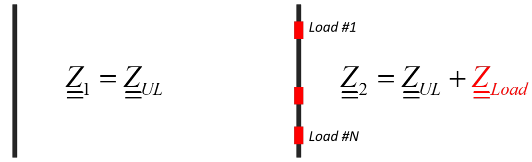


(c)

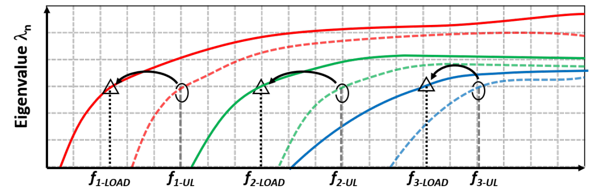


(d)

Fig. 5: Requirements on a pair of linearly polarized characteristic modes to achieve circular polarization at frequency f_0: (a) difference of 90° between the two characteristic angles; (b) equal valued modal significance; (c) achieved axial ratio; (d) examples of non-trivial CP antenna design obtained by exploiting CMA insights (top left and bottom right in [34], bottom left in [35], top right in [39]).



(a)



(b)

Fig. 6: Example of the effects of adding reactive loads to an antenna structure: (a) the initially unloaded antenna with an impedance matrix Z_1 and three resonant modes (left) and the loaded one described by an impedance matrix Z_2; (b) the resonant modes of the unloaded antenna (dashed line) are shifted into the loaded ones (continuous line) due to the loading effect and the new resonances are $f_{i-LOAD}, i = 1, 2, 3$.

inductive exciters but the required phase difference is obtained by a phase-shift applied to the exciters. Characteristic modes have also been adopted to study the current modes excited on a finite metasurface and find the proper exciter to convert a linearly polarized source into a circular radiated field [37], [38]. Finally, CMA has also been exploited for the design of three-dimensional structures [39], [40] able to provide CP.

Although the conditions to obtain CP of the field radiated by an antenna can be easily defined in terms of characteristic modes, the designer has to choose the most suitable excitation scheme through the implementation of inductive or capacitive exciters such as slots, vias or loops [6], [34], [36], [41].

C. Reconfigurable Antennas

The reconfigurability feature of a radiator may involve its radiation pattern shape and polarization as well as its working frequencies. The modal resonance can be controlled by applying reactive loads in view of obtaining the desired reconfigurability function. A first approach using CMA was presented in [29], [42] where the modal resonances are calculated from a matrix which is the sum of the impedance matrix of the unloaded structure, which has to be evaluated just once, and a diagonal load matrix that can be optimized to achieve the requested radiator performance. The main concept is illustrated in Figure 6(a) where an unloaded antenna with an impedance matrix Z_1 is initially considered. This radiator has three resonant modes corresponding to the frequencies, $f_{i-UL}, i = 1, 2, 3$, where the eigenvalues cross the frequency axis (see Figure 6(b)). The modal resonances can be shifted through the use of lumped elements (i.e. $Load\ \sharp i$, $i = 1, 2, ...N$) placed on the radiator, thus obtaining a new impedance matrix, Z_2, that is the sum of the unloaded antenna matrix and the matrix containing the loading impedances. The introduction of the loads alter the original characteristic modes of the unloaded structure but the current modes on the new structure can be expressed in terms of the original current modes to ease the design process. An example of this method of using concentrated reactive loads for shaping the characteristic mode eigenvalues is illustrated in [43] where it is employed for increasing the antenna bandwidth. An approach relying on reactively-loaded antenna design is adopted in [44] where the resonance frequency can be tuned at six different states within the addressed bandwidth (i.e., 50 MHz-350 MHz). Another example of frequency-reconfigurable antenna is reported in [45] where two tunable capacitors load a slotted patch antenna to achieve a variable resonance frequency in the range between 2.0 GHz and 3.0 GHz. The use of reactively-loaded elements has also been used to achieve main lobe reconfigurability. In this case, the loads are introduced to change the resonance frequencies of the characteristic modes and therefore the radiated field. This approach has been firstly introduced in [46] but recent applications of this concept can be found in [47]–[49]. In [50] reactive elements are used to provide the proper phase shift between the mode exciters with the aim of achieving a full 3D null-pattern control whereas reactive loading is exploited for polarization reconfigurability in [51], [36] and [52].

D. Wideband Antennas

During the last decade, CMs have been used to design a great variety of wideband antennas for applications ranging from WLAN to 5G, on-body implantable devices or AiP designs for mm-wave. In other cases, CMA has been used to analyze the behavior of some wideband antennas so as to get deeper physical insight into the operation performance, and draw clear guidelines for their design [63].

From a modal perspective, there are two prevalent strategies behind the design of wideband antennas: A) Excitation of wideband mode(s), or B) combination of multiple close narrowband modal resonances. In the first case (A), the geometry

TABLE I: Comparison of innovative wideband and bandwidth-enhanced antennas using CMA

Ref.	Antenna type	Freq. (GHz)	Size (λ_0)	FBW (%)	Design strategy	Peak gain (dBi)	Application
[53]	Miniaturized patch	5.8	1.31 × 1.31 × 0.03	8.5	A & B	10.5	Wifi
[54]	Stub-loaded Printed Dipole	3.5	1.03 × 0.82 × 0.014	11.2	B	4.4	5G communications
[55]	Dipole and loop combination	2.4	0.63 × 0.32 × 0.006	44.2	B	2	Wireless communications with stable radiation pattern
[56]	Groups of patches	28	1.12 × 1.12 × 0.04	37.5	B	10.7	Mm-wave antenna-in-package
[57]	2x4 dual-patch array	25.3	0.36 × 0.37 × 0.05	20.6	A & B	14.2	Mm-wave antenna-in-package
[58]	Dual-mode SIW & PIN-loaded	4	1.33 × 1.33 × 0.09	22	B	11.9	High-gain communications
[32]	Patch Antenna	5	1.3 × 1.3 × 0.05	6	B	9.6	5G communications
[59]	Metasurface-inspired Omni Antenna	5.5	1.13 × 1.13 × 0.06	19.9	A	1.8	5G wireless local area network
[60]	Metallic planar loop	3.1	0.58 × 0.58 × 0.013	51.6	B	4	Wireless Communications
[61]	U-Slot-Loaded Microstrip Patch	3.1	0.58 × 0.58 × 0.013	51.6	B	4	Wireless Communications
[62]	CPW-fed biocompatible planar monopole	2.5	0.42 × 0.42 × 0.0072	93.5	B	-6.8	Body implantable applications

of the radiating structure can be modified or the excitation mechanism can be optimized to excite CMs that provide large bandwidth or low Q factor [64]. Following this strategy, some recent designs propose the proper excitation of wideband higher-order modes instead of narrowband lower-order modes to achieve wideband performance, and the optimization of the feeding mechanism to excite wideband modes [59], [61]. For instance, this technique helped to enhance the bandwidth of a microstrip patch up to 46% by optimizing the feed structure in [61].

In the second case (B), the strategy consists of appropriately exciting two or more modes that resonate at closely spaced frequencies so the overall bandwidth of the antenna is increased. Close resonant modes can be obtained in different ways: by combining resonances of two different structures by loading the antenna with Foster or non-Foster elements, or by modifying the geometry of the antenna to shift the resonant frequency of some mode(s) close to other modes, thus increasing the total bandwidth [54]- [56], [58], [32], [60]- [62]. To indicate the potential of this technique, in a recent design [54] the bandwidth of a simple dipole has been increased from 1.8% to 11.2%.

Additionally, the two strategies can be combined to significantly enhance the bandwidth of radiating structures, as in [53] and [57]. Moreover, excitation of multiple modes in PIFA antennas has also been very popular for creating multiband or wideband antennas [65], [66].

Table I summarizes the main characteristics of very recent wideband antenna designs for several innovative applications, employing the aforementioned techniques. Note that some of these designs aim to obtain large bandwidth together with high-gain and/or stable radiation pattern.

IV. New Directions in CMA for Antennas

Beyond the topics discussed in Section III, several new directions are growing in the literature on CMA. The first is dielectric resonator antenna (DRA) design. Compared to metallic antennas, limited research efforts have been made in applying CMA to DRAs, partly due to the existence of spurious modes [67] in surface equivalence based formulations and the heavy computation in volume equivalence based formulations. Interested readers can refer to [PAPER1] for discussion on CMA formulations for DRAs. Though DRAs based on canonical geometries can be readily analyzed through approximate analytical formulas, CMA brings similar physical insight to more general DRAs of more complex geometries where analytical solutions are impossible. CMA also enables direct study of stored energies and quality factors of DRAs without a feed [68], again bringing useful new design techniques to non-canonical DRAs. Some of the CMA techniques described in Section III can be applied to DRAs. For example, recent CMA-based DRA shape synthesis work has explored new DRA concepts beyond simple shapes [23], [69] for MIMO and wideband applications.

Beyond just dielectrics, another interesting direction is the development of new CMA techniques capable of handling complex materials that can solve more practical engineering problems. Recent developments in this direction include both new CM formulations for lossy [70] and magnetodielectric materials [71] and new applications to the design of antennas resistant to hand loading [72].

Finally, CMA has been applied to other emerging antenna design topics. For example, the study of metasurfaces and other periodic structures with CMA is increasingly popular and is covered in more detail in this special issue [PAPER5]. Also, applying duality, CMA has recently been used for the design of aperture antennas [73] to directly study the equivalent magnetic current modes. CMA has also been recently applied to orbital angular momentum (OAM) structures [74], [75].

V. Conclusion

In recent years, researchers have recognized the important role of CMA in antenna design due to its ability to directly investigate the fundamental building blocks of an antenna's response and to do so in a way that is unconstrained by a feed choice. The link between the radiation characteristics, equivalent circuit behavior, and the antenna structure is invaluable to improve design insight toward new antenna element designs. The past decade has shown that different aspects of the versatile CMA technique can provide value in nearly any antenna problem; today's new engineers will inevitably find a way to apply these tools to tomorrow's problems as well.

References

[1] R. Garbacz, "A generalized expansion for radiated and scattered fields," Ph.D. dissertation, Ohio State University, 1968.

[2] R. Harrington and J. Mautz, "Theory of characteristic modes for conducting bodies," *IEEE Trans. Antennas Propag.*, vol. 19, no. 5, pp. 622–628, 1971.

[3] A. Yee and R. Garbacz, "Self-and mutual-admittances of wire antennas in terms of characteristic modes," *IEEE Trans. Antennas Propag.*, vol. 21, no. 6, pp. 868–871, 1973.

[4] E. Newman, "Small antenna location synthesis using characteristic modes," *IEEE Trans. Antennas Propag.*, vol. 27, no. 4, pp. 530–531, 1979.

[5] R. Garbacz and D. Pozar, "Antenna shape synthesis using characteristic modes," *IEEE Trans. Antennas Propag.*, vol. 30, no. 3, pp. 340–350, 1982.

[6] M. Cabedo-Fabres, E. Antonino-Daviu, A. Valero-Nogueira, and M. F. Bataller, "The theory of characteristic modes revisited: A contribution to the design of antennas for modern applications," *IEEE Antennas Propag. Mag.*, vol. 49, no. 5, pp. 52–68, Oct. 2007.

[7] K. A. Obeidat, B. D. Raines, and R. G. Rojas, "Discussion of series and parallel resonance phenomena in the input impedance of antennas," *Radio Science*, vol. 45, no. 6, p. RS6012, Dec 2010.

[8] J. J. Adams and J. T. Bernhard, "Broadband equivalent circuit models for antenna impedances and fields using characteristic modes," *IEEE Trans. Antennas Propag.*, vol. 61, no. 8, pp. 3985–3994, 2013.

[9] B. Yang and J. J. Adams, "Computing and visualizing the input parameters of arbitrary planar antennas via eigenfunctions," *IEEE Trans. Antennas Propag.*, vol. 64, no. 7, pp. 2707–2718, 2016.

[10] J.-F. Lin and Q.-X. Chu, "Accurately characterizing the coupling effects of patch antennas with complex- and frequency-dependent J/K inverters," *IEEE Trans. Antennas Propag.*, vol. 67, no. 3, pp. 1554–1561, 2019.

[11] M. Khan and D. Chatterjee, "Characteristic mode analysis of a class of empirical design techniques for probe-fed, U-slot microstrip patch antennas," *IEEE Trans. Antennas Propag.*, vol. 64, no. 7, pp. 2758–2770, 2016.

[12] J. J. Borchardt and T. C. Lapointe, "U-slot patch antenna principle and design methodology using characteristic mode analysis and coupled mode theory," *IEEE Access*, vol. 7, pp. 109 375–109 385, 2019.

[13] D. Manteuffel and R. Martens, "Compact multimode multielement antenna for indoor UWB massive MIMO," *IEEE Trans. Antennas Propag.*, vol. 64, no. 7, pp. 2689–2697, 2016.

[14] N. Belmar-Moliner, A. Valero-Nogueira, M. Cabedo-Fabrés, and E. Antonino-Daviu, "Simple design for cost-effective diversity antennas," *Microw. Opt. Technol. Lett.*, vol. 49, no. 4, pp. 994–996, 2007.

[15] E. Antonino-Daviu, M. Cabedo-Fabres, M. Ferrando-Bataller, and M. Gallo, "Design of a multimode MIMO antenna using the theory of characteristic modes," *Radioengineering*, vol. 18, no. 4, pp. 425–430, Dec. 2009.

[16] E. Antonino-Daviu, M. Cabedo-Fabres, B. Bernardo-Clemente, and M. Ferrando-Bataller, "Printed multimode antenna for MIMO systems," *J. Electromagn. Waves Appl.*, vol. 25, no. 14-15, pp. 2022–2032, 2011.

[17] H. V. Singh and S. Tripathi, "Compact UWB MIMO antenna with cross-shaped unconnected ground stub using characteristic mode analysis," *Microw. Opt. Technol. Lett.*, vol. 61, no. 7, pp. 1874–1881, 2019.

[18] J. Molins-Benlliure, E. Antonino-Daviu, M. Cabedo-Fabrés, and M. Ferrando-Bataller, "Four-port wide-band cavity-backed antenna with isolating X-shaped block for sub-6 GHz 5G indoor base stations," *IEEE Access*, vol. 9, pp. 80 535–80 545, 2021.

[19] J. L. T. Ethier and D. A. McNamara, "Antenna shape synthesis without prior specification of the feedpoint locations," *IEEE Trans. Antennas Propag.*, vol. 62, no. 10, pp. 4919–4934, 2014.

[20] B. Yang and J. J. Adams, "Systematic shape optimization of symmetric MIMO antennas using characteristic modes," *IEEE Trans. Antennas Propag.*, vol. 64, no. 7, pp. 2668–2678, 2015.

[21] B. Yang, J. Zhou, and J. J. Adams, "A shape-first, feed-next design approach for compact planar MIMO antennas," *Progress In Electromagnetics Research*, vol. 77, pp. 157–165, 2019.

[22] F. Jiang, C.-Y. Chiu, S. Shen, Q. S. Cheng, and R. Murch, "Pixel antenna optimization using n-port characteristic mode analysis," *IEEE Trans. Antennas Propag.*, vol. 68, no. 5, pp. 3336–3347, 2020.

[23] B. Yang, A. Eroglu, and J. J. Adams, "Shape synthesis of multi-mode dielectric resonator antennas using characteristic modes," in *Proc. 2020 IEEE Int. Appl. Computational Electromagnetics Society Symp. (ACES)*, 2020.

[24] M. Capek, J. Eichler, and P. Hazdra, "Evaluating radiation efficiency from characteristic currents," *IET Microw. Antennas Propag.*, vol. 9, no. 1, pp. 10–15, 2015.

[25] M. Capek, P. Hazdra, and J. Eichler, "A method for the evaluation of radiation q based on modal approach," *IEEE Trans. Antennas Propag.*, vol. 60, no. 10, pp. 4556–4567, 2012.

[26] J. Ethier and D. A. McNamara, "Through the looking glass: A characteristic mode view of electromagnetic modeling & design," in *Proc. 2010 14th IEEE Int. Symp. on Antenna Technology and Appl. Electromagnetics & the American Electromagnetics Conference*, 2010, pp. 1–4.

[27] H. Li, S. Sun, W. Li, M. Wu, and C. Zhou, "Systematic pattern synthesis for single antennas using characteristic mode analysis," *IEEE Trans. Antennas Propag.*, vol. 68, no. 7, pp. 5199–5208, 2020.

[28] J. Ethier and D. A. McNamara, "An interpretation of mode-decoupled MIMO antennas in terms of characteristic port modes," *IEEE Transactions on Magnetics*, vol. 45, no. 3, pp. 1128–1131, 2009.

[29] J. Mautz and R. Harrington, "Modal analysis of loaded N-port scatterers," *IEEE Trans. Antennas Propag.*, vol. 21, pp. 188–199, Mar. 1973.

[30] B. Yang and J. J. Adams, "A decoupling network based on characteristic port modes," in *Proc. 2020 IEEE Int. Symp. on Antennas and Propagation and North American Radio Science Meeting*, 2020, pp. 1667–1668.

[31] Y. Zheng and S. Yan, "A low-profile half-mode annular microstrip antenna with pattern diversity," *IEEE Antennas Wireless Propag. Lett.*, vol. 19, no. 10, pp. 1739–1743, 2020.

[32] Y. Luo, Z. N. Chen, and K. Ma, "A single-layer dual-polarized differentially fed patch antenna with enhanced gain and bandwidth operating at dual compressed high-order modes using characteristic mode analysis," *IEEE Trans. Antennas Propag.*, vol. 68, no. 5, pp. 4082–4087, 2020.

[33] Z. Han, Y. Zhang, S. Shen, Y. Li, C.-Y. Chiu, and R. Murch, "Characteristic mode analysis of ESPAR for single-RF MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 20, no. 4, pp. 2353–2367, 2021.

[34] Yikai Chen and Chao-Fu Wang, "Characteristic-mode-based improvement of circularly polarized U-slot and E-shaped patch antennas," *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 1474–1477, 2012.

[35] H. H. Tran, N. Nguyen-Trong, and A. M. Abbosh, "Simple design procedure of a broadband circularly polarized slot monopole antenna assisted by characteristic mode analysis," *IEEE Access*, vol. 6, pp. 78 386–78 393, 2018.

[36] F. A. Dicandia and S. Genovesi, "A compact CubeSat antenna with beamsteering capability and polarization agility: Characteristic modes theory for breakthrough antenna design," *IEEE Antennas Propag. Mag.*, vol. 62, pp. 82–93, Aug. 2020.

[37] C. Zhao and C.-F. Wang, "Characteristic mode design of wide band circularly polarized patch antenna consisting of H-shaped unit cells," *IEEE Access*, vol. 6, pp. 25 292–25 299, 2018.

[38] X. Gao, G. Tian, Z. Shou, and S. Li, "A low-profile broadband circularly polarized patch antenna based on characteristic mode analysis," *IEEE Antennas Wireless Propag. Lett.*, vol. 20, pp. 214–218, Feb. 2021.

[39] S. V. Kumar and A. R. Harish, "Generation of circularly polarized conical beam pattern using torus knot antenna," *IEEE Trans. Antennas Propag.*, vol. 65, no. 11, pp. 5740–5746, Nov. 2017.

[40] S. Liu, D. Yang, Y. Chen, S. Huang, and Y. Xiang, "Broadband dual circularly polarized dielectric resonator antenna for ambient electromagnetic energy harvesting," *IEEE Trans. Antennas Propag.*, vol. 68, no. 6, pp. 4961–4966, 2020.

[41] M. Han and W. Dou, "Compact clock-shaped broadband circularly polarized antenna based on characteristic mode analysis," *IEEE Access*, vol. 7, pp. 159 952–159 959, 2019.

[42] R. Harrington and J. Mautz, "Control of radar scattering by reactive loading," *IEEE Trans. Antennas Propag.*, vol. 20, no. 4, pp. 446–454, 1972.

[43] E. Safin and D. Manteuffel, "Manipulation of characteristic wave modes by impedance loading," *IEEE Trans. Antennas Propag.*, vol. 63, no. 4, pp. 1756–1764, 2015.

[44] K. A. Obeidat, B. D. Raines, R. G. Rojas, and B. T. Strojny, "Design of frequency reconfigurable antennas using the theory of network characteristic modes," *IEEE Trans. Antennas Propag.*, vol. 58, pp. 3106–3113, Oct. 2010.

[45] Z. Mahlaoui, E. Antonino-Daviu, A. Latif, and M. Ferrando-Bataller, "Design of a dual-band frequency reconfigurable patch antenna based on characteristic modes," *Int. J. Antennas Propag.*, vol. 2019, p. e4512532, Mar. 2019.

[46] R. Harrington, "Reactively controlled directive arrays," *IEEE Trans. Antennas Propag.*, vol. 26, pp. 390–395, May 1978.

[47] W. Wu, B. Wang, X. Yang, and Y. Zhang, “A pattern-reconfigurable planar fractal antenna and its characteristic-mode analysis,” *IEEE Antennas Propag. Mag.*, vol. 49, pp. 68–75, Jun. 2007.

[48] K. K. Kishor and S. V. Hum, “A pattern reconfigurable chassis-mode MIMO antenna,” *IEEE Trans. Antennas Propag.*, vol. 62, no. 6, pp. 3290–3298, 2014.

[49] Y. Chen and C.-F. Wang, “Synthesis of reactively controlled antenna arrays using characteristic modes and de algorithm,” *IEEE Antennas Wireless Propag. Lett.*, vol. 11, pp. 385–388, 2012.

[50] F. A. Dicandia, S. Genovesi, and A. Monorchio, “Advantageous exploitation of characteristic modes analysis for the design of 3-D null-scanning antennas,” *IEEE Trans. Antennas Propag.*, vol. 65, no. 8, pp. 3924–3934, 2017.

[51] K. Li, Y. Shi, H. Shen, and L. Li, “A characteristic-mode-based polarization-reconfigurable antenna and its array,” *IEEE Access*, vol. 6, pp. 64 587–64 595, 2018.

[52] A. Li, W. Jiang, and S. Gong, “Dual-linear polarisation reconfigurable broadband omnidirectional antenna,” *IET Microw. Antennas Propag.*, vol. 14, no. 9, pp. 928–933, 2020.

[53] A. Bhattacharyya, J. Pal, K. Patra, and B. Gupta, “Bandwidth-enhanced miniaturized patch antenna operating at higher order dual-mode resonance using modal analysis,” *IEEE Antennas Wireless Propag. Lett.*, vol. 20, no. 2, pp. 274–278, 2021.

[54] Y. Luo, Z. N. Chen, and K. Ma, “Enhanced bandwidth and directivity of a dual-mode compressed high-order mode stub-loaded dipole using characteristic mode analysis,” *IEEE Trans. Antennas Propag.*, vol. 67, no. 3, pp. 1922–1925, 2019.

[55] D. Wen, Y. Hao, H. Wang, and H. Zhou, “Design of a wideband antenna with stable omnidirectional radiation pattern using the theory of characteristic modes,” *IEEE Trans. Antennas Propag.*, vol. 65, no. 5, pp. 2671–2676, 2017.

[56] M. Xue, W. Wan, Q. Wang, and L. Cao, “Low-profile wideband millimeter-wave antenna-in-package suitable for embedded organic substrate package,” *IEEE Trans. Antennas Propag.*, pp. 1–1, 2021.

[57] L. Wang, J. Shi, K. Xu, and Z. W. Yin, “Compact dual-strip coupled dual-patch antenna for millimeter-wave aip applications,” *IEEE Antennas Wireless Propag. Lett.*, vol. 20, no. 4, pp. 577–581, 2021.

[58] X. Zhang, T.-Y. Tan, Q.-S. Wu, L. Zhu, S. Zhong, and T. Yuan, “Pin-loaded patch antenna fed with a dual-mode SIW resonator for bandwidth enhancement and stable high gain,” *IEEE Antennas Wireless Propag. Lett.*, vol. 20, no. 2, pp. 279–283, 2021.

[59] X. Yang, Y. Liu, and S.-X. Gong, “Design of a wideband omnidirectional antenna with characteristic mode analysis,” *IEEE Antennas Wireless Propag. Lett.*, vol. 17, no. 6, pp. 993–997, 2018.

[60] D. Wen, W. Hao, H. Wang, and H. Zhou, “Design of a wideband antenna by manipulating characteristic modes of a metallic loop,” *Microw. Opt. Technol. Lett.*., vol. 61, no. 2, 2019.

[61] M. Khan and M. Chowdhury, “Analysis of modal excitation in wideband slot-loaded microstrip patch antenna using theory of characteristic modes,” *IEEE Trans. Antennas Propag.*, vol. 68, no. 11, pp. 7618–7623, 2020.

[62] S. Bhattacharjee, S. Maity, S. R. B. Chaudhuri, and M. Mitra, “Metamaterial-inspired wideband biocompatible antenna for implantable applications,” *IET Microw. Antennas Propag.*, vol. 12, no. 11, p. 1799–1805, 2018.

[63] W. Wu and Y. P. Zhang, “Analysis of ultra-wideband printed planar quasi-monopole antennas using the theory of characteristic modes,” *IEEE Antennas Propag. Mag.*, vol. 52, no. 6, pp. 67–77, 2010.

[64] E. Antonino-Daviu, M. Cabedo-Fabres, M. Ferrando-Bataller, and A. Valero-Nogueira, “Wideband double-fed planar monopole antennas,” *Electronics Letters*, vol. 39, no. 23, p. 1635–1636, 2003.

[65] R. Varma and J. Ghosh, “Analysis and design of compact triple-band meandered PIFA for 2.4/5.2/5.8 GHz WLAN,” *IET Microw. Antennas Propag.*, vol. 13, no. 4, pp. 505–509, 2019.

[66] N. L. Bohannon and J. T. Bernhard, “Design guidelines using characteristic mode theory for improving the bandwidth of PIFAs,” *IEEE Trans. Antennas Propag.*, vol. 63, no. 2, pp. 459–465, 2015.

[67] H. Alroughani, J. Ethier, and D. McNamara, “Observations on computational outcomes for the characteristic modes of dielectric objects,” in *Proc. 2014 IEEE Antennas and Propagation Society Int. Symp. (APSURSI)*. IEEE, 2014, pp. 844–845.

[68] B. Yang and J. J. Adams, “Quality factor calculations for the characteristic modes of dielectric resonator antennas,” in *Proc. 2017 United States National Committee of URSI National Radio Science Meeting (USNC-URSI NRSM)*, 2017.

[69] H. Alroughani and D. A. McNamara, “The shape synthesis of dielectric resonator antennas,” *IEEE Trans. Antennas Propag.*, vol. 68, no. 8, pp. 5766–5777, 2020.

[70] P. Ylä-Oijala, H. Wallén, and S. Järvenpää, “Theory of characteristic modes for lossy structures: Formulation and interpretation of eigenvalues,” *Int. J. of Numerical Modelling: Electronic Networks, Devices and Fields*, vol. 33, no. 2, p. e2627, 2020.

[71] S. Huang, J. Pan, and D. Yang, “A novel electromagnetic power-based characteristic mode for magnetodielectric materials,” *Radio Science*, vol. 53, no. 4, pp. 458–471, 2018.

[72] R. Luomaniemi, P. Ylä-Oijala, A. Lehtovuori, and V. Viikari, “Designing hand-immune handset antennas with adaptive excitation and characteristic modes,” *IEEE Trans. Antennas Propag.*, vol. 69, no. 7, pp. 3829–3839, 2021.

[73] P. Liang and Q. Wu, “Duality principle of characteristic modes for the analysis and design of aperture antennas,” *IEEE Trans. Antennas Propag.*, vol. 66, no. 6, pp. 2807–2817, 2018.

[74] W. Li, L. Zhang, J. Zhu, Z. Tao, and Q.-H. Liu, “Constructing dual-frequency OAM circular patch antenna using characteristic mode theory,” *J. of Appl. Physics*, vol. 126, no. 6, p. 064501, 2019.

[75] G. Yang, Y. Liu, Y. Yan, and W. Li, “Vortex wave generation using single-fed circular patch antenna with arc segment,” *Microw. Opt. Technol. Lett.*, vol. 63, no. 6, pp. 1732–1738, 2021.