Selecting the best risk measure in multiobjective cash management

Francisco Salas-Molina*[1]

[1]Universidad Politécnica de Valencia, Ferrándiz y Carbonell, 03801, Alcoy, Alicante, Spain

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Abstract

In this paper, we consider cash management from a multidimensional perspective in which cost and risk are desired goals to minimize. Cash managers interested in minimizing risk need to select the most appropriate risk measure according to their particular needs. In order to assess the quality of alternative risk measures, we empirically compare eight different risk measures in terms of the combined cost-risk performance of a cash management model. To this end, we rely on goal programming to derive optimal solutions for cash management models. Our results show that risk measures based on cost deviations better capture risk in comparison to those based on a reference cash balance. The methodology proposed in this paper allows cash managers to propose and evaluate new risk measures.

Keywords: Multidimensional finance; data-driven models; risk analysis; goal programming

1 Introduction

When facing cash management, we usually assume that risk control is implicit in decision-making by considering much higher penalty costs for negative cash balances than holding costs for positive ones. Under the usual assumption of linear holding costs (see e.g. Gormley and Meade (2007); da Costa Moraes and Nagano (2014)), the lower the balance, the lower the cost. However, low balances may lead to high overdraft costs due to the uncertainty associated to future cash flows. This situation can be partially solved by setting minimum cash balances for precautionary purposes (Ross et al., 2002). However, cash managers can also derive better cash policies by including risk analysis in their decision-making processes as recently proposed by Salas-Molina et al. (2016) and Salas-Molina et al. (2018).

The cash management problem (CMP) is defined as an optimization problem that aims to find the best sequence of control actions over a given planning

*Corresponding author. E-mail addresses: *frasamo@upv.es*

horizon, what is called a policy, that minimizes the cost of idle balances (holding/penalty costs) and the cost of controlling balances (transaction costs). Since Baumol (1952) and Miller and Orr (1966), the common two-assets setting prevailed in many research works as surveyed in Gregory (1976); Srinivasan and Kim (1986) and da Costa Moraes et al. (2015). This framework assumes the existence of a main bank account for operational purposes, and a second account summarizing short-term assets such as treasury bills or marketable securities ready to be converted in cash when needed.

Most cash management models in the literature focused on a single objective, namely, minimizing holding and transaction costs (Premachandra, 2004; Gormley and Meade, 2007; Baccarin, 2009; Righetto et al., 2016). Recently, Salas-Molina et al. (2016) introduced risk analysis in cash management by measuring both the cost and the risk of alternative policies. The authors measured cost by the average daily cost and risk by the standard deviation of daily cost over a given planning horizon. A different approach to cash management was proposed by Herrera-Cáceres and Ibeas (2016) by minimizing the sum of squared deviations from a cash balance reference signal, but without considering costs. However, there is a lack of research about the goodness of such risk metrics in the context of cash management.

Risk assessment is an ongoing issue in many scientific fields. On the one hand, risk has to be defined from a qualitative point of view. Kaplan and Garrick (1981) define risk as the combination of uncertainty and damage. The possibility of an unfortunate occurrence or the deviation from a reference value and associated uncertainties are some of the additional definitions recently proposed by Aven (2016). On the other hand, risk has to be quantitatively defined. In other words, some particular metric has to be proposed in order to facilitate risk assessment. Furthermore, this metric is usually domain specific. In this paper, we focus on this second aspect of risk analysis in an attempt to evaluate alternative risk measures. First, we accept the fact that decision-making in cash management is enriched by considering multiple criteria (Steuer, 1986; Ballestero and Romero, 1998; Branke et al., 2008; Aouni et al., 2014) such as cost and risk. Then, we propose a method to select the most appropriate risk measure to be used as a key input to a multiobjective cash management model. As a result, the main purpose of this paper is to provide a method to empirically compare different risk measures within a multiobjective framework in which cost and risk are desired objectives to minimize.

To this end, we first represent the common two-assets framework as a simple cash management system with: (i) two accounts; (ii) two possible transactions; and (iii) a given cost structure with holding and transaction costs. Once a particular cash management system is defined, we formulate the CMP as a multiobjective goal program in order to ensure the optimality of solutions. By establishing an achievement objective function with both a cost and a risk measure, we aim to minimize a loss function expressed in terms of aggregated Manhattan distances to an ideal point where cost and risk are minimum (Zeleny, 1982; Yu, 2013; Ballestero and Romero, 1998; Ballestero and Pla-Santamaría, 2003; Jones et al., 2010). We consider linear and quadratic risk functions for computational reasons to ensure the optimality of solutions as a desirable feature from a cash manager point of view. It is important to say that the methodology presented in this paper can be extended to consider additional measures of risk. This extension can be done either by linearizing non-linear risk functions or by using heuristics such as evolutionary algorithms to obtain solutions. Here, we focus on two kinds of risk measures:

- Risk measures based on a reference cash balance as in Herrera-Cáceres and Ibeas (2016).
- Risk measures based on cost deviations as in Salas-Molina et al. (2016).

In order to perform an empirical evaluation of alternative risk measures, we use a data set with real daily cash flows from 54 small and medium companies in Spain, with annual revenue up to €10 million each. The whole data set contains 58005 daily observations, with a minimum, average and maximum time range of 170, 737, 1508 working days, respectively. After defining a number of alternative risk measures, we formulate and solve multiple instances of a general mathematical program to derive an optimal policy for a given planning horizon. Finally, we compute the empirical average and standard deviation of both the cost and risk from the optimal policy with respect to a benchmark model. As a benchmark, we use the Miller and Orr (1966) model due to its relevance and simplicity.

The results of our empirical analysis show that the set of risk measures based on cost deviations better capture risk in comparison to those based on a reference cash balance. In addition, no significant difference is found between cost-based risk measures. Since we compare both linear and quadratic risk measures, our results imply that linear risk measures are recommended because of the less computational burden than in the case of quadratic measures.

Summarizing, we propose a novel method to elicit the most appropriate risk measure to be used as a key input to a multiobjective cash management model that extends the multidimensional approach proposed in Salas-Molina et al. (2016). We suggest additional measures of risk in cash management. Instead of a complete search of solutions used in Salas-Molina et al. (2016), the method described in this paper ensures optimality by relying on mathematical programming. We propose and compare additional measures of risk providing empirical evidence in favor of a particular type of risk measures. Finally, it is important to highlight that the experiments in Salas-Molina et al. (2016) are based on data for single company. Our results are based on a comprehensive data set with cash flows from 54 different companies, which make this data set publicly available for further research. [1]

In what follows, we first formulate the cash management problem in Section 2. Next, we introduce a number of alternative risk measures in Section 3, which we empirically compare in Section 4. Finally, we conclude in Section 5, suggesting natural extensions of this work.

2 Formulation of the problem

In this section, we provide a mathematical formulation of the CMP, which we build on top of the multiobjective cash management model by Salas-Molina et al. (2016). To this end, consider the common two-assets setting of the CMP with two bank accounts as depicted in Figure 1.

[1] `http://www.iiia.csic.es/~jar/54datasets3.csv`

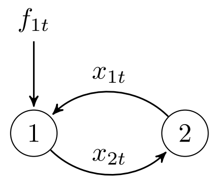

Figure 1: The common two-assets setting in the cash management problem.

Account 1 receives payments from customers (inflows) and it is also used to send payments to suppliers (outflows). Both inflows and outflows are summarized through net cash flow f_{1t}, which we assume to be a random variable represented by either a theoretical probability distribution or by a data set of previous observations. If further information about cash flows is available, these forecasts can be used as a key input to the model in order to reduce the uncertainty about the near future. Otherwise, random draws from the empirical/theoretical distribution can be used for comparative purposes as we will describe below. The discrete-time evolution of the system in Figure 1 with two transactions between two accounts is represented by the following set of linear equations:

$$\begin{bmatrix} b_{1t} \\ b_{2t} \end{bmatrix} = \begin{bmatrix} b_{1t-1} \\ b_{2t-1} \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} f_{1t} \\ 0 \end{bmatrix} \tag{1}$$

that can be expressed in matrix notation as follows:

$$\boldsymbol{b}_t = \boldsymbol{b}_{t-1} + A \cdot \boldsymbol{x}_t + \boldsymbol{f}_t. \tag{2}$$

In the usual case of linear transaction costs between accounts with a fixed part γ_0, and a variable part γ_1, the transaction cost function $\Gamma(\boldsymbol{x}_t)$ at time t can be expressed as:

$$\Gamma(\boldsymbol{x}_t) = \boldsymbol{\gamma}_0' \cdot \boldsymbol{z}_t + \boldsymbol{\gamma}_1' \cdot \boldsymbol{x}_t \tag{3}$$

where $\boldsymbol{z}_t$ is a 2×1 binary vector with element z_{it} set to one if the i-th element of $\boldsymbol{x}_t$ is not null, and zero otherwise; $\boldsymbol{\gamma}_0$ is a 2×1 vector of fixed transaction costs for each transaction; and $\boldsymbol{\gamma}_1$ is a 2×1 vector of variable transaction costs. In order to link $\boldsymbol{z}_t$ to $\boldsymbol{x}_t$, the following constraint must hold:

$$m \cdot \boldsymbol{z}_t \leq \boldsymbol{x}_t \leq M \cdot \boldsymbol{z}_t \tag{4}$$

where $M(m)$ is a very large (small) number. Furthermore, we avoid that both transactions x_{1t} and x_{2t} can simultaneously occur by placing the following constraint:

$$z_{1t} + z_{2t} \leq 1. \tag{5}$$

On the other hand, the expected holding cost function at time t is usually expressed as:

$$\Delta(\boldsymbol{b}_t) = \boldsymbol{v}' \cdot \boldsymbol{b}_t \tag{6}$$

where $\boldsymbol{v}$ is an 2×1 column vector with the j-th element set to the holding cost per money unit for account j. As a result, given a cash planning horizon of τ time steps and an initial cash balance $\boldsymbol{b}_0$, the solution to the problem is the $2 \times \tau$

policy matrix $X = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_\tau]$, obtained through vector concatenation, that minimizes both a cost function $C(X,\tau)$ and a risk function $R(X,\tau)$ according to some preferences w_1 and w_2, with $w_1 + w_2 = 1$, defined by cash managers:

$$\min \quad \frac{w_1}{C_{max}} \cdot C(X,\tau) + \frac{w_2}{R_{max}} \cdot R(X,\tau) \tag{7}$$

subject to equation (2) and $\boldsymbol{b}_t \geq 0$, and where C_{max} and R_{max} are normalization or benchmarking factors used to avoid meaningless comparison between goals and also for comparative purposes. Note that by considering the sum of a cost measure and a risk measure in a normalized cost-risk space we are minimizing the Manhattan distance to an ideal point with zero cost and zero risk, which is usually infeasible. The rationale behind this selection is guaranteeing the optimality of solutions derived from multiobjective cash management models. In addition, since cash managers usually discard policies with negative balances due to high penalty costs, we force cash balances to be non-negative. In what follows, we measure cost as the sum of daily cost $c(\boldsymbol{x}_t)$ over planning horizon τ:

$$C(X,\tau) = \sum_{t=1}^{\tau} c(\boldsymbol{x}_t) = \sum_{t=1}^{\tau} \left(\Gamma(\boldsymbol{x}_t) + \Delta(\boldsymbol{b}_t)\right). \tag{8}$$

In addition, we consider alternative risk measures denoted by $R(X,\tau)$ as detailed in the next section, where we omit τ from $C(X,\tau)$ and $R(X,\tau)$ when referring to cost and risk measures for ease of notation.

3 Alternative measures of risk

Consider an hypothetical a cash manager trying to solve a CMP problem. She/he is interested in minimizing cost but also in controlling risk. Intuitively, risk is associated to any event or action that may adversely affect an organization's ability to achieve its objectives and execute its strategies (McNeil et al., 2005). Quantitatively, our hypothetical cash manager would face a number of alternative choices to measure risk. Markowitz (1952) proposed the use of variance of returns over a given period of time in the past as a measure of risk in his well-known mean-variance portfolio selection model. Later on, Artzner et al. (1999); Szegö (2002); Rockafellar et al. (2002) and Rockafellar and Uryasev (2002) discussed about the coherence of different risk measures. As a result, decision makers have to select the risk measure that better captures their attitude towards risk. However, this choice is by no means straightforward and we here aim to facilitate this task.

Within the particular context of cash management, randomness is introduced by cash flow variability. The higher the variability, the higher the risk. Since variability of a random variable is usually measured by variance or standard deviation in many contexts, we consider variance as a possible risk measure in cash management. Indeed, Herrera-Cáceres and Ibeas (2016) proposed to control cash balances by minimizing the sum of squared deviations from a cash balance reference signal, which they assumed to be optimal. However, chances are that our hypothetical cash manager is more interested in cost variability rather than balance variability. Salas-Molina et al. (2016) measured risk by the standard deviation of daily costs over a given planning horizon. As a result, we here focus on two ways of measuring risk:

- Risk measures based on a reference cash balance as in Herrera-Cáceres and Ibeas (2016) such as the sum of squared deviations from a given cash balance reference.
- Risk measures based on cost deviations as in Salas-Molina et al. (2016) such as the sum of squared deviations of cost, the sum of positive deviations from the average cost, and the sum of positive and negative deviations from the average cost.

For computational reasons, we limit our analysis to either linear or quadratic risk functions. In order to check the goodness of a range of cash balance references, let us consider a first group of functions by varying a non-negative parameter p in the following risk measure based on the sum of squared deviations from a given cash balance reference $\boldsymbol{b}_{ref}$:

$$R_1(X,p) = \sum_{t=1}^{\tau} (\boldsymbol{b}_t - p \cdot \boldsymbol{b}_{ref})' Q (\boldsymbol{b}_t - p \cdot \boldsymbol{b}_{ref}) \tag{9}$$

where Q is a 2×2 matrix whose main diagonal determines which accounts need to be controlled (or even to weight such a control). For instance, by setting:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tag{10}$$

we control balances for account 1, but not for account 2 in Figure 1. Following with our example, if we assume a Gaussian cash flow $\mathcal{N}(\mu, \sigma)$ for account 1 and we set $\boldsymbol{b}_{ref} = [3 \cdot \sigma, 0]'$ as a cash balance reference for precautionary purposes, we can evaluate the overall cost-risk performance of a cash management model for a range of p-references by varying parameter p.

As suggested in Salas-Molina et al. (2016), cash managers may be more interested in cost variability rather than in balance deviations as a measure of risk. To this end, either the standard deviation or the upper semideviation of daily cost are suitable measures of risk in cash management. An equivalent measure of risk to the standard deviation, which is more suitable for optimization purposes, is the sum of squared deviations from the average daily cost:

$$R_2(X) = \sum_{t=1}^{\tau} (c(\boldsymbol{x}_t) - \bar{c})^2 \tag{11}$$

where $\bar{c}$ is the average daily cost over the planning horizon τ. A common criticism made on the sum of squared deviations is that both positive and negative deviations are equally considered in this measure. Since cash managers are usually more interested in reducing positive deviations of cost, we next consider an additional measure of risk equivalent to the upper semideviation of cost:

$$R_3(X) = \sum_{t=1}^{\tau} \max(c(\boldsymbol{x}_t) - \bar{c}, 0). \tag{12}$$

For computational reasons, we may be interested in avoiding the use of a quadratic risk function as in equation (11). The following expression is an equivalent risk measure that can be easily linearized within a mathematical programming framework as we will see below:

$$R_4(X) = \sum_{t=1}^{\tau} |c(\boldsymbol{x}_t) - \bar{c}|. \tag{13}$$

By minimizing objective function (7) when the cost of policies is measured by equation (8) and the risk is measured by any of the alternative functions introduced in this section, we derive policies that ultimately depend on the risk measure used. As a result, we are in a position to compare the cost-risk performance of polices with respect to a given benchmark model. We use this combined performance as a proxy to compare risk measures.

4 Selecting the best risk measure

This section is the core of this paper. Here, we aim to empirically evaluate a number of alternative risk measures by comparing the combined cost-risk performance of policies derived from the use of these measures. This evaluation procedure allows cash managers to determine which measure better represents the way they face risk in a multiobjective framework where the ultimate goal is minimizing both cost and risk. To this end, we experiment on 54 real cash flow data sets from small and medium companies in Spain as a representative sample of the most common type of companies in Europe. Indeed, small and medium companies contribute to 99.8% of all enterprises, 57.4% of value added, and 66.8% of employment across the EU28 (Muller et al., 2015). In what follows, we first describe the assumptions taken in this empirical study, the methodology used, and we finally discuss the results obtained.

4.1 Assumptions

For simplicity, we assume that cash managers are neutral to risk, i.e., we set $w_1 = w_2 = 0.5$ in objective function (7). We also consider a representative cost structure summarized in Table 1. These costs are adjusted to current bank practices in Spain and selected between those recently proposed by da Costa Moraes and Nagano (2014) in a similar experimental context. Since we use as a benchmark the Miller and Orr (1966) model under the same cost structure, we argue that the influence of these costs in the results of this empirical study is very low. Indeed, any cost variation may affect both models resulting in a similar relative performance.

The costs from Table 1 establish the economic context within the common two-assets setting depicted in Figure 1. Temporary idle cash balances in account 1 can be invested in short-term marketable securities through investment account 2 with an average yearly return of 7%, equivalent to 0.02% per day. We translate this return into a cost by setting a holding cost of 0.02% per day for account 1. Both positive (x_{1t}) and negative (x_{2t}) control actions with respect to account 1 are charged with a fixed (γ_0) and a variable (γ_1) cost. This cost structure respects the conditions for non-triviality pointed out by Constantinides and Richard (1978) that require a variable cost for transaction 2 lower than the holding cost for account 1, and a variable cost for transaction 1 lower than the penalty cost for negative cash balances in account 1. Otherwise, it is never optimal to transfer money between accounts. We here assume that the penalty cost for negative cash balances is infinite by restricting feasibility to non-negative cash balances.

Table 1: Cost structure data used in the empirical study.

Transaction	γ_0 (€)	γ_1 (%)	Account	v (%)
1	20	0.01	1	0.02
2	20	0.01	2	0

The data set used in our experiments contains daily cash flows from 54 different small and medium companies for different sectors with annual revenues of up to €10 million each, covering a date range of about 8 years. In Table 2, we summarize minimum and maximum values, means and standard deviations, and the number of available observations for each company.

As a benchmark, we use the Miller and Orr (1966) model based on a set of three bounds. Recent works in cash management (Premachandra, 2004; da Costa Moraes and Nagano, 2014; Salas-Molina et al., 2016) use this model as a framework for experimental purposes. To obtain control bounds, we follow the recommendations in Ross et al. (2002) by setting a lower bound for precautionary purposes. Cash balance is allowed to wander around between lower bound L and upper bound H. When any of these bounds is reached a control action is made to restore the balance to a target level Z. Formally, the transfer x_t occurring at time t is elicited by comparing the current cash balance, $b_{t-1} + f_t$, to the lower and upper bounds:

$$x_t = \begin{cases} Z - b_{t-1} - f_t, & \text{if} \quad b_{t-1} + f_t > H \\ 0, & \text{if} \quad L < b_{t-1} + f_t < H \\ Z - b_{t-1} - f_t, & \text{if} \quad b_{t-1} + f_t < L. \end{cases} \tag{14}$$

Once the cash manager has set a lower limit L for precautionary purposes, Miller and Orr (1966) show that the optimal policy parameters Z and H for a random walk cash flow process are given by:

$$Z = L + \left(\frac{3\gamma\sigma^2}{4v} \right)^{1/3} \tag{15}$$

and

$$H = 3Z - 2L. \tag{16}$$

Let us assume that each of our 54 data sets $\{\boldsymbol{f}_j : j = 1, 2, \ldots 54\}$ is the best available description of the real cash flow distribution for each of the companies in this study. Following a similar approach to Ben-Tal and Nemirovski (1999) and Ben-Tal et al. (2009) in robust optimization, we set lower control bounds as follows:

$$L_j = \xi \cdot \text{std}(\boldsymbol{f}_j). \tag{17}$$

The operator std computes the standard deviation of the elements of a given vector, and parameter $\xi \in \mathbb{R}_+$ is a subjective value chosen by the cash manager to reflect her/his attitude towards risk. The larger the value of ξ, the more averse to risk she/he is. For instance, assuming Gaussian cash flows, setting

Table 2: Data sets description. Figures in thousands of €.

Company	Min	Max	Mean	Std Dev	Length
1	-90,66	902,69	0,22	39,71	622
2	-565,06	626,55	3,09	65,35	544
3	-6.631,47	6.710,41	-7,44	414,08	935
4	-2.233,81	727,63	-0,88	170,28	893
5	-182,62	164,20	0,01	18,41	709
6	-689,70	562,69	-0,41	72,83	688
7	-300,09	829,05	0,26	65,52	555
8	-242,06	113,14	-0,14	31,89	789
9	-4.703,91	4.733,65	-3,10	658,57	754
10	-1.115,80	787,24	-1,89	83,55	788
11	-1.915,34	307,44	1,33	107,38	428
12	-615,77	7.713,80	-0,11	338,26	555
13	-1.183,62	2.274,46	-0,39	287,26	549
14	-769,28	927,11	-1,00	142,23	606
15	-551,11	556,13	0,39	114,95	696
16	-220,49	226,11	-0,50	18,25	577
17	-2.253,22	2.501,26	0,63	175,38	991
18	-287,58	263,61	-0,09	26,41	610
19	-161,73	154,82	-3,08	25,47	640
20	-150,00	160,38	-0,37	15,40	632
21	-700,00	531,66	-0,54	65,06	730
22	-2.442,94	1.388,74	-2,15	280,20	509
23	-2.898,68	2.898,68	-2,54	336,42	586
24	-3.025,05	3.178,51	-4,05	247,62	1285
25	-1.969,42	2.011,31	-0,39	174,53	600
26	-107,28	155,63	-0,05	18,64	708
27	-70,99	118,38	2,75	16,87	340
28	-324,81	390,08	-0,79	48,56	901
29	-900,41	558,88	-0,34	65,59	574
30	-188,79	198,15	-0,46	17,59	536
31	-1.344,75	349,45	-2,75	119,68	336
32	-359,16	245,04	2,71	48,77	860
33	-943,25	955,89	-1,18	78,27	670
34	-1.149,40	496,55	-1,39	108,36	1490
35	-410,71	291,91	-0,55	57,86	600
36	-78,72	118,40	4,45	18,64	357
37	-2.288,85	2.184,18	-10,16	180,89	497
38	-619,33	196,64	-11,18	67,60	193
39	-64,71	65,67	-0,11	11,66	829
40	-256,27	369,14	0,24	103,05	291
41	-626,65	643,39	-5,55	96,41	300
42	-370,21	368,46	0,47	23,11	749
43	-658,44	733,95	-0,37	131,40	832
44	-1.187,40	1.203,41	-1,83	115,28	378
45	-1.071,96	1.128,00	0,58	127,81	881
46	-511,63	738,32	10,06	75,56	411
47	-10.374,88	4.782,62	-22,94	723,62	532
48	-2.070,38	2.030,93	-5,58	255,32	581
49	-107,84	127,25	-2,07	19,96	573
50	-2.625,18	2.219,57	-2,45	351,19	374
51	-4.198,83	4.816,62	151,28	970,81	1222
52	-3.254,65	7.006,59	89,72	494,93	1220
53	-1.968,77	384,84	7,76	117,51	738
54	-10.213,56	15.321,00	9,61	1.124,10	589

$\xi = 3$ would be approximately equivalent to ensure a positive cash balance with probability 0.99 if the current cash balance is L. Then, we first set L_j for each company j according to equation (17) with $\xi = 3$. Next, we set bounds Z and H using equations (15) and (16). We set C_{max} to the cost computed using equation (8) that derives from applying a Miller-Orr policy according to equation (14). Finally, we set R_{max} to the risk derived from the same Miller-Orr policy computed using each one of the risk measures introduced in Section 3.

4.2 Methodology

Recall from the introduction that we aim to empirically compare different risk measures to ultimately select the most appropriate for a given cash manager. To this end, we solve multiple instances of the general mathematical program encoded in equation (7) subject to equation (2) and $\boldsymbol{b}_t \geq 0$. In all instances, we measure cost by equation (8). Then, we estimate the relative cost-risk performance of policies derived from the application of one of the alternative risk measures described in Section 3. More precisely, we proceed as follows:

1. We define both a cost and a risk measure.
2. We formulate a goal program to derive an optimal policy by minimizing a weighted cost-risk objective function.
3. For a number of replicates, we draw a random sample of cash flows for each data set to obtain the best cost-risk policy for this sample.
4. We compute averages and standard deviations of the combined cost-risk performance for models using different risk measures with respect to a benchmark model.

In order to formulate a cost-risk minimization program, we rely on Goal Programming (GP) due to its generality and ease of formulation. GP has been used in a large number of applications from its introduction by Charnes and Cooper (1959) and Charnes and Cooper (1961) up to recent dates (Tamiz et al., 1998; Ballestero et al., 2012; Aouni et al., 2014). GP aggregates multiple objectives to obtain the solution that minimizes the sum of deviations between the achievement and the aspiration levels of the goals (Lee et al., 1972; Ignizio, 1976; Romero, 1991; Jones et al., 2010). Then, for each goal g_i, indexed by $1 \leq i \leq q$, it is necessary to specify a target $G_i \in \mathbb{R}$, and both positive (δ_i^+) and negative (δ_i^-) deviation variables that are always non-negative. The particular preferences of decision-makers are set through weights w_i^+ and w_i^-. Then, we express a general weighted goal program as follows:

$$\min \sum_{i=1}^{q} (w_i^+ \delta_i^+ + w_i^- \delta_i^-) \tag{18}$$

subject to:

$$g_i(X) + \delta_i^- - \delta_i^+ = G_i \tag{19}$$

$$\delta_i^-, \delta_i^+ \geq 0 \tag{20}$$

$$i = 1, 2, \ldots, q \tag{21}$$

$$X \in S \tag{22}$$

where each $g_i(X)$ is a particular goal defined by cash managers that ultimately depends on the policy X, and S is the set of all feasible policies. Since we are dealing with two goals, namely, cost and risk, we set $q = 2$. Normalization is an important question when applying GP in practice. By comparing performances in a normalized cost-risk space in which the point $(0, 0)$ represents an ideal, but usually infeasible, model with zero cost and zero risk, we are in position to identify which risk measure is able to better capture risk with respect to a benchmark model. In addition, we use normalization factors C_{max} and R_{max} to avoid meaningless numerical comparisons between goals. Following the recommendations in Ballestero and Romero (1998), we use two normalized indexes. First, a cost index θ_1:

$$\theta_1 = \frac{C(X)}{C_{max}} \tag{23}$$

where $C(X)$ is computed using equation (8) and C_{max} is the cost derived from the deployment of the Miller-Orr policy. Second, a risk index θ_{2i}:

$$\theta_{2i} = \frac{R_i(X)}{R_{i,max}} \tag{24}$$

where $R_i(X)$, with $i = 1, \ldots 8$, is one of the eight risk measures proposed in Section 3, and $R_{i,max} = R_i(X_{max})$ is the risk derived from policy X_{max} of the Miller-Orr type and measured using the same risk function.

Our first objective is minimizing cost $g_1(X) = C(X)/C_{max}$ from equation (8). Within a GP approach, minimizing cost is equivalent to minimizing the sum of positive deviations δ_{1t}^{+} above a zero-cost target by setting $G_1 = 0$. We reasonably assume that cost functions can only take non-negative values by setting $\delta_{1t}^{-} = 0$. Our second goal is risk, measured by means of the functions introduced in Section 3. Then, we consider $g_2(x) = R_1(X)/R_{1,max}$ from equation (9) as a measure of risk. Similarly to cost, we are interested in minimizing positive deviations δ_{2t}^{+} from a zero-risk target $G_2 = 0$ when $\delta_{2t}^{-} = 0$ at each time step t. As a result, a particular case of the GP program encoded from equation (18) to (22) to obtain an optimal policy for a sample $\{f_t : t = 1, 2, \ldots, \tau\}$ is the following quadratic program:

$$\min \quad \left[\frac{w_1}{C_{max}} \sum_{t=1}^{\tau} c(\boldsymbol{x}_t) + \frac{w_2}{R_{i,max}} \sum_{t=1}^{\tau} (\boldsymbol{b}_t - p \cdot \boldsymbol{b}_{ref})' Q (\boldsymbol{b}_t - p \cdot \boldsymbol{b}_{ref}) \right] \tag{25}$$

subject to:

$$\boldsymbol{b}_t = \boldsymbol{b}_{t-1} + A \cdot \boldsymbol{x}_t + \boldsymbol{f}_t \tag{26}$$

$$m \cdot \boldsymbol{z}_t \leq \boldsymbol{x}_t \leq M \cdot \boldsymbol{z}_t \tag{27}$$

$$z_{1t} + z_{2t} \leq 1 \tag{28}$$

$$\boldsymbol{x}_t, \boldsymbol{b}_t \in \mathbb{R}^2_{\geq 0} \tag{29}$$

$$\boldsymbol{z}_t \in \{0, 1\}^2 \tag{30}$$

$$t = 1, 2, \ldots, \tau \tag{31}$$

with Q defined as in equation (10), and p determining a particular instance of the model. Since we use the Miller-Orr model as a benchmark, we set b_{ref} to the target level Z. By varying parameter p in $R_1(X,p)$, we experiment on the effectiveness of different balance references to control risk in cash management. First, let us set $p = 1$ in $R_1(X,p) = R_1(X,1)$. Second, in order to cover a wider range of possible situations, we consider four additional risk measures as follows: $R_5(X,0.5)$, $R_6(X,0.75)$, $R_7(X,1.25)$ and $R_8(X,1.5)$.

Similarly, we formulate a program to consider risk measure $R_2(X)$ from equation (11) as follows:

$$\min \left[\frac{w_1}{C_{max}} \sum_{t=1}^{\tau} c(\boldsymbol{x}_t) + \frac{w_2}{R_{2,max}} \sum_{t=1}^{\tau} (c(\boldsymbol{x}_t) - \bar{c})^2 \right]. \tag{32}$$

subject to the same set of constraints encoded from equations (26) to (31).

For risk measure $R_3(X)$ from equation (12), we are interested in minimizing positive deviations δ_{2t}^+ above target $G_2 = \bar{c}$ with $\delta_{2t}^- = 0$ at each time step t. Finally, for risk measure $R_4(X)$ from equation (13), we are interested in minimizing both positive δ_{2t}^+ and negative deviations δ_{2t}^- above target $G_2 = \bar{c}$. As a result, for both $R_3(X)$ and $R_4(X)$, we aim to solve the following program:

$$\min \left[\frac{w_1}{C_{max}} \sum_{t=1}^{\tau} c(\boldsymbol{x}_t) + \frac{w_2}{R_{i,max}} \sum_{t=1}^{\tau} (\delta_{2t}^+ + \delta_{2t}^-) \right] \tag{33}$$

subject to:

$$\boldsymbol{b}_t = \boldsymbol{b}_{t-1} + A \cdot \boldsymbol{x}_t + \boldsymbol{f}_t \tag{34}$$

$$c(\boldsymbol{x}_t) + \delta_{2t}^- - \delta_{2t}^+ = \bar{c} \tag{35}$$

$$m \cdot \boldsymbol{z}_t \leq \boldsymbol{x}_t \leq M \cdot \boldsymbol{z}_t \tag{36}$$

$$z_{1t} + z_{2t} \leq 1 \tag{37}$$

$$\boldsymbol{x}_t, \boldsymbol{b}_t \in \mathbb{R}_{\geq 0}^2 \tag{38}$$

$$\boldsymbol{z}_t \in \{0,1\}^2 \tag{39}$$

$$\delta_{2t}^-, \delta_{2t}^+ \geq 0 \tag{40}$$

$$t = 1, 2, \ldots, \tau \tag{41}$$

where $\delta_{2t}^- = 0$ in the case of risk measure $R_3(X)$. Note that this setting means that cash managers are concerned with costs above some reference $\bar{c}$ that should be viewed as a threshold rather than a target. Summarizing, we here compare the set of risk measures detailed in Table 3.

A final comment must be done regarding the uncertainty introduced by cash flows. Cash management is a short-term planning task and, usually, a remarkable percentage of the total expected cash flow can be predicted with high accuracy as it is the case of major cash flows in the sense of Stone and Miller (1987). Examples of major cash flows are taxes, payments to employees or loan payments with amount and due date previously agreed. In our experiments, we consider uncertainty represented by the cash flow data set itself. However, the methodology described below can be also used when uncertainty is represented by a data set with historical predictive errors after applying some forecasting technique.

Since short-term planning horizons in finance are usually assumed to be no longer than a month, in this study we set a planning horizon of $\tau = 20$ working

Table 3: Alternative risk measures used in the empirical study.

Risk measure	Description	Type	Label
$R_1(X)$	Sum of squared deviations from cash balance reference Z	Reference	1-ref
$R_2(X)$	Sum of squared deviations from average cost	Cost	Sq-cost
$R_3(X)$	Sum of positive deviations from average cost	Cost	Pos-cost
$R_4(X)$	Sum of positive and negative deviations from average cost	Cost	Dev-cost
$R_5(X)$	Sum of squared deviations from cash balance reference $0.5 \cdot Z$	Reference	0.5-ref
$R_6(X)$	Sum of squared deviations from cash balance reference $0.75 \cdot Z$	Reference	0.75-ref
$R_7(X)$	Sum of squared deviations from cash balance reference $1.25 \cdot Z$	Reference	1.25-ref
$R_8(X)$	Sum of squared deviations from cash balance reference $1.5 \cdot Z$	Reference	1.5-ref

days. It is also important to say that the selection of an initial cash balance does not interfere in our experimental results since the cash management model immediately adjusts balances (da Costa Moraes and Nagano, 2014). However, in order to minimize its influence, we set an initial cash balance equal to Z as a stable initial condition for both our GP model and the Miller-Orr benchmark model. Furthermore, in an attempt to achieve a reliable estimate of the average cost-risk performance obtained by each risk measure, we replicate the optimization problem 25 times for each company, resulting in a total horizon of 500 days, equivalent to two working years. This experimental framework was enough to achieve low variations in the results for the eight risk measures as we discuss in the next section.

4.3 Results and discussion

The results from the empirical evaluation for all 54 companies are depicted in Figure 2. We represent our estimate of the goodness of a particular risk measure as a point in a normalized cost-risk space. Recall that we normalize our results by using the Miller-Orr model as a benchmark through factors C_{max} and $R_{i,max}$. Each point in the normalized cost-risk space is the average combined cost-risk performance of our linear-quadratic model using one of the eight risk measures proposed above. We estimate performances by 10,800 different experiments covering a temporal range of 216,000 days. We represent the variability of the results for each risk measure by plotting an ellipse centered in the average cost-risk point. The horizontal semiaxis of the ellipse equals three standard deviations of the cost index and the vertical semiaxis equals three standard deviations of the risk index. We use this method to emphasize the low variability of some of the results. Indeed, the ellipses showing the variability of models with risk measures Sq-cost, Dev-cost, Pos-cost and 1-ref are hardly visible due to their reduced size.

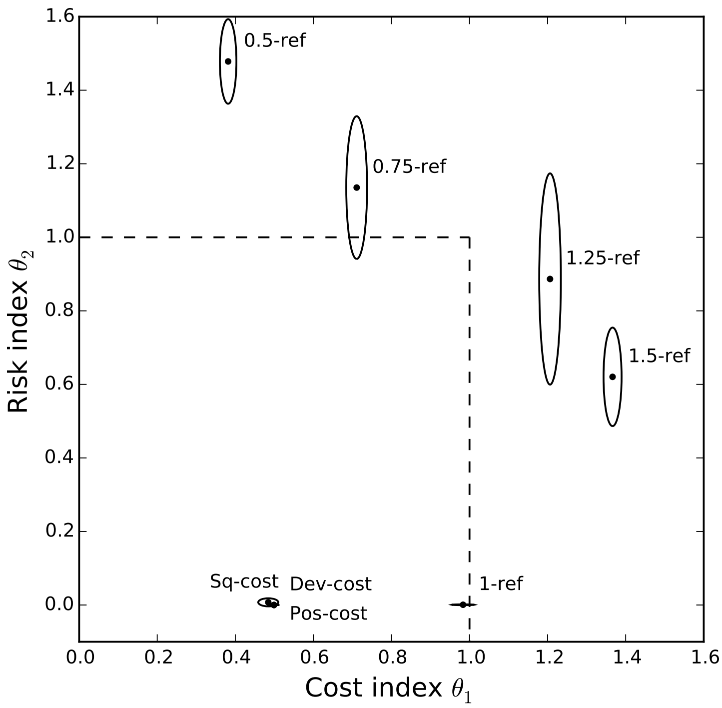

Figure 2: Expected performance of alternative risk measures in a normalized cost-risk space.

From the analysis of Figure 2, we next highlight some useful points in order to select a given risk measure. Note first that the area inside the unity dashed square includes models with better performance than the Miller-Orr benchmark. All models outside this area are worse than the benchmark in terms of either cost or risk, or even in terms of both cost and risk. Risk measures 0.5-ref and 0.75-ref, using cash balance references lower than the target level Z, resulted in a better cost performance but a worse risk control than the benchmark. This fact is mainly caused by low average balances and it seems to be confirmed by the proportionality of the cost reduction with the cash balance reference. On the other hand, risk measures 1.25-ref and 1.50-ref, using cash balance references higher than target level Z, resulted in a better risk performance but a worse cost behavior. Again, higher cash balances references reduced risk but increased costs implying a cost-risk trade-off.

Risk measure 1-ref deserves special attention due to the fact that it is able to reduce the risk index almost to zero while producing a very similar cost to the benchmark. Recall that we here use the Miller-Orr model as a benchmark. This model is not designed for reducing risk, hence explaining the dramatic reduction in the risk index. Note, however, that one may expect a similar behavior to those other risk measures based on cash balance references. This expected behavior would locate the performance of the model using the 1-ref risk measure around point $(1, 1)$ in the normalized cost-risk space. On the contrary, the use of this risk measure allows to reduce almost all risk with respect to the benchmark model with target level Z. Another interesting point is that the resulting experimental variability of this risk measure in comparison to the rest of reference-based risk measures is remarkably lower. This fact reflects the goodness of 1-ref to control risk when a reference is given. However, a change in the balance reference results in a remarkable variation in both the expected cost-risk performance and the variability of this performance with respect to the benchmark.

Within a normalized cost-risk space, point $(0, 0)$ represents the performance of an ideal model with both zero cost and zero risk. This ideal point is usually infeasible. However, the closer to the ideal point, the better the model and, ultimately, the better the risk measure in terms of both cost and risk reduction with respect to the benchmark. In this sense, the set of risk measures based on cost (Sq-cost, Dev-cost and Pos-cost) showed a very similar cost-risk performance. These results are consistent with recent results presented by Salas-Molina et al. (2016), in which no significant difference between the standard deviation and the upper semideviation of daily cost as a measure of risk was reported. One may conclude that the common criticism against the standard deviation as a measure of risk due to considering both positive and negative deviations is not supported by empirical results. This fact is probably caused by a symmetric empirical cost distribution.

Summarizing, by using these risk measures Sq-cost, Dev-cost and Pos-cost, we are able not only to drastically reduce the risk index but also the cost index with respect to the benchmark. Furthermore, the variability of the results for these cost-based risk measures is negligible. From our analysis, we can draw a number of interesting recommendations to select risk measures for multiobjective cash management:

1. Cost-based risk measures better capture risk than reference-based ones.
2. There is a cost-risk trade-off when increasing/decreasing cash balance references when using reference-based risk measures.
3. Cost-based risk measures are in general more stable than reference-based ones.
4. Cost-based risk measures computed by considering squared deviations, positive and negative deviations, and only positive deviations are not significantly different.

By using the procedure described in this section, both cash managers and researchers are empowered to suggest additional measures of risk that can be selected in terms of distances to an ideal point as an approximation of combined utility for cash managers.

5 Concluding remarks

Multiple criteria decision-making is a well established framework in decision sciences. Cash managers may be interested not only in minimizing cost but also risk when obtaining the best cash policies. Indeed, decision-making in cash management improves by analyzing the risk of the policies proposed by a cash management model to avoid an overdraft that is usually charged with high penalty costs. Recent works on cash management propose the use of risk as an additional goal. Then, a multiple-criteria decision-making framework is necessary. This framework requires the definition of a risk measure to be minimized. However, it is not clear whether a risk measure is better than another. In this paper, we propose a method to estimate the best risk measure within a given set of alternatives. To this end, we describe a general goal program that is able to accommodate a number of alternative risk measures and cash management models.

We evaluate two main classes of risk measures: those based on deviations from a given cash balance reference and those based on cost deviations from the average daily cost. We base our empirical study on 54 cash flow data sets that are publicly available for future research. Our results show that risk measures based on cost deviations achieve a better combined cost-risk performance in comparison to those based on a reference cash balance. In addition, no significant difference is found between the use of squared deviations, positive and negative deviations and only positive deviations of cost. These results imply that the sum of linear deviations is recommended because of the less computational burden. Furthermore, the common criticism against the standard deviation as a measure of risk due to considering both positive and negative deviations is not supported by our empirical results.

Summarizing, cash management researchers are now empowered to propose and evaluate new risk measures by means of the methodology presented in this paper. To this end, our general cash management model can be easily adapted to consider alternative risk measures. In addition, it can be extended to the analysis of cash management systems with multiple bank accounts. A further advantage of this method is that the optimality of the policies used to evaluate these measures is guaranteed.

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