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# Femtocell operator entry decision with spectrum bargaining and service competition

Luis Guijarro, *Member, IEEE*, Vicent Pla, *Member, IEEE*, Jose R. Vidal, and Jorge Martinez-Bauset

**Abstract**—This paper analyzes the effect of the entry of a femtocell operator into a mobile communications market where a macrocell operator exists. The analysis is conducted using a game-theory-based model, specifically a three-level multi leader-follower game, where different solution concepts are applied: Shapley value, Nash equilibrium and Wardrop equilibrium. It aims to answer the question of which benefit mobile communication users get from the entry of a femtocell operator into the market. The equilibrium is assessed from the point of view of each agent (e.g. profits and utilities), and of the whole (e.g. social welfare). A case for regulatory intervention is made.

**Index Terms**—Oligopoly, game theory, wireless communications, femtocell.

## I. INTRODUCTION

THIS paper analyzes the effect of the entry of a femtocell operator into a mobile communications market where a macrocell operator exists.

The scenario consists of one Macrocell Operator (MO) and one Femtocell Operator (FO), which compete for the provision of service to the end users. The FO has the same service coverage as the MO, and each user can access at every point in space to both MO and FO services [1]. This overlapping service coverage may be the case of residential areas, corporate headquarters, university campuses and hot spots, where there is a large population of users within a limited geographical area. The MO is entitled to use a certain spectrum band and leases an amount  $b$  of the band to the FO, keeping for itself the rest, up to  $W$ . The FO pays a price  $p$  m.u. (monetary units) per unit of spectrum. The MO leases spectrum to the FO, which can use it more efficiently, in order to increase its revenue. Finally, a user would pay  $p_1$  m.u. if she subscribed to MO service, or  $p_2$  m.u. if she subscribed to FO service. All three prices ( $p$ ,  $p_1$  and  $p_2$ ) are referring to the same time period.

As regards the femtocell deployment, the following assumptions are made [2]. First, the femto base stations (femto-BSs) are deployed by the FO and no capital nor operational expenses are levied onto the users. The MO is not involved in the femto-BS deployment, either. Second, the femto-BSs operate in open-access mode, as this mode allows the FO to offer service to users. And third, an orthogonal spectrum assignment is performed through MO-FO coordination, so that cross-tier interference is completely eliminated.

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L. Guijarro, V. Pla, J. R. Vidal and J. Martinez-Bauset are with the Universitat Politècnica de València, Spain (e-mail: {lguijar, vpla, jrvidal, jmartinez}@dcom.upv.es)

As regards the economic interaction, we assume that the operators compete *à la Bertrand*, that is, playing a one-shot simultaneous game where MO and FO strategies are  $p_1$  and  $p_2$ , respectively. Both the price  $p$  and the leased spectrum amount  $b$  are partially the result of a bargaining process between the MO and the FO. Finally, each user subscribes to the service providing the highest utility. The subscription period matches the time period of the bargain price  $p$  and the service prices  $p_1$  and  $p_2$ .

There have been recent works on economic modeling of femtocell service provision. Reference [3] covers a similar setting as our work, since it analyzes the spectrum leasing between a macrocell service provider and a femtocell service provider. It is concerned not only with the amount of spectrum which is leased, but also with the amount of leased spectrum that the femtocell service provider is willing to share with the users of the macrocell service provider. However, it builds a non-standard model according to microeconomics, since it does not incorporate the users demand into the model, which we indeed do; instead, it inserts a proxy for the users utility in the service providers utility. Reference [4] models a scenario where one macrocell operator leases spectrum to one femtocell operator, and it derives the conditions under which the former has incentives to lease. It assumes, as in our work, that the femtocell operator provides the same coverage as the macrocell operator. However, it assumes that the leasing (wholesale) price and the service (retail) price are the same and it bases the assumption on the need to avoid arbitrage. We do not base our model on this assumption; on the contrary, we model the leasing price and the subscription price as different and independent variables.

Spectrum leasing has been part of several proposals aimed at enhancing the efficiency of the spectrum exploitation by providing flexibility to users and operators, e.g., reference [5] analyzes primary spectrum leasing in exchange for secondary user cooperation.

This paper aims to answer the following question: which benefit do mobile communication users get from the entry of a femtocell operator into the market? The main contribution of the paper is two-fold. Firstly, it models a scenario where there is an incumbent operator and where an entry decision should be made by an entrant operator; this scenario is more realistic than static duopoly scenarios frequently presented in the literature. And secondly, it incorporates a bargaining before the competition takes place, which is not common in the literature.

Our analysis may also be applied in a wider context where a new technology—providing greater efficiency than the current

one—is introduced by an entrant. However, for the sake of conciseness we have focused on a femtocell scenario.

The paper is structured as follows. In the next section, the game-theory-based model for the scenario is described and the equilibrium concepts that we apply are presented and derived. The analysis and the results are presented and discussed in section III, and conclusions are drawn in Section IV.

## II. MODEL

The strategic interaction between the two operators and the  $n$  users is modeled as a three-level multi leader-follower game that we analyze through backward induction.

### A. Subscription game—third phase

In this phase, a pair of values  $b$  and  $p$  has been agreed and the prices  $p_1$  and  $p_2$  have been announced.

The utility that the users receive from each operator depends on two factors:

**Quality of service:** Each operator exploits, during each subscription period, an amount of spectrum which is agreed at the end of the first phase:  $W_2 = b$  for the FO and  $W_1 = W - b$  for the MO. Channel conditions from the different locations within a macrocell are detrimental to the data rate that the users of the macrocell service obtain from a given amount of bandwidth. We model this fact by a *macrocell spectrum efficiency*  $\theta$ . On the other hand, since femto-BSs are deployed indoors and are very close to the users' phones, we assume that all users of the femtocell service have equal good channel conditions and achieve the same maximum spectrum efficiency. Then the transfer rate that is offered to each user is  $\theta_i W_i / n_i$  [6], where  $n_i$  is the number of subscribers of operator  $i$ . We propose to use this transfer rate as the main quality factor which contributes to the user utility. Specifically,  $Q_i = \log(1 + \theta_i W_i / n_i)$ , where  $\theta_2 = 1$  for the FO, and  $\theta_1 < \theta_2$  for the MO. There is an increasing evidence that user experience and satisfaction in mobile broadband scenarios follow logarithmic laws [7].

**Price:** the higher the subscription price, the lower the user utility.

Based on the above discussion, we propose a quasi-linear expression for the user utility. Specifically, for operator  $i$ 's users, it is  $U_i = Q_i - p_i$ . Each user will subscribe to the operator providing the service with the higher utility. Assuming that the number of users is high enough, the individual subscription decision of each individual user will not affect the utility received by the rest. Then, the equilibrium notion is the Wardrop equilibrium [8]. We assume that the users who do not subscribe have a utility equal to 0. Applying the Wardrop equilibrium concept, we may state the following:

If some users decide not to subscribe to either the MO or the FO, then  $U_1 = U_2 = 0$  (i.e., the users are indifferent between subscribing or not). Given the expression for the utility, it can be shown that this scenario cannot occur in the equilibrium [9].

Alternatively, if every user subscribes to either the MO or the FO then  $U_i \geq 0$ . Here, the case  $U_1 > U_2$  (respectively,  $U_2 > U_1$ ) where no user subscribes to the FO (MO) cannot be an equilibrium, apart from the trivial case  $b = 0$  ( $b = W$ ).

Therefore, in the next sections, we will restrict to the case where all users distribute between the MO and the FO (i.e.,  $U_1 = U_2 \geq 0$ ).

### B. Price competition game—second phase

In this phase, a pair of values  $b$  and  $p$  has been agreed on, and each operator chooses its price so as to maximize its profits. The outcome of the subscription game is assumed to be anticipated by both operators, and taken into account in the pricing decisions.

The profits of the MO and the FO can be expressed, respectively, as  $\Pi_1 = n_1 \cdot p_1 + p \cdot b - C_1$  and  $\Pi_2 = n_2 \cdot p_2 - p \cdot b - C_2$ , where  $C_i$  is the cost born by Operator  $i$

When solving the equilibrium equations for the second and the third phase,  $n_1$  and  $n_2$  may be expressed as functions of  $p_1$  and  $p_2$ , so that operator profits are functions of  $p_1$  and  $p_2$  only:  $\Pi_1 = \Pi_1(p_1, p_2)$ ,  $\Pi_2 = \Pi_2(p_1, p_2)$ .

Now, turning our attention to the pricing game, the equilibrium strategies  $p_1^*$  and  $p_2^*$  are given by the Nash equilibrium conditions [10]:  $\Pi_1(p_1^*, p_2^*) \geq \Pi_1(p_1, p_2^*)$ ,  $\forall p_1$ ;  $\Pi_2(p_1^*, p_2^*) \geq \Pi_2(p_1^*, p_2)$ ,  $\forall p_2$ ; meaning that no operator can unilaterally increase its profits by a price change.

These equilibrium conditions can be formulated as a pair of optimization problems  $i = 1, 2$

$$\begin{cases} \max_{\alpha, p_i} & f_i(\alpha, p_i) = \alpha p_i \\ \text{subject to} & g_i(\alpha, p_i) = U_1 - U_2 = 0, \quad h_i(\alpha, p_i) = U_i \geq 0 \end{cases}$$

Note that we have introduced  $\alpha$  as an auxiliary optimization variable. It is defined as  $\alpha \triangleq n_1/n$ , and it denotes the fraction of users subscribing to the MO.

The Karush-Kuhn-Tucker (KKT) conditions for the two optimization problems ( $i = 1, 2$ ) are

$$\nabla f_i + \lambda_i \nabla g_i + \mu_i \nabla h_i = 0 \quad (1)$$

$$g_i = 0; \mu_i h_i = 0; \mu_i \geq 0; h_i \geq 0. \quad (2)$$

We distinguish between two cases. If  $U_1 = U_2 > 0$ , then the inequality constraints are not active. Under the assumption that the partial derivatives of  $\Pi_1$  and of  $\Pi_2$  with respect to  $p_1$  and  $p_2$  exist, the following equation is obtained for  $\alpha$  [9]:

$$f(\alpha, r_1, r_2) = f(1 - \alpha, r_2, r_1), \quad (3)$$

where  $f(x, a, b) = \log(1 + \frac{a}{x}) - \alpha \left( \frac{1}{x^2/a+x} + \frac{1}{(1-x)^2/b+1-x} \right)$  and  $r_i = \theta_i W_i / n$ ,  $i = 1, 2$ .

It can be easily shown that there exists a unique value  $\alpha^* \in (0, 1)$  which satisfies (3) [9]. The prices  $p_1^*$  and  $p_2^*$  are obtained from  $\alpha^*$  by recalling that  $U_1 = U_2$ .

Else, if  $U_1 = 0$  or  $U_2 = 0$ , the following reasoning applies. Let  $\alpha_1$  and  $\alpha_2$  be, respectively, the only solutions in  $(0, 1)$  of  $f(\alpha, r_1, r_2) = 0$  and  $f(1 - \alpha, r_2, r_1) = 0$ . If  $U_1 = 0$ , from (1)–(2) we obtain the condition C1:  $f(\alpha, r_1, r_2) \leq 0$ , which holds if, and only if,  $\alpha \geq \alpha_1$ . Likewise, if  $U_2 = 0$  we obtain the condition C2:  $f(1 - \alpha, r_2, r_1) \leq 0$ , which holds if, and only if,  $\alpha \leq \alpha_2$ . The detailed derivation of the expressions may be found in [9].

Depending on the relative position of  $\alpha_1$  and  $\alpha_2$  we have one of the following three situations: *i*) If  $\alpha_1 > \alpha_2$ , it is not

possible to meet C1 and C2 simultaneously. Hence, an equilibrium such that  $U_1 = U_2 = 0$  does not exist. Furthermore, the solution to (3)  $\alpha^* \in (\alpha_2, \alpha_1)$  yields  $U_1 = U_2 > 0$ . *ii*) If  $\alpha_1 < \alpha_2$ , any  $\alpha \in (\alpha_1, \alpha_2)$  satisfies C1 and C2; there exists an infinite and non-denumerable set of equilibrium points, and the analysis fails to predict an outcome [11]. *iii*) If  $\alpha_1 = \alpha_2$ , then  $\alpha^* = \alpha_1 = \alpha_2$  is both the only solution to (3) and the only value that satisfies C1 and C2 simultaneously.

### C. Operators bargaining—first phase

As stated at the beginning of this section, the price  $p$  and the amount of spectrum  $b$  are subject to a bargaining process between the MO and the FO, which is conducted before the subscription prices are advertised by the operators and the subscription decision is made by the users.

We model the bargaining as a non-cooperative game where the incumbent operator (the MO) has full bargaining power and therefore offers a take-it-or-leave-it offer to the entrant operator (the FO) [12]. For the sake of simplicity, we stand by the full bargaining power case, although alternative assumptions are possible, as discussed later.

The game is analyzed as a dynamic game in an extensive form. Following backward induction, depending on the values of  $(b, p)$  which characterize the MO offer, the bargaining outcome is as follows: 1) The MO will make an offer only if it prefers the competition outcome compared to the monopolistic outcome, that is,  $\Pi_1 \geq \Pi_m$ . 2) If the offer made by the MO induces  $\Pi_2 \geq 0$ , the FO will accept the offer. Otherwise,  $\Pi_2 < 0$ , and the FO will refuse it.

To compute  $\Pi_m$ , the problem should be stated as an optimal decision problem, such that the optimal price  $p_m^*$  should fulfill  $\Pi_m(p_m^*) \geq \Pi_m(p_m)$ ,  $\forall p_m$ .

From the condition  $\Pi_2 \geq 0$  it follows that  $p \leq U(b) \triangleq nb^{-1}((1 - \alpha)p_2 - C_2/n)$ , and from the condition  $\Pi_1 \geq \Pi_m$ , we get  $p \geq L(b) \triangleq nb^{-1}(\log(1 + \theta_1 W/n) - \alpha p_1)$ . Again, the detailed derivation of the expressions may be found in [9]. Therefore, a non-empty feasibility region (FR) will exist if  $L(b) \leq U(b)$  and a point  $(b, p)$  will be in the FR iff  $\max(0, L(b)) \leq p \leq U(b)$ . Our numerical experiments — not shown here due to the lack of space— revealed that for each configuration there exists a threshold value  $b_{min}^f$  such that  $L(b) \leq U(b)$  if  $b \geq b_{min}^f$ . In other words, for values of  $b$  greater than a  $b_{min}^f$ , corresponding values for  $p$  can be found such that competition results in an equilibrium.

The final bargaining outcome will depend on the specific assumptions made. If the incumbent has full bargaining power, it will ask for a profit-maximizing price  $p$  —i.e. such that the  $p = U(b)$ . Given that  $\Pi_1$  is monotonically increasing on  $p$ , this would provide the incumbent with maximum profits.

Nevertheless, other solution concepts can be borrowed from the cooperative game theory for choosing the value of  $p$ . We can transform the problem of setting the price  $p$  at which the MO sells bandwidth to the FO, to an equivalent one of deciding how the extra profit  $\Delta$  is shared between the MO and the FO. For this equivalent problem, the *Shapley value* [10] provides a *fair* allocation of the payoff obtained by the MO–FO coalition, such that each operator (MO or FO) will receive

a share of the profits proportional to its contribution to the total profits. In our case, the Shapley value allocation yields  $\Pi_1 = \frac{1}{2}\Pi_m + \frac{1}{2}(\Pi_m + \Delta - 0) = \Pi_m + \frac{\Delta}{2}$ ,  $\Pi_2 = \frac{1}{2}0 + \frac{1}{2}(\Pi_m + \Delta - \Pi_m) = \frac{\Delta}{2}$  which correspond to  $p = (U(b) + L(b))/2$ .

Alternatively, the problem of agreeing a value for  $p$  can be casted into a two person bargaining problem in which the disagreement point is  $(\Pi_m, 0)$  and the players' strategies are their offers about  $p$ . In this setting, both the *Nash bargaining solution* and the *Kalai-Smorodinsky bargaining solution* [10] can be computed; it can be shown that both yield the same results as the provided by the Shapley value.

Note, however, that the value  $b$  is not determined by the bargaining, but only constrained by  $L(b) \leq U(b)$ . As shown in the next section, welfare can determine the value  $b$ .

## III. RESULTS AND EQUILIBRIUM ANALYSIS

In order to evaluate the different competitive equilibria, we propose to use the following indicators: operators profits  $\Pi_1$  and  $\Pi_2$ ; user utilities,  $U_1 = U_2$ ; and social welfare,  $SW$ , computed as the sum of the users utilities and the operators profits  $SW = n_1 \cdot U_1 + n_2 \cdot U_2 + \Pi_1 + \Pi_2$ . We have conducted a series of numerical experiments in order to obtain a better understanding of the scenario from the point of view of the economic interactions. The values for the parameters are:  $n = 10000$  users;  $W = 100$  kHz;  $C_1 = 20$  m.u.;  $C_2 = 10$  m.u.;  $\theta_1 = 0.5$  bit/s/Hz;  $\theta_2 = 1$  bit/s/Hz. The values do not match any specific real scenario. In order to do so, appropriate coefficients must be inserted in the model equations.

### A. On the value of the leased spectrum

The objective of this experiment is to evaluate the effect of varying the amount of leased spectrum  $b$ . We assumed that  $p$  is agreed so that the Shapley values result for the profits. The following values are simultaneously represented as functions of  $b$  on Fig. 1:  $\alpha$ ,  $U_1$  and  $U_2$ ,  $\Pi_1$  and  $\Pi_2$ , and  $SW$ ; the left axis is for  $\alpha$  and  $U_i$ , and the right one is for  $\Pi_i$  and  $SW$ . For the values  $b$  such that multiple equilibria would result, i.e.,  $b \notin [b_{min}^u, b_{max}^u]$ , no value is represented. For the values of  $b$  such that  $(b, p)$  does not yield competitive equilibrium, i.e.,  $b < b_{min}^f$ , results for the MO correspond to the monopoly scenario.

We see that, with respect to the number of subscribers, throughout the interval  $b < b_{min}^f$ , the MO remains as the monopolistic operator, i.e.  $\alpha = 1$  holds. When  $b \geq b_{min}^f$ , the FO enters the market, and  $\alpha$  decreases as  $b$  increases.

With respect to profits  $\Pi_1$  and  $\Pi_2$ , both operators increase their respective profits when the FO enters the market. Furthermore, they keep increasing as the FO gets more spectrum for providing service to its users. The FO's higher efficiency and the increasing payment to the MO explains this behavior.

With respect to utilities  $U_1$  and  $U_2$ , the range of values  $b \in [b_{min}^u, b_{max}^u]$ , where a unique equilibrium results, corresponds to positive values of  $U_1 = U_2$ , as stated in section II-B. The entry of FO is beneficial for the users, as the utility increases as  $b$  increases up to  $b_{UW} \approx 0.5W$ . From this value, the utility decreases down to zero. We may conclude from this behavior that a symmetric equilibrium, where the FO and the



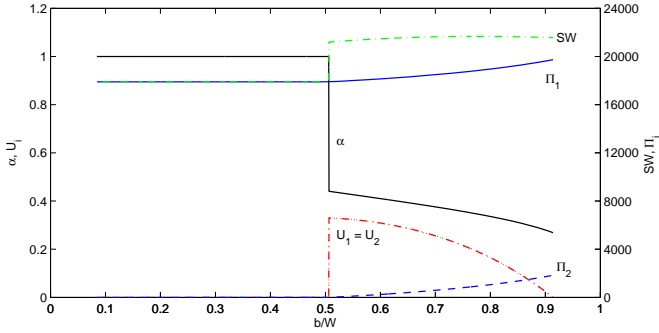


Fig. 1. Effect of variation of  $b/W$

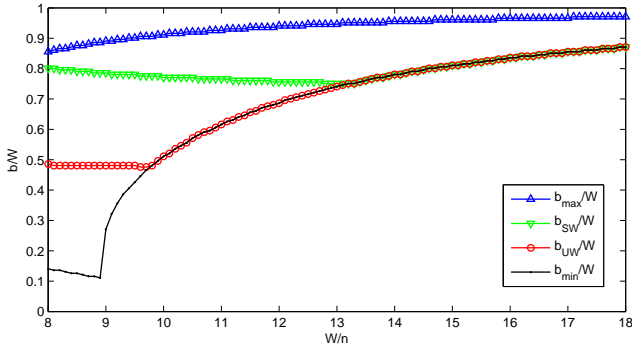


Fig. 2.  $b/W$  that yield maximum user utility and maximum  $SW$

MO have a similar amount of spectrum, precludes them from exercising any kind of market power. In other words, the level of competition is maximum and the users derive the maximum utility from the service.

With respect to the social welfare  $SW$ , a maximum is reached for a value  $b_{SW}$ , which is greater than  $b_{UW}$ . Bearing in mind that the social welfare adds up the users utility and the operators profits, this maximum is a trade off between the maximum utility reached at  $b_{UW}$  and the increasing profits.

In this experiment, the value  $p$  that is agreed provides the Shapley values for the profits. For any other value of  $p$ , specifically for  $p = U(b)$  —i.e., the incumbent has full bargaining power—the profits  $\Pi_1$  and  $\Pi_2$  would obviously be different. Nevertheless, the aggregate profits and the equilibrium prices would remain the same as above, and consequently the user utility, and the social welfare.

### B. Welfare-maximizing values for the leased spectrum

We proceed to evaluate the optimum values  $b$  of leased spectrum from the point of view of the welfare. Specifically we have computed and represented the following values in Fig. 2: maximum (minimum) value of  $b/W$  in the FR which results in a unique competitive equilibrium ( $b_{max}/W$  [ $b_{min}/W$ ]); and value of  $b/W$  between  $b_{min}/W$  and  $b_{max}/W$  such that users utility (social welfare) is maximized ( $b_{UW}/W$  [ $b_{SW}/W$ ]). We have performed different experiments varying  $W$ , which is represented in the x-axis relative to the number of users  $n$ .

We see that the value  $b_{max}/W$  tends to the value 1, which is the case where the whole spectrum  $W$  is leased to the

FO and the profits are maximized. The constraint is put by the uniqueness of the equilibrium,  $b < b_{max}^u$ . As regards  $b_{min}/W$ , there is a lower range of values of  $W$  where the uniqueness criteria constrains the possibility of a competitive equilibrium,  $b_{min} = b_{min}^u > b_{min}^f$ , and a higher range of values of  $W$  where the bargaining constrains the possibility,  $b_{min} = b_{min}^f > b_{min}^u$ . Finally,  $b_{UW}$  is lower than  $b_{SW}$ , as expected, but eventually  $b_{min}$  increases and precludes any interior maximum for  $b_{UW}$  and  $b_{SW}$  to occur.

The above results mean that the degenerate case  $b/W \approx 1$ , which would be the optimum from the point of view of the operators, is not always the optimum from the point of view of either users utility or social welfare. We would argue then that a regulator would have strong arguments —i.e., welfare enhancement—to set a maximum value  $b/W < 1$  of leased spectrum. And our argument is independent on the procedure that implements the bargaining on  $p$ .

## IV. CONCLUSIONS

From our analysis and results we can conclude that every actor, that is, the users and the two operators, are better off when the FO enters the market. This entry (and the competitive equilibrium that results) requires that the bargain outcome lies inside the feasibility region and the uniqueness region, both of which have been characterized. Furthermore, the regulator intervention is deemed necessary in order to restrain the incumbent operator from leasing the whole amount of the spectrum to the entrant operator, which will harm the users.

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