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PREDICTING COCAINE CONSUMPTION IN SPAIN. A MATHEMATICAL MODELING APPROACH

Abstract

In this article, we analyze the evolution of cocaine consumption in Spain and we predict consumption trends over the next few years. Additionally, we simulate some scenarios which aim to reduce cocaine consumption in the future (sensitivity analysis). Assuming cocaine dependency is a socially transmitted epidemic disease, this leads us to propose an epidemiological-type mathematical model to study consumption evolution. Model sensitivity analysis allows us to design strategies and analyze their effects on cocaine consumption.

The model predicts that 3.5% of the Spanish population will be habitual cocaine consumers by 2015. The simulations carried out suggest that cocaine consumption prevention strategies are the best policy to reduce the habitual consumer population.

In this paper, we show that epidemiological-type mathematical models can be a useful tool in the analysis of the repercussion of health policy proposals in the short-time future.

Keywords: Cocaine consumption, mathematical model, simulation, short-time predictions.

PREDICTING COCAINE CONSUMPTION IN SPAIN. A MATHEMATICAL MODELING APPROACH

1. Introduction.

Cocaine consumption is growing at a worrying rate in developed and developing countries (EMCDDA, 2007; UNODC, 2008). In Spain it is becoming a serious problem not only from an individual health point of view but also from the public socioeconomic one (NPD, 2000; NPD, 2007). We note that cocaine consumption is increasing (See Table 1).

<Insert Table 1>

Thus it is in the interest of public health to study the dynamics of cocaine consumption. In this article, we analyze the evolution of people with habitual cocaine consumption in Spain and simulate some health policy proposals and their effect in reducing this population.

Spanish Government strategy on drug abuse appears in the Plan Nacional sobre Drogas (Drugs National Plan) (NPD, 2000; BOE, 2009), issued by the Spanish Health Ministry.

The objectives mentioned in this document are:

1. The prevention of drug consumption, pointing out the health concerns produced by their consumption, delaying the age of the first contact with drugs, education programs and legal fight against drugs dealing.
2. To improve quantitative and qualitative research, implement new treatments, evaluate current therapy programs and training to increase professional competence of the people who work with drug abusers.

The paper is organized as follows. In the Method Section, the mathematical model is presented and the parameters of the model are estimated. In the Results Section, we present the predicted evolution in the next few years of cocaine consumption in Spain, furthermore, some public health proposals in order to reduce habitual cocaine consumption population are discussed. The last section is devoted to conclusions.

2. Methods.

In this paper, we take cocaine consumption to be as socially transmitted epidemic disease. We treat cocaine consumption as a disease that spreads through social peer pressure or social contact. These social contacts have an influence on the probability of transmission of cocaine consumption. These facts lead us to propose an epidemiological-type model to study the evolution of this consumption. These types of mathematical models also have been used in the study of other drug addictions, such as alcohol, tobacco, ecstasy or heroin addiction (Gorman et al., 2006; Sharomi & Gumel, 2008; Song et al., 2006; White & Comiskey, 2007) and in the approach to another **sociological topics** that are spread by social contact as obesity or extreme ideological behaviors (Jódar et al., 2008; Santonja et al., 2008).

2.1 Mathematical model.

In order to build the mathematical model, the 15-64 years old Spanish population is divided into four subpopulations: $N(t)$: Non-consumers, individuals that have never consumed cocaine. $C_o(t)$: Occasional consumers, individuals that have consumed sometimes in their life. $C_r(t)$: Regular consumers, individuals that have consumed in the last year. $C_b(t)$: Habitual consumers, individuals that have consumed in the last month. These subpopulations are defined by the Spanish Health Ministry. This classification is

frequently used in the surveys of the Spanish Health Ministry (Spanish Health Ministry, 2008; NPD, 2000, 2007).

Furthermore, we consider the following assumptions:

1. Let us assume homogeneous population mixing, i.e., each individual can contact with any other individual (Murray, 2002).
2. The transitions between the different subpopulations are determined as follows:
 - a. Let us consider that the newly recruited 15 years old individuals become members of the $N(t)$ subpopulation, i.e., we consider that they never consumed cocaine before.
 - b. Once an individual begins cocaine consumption he/she becomes a occasional consumer, $C_o(t)$. If this person increases cocaine consumption he/she may become a regular consumer, $C_r(t)$. If this individual continues with his/her consumption he/she may become a habitual consumer, $C_b(t)$.
 - c. An individual of subpopulation $C_b(t)$ becomes a member of subpopulation $N(t)$, non-consumer subpopulation, if he/she decides to give up cocaine consumption and go into therapy.
 - d. An individual in $N(t)$ transits to $C_o(t)$ because people in $C_o(t)$, $C_r(t)$ or $C_b(t)$ transmit cocaine consumption habit by social contact at rate β . Therefore, this is a nonlinear term modeled by $\beta N(t)(C_o(t)+C_r(t)+C_b(t))$. The remainder transits are governed by terms proportional to the sizes of the subpopulations:
 - i. $\gamma C_o(t)$ to transit from $C_o(t)$ to $C_r(t)$,
 - ii. $\sigma C_r(t)$ to transit from $C_r(t)$ to $C_b(t)$,
 - iii. $\varepsilon C_b(t)$ to transit from $C_b(t)$ to $N(t)$.

Under the above assumptions, a dynamic cocaine consumption model for Spanish population is given by the following nonlinear system of ordinary differential equations:

<Insert Figure 1>

where the parameters of the model are:

- μ , birth rate in Spain.
- d , death rate in Spain.
- β , transmission rate due to social pressure to consume cocaine.
- ε , rate at which a habitual consumer goes to therapy and becomes a non-consumer.
- d_c , augmented death rate due to cocaine consumption.
- γ , rate at which an occasional consumer transits to the regular consumption subpopulation.
- σ , rate at which a regular consumer transits to the habitual consumption subpopulation.

Figure 2 shows the diagram of the dynamic cocaine consumption model. The boxes represent the subpopulations and the arrows represent the transitions between the subpopulations. Arrows are labeled by their corresponding parameters of the model.

<Insert Figure 2>

Data in Table 1 is related to the percentages of population whereas model in Figure 1 is related to the number of individuals. It leads us to transform (by scaling) the model into the same units as data. Hence, following ideas developed in (Santonja et al., 2008; Martcheva & Castillo-Chávez, 2003; Mena-Lorca & Hethcote, 1992) we scale the model in order to estimate the unknown parameter by fitting it with data in the following section.

2.1.1 Estimation of parameters.

We give an estimation of parameters μ , ε , d_c , d using the following sources:

1. Spanish birth and death time series from Spanish Statistic Institute (INE, 2008).
2. Some papers where the authors study the therapy success rate (Dutra et al., 2008; Schmitz et al., 2001; Johnson et al., 2006; Stotts et al., 2007; Levin et al., 2007). We use the average value presented in these papers as the estimation of the therapy success rate.
3. In the articles (Dutra et al., 2008; Caballero, 2005; Budney et al., 1998; Mercer & Woody, 1998), the authors state the appropriate therapy duration and the years of consumption before going to therapy. These values are estimated by the average of the values presented in these works.
4. The technical report published by the Spanish Health Ministry where the profile of the drug users admitted to treatment is described (Spanish Health Ministry, 2008).

The estimation of these parameters, time t in years, is presented:

- $\mu = 0.01 \text{ years}^{-1}$. This is the average Spanish birth rate between years 1995-2007 (INE, 2008).
- $\varepsilon = 0.0000456 \text{ years}^{-1}$. From official data (Spanish Health Ministry, 2008) 4.25% of habitual consumers begin a therapy program every year. Furthermore, using data from Table 1 corresponding to National Drug Observatory Reports (Spanish Health Ministry, 2008), the average value of population with habitual consumption is 0.93%. Moreover, the conclusion specified (Spanish Health Ministry, 2008; Caballero, 2005; Dutra et al., 2008) that a habitual consumer takes about nine years before going to therapy. Therefore, the percentage of habitual consumers in therapy per year is 0.00439%. To be precise, $0.0093 * 0.0425 * 1/9 = 0.0000439$.

Additionally, **in the references** (Budney et al., 1998; Mercer & Woody, 1998; Schmitz et al., 2001; Johnson et al., 2006; Stotts et al., 2007; Levin et al., 2007; Dutra et al., 2008) **the authors** conclude that around 52% of the individuals on therapy recover with an average of six months. Then, we obtained $\varepsilon = 0.0000439 * 0.52 * 1/0.5 = 0.0000456$, i.e. $\varepsilon = \varepsilon_1 * \varepsilon_2 * \varepsilon_3 * \varepsilon_4 * \varepsilon_5 = 0.0093 * 0.0425 * 1/9 * 0.52 * 1/0.5 = 0.0000456$. **Notice that the value $\varepsilon_1 = 0.0093$ is related to the average percentage of the subpopulation of habitual consumers, $\varepsilon_2 = 0.0425$ is the percentage of habitual consumers that begin therapy every year, $\varepsilon_3 = 1/9$ means that a habitual consumer takes 9 years before to go to therapy, $\varepsilon_4 = 0.52$ is the average percentage of success for therapy programs and $\varepsilon_5 = 1/0.5$ indicates that the success in therapy programs is reached after half a year.**

- $d = 0.008388 \text{ years}^{-1}$ is the average Spanish death rate between years 1995-2007 (INE, 2008).
- $d_c = 0.01636 \text{ years}^{-1}$ is the augmented death rate due to drug consumption. In Spain, approximately 6.8% of mortality is due to drugs consumption (Spanish Health Ministry, 2008).

Taking as the initial conditions of the model (year 1995, i.e., $t=0$), $N(t=0)=0.944$, $C_o(t=0)=0.034$, $C_r(t=0)=0.018$ and $C_b(t=0)=0.004$, the parameters β , γ and σ have been estimated by fitting the scaled model with data from Table 1.

In order to compute the best fitting, we carried out computations with *Mathematica* (Wolfram Co., 2009) and we implemented a function F , defined in \mathbb{R}^3 whose entries are values for parameters (β, γ, σ) and with values in \mathbb{R} , $F(\beta, \gamma, \sigma)$, so that:

1. Solve numerically (NDSolve []) the scaled system of differential equations with initial values $N(t=0)=0.944$, $C_o(t=0)=0.034$, $C_r(t=0)=0.018$ and $C_b(t=0)=0.004$.
2. For $t = \text{year 1995, year 1997, year 1999, year 2001, year 2003 and year 2005}$, corresponding to the biannual drug use surveys, evaluate the computed numerical solution for each subpopulation $N(t)$, $C_o(t)$, $C_r(t)$ and $C_b(t)$.
3. Compute the mean square error between the values obtained in Step 2 and the real data presented in Table 1.

Function F takes values in \mathbb{R}^3 , (β, γ, σ) , and returns a real values. Hence, we minimize this function using the Nelder-Mead algorithm (Nelder & Mead, 1964) that does not need the computation of any derivate or gradient, this being impossible to know in this

case. The values of β , γ and σ that minimize the function F , i.e., fit the model with data, are $\beta=0.09614$, $\gamma=0.0596$, $\sigma=0.0579$.

3. Results.

3.1 Predictions

The graphical representation of the model fitting and the predictions in the next few years can be seen in Figure 3. Points represent data from Table 1.

<Insert Figure 3>

We noted a decreasing trend in non-consumer subpopulation, $N(t)$. Also, there is an increasing trend in the occasional, $C_o(t)$, regular, $C_r(t)$, and habitual consumers subpopulation, $C_b(t)$. In Table 2, some of the numerical values depicted in Figure 3 are presented.

<Insert Table 2>

If there are no changes in current cocaine consumption policies in the next few years, the model predicts that 78.5%, 12.5%, 5.5% and 3.5% of 15-64 years old individuals in Spain will be, by year 2015, non-consumer, occasional consumer, regular consumer and habitual consumer, respectively.

This prediction seems to be very alarming but the survey of 2007 recently published gives the following data for the considered subpopulations: Non-consumers = 87.4%; Occasional consumers = 8%; Regular consumers = 3% and Habitual consumers = 1.6%

these being very similar to the ones predicted by the proposed model (see Table 2). A possible explanation of this increase would be that, even though there is an intensive effort put into anti-drugs campaigns, this is accompanied by legal permissibility to consumers (they are viewed as victims) and to small dealers.

3.2 Sensitivity analysis

We performed several simulations varying the parameters of the model in order to find out what the influence of the changes on the final solution is (cocaine consumption). We carry out these variations (sensitivity analysis) to analyze the strategies of Spanish Government against drug abuse.

The five health policies simulated here are (related with the ones mentioned in the introduction):

1. Variation on the time that a habitual consumer takes before going into therapy. It involves a variation in parameter ε , to be precise in ε_3 . This parameter ε is associated with the implementation of new treatments, evaluation of current therapy programs, training plans to increase professional competence of the people who work with drug consumers, etc.
2. Variation on the percentage of habitual consumers in therapy. It also involves a variation in parameter ε , to be precise in ε_2 .
3. Variation on the success rate of therapy programs. It involves a variation in parameter ε , to be precise in ε_4 .
4. Variation on the duration of therapy programs. It involves a variation in parameter ε , to be precise in ε_5 .

5. Variation on the transition rate from non-consumers to occasional consumers. It involves a variation of parameter β . This parameter is associated with preventions policies.

Recall that parameter $\varepsilon = 0.0000456 \text{ years}^{-1}$ is computed as $\varepsilon_1 * \varepsilon_2 * \varepsilon_3 * \varepsilon_4 * \varepsilon_5 = 0.0093 * 0.0425 * 1/9 * 0.52 * 1/0.5 = 0.0000456$. Where the value 0.0093 is related to the average percentage of the subpopulation of habitual consumers, 0.0425 is the percentage of habitual consumers that begin therapy every year, 1/9 means that a habitual consumer takes 9 years before to go to therapy, **0.52 is the average percentage of success for therapy programs** and 1/0.5 indicates that the success in therapy programs is reached after half a year.

In order to perform the sensitivity analysis, let us use the technique called Latin Hypercube Sampling (LHS) to vary parameter values in the proposed model. Latin Hypercube Sampling, a type of stratified Monte Carlo sampling, is a sophisticated and efficient method for achieving equitable sampling of all input parameters simultaneously (Blower & Dowlatabadi, 1994; Olsson et al., 2003).

Each parameter for a model can be defined as having an appropriate probability density function associated with it. It is usual to use the uniform distribution centred at deterministic parameters estimators in absence of data to inform on the distribution for a given parameter (Marino et al., 2008; Hoare et al., 2008). Then, the model can be simulated by sampling a single value from each parameter distribution. Many samples should be taken and many simulations should be run, producing variable output values.

To vary ε_2 , ε_3 , ε_4 , ε_5 and β , we assume that all of them follow a uniform probability distribution with support on the intervals $[0, 0.085]$, $[0.06, 0.2]$, $[0.32, 0.72]$, $[0, 4]$ and $[0, 0.19]$ respectively. The intervals for ε_2 and β are chosen assuming that the value of the parameter may have a perturbation not greater than 100%. The interval for ε_5 (time of treatment) is chosen assuming a perturbation not greater than 100% to consider all of the treatments studied in Dutra study (Dutra et al., 2008). The interval for ε_4 (success rate of therapy programs) is chosen taking into account all of the programs analyzed in Dutra study. This percentage takes into account all of the programs analyzed in the studies (Budney et al., 1998; Mercer & Woody, 1998; Schmitz et al., 2001; Johnson et al., 2006; Stotts et al., 2007; Levin et al., 2007; Dutra et al., 2008). The interval for ε_3 (years of cocaine consumption before therapy) is chosen assuming the years of cocaine use before therapy presented in Dutra study (Dutra et al., 2008), 5-15 years.

LHS was used to generate 5,000 different values of the parameters ε_2 , ε_3 , ε_4 , ε_5 and β (input). Then we used these samples to run 5,000 evaluations of the model. The results of these evaluations allow us to determinate the 90% confidence intervals to the consumption predictions. The obtained predictions (regular consumption and habitual consumption) for year 2011 and year 2015 after the variation of the parameters can be observed in Table 3 and Table 4. **Note that the variation of the epsilons produces a variation in the 90% confidence interval smaller than 10^{-2} .**

<Insert Table 3>

<Insert Table 4>

The results in Table 3 and Table 4 allow us to say that health prevention policies (the ones related with parameter β) may have a noticeable effect on the reduction of drug consumption. Alternatively, if prevention policies are disregarded, drugs consumption will increase.

4. Conclusion.

In this paper, we propose type-epidemiological mathematical models applied to cocaine consumption in Spain. This model considers the habit of cocaine consumption as a communicable disease that is spread by social transmission. The Spanish population is divided into subpopulations of interest where certain parameters determine the transitions among these subpopulations. Furthermore, these parameters are associated with health policies of the Spanish Health Ministry.

Parameter β is associated with prevention policies and parameter ε with treatment policies. After the simulation of different hypothetical scenarios where different health policies are performed, we can conclude that prevention policies seem to be the best effective strategy to reduce the population of regular and habitual consumers. We note that taking into account random perturbations on β , the 90% confidence interval prediction presents the most important variability, i.e., modifications in prevention programs (variations of β) are the best option to modify consumption (confidence interval). In the other cases, **the ones related to parameters ε_2 , ε_3 , ε_4 and ε_5** , the variations on the parameters do not produce noticeable variations on the confidence intervals (cocaine consumption prediction). **This happens because the percentage of habitual consumers that begin therapy every year is around 4.25% as we mentioned in Section**

3.2, and it is a very small amount of the total population. Therefore, the improvement of the success of treatments hardly has impact in the model predictions.

In this paper, we show how such **type-epidemiological** mathematical models can be a useful tool to experiment with health policy proposals. Using this type of mathematical approach, we are able to simulate different situations and analyze the effect of changes in health policies on the dynamics of drug consumption.

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Figure 1. Mathematical Model

$$N'(t) = \mu P(t) - d N(t) - \beta \frac{N(t)(C_o(t) + C_r(t) + C_b(t))}{P(t)} + \varepsilon C_b(t)$$

$$C_o'(t) = \beta \frac{N(t)(C_o(t) + C_r(t) + C_b(t))}{P(t)} - d_c C_o(t) - \gamma C_o(t)$$

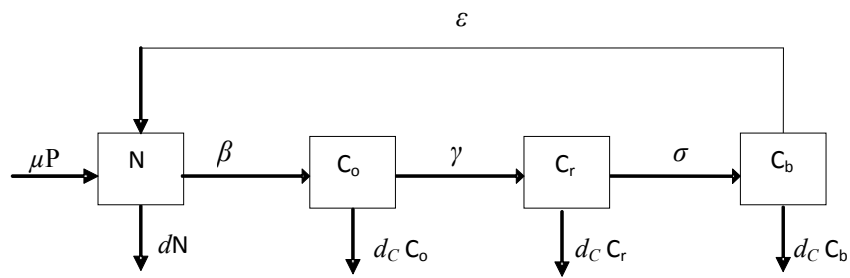
$$C_r'(t) = \gamma C_o(t) - d_c C_r(t) - \sigma C_r(t)$$

$$C_b'(t) = \sigma C_r(t) - d_c C_b(t) - \varepsilon C_b(t)$$

$$P(t) = N(t) + C_o(t) + C_r(t) + C_b(t)$$

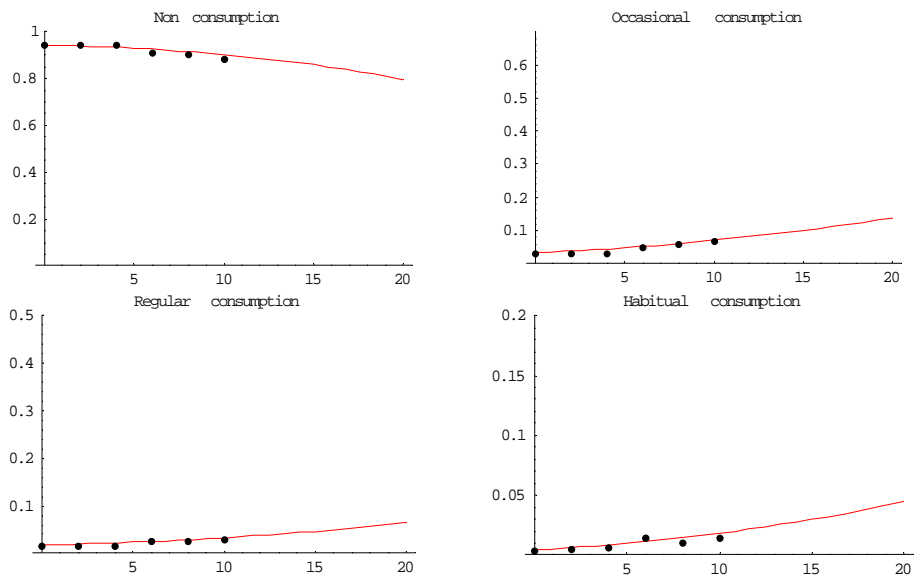
Mathematical model for drug consumption dynamics

Figure 2. Diagram model



Flow diagram of the mathematical model for the dynamics of cocaine consumption in Spain. The boxes represent the subpopulations and the arrows represent the transitions between the subpopulations. Arrows are labeled by the parameters of the model

Figure 3. Model fitting



Numerical simulations of the fitted mathematical model. $t=0$ corresponds to year 1995 and $t=20$ to year 2015. Points are data from Table 1. Moreover, the predictions for the next few years until 2015 are included. Note the increasing trend of cocaine consumption in Spain.

Table 1. Evolution of cocaine consumption.

Percentages/years	1995	1997	1999	2001	2003	2005
% Non-consumer	0.944	0.948	0.948	0.911	0.903	0.884
% Occasional consumers	0.034	0.032	0.031	0.049	0.059	0.070
% Regular consumers	0.018	0.015	0.015	0.026	0.027	0.030
% Habitual consumers	0.004	0.005	0.006	0.014	0.011	0.016

Evolution of the proportion of Non-consumers, Occasional consumers, Regular Consumers and Habitual Consumers subpopulations for different years. The data have been obtained from the Drug National Observatory Reports (NPD, 2000, 2007).

Table 2. Model predictions

	Non-consumers	Occasional consumers	Regular consumers	Habitual consumers
2007 (t=12)	0.869	0.078	0.034	0.019
2010 (t=15)	0.842	0.093	0.040	0.025
2012 (t=17)	0.821	0.105	0.046	0.028
2015 (t=20)	0.785	0.125	0.055	0.035

Model predictions for years 2007, 2010, 2012 and 2015 of percentage of non-consumers, occasional consumers, regular consumers and habitual consumers.

Table 3. Sensitivity analysis. Regular consumption

Strategy	Parameter (new value)		Regular consumers Population (%) (year 2011)	Regular consumers Population (%) (year 2015)
Variation of the habitual consumers in therapy.	$\varepsilon_2 \in [0, 0.085]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[4.34%, 4.34%] 4.34% 4.3%	[5.54%, 5.54%] 5.54% 5.5%
Variation of the time before going into therapy.	$\varepsilon_3 \in [0.06, 0.2]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[4.34%, 4.34%] 4.34% 4.3%	[5.54%, 5.54%] 5.54% 5.5%
Variation of the rate of success of therapy programs.	$\varepsilon_4 \in [0.32, 0.72]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[4.34%, 4.34%] 4.34% 4.3%	[5.54%, 5.54%] 5.54% 5.5%
Variation of the duration of therapy programs.	$\varepsilon_5 \in [0, 4]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[4.34%, 4.34%] 4.34% 4.3%	[5.54%, 5.54%] 5.54% 5.5%
Variation of the transition rate to occasional consumers.	$\beta \in [0, 0.19]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[1.66%, 9.80%] 4.89% 4.3%	[1.49%, 14.36%] 6.51% 5.5%
Variation of ε_2 , ε_3 , ε_4 , ε_5 and β (all together)	$\varepsilon_3 \in [0.06, 0.2]$ $\varepsilon_2 \in [0, 0.085]$ $\varepsilon_4 \in [0.32, 0.72]$ $\varepsilon_5 \in [0, 4]$ $\beta \in [0, 0.19]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[1.66%, 9.80%] 4.89% 4.3%	[1.49%, 14.36%] 6.51% 5.5%

Results obtained after sensitivity analysis applying 5,000 realizations with Latin Hypercube Sampling technique. The values of the parameters not perturbed are the ones described in Section 2.1.1. In the last two columns there is: the 90% confidence interval, the mean value of the 5,000 realizations and the model prediction. The perturbation of parameter β leads to larger variations in output than the rest of perturbed parameters. This fact allows us to say that prevention policies (the ones related to parameter β) may have a noticeable effect in the reduction of drug consumption.

Table 4. Sensitivity analysis. Habitual consumption

Strategy	Parameter (new value)		Habitual consumers Population (%) (year 2011)	Habitual consumers Population (%) (year 2015)
Variation of the habitual consumers in therapy.	$\varepsilon_2 \in [0, 0.085]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[2.62%, 2.62%] 2.62% 2.6%	[3.56%, 3.56%] 3.56% 3.5%
Variation of the time before going into therapy.	$\varepsilon_3 \in [0.06, 0.2]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[2.62%, 2.62%] 2.62% 2.6%	[3.56%, 3.56%] 3.56% 3.5%
Variation of the rate of success of therapy programs.	$\varepsilon_4 \in [0.32, 0.72]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[2.62%, 2.62%] 2.62% 2.6%	[3.56%, 3.56%] 3.56% 3.5%
Variation of the duration of therapy programs.	$\varepsilon_5 \in [0, 4]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[2.62%, 2.62%] 2.62% 2.6%	[3.56%, 3.56%] 3.56% 3.5%
Variation of the transition rate to occasional consumers.	$\beta \in [0, 0.19]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[1.80%, 4.09%] 2.75% 2.6%	[2.03%, 6.58%] 3.85% 3.5%
Variation of ε_2 , ε_3 , ε_4 , ε_5 and β (all together)	$\varepsilon_3 \in [0.06, 0.2]$ $\varepsilon_2 \in [0, 0.085]$ $\varepsilon_4 \in [0.32, 0.72]$ $\varepsilon_5 \in [0, 4]$ $\beta \in [0, 0.19]$	90% confi. Interv: Mean 5000 realiz: Model estimation:	[1.80%, 4.09%] 2.75% 2.6%	[2.03%, 6.58%] 3.85% 3.5%

Results obtained after sensitivity analysis applying 5,000 realizations with Latin Hypercube Sampling technique. The values of the parameters not perturbed are the ones described in Section 2.1.1. In the last two columns there is: the 90% confidence interval, the mean value of the 5,000 realizations and the model prediction. The perturbation of parameter β leads to larger variations in output than the rest of perturbed parameters. This fact allows us to say that prevention policies (the ones related to parameter β) may have a noticeable effect in the reduction of drug consumption.