Use of Thermal Conductivity from Thermal Response Test for Estimating Steady State Temperatures in Rock and Stratified Soil near Line Source of Heat

ABSTRACT

This paper addresses anisotropic dependence of effective thermal conductivity measured by a field thermal response test (TRT). That is a key parameter in the design of Ground-Coupled Heat Pumps (GCHP) to heat and cool buildings.

First, the paper provides a brief overview of the current technique of estimating thermal conductivity from a data obtained in TRT based on predictions for temperature from line source of heat in an isotropic ground. Then, the solutions for isotropic medium are used to develop this temperature transient method for stratified medium, where the angle between the ground surface and the sedimentary strata is arbitrary. In addition, the paper provides a new analytical exact solution for temperatures around finite line source (FLS) of heat in an anisotropic semi-infinite medium. Approximate expressions for the temperature evolution during the test duration and for the steady state temperature are presented.

The limitations of the FLS method in stratified medium and recommendations for layout of multiple vertical or horizontal ground coupled heat exchangers or waste canisters in repository rock are discussed.

INTRODUCTION

Thermal conductivity of the ground is a key property when sizing of the ground coupled heat pump (GCHP) air-conditioning systems. For large commercial installations it is measured on a field borehole in a thermal response test (TRT) the scheme of which is shown in Figure 1. Figure 1 represents a typical TRT test to measure the temperature response of the borehole heat exchanger (BHE) to a constant heat injection or extraction. A U-tube loop, through which a heat carrier fluid circulates, is inserted inside the borehole to approximately the same depth as the BHE planned for the site. The outputs of the TRT are the inlet (T_{in}) and outlet (T_{out}) temperatures of the heat carrier fluid as a function of time (see Figure 1). The average change of fluid temperature is directly related to the rock/soil thermal conductivity around the well. To determine the rate at which heat is transferred into the ground its model is necessary that may account for underground water flows, temperature dependency of thermal conduction, variable thickness of the strata , d_i see Figure 1. The temperatures T_{in} and T_{out}, measured at the end points of the U-tube, are used to determine a mean value of thermal conductivity, averaged over the length of shallow BHE. The effective thermal conductivity represents a number of the model parameters, when fitting the TRT data.

From the experimental data, and with an appropriate model describing the heat transfer between the fluid and the ground, the effective thermal conductivity of the surroundings is inferred. Thermal conduction of ground from a TRT data can be estimated with different models. The measured thermal conductivity of the ground depends on parameters of the model for the ground chosen for analysis through the effective thermal conductivity.

The Kelvin's solution for temperature of the ground surrounding the borehole heat exchanger (BHE) modeled as an infinite line source (ILS), is the basis for the TRT in estimating the thermal properties of the ground. This approach is used further in the GCHP design standards of the International Ground Source Heat Pump Association (Bose et al. 1985). The cylinder heat source and line heat source (Carslaw and Jaeger 1959) model for BHE with parameter-estimating techniques are commonly applied for the design and analysis of vertical ground coupled systems (Bernier 2001).

The Kelvin's concept assumes a homogeneous isotropic media surrounds the heat line source of a constant heat rate. However, vertical BHE systems are often installed in ground of multiple dipping layers (of rock or soil) with different thermal conductivities. For stratified media effective thermal conductivity in the ILS theory represents average thermal conductivity.

An algorithm proposed in (Sutton et al. 2001) for the performance of vertical BHE is based on analytical solution of an infinite cylindrical heat source model for horizontally stratified geologic formations. These models for the BHE describe radial heat flow that implies only transverse conductivity to its axis.

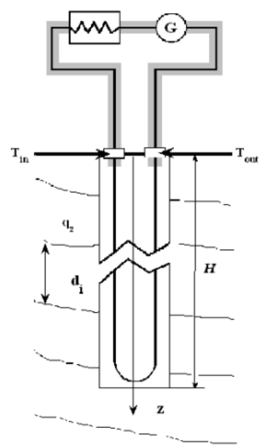

Figure 1. In-situ TRT schematic and formation layers.

In general, the ground is an anisotropic medium whose thermal conductivity depends on the direction. Typically, the sedimentary soil or rock formations have the conductivity in one direction greater than in another: the heat flow passes more easily along the planes of deposition than across them and, thus, direction of heat flow does not coincide with the direction of the imposed temperature gradient. The heat flow and temperature gradient are vector quantities related by the thermal conductivity tensor in anisotropic media instead of scalar thermal conductivity in isotropic one.

It is necessary to determine thermal conductivity tensor of the ground with application to the estimation of temperature field. On a large scale it depends upon the average thermal conductivities parallel and perpendicular to bedding and its spatial orientation to the surface.

A geophysical logging of wells is one of the methods presently used for identification of type of the ground, and establishment of thermal conductivity distribution in depth (Pribnow and Sass, 1995; Davis et al., 2007). In-plane thermal conductivity $\lambda_{\parallel}$ and thermal conductivity $\lambda_{\perp}$ normal to the bedding can be determined from parallel and series models, applicable for bedded sediments.

Data on thermal conductivities and anisotropy values, assembled from different areas around the world are available from literature (Deming, 1994). These data are classified by rock name and origin. Thermal properties of samples extracted from identified layers are available from review articles (Pribnow and Sass, 1995).

There are areas where only data on borehole cuttings are available. The typical approach to the estimation of thermal conductivities is to carry out measurements in the laboratory on samples. The thermal conductivity tensor and the anisotropy (defined as $a^2 = \frac{\lambda_{\parallel}}{\lambda_{\perp}}$) can be obtained on oriented core, when measuring by line source probe on the same sample face at multiply angles to bedding (Pribnow and Sass, 1995; Popov et al., 1999).

Laboratory results are normally combined with in situ thermal conductivity measurements. Assessments of the thermal conductivities by laboratory methods are difficult to extrapolate to in-situ conditions for deep boreholes (Pribnow and Sass, 1995).

The line source method, used for thermal conductivity determination in both field and laboratory, provides ground thermal conductivity in the direction perpendicular to the line.

In addition, the mean dip angle between bedding and surface is required for practical applications to define the average thermal conductivity tensor. Small variations of dip in wells can be viewed in borehole imaging logs (Borehole Televiewer, Formation Micro Scanner) (Pribnow and Sass, 1995). In practice, the value of dip angle can be estimated simply by examining the in situ cross section.

When no data are available on the dip angle between the bedding formations and the earth's surface, assessment of the temperature in the BHE surroundings may be useful in the limiting cases of horizontal and vertical stratification. Such estimation defines upper and lower limits for average temperature field for the intermediate angle values from 0 to $\frac{\pi}{2}$.

For geologic applications, to measure thermal conductivity in vertical direction, normal to the earth's surface, the model was developed for arbitrary angle between the ILS and the principal direction of heat flow in an infinite anisotropic medium (Grubbe et al. 1983).

However, infinite-source models have some limitations. For long time periods the finite size effects need to be taken into account; otherwise the ILS models predict unlimited increase of the temperature when time tends to infinity. The very introduction of the surface boundary has the effect of setting a steady-state (Bandos et al., 2009); this is beyond the scope of the infinite line-source models either for isotropic or anisotropic media. Three dimensional finite line source (FLS) model of the BHE in a semi-infinite medium (Carslaw and Jaeger, 1959) does account for vertical heat transfer with both the soil surface and deep earth.

Design tools use the so called "*g-function*" introduced by Eskilson (1987), which represents the thermal response factor of the borehole to a constant heat pulse at the borehole periphery, i.e. $r = r_b$. It is estimated at the BHE mid-point in simulations of GCHP systems because ILS method implies the temperature at the point far away from the BHE ends. After Zeng et al. (2002), Lamarche and Beauchamp (2007) extended the *g-function* concept of Eskilson to analytical integral average g-*function.* Further, the mean g-function has been approximated for a wide time range, providing its explicit steady state limit at any point and the finite size corrections during the test for $H >> r \geq r_b$, i.e. in the borehole vicinity (Bandos et al. 2009). The edge effects are due to the vertical heat flow along the borehole from the deep earth and its surface. The thermal response of a borehole is proportional to the ratio of $\frac{q_z}{\lambda_{eff}}$ of two significant factors in almost all analytical g-functions for the short and long term time analysis of the BHE response (Javed et al., 2009).

These FLS models have been limited in application to the infinite ground of either isotropic or anisotropic thermal properties, whereas to the best of author knowledge, solution for the temperature in the semi-infinite anisotropic medium has never been known. To assess properly the steady state temperature of the underground installation one needs to account for principal directions of the heat flow in the ground. It may be relevant to guarantee stability of operating the ground coupled installation as well as the time of investment return. The financial reward of installing a geothermal system comes after the long term. The anisotropy effect on temperature in borehole surroundings also becomes significant for very long time values.

In this context, line-source methods to estimate thermal conductivity include conducting laboratory experiments on rock and soil samples and/or performing field tests (Davis et al. 2007; Popov et al. 1999). It should be noted, however, that the ILS based method was developed for rocks layered non perpendicular to the ILS (Grubbe et al. 1983). In geologic applications it is widely used for calculation of terrestrial heat flow density, while in geothermal applications it is necessary to determine the borehole temperature for the design purposes. However, for both applications of this method do not account for the fact that the earth's surface can make arbitrary angle with the sedimentary bedding. The ILS method in an infinite anisotropic medium was proposed to determine the vertical component of thermal conductivity along the ILS embedded in rocks layered non perpendicular to it (Grubbe et al. 1983). However, this method cannot describe exactly the temperature field in an anisotropic half-space without accounting for a boundary condition on the ground surface.

Further refinement of the FLS approach is desirable for anisotropic semi-infinite medium; anisotropic corrections to the g-function reveal how rocks are layered to the surface. Moreover, bedding angle dependence on temperature response is of significant importance for long-term underground energy systems. It may be important, when estimating maximum temperatures tolerated in nuclear waste repositories or aquifer thermal energy storages (Hörmark and Claesson 2005; Sundberg and Helström 2009).

The effect of anisotropy of heat flow in a multi-layer geological formation on the temperature around the vertical line heat source at an arbitrary dip angle of the strata to the earth surface is the subject of this paper. It has practical implications for the estimate of test data, the steady-state temperature field and for the selection of orientation of vertical bore field.

This paper presents (I) exact solution for the FLS thermal response function of a borehole that takes into account the geometrical disposition of the earth surface and the sedimentary bedding; (II) approximate expressions for the mean temperature of the vertical BHE for the times corresponding to the TRT duration as well as to the long times in the limiting cases of horizontal and vertical stratification to the earth's surface. Results on the time-series expansion for the temperature around the finite line-source in an anisotropic semi-infinite medium - including the existence of a steady-state limit – are also discussed.

PROBLEM STATEMENT

For the line-source analysis of TRT data, the ground is assumed to be a homogeneous isotropic medium characterized by scalar thermal conductivity λ. For the stratified geologic regime, this assumption is extended to the thermal conductivity tensor that characterizes anisotropic medium. The heat flow and temperature gradient are vector quantities related by the thermal conductivity tensor Λ_{ik} (Carslaw and Jaeger, 1959). The heat flow in the i-th direction Q_i at a given point of the anisotropic medium is given by

$$Q_i = -\sum_{k=1}^{3} \Lambda_{ik} \partial T / \partial x_k$$

It is assumed that the heat flow in the stratified ground proceeds as if the media were homogeneous, i.e. the thermal conductivity tensor is homogeneous, but anisotropic.

This paper considers heat flow along the vertical z- axis, which is perpendicular to the surface of the semi-infinite region, as shown in Figure 1. The heat is realised at a constant rate along the z-axis of the Borehole Heat Exchanger (BHE), modelled as the Finite Line- Source (FLS), and is transferred by thermal conduction along the preferential directions in the semi-infinite region. In the anisotropic model the equation of heat diffusion, generally, is not invariant under spatial rotation about the z-axis of the vertical BHE. The subsurface temperature, T, is governed by the heat conduction equation:

$$C\frac{\partial T(\vec{x},t)}{\partial t} = \sum_{i=1}^{3}\sum_{k=1}^{3} \Lambda_{ik} \frac{\partial^2 T(\vec{x},t)}{\partial x_k \partial x_i} + q_z \delta(x)\delta(y)(\theta(z) - \theta(z-H)), \qquad for \;\; t \ge 0, \; z \ge 0 \tag{1}$$

where $\vec{x} = (x_1, x_2, x_3) = (x, y, z)$ is the coordinate vector, and q_z is the heat flux density per length unit of the BHE of radius r_b, where $\delta(x)$ is the Dirac delta function characterized by the property $\int_{-\infty}^{\infty} \delta(x) f(x) = 0$ for all functions f, $\theta(z)$ is the unit step function, which is zero for z <0 and unity for z >0 . The initial condition and boundary condition on the surface are given by:

$$T(\vec{x}, \quad t = 0) = T_0, \qquad\qquad T(x, y, z = 0, \quad t) = T_0 \tag{2}$$

Typically, the line-source of heat is introduced as boundary condition on the cylindrical surface

$$\frac{q_z}{2\pi r_b} = \left[-\Lambda_{11}\partial T/\partial x - \Lambda_{22}\partial T/\partial y - \Lambda_{13}\partial T/\partial z\right]_{\Sigma}$$

and not as a heat generation term of Equation (1) in this equivalent formulation of problem (Carlaw and Jaeger, 1959).

We address the simplest case of anisotropy in which the thermal conductivity is the same for all directions of a plane X'Y and differs in the Z' direction noted in Figure 2. Two components of the thermal conductivity for heat flow through the ground in a direction perpendicular and parallel to the bedding plane are denoted by $\lambda_{\perp}$ and $\lambda_{\|}$, respectively. The in-plane thermal conductivity $\lambda_{\|}$ is larger than orthogonal component of thermal conductivity tensor $\lambda_{\perp}$ (Davis et al. 2007; Popov et al. 1999), but this study is valid for any anisotropy ratio.

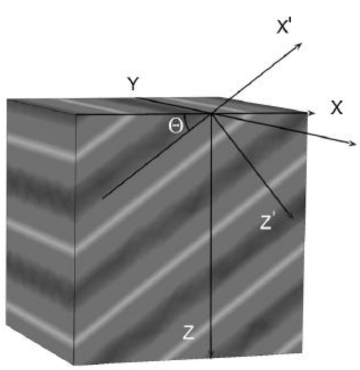

Figure 2. Direction of X', Y, Z' principal axes of the thermal conductivity tensor. The XY plane represents the ground surface at the dip angle Θ from the X'Y plane.

In order to formulate the problem around the BHE inserted into the ground so that its surface is at angle Θ to the bedding plane one needs to find the conductivity tensor in the chosen axes. The thermal conductivity tensor is diagonal in the X', Y, Z' coordinates shown in Figure 2. These three directions are called the principal axes of the thermal conductivity tensor:

$$\sum_{k=1}^{3} \lambda_k \delta_{ik} \qquad (3)$$

with the $\lambda_1 = \lambda_{\|}$, $\lambda_2 = \lambda_{\|}$ and $\lambda_3 = \lambda_{\perp}$ components.

To find the conductivity tensor Λ_{ij} in the chosen axes X, Y and Z (so that the BHE axis is at angle Θ to the Z' axis) one performs a rotation of tensor in Equation 3:

$$\Lambda_{ij} = \sum_{m=1}^{3} \sum_{k=1}^{3} R_{ki} \lambda_k \delta_{mk} R_{mj} \tag{4}$$

by the orthogonal matrix R_{mj} describing the rotation between two sets of axes shown in Figure 2. This is given by (Hastie et. al 2001):

$$R = \begin{pmatrix} \cos\Theta & 0 & \sin\Theta \\ 0 & 1 & 0 \\ -\sin\Theta & 0 & \cos\Theta \end{pmatrix} \tag{5}$$

Making use of the transformation defined above one gets

$$\Lambda = \begin{pmatrix} \lambda_{\perp} sin^2\Theta + \lambda_{\|} cos^2\Theta & 0 & (-\lambda_{\perp} + \lambda_{\|}) sin2\Theta / 2 \\ 0 & \lambda_{\|} & 0 \\ (-\lambda_{\perp} + \lambda_{\|}) sin2\Theta / 2 & 0 & \lambda_{\perp} cos^2\Theta + \lambda_{\|} sin^2\Theta \end{pmatrix} \tag{6}$$

The coefficient Λ_{11} is the thermal conductivity coefficient for the heat flow in the X direction due to a gradient in the direction X. It also gives rise to a heat flow in the vertical direction due to the presence of the off-diagonal coefficient Λ_{13}.

$$Q_3 = -\Lambda_{31} \partial T / \partial x - \Lambda_{33} \partial T / \partial z$$

The anisotropy factor causes the distortion of the temperature gradient at the surface of multilayered ground and around the BHE bottom, because the heat flow is not normal to the isotherms.

Throughout the paper the following normalization was used

$$\varepsilon_{ij} = \Lambda_{ij} / \Lambda_{33}, \quad i, j = 1, 2, 3 \tag{7}$$

The temperature field is thus defined by the solution of Equations 1, 2 with the above Λ matrix in Equation 6. If the angle $\Theta = 0$ (or $\Theta = \pi / 2$), the axes coincide with appropriate symmetry directions of a multilayered ground, this matrix Λ becomes diagonal one, where $\Theta = 0$ (or $\Theta = \pi / 2$) correspond to horizontal (or vertical) stratification of the ground.

In the following the grounds of these types will be considered as well as the ground strata at any dip angle to the surface.

ANISOTROPIC DIFFUSION IN SEMI-INFINITE MEDIUM. LINE HEAT SOURCE THEORY

This section is focused on the generalization of the analytical solutions of the thermal conduction problem for isotropic medium to the solutions for anisotropic semi- infinite medium representing multi-layered ground.

We introduce the common methods for TRT estimations and highlight their limitations due to the isotropy assumption.

Solution for Finite Line Heat Source in Isotropic Medium

The exact solution for temperature response from the isotropic ground, where the thermal tensor is diagonal, $\Lambda_{ik} = \lambda\delta_{ik}$, can be written (Bandos et. al 2009):

$$T(r,z,t) - T_0 = \frac{q_z}{4\pi\lambda} \int_{r/\sqrt{4\alpha t}}^{\infty} \{2erf(\frac{z}{r}) - erf(\frac{H+z}{r}) + erf(\frac{H-z}{r}u)\} e^{-u^2} u du \qquad (8a)$$

and its integration over the length of the BHE gives:

$$< T(r,t) - T_0 > = \frac{q_z}{2\pi\lambda} \hat{g}(h,t), \qquad < T > = \frac{1}{H}\int_0^H T(z)\, dz$$

$$\hat{g}(h,t) = \frac{1}{2} \int_{r/\sqrt{4\alpha t}}^{\infty} \{4erf(hu) - 2erf(2hu) - (3 + e^{-4h^2u^2} - 4e^{-h^2u^2}) \frac{1}{\sqrt{\pi}hu}\} \frac{e^{-u^2}}{u} du \qquad (8b)$$

Both the exact solution and its average represented in such a form recover straightforwardly the ILS result in the limit $H \to \infty$

$$-\frac{q_z}{4\pi\lambda} Ei(-\frac{r^2}{4\alpha t}) \approx \frac{q_z}{4\pi\lambda} \{\ln \frac{4\alpha t}{r^2} - \gamma\}, \qquad for \quad 5\frac{r^2}{\alpha} << t << \frac{H^2}{\alpha} \qquad (9)$$

There are some approaches of deriving analytical expressions for Equation 8a (Eskilson, 1987) to overcome time consuming numerical calculation of the above integrals and to get insight on physical interpretation of the heat transfer processes. It can be seen that there are two characteristic scales of time, namely, $t_z = H^2/\alpha$, $t_r = r_b^2/\alpha$. Early time values (i.e. $t < 5t_r$) are of the order of one day, whereas typical thermal test durations range from 40 to over 200 hours (Sutton et al., 2001). Thus, the duration of TRTs conform to what are called intermediate times ($t_r < t < t_z$) to distinguish them from very long times ($t > t_z$) that would approach those corresponding to steady-state conditions. Time of steady-state attainment is infinite and finite for the ILS and FLS, respectively. Furthermore, the approximation of the average ground temperature for the times corresponding to the TRT (i.e., for $5t_r < t < t_z$) is given by:

$$< T(r,t) - T_0 > = \frac{q_z}{4\pi\lambda}\left(ln\frac{4\alpha t}{r^2} - \gamma - 3\left\{ \sqrt{\frac{4t}{\pi t_z}} - \frac{r}{H} + \frac{r^2}{H^2}\sqrt{\frac{t_z}{4\pi t}} \right\} \right) \quad for\ r_b \le r << H \qquad (10)$$

This expression for the average temperature of the BHE differs from the classical one by the finite-size corrections, which vanish in the limiting case of $H \to \infty$. The comparison between the numerical results of FLS and ILS models applied to the same experimental data showed that, as predicted by Bandos et al. (2009), the thermal conductivity value of the ground is overestimated by the ILS model (Bandos et al., 2011). In addition, error in estimating the thermal conductivity between two models can be found analytically.

Evaluating TRT is based on the linear logarithmic time dependence for the temperature from the ILS theory. From Equations 9 and 10 one can find

$$\frac{\lambda^{FLS}}{\lambda^{ILS}} = 1 - \frac{3}{\sqrt{\pi}}\sqrt{\frac{t}{t_z}}\left(1 - \frac{r_b^{\ 2}}{4\alpha t}\right) \qquad for \quad \frac{r_b}{H} << 1, \quad 5t_r \le t << t_z$$

$$\lambda^{FLS} = \frac{q_z}{4\pi}\frac{\partial \ln t}{\partial < T(r_b,t) >} \quad , \qquad \lambda^{ILS} = \frac{q_z}{4\pi}\frac{\partial \ln t}{\partial T(r_b,t)} \qquad (11)$$

Here λ^{ILS} and λ^{FLS} are the effective thermal conductivities estimated with the ILS and the approximation of the mean FLS models, respectively. Therefore, the estimate from the TRT with the mean FLS model gives a lower value for the log-derived thermal conductivity than the one predicted by the ILS model; the relative error is proportional to the square root of the small parameter $\frac{t}{t_z} << 1$ for test durations.

The explicit steady state borehole temperature was derived amid the approximate expressions for the mean ground temperature over a wide range of time values (Bandos et al. 2009).

$$\lim_{t\to\infty} < T(r,t) - T_0 > = \frac{q_z}{2\pi\lambda}\hat{g}_S(\frac{H}{r}), \qquad \lim_{t\to\infty}\hat{g}(h,t) = \hat{g}_S(h)$$

$$\hat{g}_S(h) = \ln\frac{\left(h+\sqrt{1+h^2}\right)^2}{2h+\sqrt{1+4h^2}} - \frac{1}{2h}\left(3+4\sqrt{1+h^2}-\sqrt{1+4h^2}\right) \tag{11}$$

To proceed further, the anisotropy effect on the TRT estimate and the long time temperature profile are considered for horizontally and vertically alternating formations and in general case for layers non-parallel to the ground surface.

Solution for Finite Line Heat Source in Anisotropic Medium

This section, firstly, addresses to the simple case when a main direction of the thermal conductivity coincides with the vertical direction, perpendicular to the surface, while in-plane bedding plane is horizontal, i.e. parallel to the surface. Secondly, we introduce mean temperature method for the horizontal strata, $\Theta = 0$, i.e. parallel to the surface, present the closed form temperature solution around FLS for strata at any dip angle to it and conclude with the limiting case of vertical stratification, $\Theta = \pi/2$.

Mean temperature approximations at horizontal stratification

It is assumed that the thermal conductivity of horizontally stratified ground takes on different values in the horizontal (in-plane) direction, $\lambda_1 = \lambda_2 = \lambda_{||}$, and in the vertical z direction, $\lambda_3 = \lambda_{\perp}$, which are the diagonal components of the thermal conductivity tensor with zero off-diagonal elements, $\Lambda_{ik} = \lambda_k \delta_{ik}$, $k = 1,2,3$. The problem of heat diffusion in horizontally stratifed geologic regime ($\Theta = 0$) is subject to the conditions specified in Equation 2. Its solution is invariant under spatial rotation about the axis of the vertical BHE as it is in the case of the isotropic medium. Furthermore, after a transformation $z \to z\sqrt{\lambda_{||}/\lambda_{\perp}}$, Equation 1 takes the same form as the equation for the isotropic homogeneous ground with the thermal conductivity $\lambda_{||}$ as for the primary line source model. This transformation reduces the heat conduction problem in the horizontally stratified anisotropic ground to the one in the isotropic semi-infinite medium of the thermal conductivity $\lambda_{||}$ and diffusivity $\alpha_{||} = \lambda_{||}/C$. Thus the solutions for the anisotropic ground can be obtained from Equations 8a, 8b for the isotropic soil by substituting $z \to z\sqrt{\lambda_{\|} / \lambda_{\perp}}$ supplemented by the $H \to H\sqrt{\lambda_{\|} / \lambda_{\perp}}$ rescaling; hence, the resulting depth is stretched for the horizontal stratification for $\lambda_{\|} \geq \lambda_{\perp}$.

Approximate expressions for ground temperature, averaged over the BHE depth, were derived to use (instead of temperature at the mid-point) over a wide range of time values (Bandos et al. 2009). Then, after applying the above described transformations, the average ground temperature response for the time in the interval corresponding to the TRT (i.e. for $r^2 / 4\alpha_{\|} << t << H^2 a^2 / 4\alpha_{\|}$) can be written as:

$$< T(r,t) - T_0 >= \frac{q_z}{4\pi\lambda_{\|}}\left[ln(4Fo_{\|}) - \gamma - \frac{3r}{\sqrt{\pi}aH}\left\{ \sqrt{4Fo_{\|}} - \sqrt{\pi} + \frac{1}{\sqrt{4Fo_{\|}}} \right\} \right] \quad for \quad r << aH \tag{12}$$

where $a^2 = \lambda_{\|} / \lambda_{\perp}$ is the in-plane conductivity scaled by the normal conductivity, $Fo_{\|} = \frac{t\alpha_{\|}}{r^2}$ is the Fourier number that refers to a radial distance r from the borehole center, not to the borehole radius r_b , which defines characteristic time t_r . The TRT measures a multiplier for logarithm of time that is a function of model parameters. Effective thermal conductivity is such a function that is inversely proportional to logarithmic derivative from the temperature in the intermediate-time interval. From the above equation effective thermal conductivity λ_{eff} measured by the line source method equals to the thermal conductivity in direction parallel to bedding (to the ground surface for horizontal stratification) $\lambda_{\|}$. Note that the log-derived thermal conductivity is equal to the only parameter of the isotropic model of the ground: $\lambda_{eff} = \lambda$.

The effects of the finite source size (described by the last three terms in the right hand side of Equation 12 for intermediate time values) depend on the anisotropy a , vanish in the limiting case $H \to \infty$ and are smaller than those in the isotropic model ($a = 1$) for $\lambda_{\|} > \lambda_{\perp}$.

Application of the same scale transformations to the approximation derived for the long times (Bandos et al. 2009), when approaching the steady-state conditions, the integral average temperature response at the radial distance r from the borehole center is given by:

$$< T(r,t) - T_0 >= \frac{q_z}{2\pi\lambda_{\|}}\left\{ \hat{g}_S(\frac{aH}{r}) - \frac{(t_{\|}a^2 / t)^{3/2}}{24\sqrt{\pi}}\left[1 - 3t_{\|}a^2 \frac{1 + r^2 / (Ha)^2}{20t} \right] \right\}, \quad t_{\|} = \frac{H^2}{\alpha_{\|}}, \quad t >> \frac{max(a^2H^2, r^2)}{\alpha_{\|}} \tag{13}$$

This equation provides time-asymptotic approach to the steady-state of the designed geothermal system, whereas Equation 12 is applicable to analysis of the TRT data in the intermediate-time interval. It is noteworthy to mention that the effective thermal conductivity $\lambda_{eff} = \lambda_{\parallel}$ defines thermal response of the BHE embedded in horizontally stratified ground in the the intermediate and the long-time intervals. Furthermore, both above approximations for the mean BHE response depend on anisotropy through the ratio $\frac{aH}{r}$. There are two characteristic times $\frac{r^2}{\alpha_{\parallel}}$ and $\frac{H^2a^2}{\alpha_{\parallel}} = \frac{H^2C}{\lambda_{\perp}}$ for anisotropic diffusion in the radial and axial directions, respectively; these directions coincide with the Λ principal axes for horizontal stratification.

Exact solution for the mean steady-state temperature in the dimensionless form of $\hat{g}_S(\frac{aH}{r})$ reveals anisotropy effect at any radial distance from borehole center. Using the expansion we arrive at the following result for anisotropy correction to the steady state temperature from isotropic and anisotropic models, which can be used in the vicinity of borehole, i.e. $r \ll H$

$$\left\langle T(r)_{|a=1} - T(r)_{|a\neq 1} \right\rangle = \frac{q_z}{2\pi\lambda_{eff}}\left[\hat{g}_S(\frac{H}{r}) - \hat{g}_S(\frac{Ha}{r})\right] = \frac{q_z}{2\pi\lambda_{\parallel}}\left[-2\ln a + \frac{3r}{H}(1-a^{-1}) + O(\frac{r}{H})\right] \quad for \quad \frac{H}{r} \to \infty \quad (HS)$$

The symbol O(x) denotes terms proportional to x and higher powers of x. This comparison is done for $\lambda_{eff} = \lambda = \lambda_{\parallel}$.

Throughout the paper the following parameters were used in the numerical calculations: $a = 1.4$ for the anisotropy case (Davis et. al 2007) and $\alpha = 1.16 \times 10^{-6}$ m²/s, $\lambda = 4.3$ W/(mK). How anisotropy of the ground thermal conductivity influences the time dependence of the temperature distribution around the vertical BHE penetrating strata is shown in Figure 3. Exact temperature profiles along the borehole calculated for the horizontal stratification are presented in Figure 3a at various time values from 1.5 months to 12 years. Figure 3a shows that maximum temperature along the BHE for $\lambda_{\parallel} / \lambda_{\perp} > 1$ (anisotropic case) becomes noticeably higher than that for $a = 1$ (isotropic case) as the time increases. That is due to decreased heat transfer from the bottom of the borehole.

(a) (b)

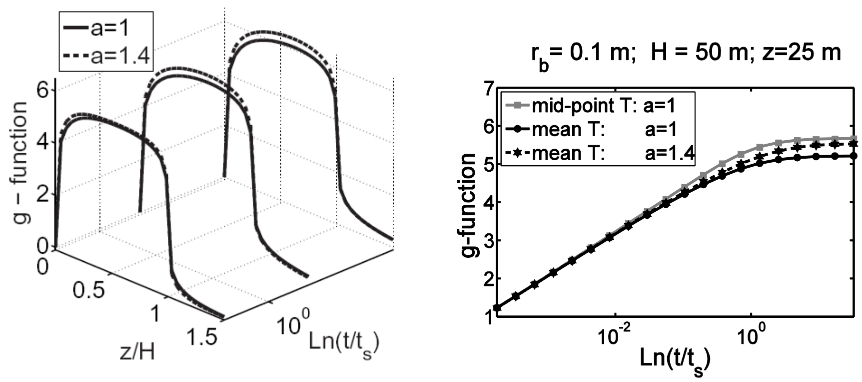

Figure 3. Comparison between thermal response g-functions at $r = r_b$ =0.1m around the borehole penetrating horizontal strata $(\Theta = 0)$ from two models: isotropic model ($\lambda_{\|} = \lambda_{\perp} = \lambda$) and anisotropic model ($\lambda_{\|} = a^2 \lambda_{\perp}$, $\Theta = 0$; $\lambda = \lambda_{\|}$) of the ground. (a) Profiles of the g-function versus the natural logarithm of time t / t_s and the dimensionless coordinate z / H along the borehole; (b) Mid-point (z=H/2) (gray line) and mean g-functions from the isotropic model and mean g-function from the anisotropic model versus the natural logarithm of time. Exact solutions calculated for constant heat injection are shown in the range: $5 r_b^2 / \alpha_{\|} < t < H^2 a^2 / \alpha_{\|}$, $\alpha = \alpha_{\|}$. The time is scaled by $t_s = H^2 / (9\alpha)$ (Eskilson, 1987).

Figure 3b shows that the *g*-function estimated at the BHE mid-point (Eskilson, 1987) and averaged response function are rather close to each other for the isotropic medium ($\lambda = \lambda_{\|}$) and to the mean temperature response function for the anisotropic medium in the intermediate-time interval. There is the increase of the mean temperature evaluated from Equation 13 for the horizontal stratification of sufficiently low $\lambda_{\perp}$ value: $a = 1.4$ compared to the mean temperature , but this temperature remains lower than mid-point temperature at $z = H / 2$ for the isotropic case.

Notice that the higher the scaled thermal conductivity $\lambda_{\|} / \lambda_{\perp}$ in the horizontal direction, the later is the onset of the asymptotic behavior when attaining steady state. Therefore, evaluation of thermal conductivity from the TRTs provides primarily effective thermal conductivity in the horizontal direction, while thermal conductivity in the vertical direction noticeably manifests itself for the long time values.

Temperature solution for FLS in a half-space of axial anisotropy at arbitrary dip angle

The problem for ground layered non-parallel to the surface can be solved by using exact correspondences between the isotropic and anisotropic solutions. It is easy to check that the transformation of coordinates

$$\{x, y, z\} \rightarrow \left\{ \frac{x-\varepsilon_{13}}{\sqrt{\Delta\Lambda_{33}}}, \quad \frac{y}{\sqrt{\Lambda_{22}}}, \quad \frac{z}{\sqrt{\Lambda_{33}}} \right\}, \qquad \Delta = \varepsilon_{11} - \varepsilon_{13}{}^{2} \tag{14}$$

reduces Equation 2 to the heat conduction equation in an isotropic medium. Formulation in the new coordinates $\vec{y} = (y_1, y_2, y_3)$ is given by:

$$C\frac{\partial T(\vec{y},t)}{\partial t} = \sum_{k=1}^{3} \frac{\partial^2 T(\vec{y},t)}{\partial y_k{}^2} + q_z{}'\delta(y_1 + y_3 \tan\vartheta)\delta(y_2)(He(y_3) - He(y_3 - \frac{H}{\Lambda_{33}})), \tag{15}$$
$$for \;\; t \ge 0, \; y_3 \ge 0$$

where

$$\tan\vartheta = \sqrt{\frac{\varepsilon_{13}{}^{2}}{\varepsilon_{11}}}, \qquad\qquad \cos\vartheta = \sqrt{\frac{\Delta}{\varepsilon_{11}}}$$

with exactly the same initial and boundary conditions :

$$T(\vec{y}, t=0) = T_0, \qquad\qquad T(y_1, y_2, y_3 = 0, t) = T_0 \tag{16}$$

Equations 15, 16 formulate the problem for an inclined line-source of heat strength $q_z' = q_z\sqrt{\frac{\Lambda_{33}}{\det\Lambda}}$ in the semi-infinite medium of the unit thermal conductivity, where ϑ denotes tilting angle of the line source with the y_3 axis (Cui et al., 2006) in the mapped space (not shown in Figure 2).

Since its solution is known (Cui et al., 2006), the solution in the physical space $\vec{x}$ can be obtained directly by back transformation from the $\vec{y}$ coordinate space as:

$$T - T_0 = \frac{q_z}{4\pi\sqrt{\lambda\Lambda_{11}}} \int_0^{H/\cos\vartheta} \{erfc\frac{r_+}{\sqrt{4\alpha_{33}t}} - erfc\frac{r_-}{\sqrt{4\alpha_{33}t}}\}dz', \quad \alpha_{33} = \frac{\Lambda_{33}}{C} \tag{17}$$

where

$$r_+^{\,2} = \sum_{i,j=1}^{3} (\vec{x}-\vec{x}')_i \Lambda_{ij}^{-1} (\vec{x}-\vec{x}')_j \Lambda_{33}\,,\ r_-^{\,2} = r_+^{\,2} + 4zz'\cos\vartheta \tag{18}$$

and $\vec{x}' = \{0, 0, z'\cos\vartheta\}$ is the vector along the line-source of the length $H/\cos\vartheta$.

Note also that the tilting angle ϑ in the transformed space can be expressed through the dip angle Θ as follows

$$\vartheta = \arctan\left((\frac{1}{\lambda_\perp} - \frac{1}{\lambda_\parallel}) \frac{\sin 2\Theta}{2\sqrt{2}} \right) \tag{19}$$

For the TRT analysis the above solution $T(\vec{x}, t)$ obtained for the finite-line source in anisotropic medium was approximated and compared with that from the isotropic FLS model in a wide time range starting from the intermediate times.

We proceed to derive expression that allows the calculation of an effective thermal conductivity as a function of bedding direction.

Effective thermal conductivity, measured by the vertical line-source method: the layers are non parallel to the ground surface. To compare the results of evaluating the thermal conductivity for isotropic medium with that developed here for anisotropic medium, the ground temperature in the vicinity of the mid-point of the finite depth BHE was calculated.

Series of $T(\vec{x}, t)$ in time about a mid-point depth (up to the exponentially small correction terms) can be written as:

$$T - T_0 = -\frac{q_z}{4\pi\sqrt{\Lambda_{11}\lambda_\parallel}} Ei(-\frac{\tilde{r}^2}{4\alpha_{33}t}) \approx \frac{q_z}{4\pi\sqrt{\Lambda_{11}\lambda_\parallel}} \{\ln\frac{4\alpha_{33}t}{\tilde{r}^2} - \gamma\} \tag{20}$$

$$T - T_0 \approx \frac{q_z}{4\pi\lambda_{eff}} \left(\ln t + \ln 4\alpha_{33}/\tilde{r}^2 - \gamma \right)$$

where

$$\tilde{r}^2 = \frac{x^2}{\varepsilon_{11}} + \frac{y^2}{\varepsilon_{22}} \quad (21)$$

and

$$\lambda_{eff} = \sqrt{\Lambda_{11}\lambda_{\parallel}} = \sqrt{\lambda_{\parallel}(\lambda_{\perp} + (\lambda_{\parallel} - \lambda_{\perp})cos^2\Theta)} \quad (22)$$

This expansion is valid for time values in the interval

$$\tilde{r} << \sqrt{\alpha_{33}\, t} << L / \cos\vartheta, \quad L = \min(z, H - z) \quad (23)$$

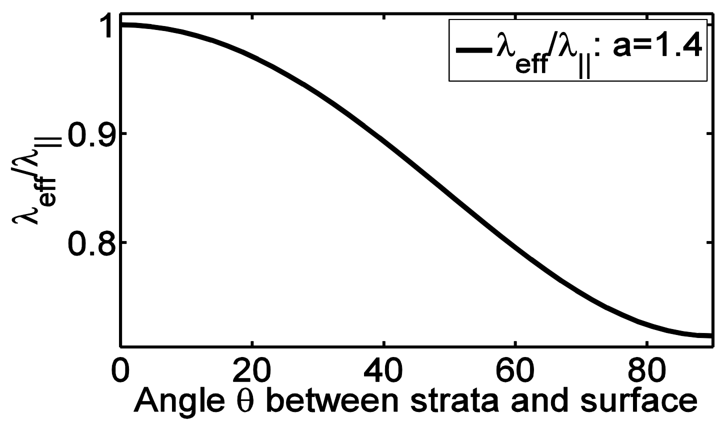

Figure 4. Effective scaled thermal conductivity versus the angle Θ between the ground surface and sedimentary planes, see Figure 2, for $a = 1.4$.

Equation 22 represents that effective thermal conductivity as a function of the thermal conductivity components and the structure of ground (dip angle). Effective thermal conductivity λ_{eff} depends on the angle Θ between the FLS and the axis of symmetry, as it is shown in Figure 4. This result for the λ_{eff} at the intermediate time values is consistent with the ILS prediction for the perpendicular thermal conductivity measured in ground bedding non-parallel to the surface (Grubbe et. al 1983). In geologic applications, the interpretation of Equation 22 enables determination of the thermal conductivity in a certain direction from the tensor components, when measuring anisotropy of rock samples in the laboratory at various angles (Pribnow and Sass, 1995).

Let us stress that the estimate of thermal conductivity is defined by the logarithmic derivative of the TRT data and should be identified with the effective conductivity from a model. So, using anisotropic model one should write

$$\lambda_{eff} = \sqrt{\Lambda_{11}\lambda_{\|}} = \frac{q_z}{4\pi}\frac{\ln(t_2/t_1)}{T(t_2)-T(t_1)} \tag{24}$$

while for isotropic model the right hand side is equal just to λ.

Figure 4 illustrates that the ground thermal conductivity from the TRT varies significantly from the maximum value $\lambda_{eff} = \lambda_{\|}$ for the horizontal stratification to the minimum value $\lambda_{eff} = \sqrt{\lambda_{\|}\lambda_{\perp}}$ as angle Θ tends from 0 to $\pi/2$.

The following shows how the anisotropy influences the steady-state temperature distribution around the line-heat source and long-term performance of underground installation due to vertical heat transfer effects with the surface and the deep earth.

Steady–state temperature field around LS penetrating layers at any angle to the ground surface. Retaining the first leading term in the expansion of the integral in Equation 17 we arrive at the following result for FLS, which can be used for the long-time values (i.e. for $\sqrt{\alpha_{33}t} >> H/\cos\vartheta$):

$$T-T_0 = \frac{q_z}{4\pi\lambda_{eff}}\{\ln[z-\eta_+ + \sqrt{\tilde{r}^2\cos^2\vartheta+(z-\eta_+)^2}\,]+ \\ \ln\frac{H+\eta_+ - z+\sqrt{\tilde{r}^2\cos^2\vartheta+(H+\eta_+-z)^2}}{H+\eta_- + z+\sqrt{\tilde{r}_-^2\cos^2\vartheta+(H+\eta_-+z)^2}}[z+\eta_- + \sqrt{\tilde{r}_-^2\cos^2\vartheta+(z+\eta_-)^2}\,]\} \tag{25}$$

Here

$$z+\eta_- = z\cos 2\vartheta + \frac{x}{2\varepsilon_{13}}(1-2\cos 2\vartheta), \qquad z-\eta_+ = z-\frac{x}{2\varepsilon_{13}}(1-2\cos 2\vartheta) \tag{26}$$

$$\tilde{r}_-^{\,2} = \tilde{r}_+^{\,2} + 4\sin^2\vartheta\, z(\frac{x}{\varepsilon_{13}} - z), \qquad \varepsilon_{13} = \frac{\lambda_{\|}-\lambda_{\perp}}{\lambda_{\perp}\cot^2\Theta+\lambda_{\|}\tan^2\Theta} \tag{27}$$

This result, describing the steady state temperature field of one borehole, embedded in multilayered semi-infinite medium at arbitrary angle of the bedding relative to the surface, agree with the result derived from the isotropic FLS model (Zeng et al. 2002). In the limiting case, when $\lambda_{\perp}$ tends to $\lambda_{\|}$, the proposed Equation 25 recovers the well known result for the steady state limit (SSL) of the temperature in the isotropic model.

One can see that the steady state temperature is proportional to the $\frac{q_z}{\lambda_{eff}}$ in addition to the BHE response function on anisotropy, dip angle and borehole depth. The effect of anisotropy manifests itself in the steady state conditions, whereas thermal conductivity values obtained by fitting the same TRT data to anisotropic solution, Equation (20), and isotropic solution, Equation (9), are identical. The steady state BHE response is strongly influenced by anisotropy.

Figure 3 shows that the steady state temperature profile along the borehole depth at the horizontal stratification exceeds that for the isotropic ground, as one might expect for $\lambda_\perp \leq \lambda_\parallel$.

The following section addresses the specific case of vertically stratified geologic regime, i.e. for $\Theta = \pi / 2$ and thus $\varepsilon_{13} = 0$.

Mean temperature approximations at vertical stratification

The effect of anisotropy on vertical temperature dependencies increases with increasing of the dip angle for typical situation: $\lambda_\perp \leq \lambda_\parallel$. Indeed, let there be strata parallel to the plane YZ depicted in Figure 2. The thermal conductivity of such vertically stratified ground takes smaller values in the X direction, $\lambda_1 = \lambda_\perp$, than in the in-plane direction, $\lambda_2 = \lambda_3 = \lambda_\parallel$ (Polubaronova-Kochina 1962). These define the principal components of the thermal tensor. The exact temperature solution is presented by Equation 17 at $\vartheta = 0$; it is not invariant under spatial rotation about the axis of the vertical BHE as at the horizontal stratification of the previous section. Notice that the effective thermal conductivity attains minimum value: $\lambda_{eff} = \sqrt{\lambda_\parallel \lambda_\perp}$. Furthermore, substituting $x \to x\sqrt{\lambda_\parallel / \lambda_\perp}$ and $r \to \tilde{r} = \sqrt{x^2 a^2 + y^2}$ into Equation 8b one can also get the solution for the temperature field integrated along the same borehole depth H . Indeed, under the conditions in Equation 23, the mean thermal response at $\vartheta = 0$ is represented by the approximation in the transient regime .

$$< T - T_0 >= \frac{q_z}{4\pi\sqrt{\lambda_\perp \lambda_\parallel}} \left(ln \frac{4\alpha t}{\tilde{r}^2} - \gamma - 3\left\{ \sqrt{\frac{4t}{\pi t_\parallel}} - \frac{r}{H} + \frac{r^2}{H^2}\sqrt{\frac{t_\parallel}{4\pi t}} \right\} \right), \quad \tilde{r} = \sqrt{x^2 a^2 + y^2} \quad for\ r_b \leq \tilde{r} << H \qquad (28)$$

Here $t_{\|} = H^2 / \alpha_{\|}$, $\alpha_{\|} = \lambda_{\|} / C$ and $\tilde{r}$ is defined by Equation 21 at $\varepsilon_{11} = 1/a^2$, $\varepsilon_{22} = 1$. Notice that isotherms of the mean thermal response function around the borehole, where $\tilde{r}$ is constant . Furthermore, due to $a > 1\,(\tilde{r} > r)$ the effective ratio of the BHE depth to $\tilde{r}$ is shortened for the vertical stratification with respect to H / r for the isotropic medium and the edge corrections become more pronounced. The steady state limit from Equation 8b or 21 for $t >> max(H^2, \tilde{r}^2) / \alpha_{\|}$ can be wriiten in the following form.

$$< T - T_0 >= \frac{q_z}{2\pi\sqrt{\lambda_{\perp}\lambda_{\|}}} \left\{ \hat{g}_S(\frac{H}{\tilde{r}}) - \frac{(t_{\|} / t)^{3/2}}{24\sqrt{\pi}} \left[1 - 3t_{\|} \frac{1 + \tilde{r}^2 / H^2}{20t} \right] \right\}, \quad t_{\|} = \frac{H^2}{\alpha_{\|}}, \quad t >> \frac{max(H^2, \tilde{r}^2)}{\alpha_{\|}} \qquad (29)$$

There are two characteristic times $\frac{\tilde{r}^2}{\alpha_{\|}}$ and $\frac{H^2}{\alpha_{\|}} = \frac{H^2 C}{\lambda_{\|}}$ for anisotropic diffusion in the radial and axial directions, respectively.

Exact solution for the mean steady-state temperature in the dimensionless form of $\hat{g}_S(\frac{H}{\tilde{r}})$ is valid at any distance from borehole center. Using the expansion of the $\hat{g}_S(\frac{H}{\tilde{r}})$ for large values of the $\frac{H}{\tilde{r}}$ we arrive at the following result for anisotropy correction to the steady state temperature from isotropic and anisotropic models

$$\left\langle T(x,y)|_{a=1} - T(x,y)|_{a\neq 1} \right\rangle = \frac{q_z}{2\pi\lambda_{eff}} \left[\hat{g}_S(\frac{H}{r}) - \hat{g}_S(\frac{H}{\tilde{r}}) \right] = \frac{q_z}{2\pi\lambda_{eff}} \left[-2\ln\frac{r}{\tilde{r}} + \frac{3(r - \tilde{r})}{H} + O(\frac{r}{H}) \right] \; for \quad \frac{H}{\tilde{r}} \to \infty \; (VS)$$

This correction is derived for $\lambda_{eff} = \lambda = \sqrt{\lambda_{\perp}\lambda_{\|}}$ and valid in the vicinity of borehole. Equations HS and VS provide anisotropy corrections for the mean borehole temperature in explicit form in the limiting cases of horizontal and vertical stratifications, respectively

Figure 5 presents the time-dependence of the exact mid-point *g*-function and average *g*-functions for isotropic ground and average *g*-functions for the vertical stratification at $\alpha_{\|} = \alpha$. Futhermore, as shown in Figure 5, point x=0.1m; y=0 has lower value of mean thermal response function than point x=0; y=0.1m in the uniform in – plane direction y. Notice that, although the physical distance is the same as derived from the line heat source, the ratio $\tilde{r}/H$ is different at these two points causing splitting mean $\hat{g}$ function curve calculated for an isotropic ground in two branches shown in Figure 5.

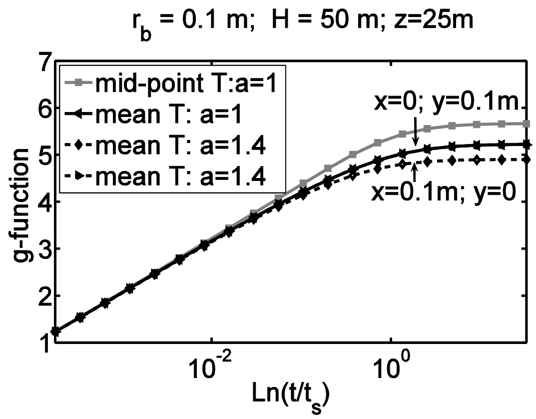

Figure 5. Comparison between the borehole response functions from two models of medium with vertical strata. Mid-point (z=H/2) (gray line) and mean g-functions from the isotropic model ($\lambda_{\|} = \lambda_{\perp} = \lambda$) and mean $\hat{g}$ -functions in the X and Y directions from the anisotropic model ($\lambda_{\|} = a^2 \lambda_{\perp}$, $\Theta = \pi/2$) versus the natural logarithm of time. Exact solutions calculated for constant heat injection are shown in the range: $5r_b^2/\alpha_{\|} < t < a^2 H^2/\alpha_{\|}$, $\alpha = \alpha_{\|}$.

This behavior attributed to the fact that the thermal conductivity parallel to the layers is higher than that perpendicular to the bedding plane, suggests that row of boreholes should be aligned along the direction X to enhance conditions of the heat exchange with a multilayered ground as comparison shows also in Figure 6.

Many sedimentary and metamorphic laminated rocks are strongly anisotropic (Davis et al., 2007): the thermal conductivity in parallel to bedding planes of these rocks is 2-3 times higher than that perpendicular to bedding (Deming, 1994; Popov et al., 1995). This proposal on layout of the borehole raw is not referred to the anisotropy values less than unity also reported (Davis et al., 2007).

Although anisotropy value: a=1.4 in the given examples is common, our solution is valid for any thermal conductivity anisotropy.

(a) (b)

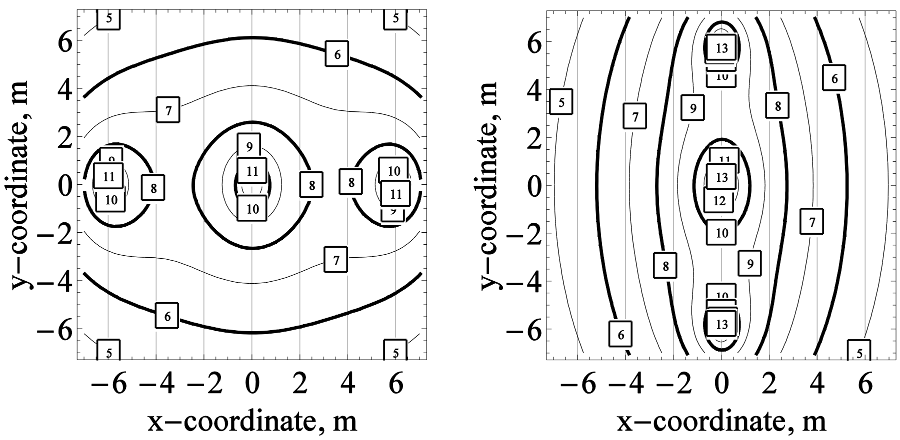

Figure 6. Comparison between isotherms curves of steady-state $\hat{g}$ -functions from two 3 borehole configurations (with an inter-borehole distance of 3 m and H=50 m) at the $\Theta = \pi/2$: (a) along the direction X; (b) along the direction Y, shown in Figure 2.

CONCLUDING REMARKS, SUMMARY AND DISCUSSION

Results have been presented of a study of the thermal response from multilayered ground modeled as an anisotropic medium to constant heat pulse from the finite line source. This study discusses anisotropic dependence of both effective thermal conductivity measured by TRT and the steady state temperature field around vertical FLS in the arbitrary oriented strata with respect to the surface of the semi-infinite medium.

What is actually measured for the intermediate time values of the TRTs is the effective thermal conductivity of the soil/rock formation in the direction perpendicular to the borehole axis. We have provided effective conductivity as a function of the inclination angle, which should prove to be useful for the geothermal applications. In addition, we have shown that the dip angle and the anisotropy factor influence the steady-state temperature field of the designed installation. Therefore, it may be a discrepancy between the real temperature spatial distribution around vertical or horizontal GCHP systems in the steady state conditions and its prediction of the isotropic model with thermal conductivity value obtained from the short time TRT, but without using the data of anisotropy and dip of the bedding.

The exact solution accounting for anisotropy, its asymptotic behavior and the steady state expression for the temperature obtained here for any dip angle between the surface and the bedding should prove to be useful for designing multiple borehole configurations in stratified medium.

Analytical formulae have been obtained for the asymptotic behavior of the average temperature in horizontally and vertically stratified ground for intermediate- and long- time scales. The suggested corrections for anisotropic effects (Equations HS and VS) may give errors, when estimating the steady state average temperature by the isotropic model.

In these limiting cases the proposed response functions in Equations 13, 29 can be easily applied to estimate maximum and minimum of the mean steady state temperature field of an arbitrary borehole configuration using the superposition principle.

Due to the fact that the thermal conductivity of ground is higher along the layers the average thermal response method provides the lowest estimation for the dimensionless response function approaching steady-state limit; that is reached in the direction across the layers at the vertical stratification. This conclusion is relevant when choosing a proper configuration to minimize temperature between vertical sources of heat from data about the geometrical disposition of the layers and the surface. To this objective the proposal consists in disposing a row of vertical heat sources normally to the lines of strata intersection with the surface rather than along them at any values of the dip angle. This recommendation regards also selection of orientation for horizontal GCHP systems. The analytical formulae for the temperature allow flexibility in the estimation of the temperatures within and around a repository of nuclear waste in anisotropic rock.

NOMENCLATURE

$a = \sqrt{\dfrac{\lambda_{\parallel}}{\lambda_{\perp}}}$ = thermal conductivity anisotropy factor

C = volumetric heat capacity of ground, $\mathrm{Jm^{-3}K^{-1}}$

Ei = exponential integral

$\hat{g} = \dfrac{2\pi\lambda_{eff}}{q_z} < T(r,\ t) - T_0 >$ = generalized thermal response function for $r \geq r_b$

$Fo_{\parallel} = \frac{t\alpha_{\parallel}}{r^2}$	= in-plane Fourier number
H	= depth of the borehole heat exchanger (BHE), m
r	= radial coordinate, m
r_b	= radius of the BHE, m
$\vec{r}$	= coordinate vector, m
q_z	= heat flow per unit length, Wm^{-1}
$\vec{Q}$	= vector of heat density per unit area, Wm^{-2}
$t_r = r_b{}^2 / \alpha$	= short time scale for the BHE, s
$t_s = \frac{H^2}{\alpha}$	= steady-state time scale, s
$t_z = H^2/\alpha$	= isotropic time scale for the BHe, s
$t_{\parallel} = H^2 / \alpha_{\parallel}$	= in-plane time scale for the BHE, s
T	= temperature of ground (K or °C)
T_0	= undisturbed ground temperature (K or °C)
z	= vertical axial coordinate, m

Greek letters

$\alpha = \lambda / C$	= isotropic thermal diffusivity, m^2/s
$\alpha_{\parallel} = \lambda_{\parallel} / C$	= in-plane thermal diffusivity, m^2/s
δ	= delta function
$\Delta = \varepsilon_{11} - \varepsilon_{13}{}^2$	= dimensionless parameter
$\varepsilon = \Lambda / \Lambda_{33}$	= dimensionless thermal conductivity tensor
γ	= Euler's constant
λ_{eff}	=Equation 20, effective thermal conductivity, W $(Km)^{-1}$
$\lambda_{\parallel}$	= in-plane thermal conductivity (parallel to bedding plane), W $(Km)^{-1}$
$\lambda_{\perp}$	= normal thermal conductivity (normal to bedding plane), W $(Km)^{-1}$
Λ	= three-dimensional thermal conductivity tensor, W $(Km)^{-1}$
$He(z)$	= unit step function
Θ	= angle between the surface and strata, °

$\vartheta = \arccos\sqrt{\frac{\varepsilon_{13}}{\Delta}}$ = dimensionless parameter

Subscripts

$\|$ = direction parallel to bedding

s = steady-state

Superscripts

$< ... > (= \frac{1}{H}\int_0^H ... \, dz)$ = integral mean

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Figure captions

Figure 1. In-situ TRT schematic and formations layers.

Figure 2. Direction of X', Y, Z' principal axes of the thermal conductivity tensor. The XY plane represents the ground surface at the dip angle Θ from the X'Y plane.

Figure 3. Comparison between thermal response g-functions at $r = r_b$ =0.1m around the borehole penetrating horizontal strata $(\Theta = 0)$ from two models: isotropic model ($\lambda_{\parallel} = \lambda_{\perp}$) and anisotropic model ($\lambda_{\parallel} = a^2 \lambda_{\perp}$) of the ground. (a) Profiles of the g-function versus the natural logarithm of time t / t_s and the dimensionless coordinate z / H along the borehole; (b) Mid-point (z=H/2) (gray line) and mean g-functions from the isotropic model and mean g-function from the anisotropic model versus the natural logarithm of time. Exact solutions calculated for constant heat injection are shown in the range: $5 r_b^2 / \alpha_{\parallel} < t < H^2 a^2 / \alpha_{\parallel}$. The time is scaled by $t_s = H^2 / (9\alpha)$ (Eskilson, 1987).

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Figure 6. Comparison between isotherms curves of generalized steady-state $\hat{g}$ -functions from two 3 borehole configurations (with an inter-borehole distance of 3 m and H=50 m) at the $\Theta = \pi / 2$: (a) along the direction X; (b) along the direction Y, shown in Figure 2.