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Exact skeletonization method in water distribution systems for hydraulic and quality models

F. Javier Martínez-Solano^{a*}, Pedro L. Iglesias-Rey^a, Daniel Mora-Meliá^{b,*} and Vicente S. Fuertes-Miquel^a

^aUniversidad Politécnica de Valencia, Dpto. Ingeniería Hidráulica y Medio Ambiente, Valencia, 46022, Spain.

^bUniversidad de Talca, Dpto Ingeniería y Gestión de la Construcción, Curicó, 3340000, Chile.

Abstract

A mathematical model is a powerful tool for simulating different scenarios that occur in a water distribution network without making physical experimentation. According to the objectives, a model can be classified into three categories: layout, design and operation. Furthermore, the level of detail is strongly related to the objective that the model tries to achieve. However, bigger amount of information does not mean better accuracy. For example, a fully detailed mathematical model of the network would lead to know every single connection. Usually, this information is so difficult to compile as imprecise. Therefore, one of the most important stages in elaborating a model consists of the model simplification, also known as skeletonization.

During the works made for model skeletonization some assumptions are made. Most of the times, these assumptions may produce significant errors. Among the different techniques for network skeletonization, series pipe removal is one of the most used. It consists of replacing several adjacent pipes with a single one which must present the same head losses than the pipes being substituted. When there are no intermediate consumptions the problem has been effectively solved. The problem arises when a demand appears in one of the pipes being removed. It has been demonstrated that methods which assume constant roughness coefficients (either Hazen-Williams or Darcy equations) produce errors in the head losses. These errors may be even higher if travel time is included as a restriction, for example in water quality models.

This paper reviews the most common techniques for serial pipes association. The error will be evaluated in both hydraulic and quality models. Finally, a method for exact substitution of serial pipes with intermediate demands is proposed. This method imposes two restrictions (head losses and travel time) and gives exact results when the flow direction is known. The method is tested with an example that highlights the results.

* Corresponding author

E-mail address: damora@utalca.cl

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1. Introduction

A water distribution system (WDS) consists of a series of connected elements that transport water from sources to customers. A mathematical model of the WDS is the starting point in the hydraulic calculation. This model allows the simulation of different scenarios in the distribution network without making physical experimentation. As a result of these simulations, some conclusions may be extracted that will be used in the planning and management of the network. The success of such decisions depends largely on the accuracy of the model.

A model of a WDS may be classified according to its size or its objectives. The level of detail is strongly related to the objective that the model tries to achieve [1]. According to the objectives, a model can be classified into three categories [2]: layout, design and operation. A very detailed model allows high hydraulic and quality accuracy, but implies a high computational effort and often complicates the understanding of the network [3]. A detailed model will need time to be built and calibrated, and the analysis of the results will also be time-consuming [4]. On the other hand, simplified models allow an easy understanding of the WDS operation and do not need so much computation time. However, the accuracy may be lower.

Nevertheless, bigger amount of information doesn't mean better accuracy. For example, include the entire network in the mathematical model lead to know every single connection, whose information is so difficult to compile as imprecise. Randomness in the water consumption of housing adds a large uncertainty in the load allocation process [1,5] state the need for model simplification when used for design or operation, since for these applications several simulations are needed and time consumption is important. That is why it is necessary to perform a certain skeletonization of the model.

There are several skeletonization techniques. Anderson and Al-Jamal [6] group them into two groups: element-by-element approaches and methods based on fitting parameters. These authors presented a methodology corresponding to the second group. Brandon [7] proposed heuristic methods to simplify a network in successive stages. First, intermediate demands in a pipe are allocated at the end of the pipe. Second, branches are eliminated and their demands are allocated in the connecting nodes. Last, adjacent nodes are joined together.

Among the simplification techniques commonly used in water networks are the following [8,9]:

- Data scrubbing. It is the removal of pipes that do not meet minimum requirements in length or diameter.
- Branch trimming. This is removing the branches with small diameters. The flow demand at the ends of the branch is accumulated at the point where the branch has been removed.
- Node aggregation. Nodes hydraulically closed whose connecting pipes are so short they barely have a change in the pressure head are joined removing the connecting pipes and adding their demands.
- Series and parallel pipe removal. Several adjacent pipes, either in series or in parallel, are replaced by a single pipe with equivalent behavior.

Of these techniques, the first three assume a certain loss of information. There are pipes or nodes that disappear from the model. However, the fourth is the replacement pipe and it is expected equivalent behavior.

In this paper, a simplification method for series pipe removal is proposed. The article reviews the most common techniques for association in series of pipes. The error will be evaluated in both hydraulic and quality models. Finally, an exact, error-free technique is proposed. The method is tested with a simple example that highlights the results.

2. Existing Methods for series pipes simplification

The simplest and most widely used method for series pipes simplification consists of substituting a series of pipes with similar characteristics with one pipe with the same characteristics and length equal to the sum of the lengths. In

fact, it is performed almost automatically when preparing a model. Note that a line in a model uses to be longer than the length of the sections commercially available. In these cases, the simplification involves taking a single pipe with the same characteristics as those that will be simplified, and a length equal to the sum of both.

If there are intermediate consumptions, there are several alternatives. The most common is to make an evenly distribution between end nodes [9]. This option, as it will be seen later, has a small error in the calculation of the pressure drops and travel times.

There are several methods for the association of pipes. Martinez-Solano et al. [10] use a method for simplification water distribution networks with both intermediate and evenly distributed consumptions. In Figure 1 an example of this method is presented. Isolated consumptions connected to node E (Q_1 in this figure) are simplified and assigned to the E. Then, this flow Q_1 is reallocated between nodes A and C. Moreover, distributed consumptions (q_2 and q_3 in Fig. 1) are also reallocated between their end nodes (A, B, C and D in Fig. 1).

Christiansen [11] introduced the use of a distribution coefficient to represent compactly a uniformly distributed consumption within irrigation pipes. This coefficient is easily extrapolated to the distribution of consumptions evenly distributed in water distribution networks. It is very useful for automatic simplification algorithms such as the one presented by Walski [9].

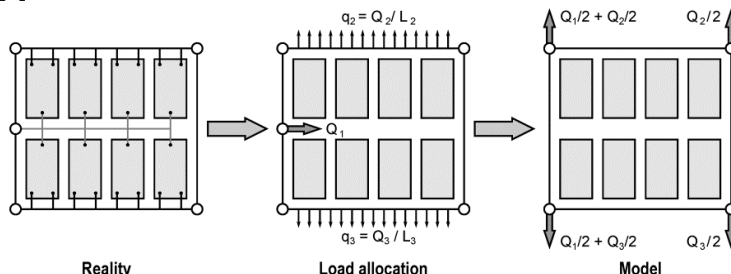


Fig. 1. Treatment of intermediate consumptions and evenly distributed.

However, the most common and reported simplification method in the literature is the simplification of pipes with heterogeneous characteristics without intermediate consumptions. Then it is necessary to determine an equivalent diameter for the new line. This option is rarely adopted, since it reaches noncommercial diameters and therefore unrealistic. Stated in its general form, the method consists of replacing several pipes arranged in series without intermediate consumption by a single pipe whose hydraulic behavior is equivalent to the series replaced. When n pipes with heterogeneous characteristics are grouped, the equivalent pipe should cause a pressure drop equal to the associated piping. Usually, this equivalence is done in hydraulic terms:

$$\Delta h_{eq} = \sum_{i=1}^n \Delta h_i \tag{1}$$

where Δh_{eq} represents the new pipe head loss and Δh_i the head loss for the pipe i . The relation between the pressure drop Δh and the flow Q is given by the pipe resistance R and an exponent b . It can be written as:

$$\Delta h = R \cdot Q^b \tag{2}$$

The resistance R depends on the pipe diameter D , the length L and a parameter that represents the roughness of the pipe (friction factor f for Darcy-Weisbach equation or coefficient C for Hazen-Williams equation). This resistance takes different expressions depending on the pressure drop equation. If using the Darcy-Weisbach equation, friction factor also depends of the Reynolds number of the flow through the pipe. On the other hand, the coefficient b is equal to 2 for Darcy-Weisbach equation and 1.85 for Hazen-Williams equation.

Combining equations (1) and (2):

$$\Delta h_{eq} = \sum_{i=1}^n \Delta h_i \Rightarrow R_{eq} Q_{eq}^b = \sum_{i=1}^n R_i Q_i^b \quad (3)$$

It is usual to accept that the roughness of the pipe (C or f depending on the equation) is almost constant in the above equation. In the case of absence of intermediate flow ($Q_i = Q_{eq}$), the equivalent pipe, using Darcy-Weisbach equation, will have a length and a diameter given by the following expressions.

$$L_{eq} = \sum_{i=1}^n L_i; \quad D_{eq} = \frac{(L_{eq})^{0.2}}{\left(\sum_{i=1}^n \frac{L_i}{D_i^5}\right)^{0.2}} \quad (4)$$

Similar equations can be obtained for Hazen-Williams head loss equation. In this sense, other authors [8,9,12] defend the use of the Hazen-Williams equation versus Darcy equation due to its ease of use and integration on calculation algorithms. This is because in the Hazen-Williams equation, the losses depend on a single coefficient (C), assumed constant, instead of the friction factor (f), which varies with the Reynolds number of the flow in the pipe.

Thus, for Hazen-Williams equation no additional assumptions are required. In this case, simplification is presented with several alternatives. Cesario [8] suggest two alternatives when the Hazen-William coefficient (C) is kept:

- Adding the lengths and determine the equivalent diameter using equations (4).
- Taking to the equivalent pipe the diameter of some of the simplified (in order to work with commercial diameters) and calculate an equivalent length.

Walski et al. [9] choose a combination of the above, fixing the equivalent diameter and length, and calculating the Hazen-Williams coefficient. However, Bombardelli and Garcia [13] point that the authors of this equation [14] limit their scope and warn about the dependency between the Hazen-Williams coefficient (C) and the pipe diameter.

In any case, whatever the choice for simplification it requires the definition of an additional criterion, such as the homogeneity of some or all the characteristics of the pipes. Furthermore, it is possible to include as a criterion of simplification a number of users or a certain length of pipe. For example, it is possible to limit the number of subscribers simplified in order to not distort too much the model, or set a minimum length of a pipe. In fact, during the calculation of quality models, the travel time is ignored in pipes shorter of a certain length [12].

The previous simplification methods have some errors due to they assume invariable friction factor or coefficient C . When using Darcy-Weisbach equation, this hypothesis aims to simplify the calculations. In the case of Hazen-William equation, the error comes from the use of the Hazen-Williams equation beyond its scope. Liou [15] estimated this error on $\pm 40\%$ for Reynolds numbers bigger than 10^6 . For this reason, the paper will focus on the use of the Darcy-Weisbach equation.

Next, a way to calculate the equivalent diameter of pipes arranged in series with the full hydraulic equivalence simplification will be analyzed. Mathematically, the approach is simple. The hypothesis of no variation of the friction factor will be ignored. It remains in any case as an additional condition that the equivalent length must be the sum of the lengths of the pipes replaced. Then, the development is done for two pipes A and B (see Fig. 2).

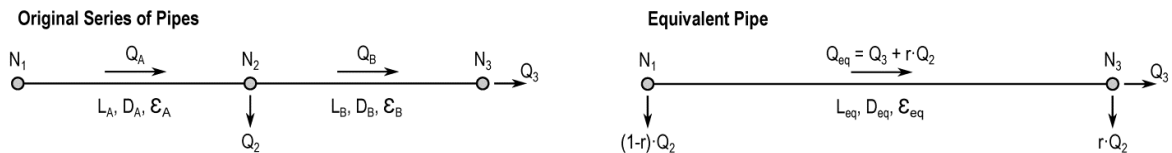


Fig. 2. Definition of a simple problem for exact equivalence.

$$\Delta h_{eq} = \Delta h_A + \Delta h_B = \frac{8f_A L_A}{\pi^2 g D_A^5} Q_A^2 + \frac{8f_B L_B}{\pi^2 g D_B^5} Q_B^2 \quad (5)$$

In this case, as in the above, it should be known how much is the total head loss, Δh_{eq} . The problem is reduced to calculate the diameter it should have a pipe with a flow Q_{eq} , which has a length L_{eq} and should cause a pressure drop Δh_{eq} . This is a basic Fluid Mechanics problem, which can be further simplified if we take the explicit Swamee and Jain approach. The equivalent friction factor considering a constant roughness height ϵ is:

$$f_{eq} = \frac{0,25}{\left[\log \left(\frac{\epsilon_r}{3,7} + \frac{5,74}{Re^{0,9}} \right) \right]^2} \quad (6)$$

where Re is the Reynolds number. The equivalent diameter, as a function of the new friction factor will be:

$$\Delta h_{eq} = \frac{8 f_{eq} L_{eq}}{\pi^2 g D_{eq}^5} Q_{eq}^2 \Rightarrow D_{eq} = \sqrt[5]{\frac{8 f_{eq} L_{eq}}{\pi^2 g \Delta h_{eq}} Q_{eq}^2} \quad (7)$$

The resolution process is iterative, but it has a fast convergence according to the diagram of Fig. 3. Note that in this calculation the distribution of intermediate flows is left as a degree of freedom. So, it can be obtained as many equivalent diameters as distribution criteria.

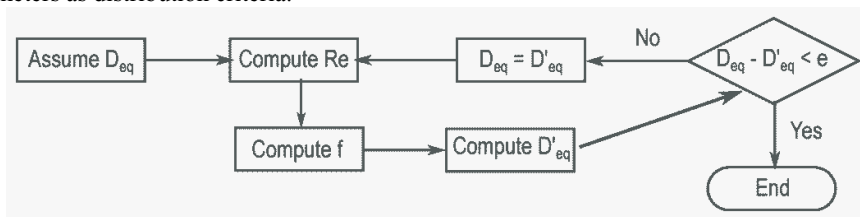


Fig. 3. Iterative process for calculating the equivalent diameter.

3. Evaluation of error in a case study.

In order to compare both methods (considering no variation of f and correcting f) a very simple case study was used. It consists of two series pipes with an intermediate demand. The layout of such an example is shown in Figure 4.

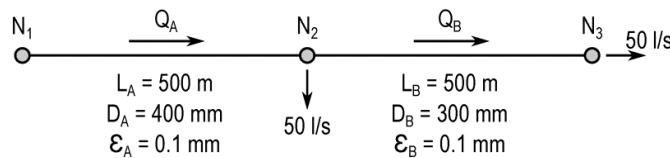


Fig. 4. Example of simplification.

Next, the error with the simplifications described above is listed. The intermediate demand in node N_2 is distributed with a distribution coefficient r . First, the calculation with no simplification was done, in order to know the exact values of travel time and head losses. Results are shown in Table 1. Then, the simplifications obtained by applying the equations (3) and (4), and the method described for equations (5) to (7) were performed.

Table 1. Hydraulic results without simplifying network.

	Q_{lin} (l/s)	v (m/s)	t_r (s)	Re	f	Δh (m)
Pipe A	100	0.7958	628.3	$2.89 \cdot 10^5$	0.01676	0.676
Pipe B	50	0.7074	706.9	$1.93 \cdot 10^5$	0.01807	0.768
Total.	-	-	1335.2	-	-	1.444

Next, the simplification admitting invariance of the friction factor is evaluated. Table 2 and Fig. 5 show the evolution of the error for both travel time and head loss as a function of the distribution coefficient r . As can be seen, for each value of r a different value of the equivalent diameter is got. Besides, two types of errors were observed:

- Despite it was a restriction, errors were observed when calculating the pressure drop when admitting no variation of f .
- The errors in the travel time, which mainly affect quality models, were observed in both cases. No cautions were taken regarding the travel time in the development of the method of simplification.

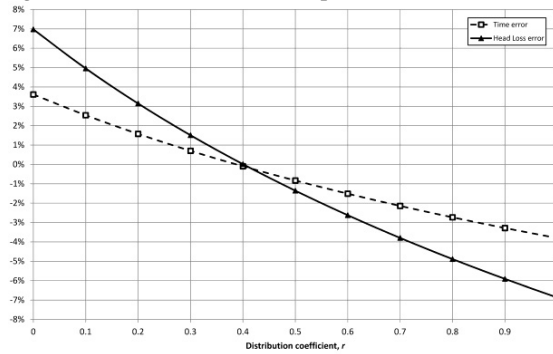


Fig. 5. Errors on travel time and head loss as a function of the distribution coefficient r .

Table 2. Hydraulic results of simplification admitting invariable f

r	Q_{eq} (l/s)	D_{eq} (mm)	t_r (s)	Error (%)	f	Δh_{eq} (mca)	Error (%)
0.0	50	301.55	1428.3	6.98	0.01807	1.497	3.65
0.1	55	313.27	1401.4	4.96	0.01788	1.481	2.56
0.2	60	324.36	1377.2	3.15	0.01771	1.467	1.59
0.3	65	334.91	1355.3	1.51	0.01755	1.454	0.70
0.4	70	344.99	1335.4	0.02	0.01741	1.443	-0.10
0.5	75	354.64	1317.1	-1.36	0.01728	1.432	-0.84
0.6	80	363.92	1300.2	-2.62	0.01716	1.422	-1.53
0.7	85	372.85	1284.5	-3.79	0.01705	1.413	-2.16
0.8	90	381.47	1269.9	-4.89	0.01695	1.404	-2.76
0.9	95	389.81	1256.3	-5.91	0.01685	1.396	-3.32
1.0	100	397.89	1243.4	-6.87	0.01676	1.389	-3.84

As noted, the assumption of invariable friction factor can cause errors in the head loss of up to 4%. For the most extended case of flow distribution (50% at each end, that is, $r = 0.5$) the error would be around 1%. If this hypothesis is corrected and an equivalent friction factor is calculated, it is possible to reach the following results according to equations (5) to (7):

Table 3. Hydraulic results of simplification with corrected f .

r	Q_{eq} (l/s)	D_{eq} (mm)	t_r (s)	Error (%)	f	Δh_{eq} (mca)	Error (%)
0.0	50	303.72	1449.0	8.52	0.01807	1.444	0.0
0.1	55	314.86	1415.6	6.03	0.01788	1.444	0.0
0.2	60	325.39	1385.9	3.80	0.01771	1.444	0.0
0.3	65	335.39	1359.2	1.80	0.01755	1.444	0.0
0.4	70	344.92	1334.9	-0.02	0.01741	1.444	0.0
0.5	75	354.05	1312.6	-1.69	0.01728	1.444	0.0

0.6	80	362.80	1292.2	-3.22	0.01716	1.444	0.0
0.7	85	371.22	1273.3	-4.63	0.01705	1.444	0.0
0.8	90	379.35	1255.8	-5.94	0.01695	1.404	0.0
0.9	95	387.20	1239.4	-7.17	0.01685	1.396	0.0
1.0	100	394.79	1224.1	-8.32	0.0167	1.389	0.0

As it is expected, the error in the calculation of the head loss is zero in this case. However, it is observed that the error in the travel time is even greater than in the previous case.

4. Method for obtaining equivalent pipes and quality hydraulic models

An alternative method for simplification is detailed below. This method adds another constraint to the constraint of equivalence in head losses of pipes: equal travel time. This new restriction involves leaving another degree of freedom. This is the distribution coefficient *r*. The constraints are, for simplifying two pipes:

- Equal head losses, according to equation (5)
- Equal travel time, according to:

$$t_{r,eq} = t_{r,A} + t_{r,B} = \frac{L_A}{v_A} + \frac{L_B}{v_B} = \frac{\pi D_A^2 L_A}{4Q_A} + \frac{\pi D_B^2 L_B}{4Q_B} \tag{9}$$

If we maintain such constraint that the equivalent length is the sum of the simplified pipes lengths, the equation (9) means to set the flow velocity in the new pipe:

$$t_{r,eq} = \frac{L_{eq}}{v_{eq}} \Rightarrow v_{eq} = \frac{L_{eq}}{t_{r,eq}} \tag{10}$$

Then it is possible to undertake an iterative process similar to that described in Fig. 3, with the difference that now the velocity is known, and the flow unknown. From an estimation of the equivalent diameter, *D_{eq}*, it is possible to obtain the Reynolds number. Then, with this value of *Re* and the diameter assumed, the friction factor is calculated (using the Colebrook-White equation or the Swamee-Jain equation if it is desired to avoid excessive iterations). Finally, Darcy-Weisbach equation is used to calculate the new equivalent diameter:

$$\Delta h_{eq} = f_{eq} \frac{L_{eq}}{D_{eq}} \frac{v_{eq}^2}{2g} \Rightarrow D_{eq} = f_{eq} \frac{L_{eq}}{\Delta h_{eq}} \frac{v_{eq}^2}{2g} \tag{11}$$

Once a value for the equivalent diameter has been reached, the flow through the pipe may be calculated and, hence, the distribution coefficient can be deducted:

$$Q_{eq} = \frac{\pi}{4} D_{eq}^2 v_{eq} \Rightarrow r = \frac{Q_{eq} - Q_3}{Q_2} \tag{12}$$

For the example shown in Fig.4, the results are presented in Table 4. As can be seen in the example, there is no error in any of the two parts considered: head losses and travel time. This will result in greater accuracy when a hydraulic and a quality model are combined.

Table 4. Hydraulic results for the simplification assuming invariability of *f*.

<i>r</i>	<i>Q_{eq}</i> (l/s)	<i>D_{eq}</i> (mm)	<i>t_r</i> (s)	Error (%)	<i>f</i>	<i>Δh_{eq}</i> (mca)	Error (%)
0.3986	69.93	344.79	1335.2	0.00	0.01741	1.444	0.00

5. Conclusions

In this paper the error made when pipes are simplified by series association has been shown. It has been found that methods that assume constant roughness coefficients (either in the Hazen-Williams or the Darcy-Weisbach equations) lead to errors that can become significant. In the same manner, an exact hydraulic equivalence using Darcy-Weisbach equation using the explicit approach of Swamee and Jain (1976) for the friction factor has been proposed. However, it has also shown that if the hydraulic model is used as a starting point for analysis of water quality, it has starting errors regarding residence times of water in the network of up to 8%.

Finally it has been presented a method to simplify pipe series with two restrictions (head loss and travel time), which gives accurate results if the flow direction of the flow is known. Method has been presented for the association of two pipes, but is easily transferable to the association of any number of pipes, without intermediate branches. The results are an equivalent diameter and load distribution for intermediate flows.

This method is especially suitable for performing automatic simplification algorithms if demand allocation is known. For a correct implementation of this method it is necessary to have demand and pipe data. The work has been completed with the application to a simple numerical example. With this work, the residence time quality model will work without error after a simplification stage. As further research, the effect of the bulk and wall reaction coefficients in a reactive component quality model should be studied.

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