A dynamic mathematical model of a shell-and-tube evaporator. Validation with pure and blend refrigerants

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SUMMARY

This work presents a mathematical model of a shell-and-tube evaporator based on mass continuity, energy conservation and heat transfer physical fundamentals. The model is formulated as a control volume combination that represents the different refrigerant states along the evaporator. Since the model is based on refrigerant and secondary fluid states prediction, it can be easily adapted for modelling any type of evaporator. The strategy of working with physical fundamentals allows the steady- and dynamic-state analysis of any of its performance variables. The paper presents a steady-state validation made with two pure refrigerants (HCFC-22, HFC-134a) and with a zeotropic blend (HFC-407C), and a dynamic validation with transient experimental tests using HCFC-22. The model prediction error is lower than 5% and it is well in accordance with actual dynamic behaviour. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: evaporator; dynamic model; shell-and-tube; refrigeration

1. INTRODUCTION

Dynamic modelling of refrigeration and air conditioning systems came about in 1978 from the work by Wedekind et al. (1978) on two-phase flow dynamic performance in heat exchangers. From this, and following the evolution of computers and the energy problem in a parallel fashion, several dynamic modelling works started to emerge. The first complete dynamic model of a vapour compression system was presented in 1979 by Dhar and Soedel (1979). This model is included in a group known as phase-dependent moving formulation. There are several models

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included in that group, Chi and Didion (1982), Sami et al. (1987), Deng (2000). Five years later in 1984, MacArthur (1984) presented a heat pump dynamic model. That model is considered to be the first work in the modelling group known as phase-independent difference methods, since the heat exchangers were represented by means of finite elements. This technique, despite not being the most computational efficient method, provides valuable information for design purposes. Using the finite elements technique, works about single refrigeration equipment elements, Renno and Aprea (2002), and works that deal with complete refrigeration systems, Rossi and Braun (1999), Braun and Bendapudi (2002), can be found.

The general objective found in the review of dynamic modelling is to obtain information about the internal variables and its behaviour in transient operation, since refrigeration systems are working in unsteady-state during most of the operation time. The information from the dynamic formulation is mainly used for improving components, MacArthur (1984); for developing control strategies, Nyers and Stoyan (1982), He et al. (1998); and for formulating fault detection and diagnosis methodologies, Clarkea et al. (2002), Wang and Xiao (2004).

This work presents a mathematical model of a shell-and-tube evaporator located in a vapour compression refrigeration system. The model is formulated from mass continuity, energy conservation and heat transfer classical equations.

The model takes the ideas presented by Deng (2000) and Finn et al. (2003) and develops a general model that could be used for modelling any type of evaporator working with any type of refrigerant. To improve the dynamic agreement with the actual performance, the model includes shell dynamics and the environmental temperature effect. In addition, the integration routine time step used is shorter than the one used in the works by Deng and Finn. This formulation allows for the study of the main variables evolution during the evaporating process, such as the heat transfer coefficients.

2. MATHEMATICAL MODEL OF THE EVAPORATOR

The evaporator considered in this work is a shell-and-tube evaporator, in which the refrigerant flows inside the tubes and the secondary coolant flows on the shell side. It is modelled using mass continuity, energy conservation and heat transfer classical equations.

The mathematical formulation calculates the control volume temporal evolutions that symbolize the different refrigerant states along the evaporator, Figure 1. It considers average properties in all the time steps in each control volume. The considered control volumes are as follows: one for the superheated refrigerant zone, another for the evaporation, one that is divided into two regions, one for the saturated liquid and another for the saturated vapour.

The main dynamics represented by the model are the refrigerant mass and energy storages in the evaporator and the tubes and the shell thermal capacities.

The mass flow rate, pressure and enthalpy of the inlet refrigerant and the mass flow rate and temperature of the inlet secondary coolant are taken as model inputs, also the environmental temperature is taken into account. The mass flow rate and enthalpy of the outlet refrigerant and the secondary coolant outlet temperature are taken as model outputs.

The model assumptions are: fluid properties in each control volume are considered constant in each time period; the heat transmitted in the two-phase region is assumed to be only taken up by the saturated liquid, thus the heat transfer only occurs in the liquid zone ($V_L$); no thermal capacity and mass storage in vapour zones are considered; the refrigerant pressure drop along
the evaporator is neglected; kinetic and potential energy variations are not taken into account; and the vapour–liquid volume relations for two-phase flow presented by Wedekind et al. (1978) are used for connecting the liquid and vapour saturated volumes in the two-phase region.

Following the equations used for modelling each control volume are presented, as shown in Figure 1.

2.1. Two-phase zone

From the model inputs, the evaporator inlet refrigerant mass vapour fraction (1) is evaluated, and from this, the inlet mass flow rates to the liquid (2) and saturated (3) volumes are evaluated.

\[
\begin{align*}
    x_{in,v} &= \frac{h_{in} - h_l}{h_v - h_l} \quad (1) \\
    \dot{m}_{in,l} &= (1 - x_{in,v}) \cdot \dot{m}_{in} \\
    \dot{m}_{in,v} &= x_{in,v} \cdot \dot{m}_{in} \\
\end{align*}
\]

The mass continuity (4) and the energy conservation (5) relations are applied in the saturated liquid volume.

\[
\begin{align*}
    \frac{dM_{V_l}}{dt} &= \dot{m}_{in,l} - \dot{m}_{lv} \quad (4) \\
    \frac{d(M \cdot U)}{dt} &= \dot{m}_{in,l} \cdot h_l - \dot{m}_{lv} \cdot h_v + \dot{Q}_l \quad (5)
\end{align*}
\]

Differential equations (4) and (5) are expanded until they are expressed as a function of refrigerant properties and control volume inputs. The variable selected to represent the control volume behaviour is the total volume of saturated liquid \((V_l)\) in the evaporator. The mass
continuity equation is expressed by (6), and the energy conservation equation by (7).

\[
\rho_i \frac{dV_i}{dt} + \frac{d\rho_i}{dt} \cdot V_i = \dot{m}_{\text{in},1} - \dot{m}_{1\text{v}}
\]  

\[
(\rho_i \cdot u_i) \frac{dV_i}{dt} + \left( u_i \cdot \frac{d\rho_i}{dt} + \rho_i \cdot \frac{du_i}{dt} \right) \cdot V_i = \dot{m}_{\text{in},1} \cdot h_i - \dot{m}_{1\text{v}} \cdot h_v + \dot{Q}_1
\]  

The boiling heat transfer for the saturated liquid refrigerant is estimated from Gungor and Winterton’s correlation (1987) for two-phase flow in smooth horizontal tubes, since it is an appropriate correlation for this heat exchange type according to several authors, Boissieux et al. (2000), Hewitt (2002). The metal temperature \(T_{\text{m,1}}\) and the saturation temperature for pure refrigerants and the average temperature between the inlet refrigerant and the vapour saturation temperatures for blend refrigerants, are considered for estimating the coefficient.

Only a mass balance is applied (8) in the saturated vapour volume since the mass storage and the thermal capacity are neglected.

\[
\dot{m}_v = \dot{m}_{\text{in},v} + \dot{m}_{1\text{v}}
\]  

The system mean void fraction relation presented by Wedekind et al. (1978) is used to relate the liquid and the vapour volumes. It has been observed that the vapour/liquid proportion practically remains constant in all the test range (Table I), using the three models presented in his work. Therefore, the Wedekind’s void fraction integral relation has not been added to the model and its value has been assumed to be constant and estimated at 86% of vapour, a proportion that is very close to the values presented in his work.

### 2.2. Superheating zone

Since mass and energy storages are neglected in vapour zones, the steady-state mass (9) and energy (10) relations represent the superheating volume behaviour.

\[
\dot{m}_{\text{out}} = \dot{m}_v
\]  

\[
\dot{Q}_2 = \dot{m}_{\text{out}} \cdot h_{\text{out}} - \dot{m}_v \cdot h_v
\]  

The heat transferred by convection between the evaporator tubes and the superheated refrigerant is estimated through the Gnielinski’s correlation (1976) for turbulent flow in smooth straight tubes. The average vapour temperature and the metal temperature \(T_{\text{m,2}}\) in that zone are used.

### Table I. Test range.

<table>
<thead>
<tr>
<th></th>
<th>HCFC-22</th>
<th></th>
<th>HFC-134a</th>
<th></th>
<th>HFC-407C</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>Evaporating pressure (bar)</td>
<td>2.656</td>
<td>4.688</td>
<td>1.740</td>
<td>3.621</td>
<td>2.440</td>
<td>4.491</td>
</tr>
<tr>
<td>Ref. mass flow rate (kg s(^{-1}))</td>
<td>0.0298</td>
<td>0.0692</td>
<td>0.0350</td>
<td>0.0869</td>
<td>0.0309</td>
<td>0.0656</td>
</tr>
<tr>
<td>Brine mass flow rate (kg s(^{-1}))</td>
<td>0.216</td>
<td>0.712</td>
<td>0.268</td>
<td>0.460</td>
<td>0.289</td>
<td>0.720</td>
</tr>
<tr>
<td>Inlet brine temp. (K)</td>
<td>267.97</td>
<td>285.85</td>
<td>274.10</td>
<td>292.76</td>
<td>270.71</td>
<td>287.72</td>
</tr>
</tbody>
</table>

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2.3. Secondary fluid side

Heat flow rate exchanges take place to the evaporator tubes \( \dot{Q}_{w,1}, \dot{Q}_{w,2} \) and from the shell \( \dot{Q}_{s,1}, \dot{Q}_{s,2} \), therefore it is necessary to consider them all to obtain a good representation of the secondary coolant dynamic behaviour. The control volumes \( V_{w,1}, V_{w,2} \) are modelled by considering that the net convection heat exchange causes a change in the fluid stored energy (11). The secondary fluid dynamics are neglected since its total mass variation in the evaporator practically does not exist.

\[
\dot{Q}_{w_i} - \dot{Q}_{s_i} = m_{w_i}/C_{0} \cdot \frac{d}{dt} [T_{w_i}/C_{1} - (T_{in,wi} - T_{out,wi})]
\]

The heat flow rate transferred from the secondary coolant to the evaporator tubes is estimated using a simplified model based on Zhukauskas’ correlation (1972) for an ideal bundle of tubes. The leakage, bypass and windows effects are neglected, and the average secondary coolant temperature and the metal temperature of the tubes are considered. The convection heat transfer from the brine to the shell is calculated with Gnielinski’s correlation (1976) using the same velocity behaviour pattern as in the bundle of tubes. The average brine temperature and the shell temperature are used for estimating this coefficient.

2.4. Metal sides

Energy storage, as a metal temperature variation, is one of the important dynamics in refrigerant behaviour. The relation representing temperature change in the metal control volumes is expressed by an energetic balance in which all of the heat flux rates are considered, computing the temperature variation from (12).

\[
\dot{Q}_{w_i} - \dot{Q}_{s_i} = M_{m_i}/C_{1} \cdot \frac{d}{dt} [T_{m_i}/C_{1}]
\]

The shell temperature variation highly influences the secondary coolant dynamic behaviour due to its high thermal capacity. Its temperature evolution can be obtained from

\[
\dot{Q}_{env} - \dot{Q}_{s,1} - \dot{Q}_{s,2} = M_{s} \cdot c_{p,s} \cdot \frac{d}{dt} [T_{s}]
\]

in which the heat flow rate from the secondary coolant to the shell and the environmental temperature disturbance are also taken into account.

2.5. Heat transfer from environment

The heat transferred from the environment to the evaporator has a lesser effect than the rest of the dynamics. Nonetheless, it is necessary to include it in the model in order to adequately predict the secondary coolant steady-state behaviour, especially in long-time transients. It is computed as

\[
\dot{Q}_{env} = U \cdot A_{ext,s} \cdot (T_{env} - T_{s})
\]

where the overall heat transfer coefficient (15) is estimated by taking a constant value that only takes shell isolation into account since the environmental effect to the isolation convection heat transfer and the shell conduction resistance offer a very poor contribution in comparison with the isolation one. The overall heat transfer coefficient is referred to the external shell.
area. The heat transferred from the environment to the shell is evaluated using the average shell temperature.

\[ U = \frac{k_{iso}}{r_{ext} \cdot \ln \left( \frac{r_{ext} + e}{r_{ext}} \right)} \]  

(15)

2.6. Refrigerant and secondary coolant modelling

The model is developed to work with pure and blend refrigerants. To calculate the refrigerant properties, the Refprop dynamic libraries (Lemmon et al., 2002) are used. Secondary fluid thermophysical properties (mixture of water–propylene-glycol 30/70% by volume) are evaluated by means of interpolated polynomials calculated from the ASHRAE Handbook (ASHRAE, 2001).

The whole model, programmed in Fortran, has 32 algebraic equations and 5 first-order differential equations that are solved by fourth-order Runge–Kutta methods.

The dynamic response calculus starts with a model initialization based on a steady-state convergence criterion of the model outlet variables. Then the dynamic model is solved using an integration step of 0.2 s. Since the evaporator is a nonlinear system the selection of a smaller integration step in comparison with the works of Deng (2000) and Finn et al. (2003) permits to analyse in more detail the evolution of quickly dynamics in the system.

3. VALIDATION

The dynamic model validation is made using data from a test campaign performed with an experimental vapour compression plant, Figure 2.

![Diagram of Refrigeration test facility](image-url)
The refrigerant facility consists of a single-stage vapour compression plant with: an open-type compressor, driven by a 4 kW electric motor, a shell-and-tube evaporator (1-2) where the refrigerant is flowing inside the tubes, a shell-and-tube condenser (1-2) with the refrigerant condensing in the shell side and a thermostatic expansion valve. It was designed for working between 5 and 13 kW of cooling capacity.

The regulation to simulate the transient phenomenon is carried out with two independent systems, one for the evaporating circuit and another for the condensing one. The evaporator’s secondary fluid loop is equipped with electric resistors that permit sudden changes of the refrigerating load (temperature transient tests); also its volumetric flow can be varied controlling the pump rotation speed (volumetric brine transient tests). In addition, another inverter is used to control the compressor rotation speed (compressor speed transient tests). This equipment allows the simulation of any type of cooling load variation in the evaporator.

The refrigerant thermodynamic states are obtained by measuring pressure and temperature at the inlet and outlet of each basic element of the facility with 15 K-type thermocouples and 8 piezoelectric pressure gauges, and by using our own software based on Refprop routines (Lemmon et al., 2002). The pressure and temperature sensors are calibrated in our own laboratory using certified references, and a reading error of 0.1 K was obtained in the thermocouples, as well as an uncertainty of ±0.1% of the full scale range (0–700 kPa and 0–3000 kPa) for the pressure transducers. The refrigerant mass flow rate is measured by a Coriolis mass flow meter located at the liquid line, with a certified accuracy within ±0.22% of the reading, and the volumetric flows of the secondary fluids are taken with two volumetric flow meters with an uncertainty of the reading of ±0.33%. The compressor power consumption is measured using a digital wattmeter, with a calibration specified uncertainty of ±0.5%, and the compressor rotation speed is measured with a capacitive sensor with an error of ±1% in relation to the reading measure. All the signals are gathered by a National Instruments SCXI data acquisition system and handled on-line with our own LabView-based acquisition application. The sampling time rate used in all tests is 0.1 s.

The evaporator in which the model has been validated is a shell-and-tube heat exchanger (1-2). The refrigerant flows inside the tubes and the water–propylene-glycol mixture (30/70% by volume) flows along the shell. The main mechanical characteristics of this evaporator are: 76 × 0.92 m long horizontal tubes with an internal and external diameter of 8.22 × 10⁻³ m and 9.52 × 10⁻³ m, respectively. The tubes have micron fins of a thickness of 0.2 × 10⁻³ m. The shell contains five segmental baffles and is isolated.

A dynamic model for representing the behaviour of any system must comply with two fundamental requirements. Firstly, it must guarantee its own stability by means of the steady-state validation, and secondly, it must accurately represent the dynamic performance of the system, hence its validation is necessary in the unsteady-state.

To accomplish the first condition, a validation in steady-state has been carried out for two types of refrigerants, pure refrigerants (HCFC-22, HFC-134a) and one blend refrigerant (HFC-407C). This validation is carried out with data from a previous work presented by Cabello et al. (2004) for the three refrigerants and within a wide working range of the evaporator, Table I. The comparison between the predicted and the measured values for the evaporator output variables have an approximate error of 5%, Figures 3–5.

Attending to the results presented in Figures 3 and 4 it can be stated that the evaporator model works better in the refrigerant side, since prediction errors are higher in the secondary
fluid modelling. Furthermore, the model presents errors that are maintained between the three refrigerants in which the model has been tested.

In order to assure an adequate model prediction of the dynamic performance, it has been validated with the refrigerant HCFC-22 in a variety of transient tests: compressor rotation speed variations and cooling load variations (by means of inlet temperature and mass flow changes of the secondary coolant).
In Figures 6–8, the results of the dynamic model simulation against the measured values in a system-free response under compressor speed changes are presented.

Agreement with the experimental data is better in the refrigerant side (Figure 6) because all the causes that affect the refrigerant are considered except the pressure drop along the evaporator. The conclusion reached after some analyses is that this pressure drop is not significantly important in the energetic behaviour of this type of evaporators, since the effect of a small pressure drop in the heat transfer characteristics is minimum. But it needs to be considered if the model is expected to be used in a complete model of a refrigerating system.
In the secondary coolant modelling (Figure 7) the dynamic agreement is also good, but the model has steady-state errors higher than in the refrigerant side (Figure 4), although they are around 5%. It is likely that neglecting the leakage, bypass and windows effects for estimating the heat transfer coefficient is the main cause of this steady-state deviation.

In Figure 9, the secondary coolant modelled outlet temperature is presented in a secondary coolant volumetric flow variation test. The predicted values agree with the measured ones, but the model response is slightly quicker than the real system response in abrupt changes. This could be caused by neglecting some thermal storage elements in the secondary fluid side or due to the delay that introduces the immersion thermocouples.
4. CONCLUSIONS

An evaporator dynamic model built from physical equations has been developed. It considers constant thermophysical properties for the fluids, and the materials that constitute the different control volumes, where these are time-dependent variables. This model is able to predict the dynamic response of a nonlinear system such as a shell-and-tube evaporator. The main limitation of the model is that only averaged values in each control volume can be analysed.

The model has been validated for pure refrigerants (HCFC-22, HFC-134a) and for a ternary mixture (HFC-407C) in a wide range of operating conditions, and it shows a better agreement in the refrigerant than in the secondary coolant, where the steady-state errors are higher. Moreover, the shell dynamics can be neglected if only a good representation of refrigerant behaviour is needed.

Since the model prediction errors are lower than a 5% and the agreement with the dynamic response of the system is quite good, it proves to be a useful tool for being applied in control strategies and in fault detection and diagnosis, mainly for the refrigerant loop. The use of small integration time steps allows a good representation of any internal variable associated with the evaporating process, such as the heat transfer coefficients or the fast variations in the process.

NOMENCLATURE

\[ \begin{align*}
A &= \text{area (m}^2\text{)} \\
c_p &= \text{specific heat capacity (J kg}^{-1}\text{K}^{-1}\text{)} \\
e &= \text{isolation thickness (m)} \\
h &= \text{specific enthalpy (J kg}^{-1}\text{)} \\
k &= \text{thermal conductivity (W m}^{-1}\text{K}^{-1}\text{)} \\
M &= \text{mass (kg)}
\end{align*}\]
\( \dot{m} \) = mass flow rate (kg s\(^{-1}\))
\( P \) = pressure (Pa)
\( \dot{Q} \) = heat flux (W m\(^{-2}\))
\( r \) = radius (m)
\( T \) = temperature (K)
\( t \) = time (s)
\( u \) = specific internal energy (J kg\(^{-1}\))
\( U \) = internal energy (J)
\( \dot{U} \) = overall heat transfer coefficient (W m\(^{-2}\) K\(^{-1}\))
\( V \) = total volume (m\(^3\))
\( x \) = refrigerant mass vapour fraction

Greek letters

\( \rho \) = density (kg m\(^{-3}\))

Subscripts

env = environment
ext = external
in = inlet
int = intermediate
iso = isolation
l = saturated liquid
lv = liquid to vapour
m = metal
out = outlet
s = shell
v = saturated vapour
w = secondary coolant
1 = two-phase zone
2 = superheating zone

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