«Economic Analysis of Wireless Sensor-Based Services in the Framework of the Internet of Things. A Game-Theoretical Approach»

Author: Àngel Sanchis Cano
Supervisor: Dr. Luis Guijarro Coloma

Valencia, April 2018
Acknowledgments

First of all, I would like to express my gratitude to my supervisor, Dr. Luis Guijarro Coloma, for his support and guidance during all my Ph.D. period.

I also would like to thank to the Spanish Ministry of Economy and Competitiveness (MINECO) for the economic support through projects TIN2013-47272-C2-1-R and BES-2014-068998, as well as to the Universitat Politècnica de València for all the facilities and resources provided.

I would like to thank to the members of GIRBA and the traffic engineering lab for creating an excellent environment to develop our research, for their help and friendship, specially to Julian, Israel, Luis, Canek, Jairo and to all the people who have worked in the laboratory during these years.

Finally, I want to thank to my wife, my family and my friends for their love and support. Without them, this dissertation would not have been possible.
Abstract

The communications world is moving from a standalone devices scenario to a all-connected scenario known as Internet of Things (IoT), where billions of devices will be connected to the Internet through mobile and fixed networks. In this context, there are several challenges to face, from the development of new standards to the study of the economical viability of the different future scenarios. In this dissertation we have focused on the study of the economic viability of different scenarios using concepts of microeconomics, game theory, non-linear optimization, network economics and wireless networks. The dissertation analyzes the transition from a Human Type Communications (HTC) to a Machine Type Communications (MTC) centered network from an economic point of view. The first scenario is designed to focus on the first stages of the transition, where HTC and MTC traffic are served on a common network infrastructure. The second scenario analyzes the provision of connectivity service to MTC users using a dedicated network infrastructure, while the third stage is centered in the analysis of the provision of services based on the MTC data over the infrastructure studied in the previous scenario. All the scenarios are described with more detail in the following paragraphs.

Firstly, we analyze a transition scenario, where HTC and MTC share a common network infrastructure. We study the economic viability of the coexistence scenario under monopolistic and duopolistic approaches, and we compare the results against a baseline scenario, where only one operator serves Human Type Communications users (HTCu). The service provision is modeled using a two-priority queue, where the HTC traffic has a higher priority than the MTC traffic. On the other hand, the competition between operators is analyzed as a two-stage game. In the first stage, each operator announces a price and in the second stage, the users decide to subscribe or not to the service based on a utility function, which is related with the packet delay and the price charged by the operators. The HTC-MTC coexistence is shown to be profitable for all the actors when there is competition between operators serving different traffic profiles. In addition, the entry of the second operator to serve Machine Type Communications users (MTCu) is shown desirable, not only from the point of view of resource usage efficiency and from the point of view of the users, but also from the point of view of the two operators. The HTC-MTC service provision is therefore shown economically viable and more efficient than the baseline scenario.

Secondly, we model a scenario where Wireless Sensor Networks (WSN) are served by a dedicated Network Operator (OP). The WSN connectivity scenario is modeled mathematically and analyzed using capacity provision mechanisms, assuming that the price is fixed by a regulator, with the objective of maximizing the profits of the OP. The scenario has several sensor clusters gathering data. Each cluster has a sink node, which uploads the sensing data gathered in the cluster to the Internet, through the wireless connectivity of the OP. The scenario is analyzed both, as a static game and as a dynamic game, with two stages each game, using game theory. The sinks behavior is characterized with a utility function related to the mean service time and the price paid to the OP for the service. The objective of the OP is to maximize its profits modifying the network capacity. In the static game, the sinks’ subscription decision is modeled using a population game. In the dynamic game, the sinks’ behavior is modeled using an evolutionary game and the replicator dynamic, while the OP optimal capacity decision is obtained solving an optimal control problem. The scenario is shown feasible from an economic point of view if the variable costs are bounded. Therefore, it is important to reduce the costs as much as possible in order to maximize the profitability of the scenario. In addition, the dynamic capacity provision optimization is shown a valid mechanism for maximize the OP profits, as well as a useful tool to analyze evolving scenarios. On the other hand, the dynamic analysis opens the possibility to study more complex scenarios using the differential games extension.
Thirdly, we analyze a whole new business model based on a MTC scenario, or equivalently, an end-to-end MTC scenario. In the scenario, two Internet of Things Service Provider (IoT-SP) deploy their own private WSN in order to collect sensing data, which allows them to offer sensing services to final users. The sensing data is collected by the WSN and transmitted by a sink node in each WSN to its IoT-SP, through the wireless connection of an OP. This scenario allows us to have a global point of view of the full business model, from the data collection to the service provision to the final users. The feasibility of providing WSN-data-based services in an IoT scenario from an economical point of view is studied. The scenario is analyzed as two games using game theory. In the first game, the OP announces a price and the sensors decide to subscribe or not to the OP service to upload the collected sensing-data. In the second game, each IoT-SP announces a price and the users decide to subscribe or not to the sensor-data-based service of the IoT-SPs based on a Logit discrete choice model related to the quality of the data collected and the subscription price. The sinks and users subscription stages are analyzed using population games and discrete choice models, while OP and IoT-SPs pricing stages are analyzed using optimization and Nash equilibrium concepts respectively. The scenario is shown feasible from an economic point of view for all the actors if there are enough interested final users in the IoT-SPs service, and their sensitivity to the amount of data/price rate is bounded, and opens the possibility of developing more efficient models with different types of services.

Finally, we analyze the previous model dynamically. We consider different time scales in each game, given that the changes in the first game are much less frequent than in the second game, and therefore, we can use the static solution of the first game as input of the second game. The second game is analyzed dynamically, using the Logit dynamic to model the behavior of the users and a differential game to model the competition between IoT-SPs. The scenario is shown feasible and the dynamic analysis able to model competitive evolving scenarios, however, it is needed to extend the analysis to more complex scenarios, with a wider range of strategies for the users.

Thanks to the analysis of all the scenarios we have observed that the transition from HTC users-centered networks to MTC networks is possible and that the provision of services in such scenarios is viable. In addition, we have observed that the behavior of the users is essential in order to determine the viability of a business model, and therefore, it is needed to study their behavior and preferences in depth in future studios. Specifically, the most relevant factors are the sensitivity of the users to the delay and to the amount of data gathered by the sensors. We also have observed that the differentiation of the traffic in categories improves the usage of the networks and allows to create new services thanks to the data that otherwise would not be used, improving the monetization of the infrastructure and the data. In addition, we have shown that the capacity provision is a valid mechanism for providers’ profit optimization, as an alternative to the pricing mechanisms. Finally, it has been demonstrated that it is possible to create dedicated roles to offer IoT services in the telecommunications market, specifically, the IoT-SPs, which provide wireless-sensor-based services to the final users using a third party infrastructure.

Summarizing, this dissertation tries to demonstrate the economic viability of the future IoT networks business models as well as the emergence of new business opportunities and roles in order to justify economically the development and implementation of the new technologies required to offer massive wireless access to machine devices.
Resumen

El mundo de las telecomunicaciones está cambiando de un escenario donde únicamente las personas estaban conectadas a un modelo donde prácticamente todos los dispositivos y sensores se encuentran conectados, también conocido como Internet de las cosas (IoT), donde miles de millones de dispositivos se conectarán a Internet a través de conexiones móviles y redes fijas. En este contexto, hay muchos retos que superar, desde el desarrollo de nuevos estándares de comunicación al estudio de la viabilidad económica de los posibles escenarios futuros. En esta tesis nos hemos centrado en el estudio de la viabilidad económica de diferentes escenarios mediante el uso de conceptos de microeconomía, teoría de juegos, optimización no lineal, economía de redes y redes inalámbricas. La tesis analiza la transición desde redes centradas en el servicio de tráfico HTC a redes centradas en tráfico MTC desde un punto de vista económico. El primer escenario ha sido diseñado para centrarse en la transición desde redes centradas en el servicio de tráfico HTC a redes centradas en tráfico MTC desde un punto de vista económico. El primer escenario ha sido diseñado para centrarse en las primeras etapas de la transición, en la que ambos tipos de tráfico son servidos bajo la misma infraestructura de red. En el segundo escenario analizamos la siguiente etapa, en la que el servicio a los usuarios MTC se realiza mediante una infraestructura dedicada. Finalmente, el tercer escenario analiza la provisión de servicios basados en MTC a usuarios finales, mediante la infraestructura analizada en el escenario anterior. En los párrafos siguientes se describe con más detalle cada escenario.

En primer lugar, analizamos un escenario de transición, donde las comunicaciones de tipo humano (HTC) y las comunicaciones de tipo máquina (MTC) comparten una única estructura de red. En el trabajo se analiza la viabilidad económica de la coexistencia de ambos tipos de tráfico mediante una aproximación de monopolio y una de duopolio. Los resultados de ambas aproximaciones se comparan con un caso base, en donde un operador monopolista ofrece servicio únicamente a usuarios HTC. La provisión de servicio de ambos tipos de tráfico es modelada mediante una cola de dos prioridades, donde el tráfico HTC se considera prioritario sobre el tráfico MTC. Por otro lado, la competencia entre los operadores se analiza como un juego de dos etapas. En la primera etapa, cada operador elige un precio que optimiza sus beneficios y lo anuncia, mientras que, en la segunda etapa, los usuarios deciden si suscribir o no al servicio en base a una función de utilidad basada en el retardo experimentado por los paquetes y el precio cobrado por los operadores. Como resultado, se muestra que la coexistencia HTC-MTC es provechosa para todos los actores cuando existe competencia entre los operadores que sirven a cada tipo de tráfico. Además, la entrada de un nuevo operador para servir a los usuarios MTC es deseable, no solo desde un punto de vista de eficiencia en el uso de la red o de los usuarios, sino también desde el punto de vista de ambos operadores. Por tanto, podemos concluir que la coexistencia es viable desde un punto de vista económico y, además, es más eficiente con respecto al caso base.

En segundo lugar, modelamos un escenario donde la conectividad para redes de sensores inalámbricos (WSN) es proporcionada por un operador de red (OP) dedicado. El escenario es modelado matemáticamente y analizado usando mecanismos de provisión de capacidad, con el objetivo de maximizar los beneficios del OP, asumiendo que el precio es fijado por un órgano regulador. En el escenario existen varios clústeres de sensores con un nodo encargado de transmitir los datos recopilados en el clúster a Internet a través del servicio de conectividad inalámbrica del OP. El escenario es analizado mediante teoría de juegos como un juego estático y como un juego dinámico, con dos etapas cada juego. El comportamiento de los sensores es caracterizado, una vez más, por una función de utilidad relacionada con el retardo de los paquetes y el precio pagado al OP por el servicio. Por su parte, el OP busca maximizar sus beneficios ajustando el valor de la capacidad de la red. En el juego estático, la decisión de suscripción de los sensores se modela mediante juegos poblacionales, mientras que, en el juego dinámico, el comportamiento de los sensores se modela mediante un juego evolutivo y la dinámica del replicador. Por otro lado, la decisión de capacidad óptima se obtiene resolviendo un problema de optimización en el caso estático y un problema de control óptimo en el dinámico. En el trabajo se
muestra que el escenario es factible desde un punto de vista económico. Además, la optimización dinámica de la provisión de capacidad se muestra como un mecanismo válido para maximizar los beneficios del OP, así como una herramienta útil para analizar escenarios cambiantes. Por otro lado, el análisis dinámico nos permite estudiar escenarios más complejos mediante el empleo de juegos diferenciales.

En tercer lugar, analizamos un modelo de negocio completo basado en un escenario MTC, o equivalentemente, un escenario MTC end-to-end. En el escenario, dos proveedores de servicios del internet de las cosas (IoT-SPs) despliegan sus propias redes privadas de sensores WSN para poder obtener datos que les permitirán ofrecer servicios a usuarios finales. Los datos se obtienen a través de las WSN, y son transmitidos al IoT-SP por un nodo encargado de recolectar todos los datos de la WSN, a través de la conexión de un OP. Este escenario nos permite tener un punto de vista global de un modelo de negocio empleando datos de sensores, desde la recolección de los datos hasta la provisión de servicio a los usuarios finales. En análisis se estudia la viabilidad económica de proveer servicios basados en datos de WSN en un escenario IoT. El escenario es analizado como dos juegos interrelacionados empleando teoría de juegos. En el primero, el OP anuncia un precio y los sensores deciden suscribir o no para subir los datos al IoT-SP correspondiente. En el segundo juego, cada IoT-SP anuncia un precio y los usuarios deciden si suscribir o no al servicio basado en datos de los sensores de los IoT-SPs en función del modelo Logit de elección discreta, basado en la calidad de los datos de sensores y el precio del servicio. Las etapas de suscripción de sensores y usuarios se analizan mediante el uso de juegos poblacionales y modelos de elección discreta, mientras que la selección de precios del OP y los IoT-SPs se analizan mediante técnicas de optimización y el equilibrio de Nash respectivamente. Los resultados muestran que el escenario es factible desde un punto de vista económico para todos los actores, siempre que haya suficientes usuarios finales interesados en el servicio. Por otro lado, si se combina con los resultados el primer escenario, existe la posibilidad de desarrollar modelos más eficientes con diferentes tipos de servicios.

Finalmente, analizamos el modelo anterior dinámicamente. Consideramos diferentes escalas de tiempo en cada juego, dado que los cambios en el primer juego son mucho menos frecuentes que en el segundo juego, y por lo tanto, podemos usar la solución estática del primer juego para resolver el segundo juego. El segundo juego se analiza dinámicamente, utilizando la dinámica Logit para modelar el comportamiento de los usuarios y un juego diferencial para modelar la competencia entre IoT-SPs. El escenario se muestra factible y el análisis dinámico capaz de modelar escenarios competitivos cambiantes, sin embargo, es necesario extender el análisis a escenarios más complejos, con un mayor abanico de posibles elecciones para los usuarios.

Gracias al análisis de todos los escenarios, hemos observado que la transición de redes centradas en usuarios HTC a redes MTC es posible y que la provisión de servicios en tales escenarios es viable. Además, hemos observado que el comportamiento de los usuarios es esencial para determinar la viabilidad de los diferentes modelos de negocio, y por tanto, es necesario estudiar el comportamiento y las preferencias de los usuarios en profundidad en estudios futuros. Específicamente, los factores más relevantes son la sensibilidad de los usuarios al retardo en los datos recopilados por los sensores y la cantidad de los mismos. También hemos observado que la diferenciación del tráfico en categorías mejora el uso de las redes y permite crear nuevos servicios empleando datos que, de otro modo, no se aprovecharían, lo cual nos permite mejorar la monetización de la infraestructura. También hemos demostrado que la provisión de capacidad es un mecanismo válido, alternativo a la fijación de precios, para la optimización de los beneficios de los proveedores de servicio. Finalmente, se ha demostrado que es posible crear roles específicos para ofrecer servicios IoT en el mercado de las telecomunicaciones, específicamente, los IoT-SPs, que proporcionan servicios basados en sensores inalámbricos utilizando infraestructuras de acceso de terceros y sus propias redes de sensores.
En resumen, en esta tesis hemos intentado demostrar la viabilidad económica de modelos de negocio basados en redes futuras IoT, así como la aparición de nuevas oportunidades y roles de negocio, lo cual nos permite justificar económicamente el desarrollo y la implementación de las tecnologías necesarias para ofrecer servicios de acceso inalámbrico masivo a dispositivos MTC.
Resum

El món de les telecomunicacions està canviant d’un escenari on únicament les persones estaven connectades a un model on pràcticament tots els dispositius i sensors es troben connectats, també conegut com a Internet de les Coses (IoT), on milers de milions de dispositius es connectaran a Internet a través de connexions mòbils i xarxes fixes. En aquest context, hi ha molts reptes que superar, des del desenrottllament de nous estàndards de comunicació a l’estudi de la viabilitat econòmica dels possibles escenaris futurs. En aquesta tesi ens hem centrat en l’estudi de la viabilitat econòmica de diferents escenaris per mitjà de l’ús de conceptes de microeconomia, teoria de jocs, optimització no lineal, economia de xarxes i xarxes inalàmbriques. La tesi analitza la transició des de xarxes centrades en el servei de tràfic HTC a xarxes centrades en tràfic MTC des d’un punt de vista econòmic. El primer escenari ha sigut dissenyat per a centrar-se en les primeres etapes de la transició, en la que ambdós tipus de tràfic són servits davall la mateixa infraestructura de xarxa. En el segon escenari analitzem la següent etapa, en la que el servei als usuaris MTC es realitza per mitjà d’una infraestructura dedicada. Finalment, el tercer escenari analitza la provisió de servicis basats en MTC a usuaris finals, per mitjà de la infraestructura analitzada en l’escenari anterior. Als paràgrafs següents es descriu amb més detall cada escenari.

En primer lloc, analitzem un escenari de transició, on les comunicacions de tipus humà (HTC) i les comunicacions de tipus màquina (MTC) comparteixen una mateixa xarxa d’accés. Al treball s’analtiza la viabilitat econòmica de la coexistència d’ambdós tipus de tràfic per mitjà d’una aproximació de monopoli i una de duopoli. Els resultats d’ambdós aproximacions es comparen amb un cas base, on un operador monopolista ofereix servei únicament a usuaris HTC. La provisió de servei d’ambdós tipus de tràfic és modelada per mitjà d’una cua de dos prioritats, on el tràfic HTC es considera prioritari sobre el tràfic MTC. D’altra banda, la competència entre els operadors s’analtiza com un joc de dos etapes. A la primera etapa, cada operador tria un preu que optimitza els seus beneficis i ho anuncia, per altra banda, a la segona etapa, els usuaris decideixen si subscriure o no al servei basant-se en una funció d’utilitat que depèn del retard experimentat pels paquets i el preu cobrat pels operadors. Com resultat, es mostra que la coexistència HTC-MTC és factible a l’escenari duopolista. A més a més, l’entrada d’un nou operador per a servir els usuaris MTC és desitjable, no sols des d’un punt de vista d’eficiència en l’ús de la xarxa o dels usuaris, sinó també des del punt de vista d’ambdós operadors. Per tant, podem concloure que la coexistència és viable des d’un punt de vista econòmic i, a més, és més eficient respecte al cas base.

En segon lloc, modelem un escenari on la connectivitat per a xarxes de sensors inalàmbriques (WSN) és proporcionada per un operador de xarxa (OP) dedicat. L’escenari és modelatge matemàticament i analitzat usant mecanismes de provisió de capacitat, amb l’objectiu de maximizar els beneficis de l’OP, assumint que el preu és fixat per un òrgan regulador. A l’escenari hi ha diversos clústers de sensors amb un node encarregat de transmetre les dades recopilats al clúster a Internet a través del servei de connectivitat sense fil de l’OP. L’escenari és analitzat per mitjà de teoria de jocs com un joc estàtic i com un joc dinàmic, amb dos etapes cada joc. El comportament dels sensors és caracteritzat, una vegada més, per una funció d’utilitat relacionada amb el retard dels paquets i el preu pagat a l’OP pel servei. Per la seua banda, l’OP busca maximizar els seus beneficis ajustant la capacitat de la xarxa. Al joc estàtic, la decisió de subscripció dels sensors es modela per mitjà de jocs poblacionals, mentre que, al joc dinàmic, el comportament dels sensors es modela per mitjà d’un joc evolutiu i la dinàmica del replicador. D’altra banda, la decisió de capacitat òptima s’obté resolent un problema d’optimització al cas estàtic i un problema de control òptim al dinàmic. Al treball es mostra que l’escenari és factible des d’un punt de vista econòmic. A més, l’optimització dinàmica de la provisió de capacitat es mostra com un mecanisme vàlid per a maximizar els beneficis de l’OP, així com una ferramenta útil per a analitzar escenaris canviants. D’altra banda, l’anàlisi dinàmica ens permet estudiar escenaris més complexos per
mitjà de l’ocupació de jocs diferencials.

En tercer lloc, analitzem un model de negoci complet basat en un escenari MTC, o de forma equivalent, un escenari MTC end-to-end. A l’escenari, dos proveïdors de serveis d’Internet de les coses (IoT-SPs) despleguen les seues pròpies xarxes privades de sensors WSN per a poder obtingre dades que els permetran oferir serveis a usuaris finals. Les dades s’obtenen a través de les WSN, i són transmesos a l’IoT-SP per un node encarregat de recollir totes les dades de la WSN, a través de la connexió d’un OP. Aquest escenari ens permet tindre un punt de vista global d’un model de negoci emprant dades de sensors, des de la recoll-lectio de les dades fins a la provisió de servei als usuaris finals. A anàlisi s’estudia la viabilitat econòmica de proveir serveis basats en dades de WSN a un escenari IoT. L’escenari és analitzat com dos jocs interrelacionats emprant teoria de jocs. Al primer joc, l’OP anuncia un preu i els sensors decideixen subscriure o no per a pujar les dades a l’IoT-SP corresponent. Al segon joc, cada IoT-SP anuncia un preu i els usuaris decideixen si subscriure o no al servei basat en dades dels sensors dels IoT-SPs en funció del model Logit d’elecció discreta, basat en la qualitat de les dades dels sensors i el preu del servei. Les etapes de subscripció de sensors i usuaris s’analitzien per mitjà de l’ús de jocs poblacionals i models d’elecció discreta, mentres que la selecció de preus de l’OP i els IoT-SPs s’analitzen per mitjà de tècniques d’optimització i l’equilibri de Nash respectivament. Els resultats mostren que l’escenari és factible des d’un punt de vista econòmic per a tots els actors, sempre que hi haja suficients usuaris finals interessats amb el servei. D’altra banda, si es combina amb els resultats del primer escenari, hi ha la possibilitat de desenrotrllar models més eficients amb diferents tipus de serveis.

Finalment, analitzem el model anterior dinàmicament. Considerem diferents escales de temps en cada joc, atès que els canvis en el primer joc són molt menys freqüents que en el segon joc, i per tant, podem utilitzar la solució èstàtica del primer joc per a resoldre el segon joc. El segon joc s’analitza dinàmicament, utilitzant la dinàmica Logit per a modelar el comportament dels usuaris i un joc diferencial per a modelar la competència entre IoT-SPs. L’escenari es mostra factible i l’anàlisi dinàmica capaç de modelar escenaris competitius canviants, no obstant això, és necessari estendre l’anàlisi a escenaris més complexos, amb un major ventall de possibles eleccions per als usuaris.

Gràcies a l’anàlisi de tots els escenaris, hem observat que la transició de xarxes centrades en usuaris HTC a xarxes MTC és possible i que la provisió de servis en tals escenaris és viable. A més a més, hem observat que el comportament dels usuaris és essencial per a determinar la viabilitat dels diferents models de negoci, i per tant, és necessari estudiar el comportament i les preferències dels usuaris en profunditat en estudis futurs. Específicament, els factors més rellevants són la sensibilitat dels usuaris al retard en les dades recopilats pels sensors i la quantitat dels mateixos. També hem observat que la diferenciació del tràfic en categories millora l’ús de les xarxes i permet crear nous servis emprant dades que, d’una altra manera, no s’aprofitarien, la qual cosa ens permet millorar la monetització de la infraestructura. També hem demonstrat que la provisió de capacitat és un mecanisme vàlid, alternatiu a la fixació de preus, per a l’optimització dels beneficis dels proveïdors de servici. Finalment, s’ha demostrat que és possible crear rols específics per a oferir servis IoT en el mercat de les telecomunicacions, específicament, els IoT-SPs, que proporcionen servis basats en sensors inalàmbrics utilitzant infraestructures d’accés de tercers i les seues pròpies xarxes de sensors.

En resum, en aquesta tesi hem intentat demostrar la viabilitat econòmica de models de negoci basats en xarxes futures IoT, així com l’aparició de noves oportunitats i rols de negoci, la qual cosa ens permet justificar econòmicament el desenrotllament i la implementació de les tecnologies necessàries per a oferir servis d’accés inalàmbria massiu a dispositius MTC.
Keywords

game theory; evolutionary game theory; queuing theory; Nash equilibrium; Wardrop equilibrium; population games; discrete choice model; network economics; mathematical modeling; pricing; dynamic capacity optimization; optimal control; differential games; monopoly; oligopoly; competition; profit maximization; users subscription; internet of things; internet of things service provider; machine type communications; wireless sensor networks;
## Contents

List of Tables \hspace{1cm} xv

List of Figures \hspace{1cm} xvii

### 1 Introduction

1.1 Motivation \hspace{1cm} 1
1.2 Objectives \hspace{1cm} 2
1.3 Methodology \hspace{1cm} 3
1.4 Tools and Means \hspace{1cm} 3
1.5 Context \hspace{1cm} 4
1.6 Publications \hspace{1cm} 4
1.7 Dissertation Outline \hspace{1cm} 4

### 2 Background and State of the Art

2.1 Microeconomics \hspace{1cm} 7
  2.1.1 The Market \hspace{1cm} 7
  2.1.2 Utility function \hspace{1cm} 8
  2.1.3 Profits \hspace{1cm} 9
  2.1.4 Competition \hspace{1cm} 10
2.1.5 Economic Viability ......................................................... 11
2.2 Game theory ................................................................. 12
   2.2.1 Overview and basic concepts ......................................... 12
   2.2.2 Types of games ....................................................... 13
2.3 Non-linear Optimization ...................................................... 17
   2.3.1 Static optimization ................................................... 17
   2.3.2 Dynamic Optimization ................................................. 18
2.4 Network economics .......................................................... 20

3 Analyzed Scenarios ............................................................ 23
   3.1 Scenario 1: HTC and MTC Service Provision on a Common Network Infrastructure .......................... 23
   3.2 Scenario 2: Dynamic Capacity Provision for Wireless Sensors Connectivity ............................ 24
   3.3 Scenario 3: Wireless Sensor Network-Based Service Provision in a Duopoly Setting with a Monopolist Operator ......................................................... 24
   3.4 Scenario 4: Wireless Sensor Network-Based Service Provision in a Duopoly Setting with a Monopolist Operator: A Dynamic Approach ........................................ 24

4 Scenario 1: HTC and MTC Service Provision on a Common Network Infrastructure ........................................ 27
   4.1 General Model ............................................................... 27
   4.2 Game Analysis ............................................................... 29
      4.2.1 Game I: Baseline Case - HTC service provision ........................................ 30
      4.2.2 Game II: Duopoly Case - HTC and MTC service provision by two competing operators ......................... 32
      4.2.3 Game III: Monopoly Case - HTC and MTC service provision by one operator ....................... 36
   4.3 Results and discussion ..................................................... 36

5 Scenario 2: Dynamic Capacity Provision for Wireless Sensors Connectivity .................................................... 39
   5.1 General Model ............................................................... 39
   5.2 Game Analysis ............................................................... 41
      5.2.1 Game I: Static Analysis ................................................. 42
5.2.2 Game II: Dynamic Analysis .............................................. 45

5.3 Results and discussion ......................................................... 50
  5.3.1 OP optimal control and sinks’ distribution with static parameters ....... 51
  5.3.2 OP optimal control and sinks’ distribution with dynamic parameters .... 52

6 Scenario 3: Wireless Sensor Network-Based Service Provision in a Duopoly Setting with a Monopolist Operator 59
  6.1 General Model ................................................................. 59
    6.1.1 Sinks ................................................................. 60
    6.1.2 Network Operator .................................................... 61
    6.1.3 Users ................................................................. 61
    6.1.4 IoT-Service Providers ............................................... 62
  6.2 Game Analysis ................................................................. 62
    6.2.1 Game I: OP and Sinks ............................................... 63
    6.2.2 Game II: Internet of Things-Service Providers (IoT-SPs) and Users .... 66
  6.3 Results and Discussion ....................................................... 70
    6.3.1 OP Pricing and Profit ............................................... 72
    6.3.2 IoT-SP\textsubscript{1} and IoT-SP\textsubscript{2} Pricing and Profits ................. 72

7 Scenario 4: Wireless Sensor Network-Based Service Provision in a Duopoly Setting with a Monopolist Operator: A Dynamic Approach 79
  7.1 General Model ................................................................. 79
    7.1.1 Sinks ................................................................. 80
    7.1.2 Network Operator .................................................... 80
    7.1.3 Users ................................................................. 80
    7.1.4 IoT-Service Providers ............................................... 81
  7.2 Game Analysis ................................................................. 81
    7.2.1 Game I: OP and Sinks ............................................... 81
    7.2.2 Game II: Internet of Things-Service Providers (IoT-SPs) and Users .... 82
7.3 Results and Discussion ................................................. 85
  7.3.1 Scenario 1: IoT-SPs dynamic competitions with different number of sinks .... 85
  7.3.2 Scenario 2: IoT-SPs dynamic competitions with same number of sinks ....... 86

8 Conclusions and Future Work ........................................ 89
  8.1 Conclusions (English) ................................................. 89
    8.1.1 Main Contributions ............................................. 90
    8.1.2 Future Work .................................................... 92
  8.2 Conclusiones (Spanish) ............................................. 92
    8.2.1 Principales contribuciones ..................................... 93
    8.2.2 Trabajos futuros .............................................. 95

Bibliography .................................................................. 97

Appendix A Scenario 3: OP Pricing Stage - Solution of KKT Problem ............ 105

Appendix Abbreviations .................................................. 107

Appendix Nomenclature .................................................. 109
## LIST OF TABLES

4.1 Scenario 1: HTC & MTC flows: Wardrop Equilibrium Conditions .......................... 34

5.1 Scenario 2: Reference Case - Static Parameters ................................................. 51

5.2 Scenario 2: Reference Case - Dynamic Common Parameters 1 ............................. 53

5.3 Scenario 2: Reference Case - Dynamic Parameters 2 ....................................... 53

6.1 Scenario 3: Reference Case ................................................................................. 71

7.1 Scenario 4: Game I parameters ....................................................................... 85

7.2 Scenario 4: Game I solutions .......................................................................... 86

7.3 Scenario 4: Reference Case - Game II Differential Game parameters ................. 86
LIST OF FIGURES

4.1 Scenario 1: Queue model for two different traffic profiles ............................... 28
4.2 Scenario 1: Description of the games ............................................................... 29
4.3 Scenario 1: Wardrop equilibrium regions ......................................................... 34
4.4 Scenario 1: Comparison between equilibrium traffic in the duopoly case ($\rho^*_1, \rho^*_2$) and baseline case ($\rho^*$) when sensitivity to delay $\alpha$ varies. .............................. 37
4.5 Scenario 1: Comparison between equilibrium prices in the duopoly case ($p^*_1, p^*_2$) and baseline case ($p^*$) when sensitivity to delay $\alpha$ varies. .............................. 38
4.6 Scenario 1: Comparison between equilibrium profits in the duopoly case ($\Pi^*_1, \Pi^*_2$) and baseline case ($\Pi^*$) when sensitivity to delay $\alpha$ varies. .............................. 38
5.1 Scenario 2: Analyzed scenario with all the actors of the market ......................... 40
5.2 Scenario 2: Model payments flow and actors involved ...................................... 41
5.3 Scenario 2: Description of the game stages ...................................................... 42
5.4 Scenario 2: Replicator dynamic convergence when the GESS is a mixed equilibrium. ...................................................... 49
5.5 Scenario 2: OP optimal capacity in the static and dynamic cases for different values of $N$. .............................. 51
5.6 Scenario 2: Social state in the static and dynamic cases for different values of $N$. .............................. 52
5.7 Scenario 2.1: Evolution of the number of sinks $N$ as a function of $t$. ................. 54
5.8 Scenario 2.1: OP optimal capacity in the cases with static and dynamic optimization as a function of $t$. .............................................................. 54
5.9 Scenario 2.1: Social state in the three studied cases as a function of $t$.  
5.10 Scenario 2.1: Evolution of the OP profits for different strategies as a function of $t$ and total profits.  
5.11 Scenario 2.2: Evolution of the number of sinks $N$ as a function of $t$.  
5.12 Scenario 2.2: OP optimal capacity in the cases with static and dynamic optimization as a function of $t$.  
5.13 Scenario 2.2: Social state in the three studied cases as a function of $t$.  
5.14 Scenario 2.2: Evolution of the OP profits for different strategies as a function of $t$ and total profits.  
6.1 Scenario 3: Analyzed scenario with all the actors of the market.  
6.2 Scenario 3: Model payments flow and actors involved.  
6.3 Scenario 3: Description of the games stages.  
6.4 Scenario 3: Normalized OP profit ($N = 1$) for each case with $c = 1$, $r = 1$.  
6.5 Scenario 3: Best responses with different values of $\varphi$.  
6.6 Scenario 3: Distribution of the users between the strategies in the equilibrium.  
6.7 Scenario 3: Minimum value of $M$ to obtain positive profits with $c = 1$ and $\tau N r = 1/3$.  
6.8 Scenario 3: OP optimal price as a function of $L$ for different values of $c$.  
6.9 Scenario 3: Social state as a function of $L$ for different values of $c$.  
6.10 Scenario 3: OP optimal profit as a function of $L$ for different values of $c$.  
6.11 Scenario 3: IoT-SP$_1$ equilibrium price as a function of $\varphi$ for different values of $c$.  
6.12 Scenario 3: IoT-SP$_1$ equilibrium price as a function of $\varphi$ for different values of $N$.  
6.13 Scenario 3: IoT-SP$_1$ equilibrium profit as a function of $\varphi$ for different values of $c$.  
6.14 Scenario 3: IoT-SP$_1$ equilibrium profit as a function of $\varphi$ for different values of $N$.  
6.15 Scenario 3: IoT-SP$_1$ equilibrium profit as a function of $\varphi$ for different values of $M$.  
6.16 Scenario 3: IoT-SP$_1$ equilibrium profit as a function of $N$.  
6.17 Scenario 3: IoT-SP$_2$ equilibrium price as a function of $\varphi$ for different values of $c$.  
6.18 Scenario 3: IoT-SP$_2$ equilibrium price as a function of $\varphi$ for different values of $N$.  
6.19 Scenario 3: IoT-SP$_2$ equilibrium profit as a function of $\varphi$ for different values of $c$.  

xviii
CHAPTER 1

INTRODUCTION

1.1 Motivation

Communications suffered a dramatic change when the Internet appeared. The Internet was designed to link a small group of networks and devices, nevertheless nowadays billions of people are connected through it. One of the first problems appeared when the multimedia applications appeared and researchers realised the need to adopt a new service model [1]. Nowadays mobile communications are experiencing a similar situation. The world is moving from a standalone devices scenario to a all-connected scenario also known as IoT. The basic idea is to have connected almost all objects around us, being able to communicate with each other and connected to the Internet. The IoT is a key concept in the present and future of the Internet [2] with several technologies involved, possible applications and open research challenges [3–5]. This concept is not new [6], however, the wide concept of IoT that we know nowadays was not defined until the last decade [7].

The traditional usage of networks where humans are the main users is changing progressively to a thing centered model [8], and the number of devices connected is growing rapidly. In the Ericsson Mobility Report an estimation of devices connected will 26 billion, and only 6.6 billion will be HTC devices [9], and according to Cisco, there will be 5.5 billion mobile devices connected to the Internet by 2020 [10], with a wide range of applications in several areas, such as education, healthcare, industry, infrastructures, military, ecology, smart homes as well as smart cities [2,8,11,12], among others. In this paradigm, MTC, also known as Machine to Machine communications (M2M) [13], have a key role in almost all the applications, with a huge amount of devices [14] trying to transmit small size packets.

The behavior of the networks in the IoT era is a challenge that must be faced, including the development of new hardware and physical layer standards [4,15] to feasible economic and business models as well as updated pricing strategies [16,17]. However, there is a lack of studies analyzing the economic aspects of IoT, and particularly of MTC and sensor network-based services, such as pricing or economic viability [15,16,18]. Recent investigations have shown an interest from the industry verticals to integrate IoT
and 5G technologies, nevertheless it is not clear how the different telecom players could benefit from it [19, 20]. Given the huge investments required to develop the new technologies [21] it is necessary to study new business models and their economic viability as well as the emergence of new actors in the market [17]. In fact, given that the main actors in the development of new networks are the operators, it is needed to justify the IoT solutions, not only from an efficiency point of view, but also from an operator profit point of view.

We have tried to solve some of the opened challenges, focusing our work on network economics. In this dissertation we propose different network scenarios and we analyze their economical feasibility from different points of view.

1.2 Objectives

The framework described above is complex. Mobile networks have experienced a huge growing in the last years, increasing its bandwidth and their coverage to serve the growing number of devices, the new applications and the services that have been appearing. Nowadays, a new challenge is opened with the growing of IoT, M2M and WSN.

Our main objective in this dissertation is to analyze the economic viability of different network scenarios using mathematical modeling, and specifically, game theory. The scenarios include wireless communications and services that will appear in the following years to serve the IoT. Our work is focused on the study of the economic viability of all the system actors including operators, service providers and final users. For the final users the viability is studied making use of a utility function related with the service received and the price payed for this service. For operators and service providers the viability is studied through profit functions.

The main objective is divided in the following sub-objectives:

1. To analyze the transition from HTC to MTC wireless networks scenarios from an economical point of view, using microeconomics, competition between operators and users, demand and supply curves and other economic concepts needed to model and study the behavior of the markets.

2. To analyze a transition scenario, where HTC and MTC users are served on a common network infrastructure. To analyze the economical viability of the scenario using game theory and non-cooperative games.

3. To study the changes due to the competition between operators, and the effect of the entry of a new operator in the market.

4. To analyze the connectivity service provision to MTC users using a dedicated network infrastructure. To study the economical viability of the scenario using the capacity provision as optimization variable from a static and a dynamic point of view.

5. To study an end-to-end IoT scenario, from the wireless connectivity of the sensors that gather data, to the provision of services based on that data to final users. To study the economical viability of the scenario using static non-cooperative games.
1.3 Methodology

In order to achieve the objectives described above we used a methodology based on the Scientific Method [22], which is described below.

The first step is to analyze the problem and identify its limitations and needs. The analysis of the existing works is essential and gives us a global insight of the research area. It helped us to identify the weaknesses in the previous models. After defining the problem, a background research was done, which helped us to identify the different approaches used by the scientific community to solve similar problems. The research of the state of art gave us a wide view of the problem, the necessary experience and knowledge to address our specific problem.

Once the research in the state of the art was done, we built new IoT scenarios and mathematical network models [23], based on the acquired knowledge and we analyzed them. The analytic models are mainly composed by a utility function, that models the value perceived by the users, a payment scheme between all the actors in the market and profit functions for operators and service providers [24, 25].

The first task in the creation of new mathematical models is to define the system: the agents (users, operators, third part service providers, ...) and the interactions between them. Once the system was defined, different utility and profit functions were proposed, as well as the equilibrium concepts that model the behavior of the agents. The next step was related with the analytic resolution of the equilibrium of the system, using game theory concepts such as Nash and Wardrop equilibriums, Population games,Stackelberg games and barward induction and discrete choice models such as Logit [26–28]. The resolution of the described equilibriums, usually implies solving restricted optimization problems. In order to solve these problems, it was needed to use mathematical tools like Karush-Kuhn-Tucker (KKT) theorem or Lagrange multipliers [29] for static optimization, and optimal control problems and differential games for dynamic optimization [30].

Finally, the results are analyzed and conclusions are drawn.

1.4 Tools and Means

In order to conduct the required research and to write this dissertation the following tools were needed:

- Access to the libraries, papers and articles related with the research field. These fields include microeconomics, game theory, network pricing, IoT, WSN, M2M, MTC, static optimization and dynamic optimization.

- A computer with the minimum requirements to run analytic and numerical tools, including Matlab and Wolfram Mathematica for the analysis of the models and LaTeX to write the documents.

- The necessary means to make an international research stay, which helped us to collaborate with other researchers in the same field.

- The necessary means to publish and publicize the results of the work.
1.5 Context

This doctoral dissertation has been developed in the GIRBA research group and the ITACA institute of the Universitat Politècnica de València over the last three years.

The dissertation has been supported by the MINECO in the framework of the project PLASMA (TIN2013-47272-C2-1-R) and (co-supported by the Europen Social Fund) BES-2014-068998. During the development of the Ph.D. the candidate has performed a three months research stay in the Department of Informatics, King’s College of London, supervised by Mischa Dohler and Massimo Condolouci and funded through the MINECO grant EEBB-I-17-11947.

1.6 Publications

As a result of the conducted work we have generated the following publications related with the different parts of the dissertation:

Chapter 4:

Chapter 5:

Chapter 6:


1.7 Dissertation Outline

The rest of this dissertation is organized as follows:

- **Chapter 2**: The background and the current state of art are reviewed. The concepts of microeconomics, game theory and optimization used to conduct the research are explained in depth, and we introduce some applications related with the study of the economic viability of wireless communications and services. In addition, the related works in network economics are reviewed.
• **Chapter 3**: The different scenarios and models analyzed are explained. The structure of the following chapters and the steps of the analysis are explained.

• **Chapter 4**: We analyze a transition scenario between networks serving only HTC and a scenario with dedicated networks to serve MTC. The economic viability of providing service to HTC and MTC on a common network infrastructure under monopolistic and duopolistic scenarios is studied. The service provision is modeled using queueing theory, and the entry of a second operator to serve MTC is analyzed using game theory.

• **Chapter 5**: As an evolution of the coexistence scenario, we study a scenario to provide wireless sensor connectivity with its own infrastructure in the framework of IoT. The price fixed by a regulator and the capacity provision is used as control variable. The optimal dynamic capacity provision is obtained solving an optimal control problem.

• **Chapter 6**: We analyze the feasibility of providing Wireless Sensor Network-data-based services in an IoT scenario from an economical point of view. This scenario allows us to have an end-to-end economic point of view of future services. The scenario has two competing service providers with their own private sensor networks, a network operator which provides connectivity service to the sensors and final users. The scenario is analyzed as two games using game theory. In the first game, sensors decide to subscribe or not to the network operator to upload the collected sensing-data. In the second game, users decide to subscribe or not to the sensor-data-based service of the service providers.

• **Chapter 7**: In this Chapter we go one step further, analyzing dynamically the scenario studied in Chapter 6. The changes in the first game are much less frequent than in the second game, and therefore the static solution of the first game is used. On the other hand, the second game is analyzed dynamically, using the Logit dynamic to model the behavior of the users and a differential game to solve the competition between service providers.

• **Chapter 8**: We conclude the dissertation summarizing all the studied scenarios and remarking the most important results. The dissertation concludes with some recommendations for future works.
CHAPTER 2

BACKGROUND AND STATE OF THE ART

2.1 Microeconomics

One of the basic tools needed to develop the aim of this work is microeconomics. Microeconomics is an area of economics that analyzes the behavior and interactions of suppliers and consumers in a market [31]. In the following points we describe the concepts of microeconomics needed to understand this paper, based on [32]. We also develop some of the functions employed in the next chapters.

2.1.1 The Market

Firstly, it is needed to analyze the scenario, defining the actors in it and the interactions between them. In our models we will typically find basically two types of actors:

- Service Providers.
- Service Consumers.

In the first group of service providers we can include the network operators (OPs) and the providers of services based on sensing data, also known as IoT-SPs. In the second group we include any actor that pays for services or goods, such as final users of a service or sensors, which need the connectivity service of an OP to upload the gathered data to the network.

Secondly, the preferences of the actors are modeled creating simplified mathematical models of the reality. One typical assumption is that all the actors act rationally, and they choose the option which
gives them a better reward, based on their preferences. How to define the preferences of the different kind of actors is not a simple task, and it is explained in the points 2.1.2 and 2.1.3.

Finally, once all the scenario is defined and modeled mathematically, the decisions of the actors are analyzed and the equilibrium of the scenario is found. The equilibrium is a state where no actor has incentive to change its decision. There are several ways to find the equilibrium values depending on the kind of market. The different types of markets are described in the point 2.1.4.

2.1.2 Utility function

The utility function is a mathematical tool to model the preferences of the service consumers. Its objective is to give an estimation of the happiness of the consumers with a product or service, based on several indicators, such as the quality of the good or the price paid for it[28, 32, 33].

Defining the utility function is not easy. The compensated utility function [34], which is a function widely used in telecommunications [35–38] models the preferences of the consumers as the difference between the quality of the service perceived and the price charged for this service

\[ U = Q - p, \]

where \( p \) is the price charged for the service, and therefore, is an objective parameter. However \( Q \) is the quality perceived by the consumers and its a subjective parameter, or equivalently, it may vary in function of the perception of each consumer.

There are several possibilities to define the value of \( Q \), in function of the terms that you want to emphasize in your analysis and the kind of communication that we are trying to model. Some publications model the quality of the service using physical parameters, like the available bandwidth, the signal noise ratio or the interference in the wireless channel [39–41]. Other papers model the behavior of the users using higher level concepts, avoiding to model the physical layer behavior explicitly [42, 43]. These papers use concepts such as the mean service time or the amount of data collected by the sensors as indicators of the quality of the service offered to the final users. In our models we have used these functions to model the behavior of users and sensor trying to upload their data to a wireless OP. The quality function employed is based on the time required to transmit one data unit, which is obtained modeling the wireless operator as a M/M/1 queue. It has been used in several works before [34, 44–46], and has the following form for a customer \( i \):

\[ Q_i = c \left( \frac{T_i}{\bar{T}} \right)^{-\alpha}, \quad (2.1) \]

where \( c \) is a scale factor, \( T_i / \bar{T} \) is the mean packet service time, normalized by the mean packet transmission time, that is the minimum value of \( T_i \) and \( 0 < \alpha < 1 \) is the customer sensitivity to delay. Note that a utility function using the quality defined before is suitable for many IoT applications with delay requirements [47].

The functions described above usually work well for many scenarios, however, when a homogeneous population is modeled using a function that does not depend of the status of the market and the number of options is limited, it may present a discontinuity. This discontinuity provokes that, with an infinitesimal change in a parameter like the price, all the population changes its choice. This behavior is not desirable and does not model a real scenario.
In order to solve the discontinuity problem in the behavior of the costumers we used discrete choice models [48], and specifically the Logit model. The Logit model has been used before in network economics [28, 49, 50]. For a consumer \( i \) with \( N \) possible alternatives to choose, the alternative \( n \) in the Logit model has the following utility:

\[
U_{i,n} = v_i + \kappa_{i,n}, \tag{2.2}
\]

where \( v_i \) is deterministic and is related with the market parameters, while \( \kappa_{i,n} \) is treated as a random variable that models the unobserved user-specific part of the utility. The random variable \( \kappa_{i,n} \) follows a Gumbel distribution of mean 0. The human behavior is hard to predict and usually users within the same population do not have the same preferences. For instance, while some users always prefer the cheapest option others only will change their decision if the difference in the perceived utility is high enough. All these unknown effects are aggregated in the random variable \( \kappa_{i,n} \). One of the advantages of the Logit model is that, assuming a high enough number of players, the number of players choosing the strategy \( i \) is proportional to the probability of a player choosing the option \( i \), which is:

\[
\omega_i = \frac{e^{v_i}}{\sum_{j=0}^{N} e^{v_j}}. \tag{2.3}
\]

When the Logit model is used in the next chapters the utility presents the following form:

\[
U_{i,n} = \varphi \log(R_i) + \kappa_{i,n}, \tag{2.4}
\]

where \( \varphi > 0 \) is a sensitivity parameter and \( R_i \) is related with some parameters of the system, such as the amount of data collected by the sensors and the price charged for the service. The logarithmic relation between physical magnitudes and the human perception observed in (2.4) has been justified in telecommunications through the Weber-Fechner Law [51, 52].

### 2.1.3 Profits

The profits is a mathematical tool to model the preferences of the service providers, such as OPs and IoT-SPs. It can also be seen as a specific type of utility function, but mainly focused in the economical aspects, due to the involved actors are mainly companies.

The profit of the providers typically has the following structure:

\[
\Pi = pn - c_v(n) - K, \tag{2.5}
\]

where \( n \) is the number of customers, \( pn \) is the revenue, while \( c_v(n) \) and \( K \) are the costs. The revenue is typically obtained multiplying the price chosen by the provider \( p \) and the number of customers subscribed to the service \( n \). The costs are divided in two groups:

- **Variable costs** \( c_v(n) \): The variable costs, also known as investment costs [53], are the costs related with the service provision and are function of the number of customers subscribed to the service. In a wireless connectivity provision scenario the variable costs usually are related with the price of the bandwidth used by the customers or the price of the maintenance and the energy used by the telecommunications equipment. This is an important term in our analysis, given that it is directly related with the viability of a scenario.
• **Fixed costs** $K$: The fixed costs typically are related with initial investments, such as equipment, spectral licenses or infrastructures. Usually it is a parameter of a scenario and it is relevant in the Capital Expenditures (CAPEX) studios. However, it is not relevant in the scenarios viability analysis [3, 32].

There are different ways of optimizing the profits, depending on the market and the kind of competition. In monopolistic scenarios the optimization is calculated solving maximization problems with restrictions in static models, and optimal control problems in dynamic models. On the other hand, in competitive scenarios, the optimization is calculated using game theory: equilibrium concepts in static models and differential games in dynamic models. Typically, the optimization variable is the price [54–57], nevertheless, it is not strange to find works using the system capacity or leased bandwidth as optimization variables [40, 58–60].

Typically the profits are used as a measure of the viability of a scenario. However, it is also common to use the social welfare as an indicator of the viability of an scenario. The social welfare uses a pondered sum of different factors as a measure of the viability. It typically is obtained adding up the providers’ profits and the customers’ utilities. The maximization of the social welfare can be performed solving a multi-variable optimization problem.

### 2.1.4 Competition

In function of the scenario analyzed, there are different ways to solve the providers’ profit maximization problem. In economics it is common to differentiate three different types of scenarios:

• **Monopoly**: A monopoly infrastructure comprises only one provider of a service with no other providers able to provide an equivalent service. The unique provider completely controls all the industry and it is able to block the entrance of new competitors. Historically there have been two different kinds of monopolies:

  – *Public Monopoly*: There are some services which are not profitable or are basics for the daily life that are provided by a public entity. In this case, the objective of the monopolistic operator is not to maximize its profits, but to maximize the social welfare.

  – *Private Monopoly*: When the entrance in a market requires a huge initial investment it is common that the first company entering into the market reaches a monopolistic status. In this case the company has a complete control in the offer and the price of the services, and therefore, can maximize its profits at expense of the consumers.

  At the beginning of the telecommunications it was a public monopoly, which evolved to a private monopoly when it started to be profitable. Nowadays, the market has evolved to an oligopoly.

  In order to analyze this scenario it is needed to solve an optimization problem.

• **Oligopoly**: In a oligopoly, there are only a small number of providers competing for offering a service. It is common markets with high initial investments and a small number of big companies. Unlike the monopolistic scenario, here the providers compete to serve the customers, and therefore, the market equilibrium is more beneficial for the customers. However, given that the number of providers is limited, the equilibrium reached is not the best for the users, and the providers are still able to have high revenues.
This is the most common scenario in telecommunications. Nowadays, telecommunications are almost an essential good, and it is recommended the control of the market by a regulator to ensure fair prices and good quality of service to all the users.

The tool needed to solve this scenarios is game theory, which helps us to analyze the competition between providers and to obtain the market equilibrium parameters.

- **Perfect Competition**: Also known as competitive market, is a scenario with numerous small providers offering equivalent services. In this scenario there is not a market leader, and all the providers have the same power. The competition in this market is fierce, and the providers are forced to minimize its profits in order to attract customers. This is the best scenario for the customers, however, it is only possible in markets with small initial investments.

### 2.1.5 Economic Viability

In the dissertation the concept of *economic viability* or *economic feasibility* is used indistinctly as a characteristic of a scenario.

The concept of economic viability means that all the actors that decide to join a scenario (operators, users, service providers, sensors, ...) are benefited, or at least are indifferent, compared with those that do not join it. More specifically, the operators and service providers obtain non-negative profits and the users and sensors get a non-negative value in their utility function.
2.2 Game theory

Game theory is a branch of mathematics that analyzes the decisions of an actor, taking into account the decisions of the rest of the actors in the system [26]. At the beginning it was used to predict the best strategies in a game, for this reason the actors are often called players. Nowadays game theory has evolved and it has become a powerful tool to address several problems, such as the analysis of the competition between providers in a scenario. It also helps us to analyze the behavior of the users and their decisions.

2.2.1 Overview and basic concepts

Following [26] and [61] we explain the game theory concepts employed in this dissertation:

- **Information Requirements**: In function of the knowledge of the actors about the system we have two different types of games:
  - *Games with perfect information*: In these games all the players know the preferences of the other players and the payoff of all the possible actions. This is the main kind of games that we use to analyze our models, nevertheless, it is not always a valid approach, given that the assumptions about perfect information are very strong.
  - *Games with imperfect information*: These games are characterized by the uncertainty of the players about the preferences of the other players or the payoff of all the actions. These games are characterized using random components. The Logit model is an example of a function that incorporates the uncertainty about the preferences of the users, allowing us to model an heterogeneous population.

- **Rationality**: We assume that a player is rational when it always chooses the option that better satisfies its preferences. In our models it means that the providers always will try to maximize their profits and the users will maximize their utility function.

- **Equilibrium**: This is one of the most important concepts of the game theory, given that it will be the solution for many of our problems. An equilibrium is a state where the players have no incentive to change their actions unilaterally. There are several equilibrium concepts, and they are different if the played games are cooperative and non-cooperative. The most common equilibria are the following:
  - **Nash**: The Nash equilibrium is reached when two or more competitors choose simultaneously their best strategy (also known as best response function) taking into account the strategies of the competitors. The Nash equilibrium concept may be used to find the prices chosen by several operators in a competitive scenario. In a two-player game, where the competition is in prices, the Nash equilibrium can be defined as follows:
    \[
    p_1^* = \arg\max_{p_1} \Pi_1(p_1, p_2^*), \\
    p_2^* = \arg\max_{p_2} \Pi_2(p_1^*, p_2).
    \]  

- **Wardrop**: The Wardrop equilibrium is a concept born in the analysis of transportation networks. The first Wardrop principle defines that in the equilibrium a user cannot improve
its utility through an unilateral change of choice. The second principle defines that the average utility of the users in the equilibrium is the best. It is useful to model the behavior of users with common interests [62].

- **Stackelberg**: The Stackelberg equilibrium is obtained in sequential games with a leader and one or more followers. The leader can choose its optimal decision firstly, anticipating the choices of the followers. In oligopoly scenarios such as telecommunications, it is common that one provider dominates the market. The leader position gives an additional advantage to the dominant provider, allowing to increase its profit.

Another important aspect is that a game may have several equilibria, and not all are desirable. Some games may have several stages or subgames, such as a subscription stage and a pricing stage. Backward induction may be used to solve these games selecting a "good equilibrium". Backward induction consists in deducing backwards from the end of a problem to the beginning to infer a sequence of optimal actions. Extensive form games may have several Nash equilibria and backward induction helps us to pick out a good equilibrium. Any Nash equilibrium found using backward induction is also a Nash equilibrium for every subgame, or equivalently a Subgame Perfect Equilibrium [61]. Such games with several stages may also be seen as a Stackelberg game, with the providers choosing the prices as leaders and the users subscribing as followers.

### 2.2.2 Types of games

There are several ways to classify the types of games. We have decided to classify the games taking into account the number of decisions that each player can make, the order of the decisions and the changes in the system during the game.

#### 2.2.2.1 Static games

In a static game each player makes only one decision with no knowledge of the decisions made by the other players, or equivalently, the decisions are simultaneous.

The concepts that we used to solve these games are the Nash equilibrium to model the behavior of operators and providers, and population games, which help us to model the behavior of large populations of players, such as the customers.

#### 2.2.2.2 Evolutionary games

Evolutionary games are games where the players are modeled as evolving populations. The main difference with respect to static games is that the decisions of the population may evolve during the time.

In evolutionary games, players use a set of rules to update their strategies. This set of rules is known as revision protocol [63] and determines the evolutionary dynamic, which is a differential equation that models the behavior of the players. There are several families of revision protocols, but we are interested in the imitative protocols and direct selection protocols. In the imitative protocols the users updates their strategies taking into account the strategies chosen by other users. But imitative protocols admit boundary rest points that are not Nash equilibria of the underlying game [64]. On the other hand direct
selection protocols are not directly influenced by the choice of others and this characteristic prevents the boundary rest points. In our work we have chosen an imitative protocol, given that it is tractable analytically and widely used in the literature. However we need to be cautious about the boundary rest points. The main mean dynamics studied [63] are the following:

- **Smith Dynamic:** The Smith dynamic is derived from behavior of users modeled using the following revision protocol:
  \[
  \Gamma_{ij} = (U_j - U_i)^+, \tag{2.7}
  \]
  where \(\Gamma_{ij}\) is the switch rate from strategy \(i\) to strategy \(j\), \(i\) is the current strategy, \(j\) is the new strategy and \(U_k\) is the utility of selecting the strategy \(k\). This revision protocol models that each time instant, the players choose a random strategy, and if its utility with the new strategy is higher than his current strategy’s utility, the player will change its strategy to the new one with a probability proportional to the difference between utilities. As we observe the decisions of the players are not based in the decisions of other players, and therefore, it is a direct revision protocol.

  If we define the social state of a system as \(X = \{x_1, x_2, ..., x_s\}\), where \(x_i\) is the fraction of players choosing the strategy \(i\) and \(S = \{1, 2, ..., s\}\) is the set of all the possible strategies. We can define the Smith mean dynamic, derived from (2.7) as:
  \[
  \dot{x}_i = \sum_{j \in S} x_j [U_i(X) - U_j(X)]^+ - x_i \sum_{j \in S} [U_j(X) - U_i(X)]^+, \tag{2.8}
  \]
  where \(\dot{x}_i\) is the instantaneous variation of the social state \(i\), and \(U_k(X)\) is the utility perceived choosing the strategy \(k\) for the current status of the system \(X\). The Smith dynamic has the following properties based on [63]:
  - Continuity: Small variations in global behavior not lead to large changes in players’ actions.
  - Positive Correlation: When a population is not in a steady state its strategies’ growth rates are positively correlated with the utilities.
  - Nash Stationarity: All the rest points of the dynamic are Nash equilibriums of the game.
  - D2 Data requirements: \(\Gamma_{ij}\) depends only on \(U_i(X)\) and \(U_j(X)\).

The Smith dynamic is one of the most interesting dynamics, due to its convergence characteristics and information requirements. However, its mathematical expression presents a discontinuity, and therefore is not analytically tractable. Despite of that, the Smith dynamic is an excellent dynamic to use in simulation environments, which are out of the aim of this dissertation.

- **Replicator Dynamic:** The Replicator dynamic is derived from behavior of users modeled using the following revision protocol:
  \[
  \Gamma_{ij} = x_j [U_j - U_i]^+, \tag{2.9}
  \]
  This revision protocol models that each time instant, the players choose a random opponent, and if the utility of the opponent is higher than his current strategy’s utility, the player will change it’s strategy to the opponent one with a probability proportional to the difference between utilities. As we observe the decisions of the players are based on the decisions of other players, and therefore, it is an imitative revision protocol.

  The mean dynamic derived from the revision protocol (2.9) is know as replicator dynamic and is:
  \[
  \dot{x}_i = x_i (U_i(X) - U_{avg}(X)), \tag{2.10}
  \]
where $U_{avg}$ is the average utility of all the players in the system. As we observe the dynamic evolves progressively to the state with higher utility, slower when it has few players and faster when the portion of players grow. However, we also observe that when a state has no players no one will move to it, regardless of the utility of this state.

The Replicator dynamic has the following properties:

- Continuity.
- Positive Correlation.
- Dn Data requirements: $\Gamma_{ij}$ depends on the utility of all the strategies $U_k(X)$, but not on the social state $X$.

The replicator dynamic does not have Nash Stationarity. However, its mathematical expression is tractable analytically and it has been widely used in the bibliography. We have used this utility to model the behavior of sensors in Chapter 6.

- **Logit Dynamic:** When the behavior of the players is modeled following the Logit model, the revision protocol is the same defined in (2.3):

$$\Gamma_i = \omega_i = \frac{e^{v_i}}{\sum_{j=0}^{n} e^{v_j}}.$$  (2.11)

The mean dynamic derived from the revision protocol (2.11) is know as Logit dynamic and is:

$$\dot{x}_i = \Gamma_i(X) - x_i = \frac{e^{v_i}}{\sum_{j=0}^{n} e^{v_j}} - x_i.$$

where $\omega_i(X)$ is the probability of choice the strategy $i$ when the social state is $X$ and $v_i$ is the deterministic part of the player utility choosing the option $i$, $U_i(X)$.

The Logit dynamic has the following properties:

- Continuity.
- Dn Data requirements: $\Gamma_{ij}$ depends on the utility of all the strategies $U_k(X)$, but not on the social state $X$.

- **Hybrid Dynamics:** The hybrid dynamic models the behavior of a player when it uses several revision protocols with different weights. The revision protocol of a player playing the dynamics $X$ with intensity $a$ and $Y$ with intensity $b$ is:

$$\Gamma^H = a\Gamma^X + b\Gamma^Y,$$  (2.13)

and the resulting dynamic is also a combination of the original dynamics. One interesting characteristic is that if the dynamic $X$ satisfies positive correlation and Nash stationarity, but the dynamic $Y$ only satisfies positive correlation, the hybrid dynamic satisfies both properties.

The solution for an evolutionary game are the stationary points of the dynamic. The dynamic reach a stationary point when no user is willing to change its strategy, or equivalently when $\dot{x}_i = 0$. However, no all the stationary points (or steady states) are stable, and it is needed to characterize its stability. One simple way to determine the stability of a steady-state is using the concept of invasion. Consider a steady-state $x \in X$ where sinks perceive a utility $U(x)$ and an invader state $y \in X$ where some sinks move to a different strategy and they perceive a utility $U(y)$. We can affirm that $x \in X$ is a Globally Evolutionary Stable Strategy (GESS) [63] if:

$$U(y) - U(x) < 0 \quad \forall \quad y \in X - \{x\},$$  (2.14)
which means that the utility perceived by the sinks who do not switch their strategy from state $x \in X$ is higher than the utility perceived by the sinks who switched it. An equivalent definition is that the utility of sinks who switch their strategy decrease or the utility of sinks who keeps their strategy increases, while the utility of sinks who switch remains constant [65]. In addition, it has been demonstrated that in every single population game, like in our models, every GESS is unique and it is also a Nash equilibrium [63]. Furthermore, every GESS is also an Evolutionary Stable Strategy (ESS) and, as proven by Barron, it is also an asymptotically stable solution of the dynamic [61].

### 2.2.2.3 Dynamic games

Dynamic games were developed in the 1920s, however they were not widely used until the 1970s [66]. Unlike static games, in dynamic games the players decide their strategies not only in one time instant, but over a time horizon. In these games the parameters of the systems and the decisions of the actors may change over time. These games are used typically to model and analyze the behavior of operators and service providers in dynamic scenarios.

The objective of the players is to maximize their profits, however, the traditional definition of profits (2.5) is not valid in this scenario. Defining the instantaneous profits of the player $i$ in a given instant of time as $\Pi_{ins}^i(t)$, we can define the overall profits in a time interval $[a, b]$ of a dynamic game as:

$$\Pi^i = \int_a^b \Pi_{ins}^i(t) dt$$

There are several kinds of dynamic games. In this dissertation we only have employed the concept of non-cooperative differential games. The solution concept that we have employed for the differential games is the Open-Loop Nash Equilibrium (OLNE) [66]. In the OLNE the equilibrium is defined as an optimal path or strategy for each player that maximizes its profits given the OLNE strategies of the other players. The OLNE does not need any information about the state of the system and is obtained in advance. In the equilibrium no player has an incentive to deviate its strategy from its original path. However, if the state of the system is deviated, given that the OLNE does not have feedback, the OLNE looses its optimality. The Markov-Perfect Nash Equilibrium (MPNE) solves this problem, nevertheless, it is not always possible to obtain.

In order to obtain the OLNE, each player has to solve an optimal control problem, taking into account the strategies of the other players. The concept employed to solve the optimal control problems is the Pontryagin Maximum Principle [30].
2.3 Non-linear Optimization

The equilibrium concepts described in the previous section are essential to solve games with 2 or more players, however, when a game has only one player the solution of the game is an optimization problem. Another characteristic of our problems is that the objective function and the constraints are not linear with the optimization variables. For this reason, the techniques described below are focused on the nonlinear optimization.

Nonlinear optimization techniques differ if the problem solved is an static one-shoot game or a dynamic game.

2.3.1 Static optimization

When the analyzed scenario does not evolve over time and the player only decides once its strategy we are in a static optimization scenario. When the problem is constrained due to physical limitation of the parameters or due to logical concepts, such as that the price paid for a service must be positive, we have several tools available \[29\]. For problems with convex constraints we can use the following analytic tools:

- **Lagrangian multipliers**: This tool allows us to solve optimization problems with equality constraints. Assuming a two variable problem with the following structure:

\[
\max_{x,y} f(x, y) \text{ s.t. } g(x, y) = c, \tag{2.16}
\]

where \( f(x, y) \) is the objective function, \( g(x, y) \) is a the constraint function, \( x, y \) are the optimization variables and \( c \) is a constant. If \( f(x, y) \) and \( g(x, y) \) are continuous functions, at least twice differentiable and defined over all the domain, the solution to the problem

\[
\nabla_{x,y,\lambda} F(x, y, \lambda) = 0, \tag{2.17}
\]

are maximums, minimums or saddle points of the problem (2.16). Where

\[
F(x, y, \lambda) = f(x, y) + \lambda(g(x, y) - c) \tag{2.18}
\]

is an auxiliary function, \( \lambda \) is the Lagrange multiplier, \( \nabla_{x,y,z} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \) is the gradient and \( \nabla_{x,y} f(x, y) = -\lambda \nabla_{x,y} g(x, y) \).

- **KKT**: This is an extension of the Lagrange multipliers, that allows us to solve optimization problems with inequality constraints of the form \( g(x) \geq 0 \). Assuming once again that \( f(x, y) \) and \( g_i(x, y) \) \( (i = 1, \ldots, m) \) are continuous functions, at least twice differentiable and defined over all the domain, we define an optimization problem with two variables:

\[
\max_{x,y} f(x, y) \text{ s.t. } g_i(x, y) \geq 0. \tag{2.19}
\]

The KKT theorem provides us the necessary conditions of the solution of the optimization problem. However, if \( f \) and \( g_i \) are also concave functions, the KKT conditions:

\[
\begin{align*}
\lambda_i^* & \geq 0, \\
\lambda_i^* g_i(x^*, y^*) & = 0, \\
\nabla_{x,y} f(x^*, y^*) + \sum_{i=1}^{m} \lambda_i^* \nabla_{x,y} g(x^*, y^*) & = 0,
\end{align*}
\tag{2.20}
\]

17
are not only necessary, but sufficient conditions for the solution \((x^*, y^*)\) of the optimization problem (2.19).

Nevertheless, when the constraints are not "mathematically friendly", or equivalently, they are not convex, we cannot use the previous methods. In such cases it is needed to use heuristic approaches to optimization:

- **Genetic Algorithms**: The genetics algorithms try to replicate the behavior of biological populations in order to find a good numerical solution for very complex problems. At the start, a huge number of candidates to maximum are generated randomly and each candidate is evaluated using a fitness function (in our case, the objective function \(f\)) in order to see how good it is. The best candidates are selected as seeds for next generations of candidates. The new generation of candidates is obtained using mutation and crossover of the best candidates in the previous generations. Genetics algorithms repeat this process until they reach a maximum number of generations or a solution good enough.

Genetics algorithms are able to perform a global maximization, where other numerical methods fail to converge, like when we have non continuous objective functions.

### 2.3.2 Dynamic Optimization

The optimization stages are solved using optimal control theory [30], which allows us to do a dynamic optimization within a time horizon and not only in the steady states. As a result of the dynamic optimization we obtain a control function in every instant of time \(t\) that optimizes the objective function within a time horizon \(t \in [0, T]\). The problem that we typically solve is to obtain the optimal capacity or price that maximizes the profits of an OP, given that the behavior of sinks is modeled by an evolutionary game.

\[
\max_p \Pi(p) = \int_0^T e^{-\eta t} \Pi_{\text{ins}}(p, t) dt
\]

\[
s.t. \quad \dot{x}_i = x_i(U_i(X) - U_{\text{avg}}(X)), \quad X(0) = X_0 \quad \text{and} \quad p \in ]0, \mathbb{R}^+[,\]

where \(\Pi_{\text{ins}}(p, t)\) is the instantaneous profit of the OP defined in (2.5), \(\eta\) is a given discount rate, \(X_0\) is the initial state of the system and \(\dot{x}_i\) is a a mean dynamic of the evolutionary game of the users, which in this particular case is the replicator dynamic (2.10).

The previous optimal control problem can be solved using the Pontryagin’s Maximum Principle (PMP), which provides the necessary conditions to find the candidate optimal strategies. For the open-loop case the hamiltonian function of the OP can be defined as:

\[
H = \Pi_{\text{ins}}(p, t) + \sigma \dot{x}_1,
\]

where \(\lambda\) is the adjoint variable of the OP, that models the variation in the profit due to the behavior of the users modeled by the replicator dynamic. The PMP gives the necessary conditions that all candidate
to optimal strategy mush satisfy:

\[ p^*(t) = \max_{p \in [0, \mathbb{R}^+]} H, \]  
\[ \dot{x}_i = x_i(U_i(X) - U_{avg}(X)), \]  
\[ \dot{\sigma}(t) = \sigma \eta - \frac{\partial H}{\partial x_1}, \]  
\[ \sigma(T) = 0. \]  

where (2.23) is the maximality condition, (2.24) is the replicator dynamic, which models the behavior of the sinks, (2.25) is the adjoint equation and (2.26) is the transversality condition. Solving (2.23) we obtain the candidate strategy to maximum in terms of the state \( x_1 \) and the adjoint variable \( \lambda \). Replacing the candidate strategy in the remaining PMP conditions we obtain a system of Partial Differential Equation (PDE) with an initial condition and an end condition. These systems are also known as Two Point Boundary Value Problem (TPBVP) and cannot be solved using traditional methods for PDE, given that they do not have initial conditions for all its variables. The TPBVP can be solved using the shooting method [67]. The problem of the shooting method is that it requires a very good initial estimation of the value of \( \lambda(0) \), otherwise the method may be unstable. In order to obtain the initial estimation we can solve the problem in several steps, starting with very small values of \( T \) and obtaining the value of \( \lambda(0) \) that solves the TPBVP. In the next steps we increase the value of \( T \) progressively, using the value of \( \lambda(0) \) obtained in the previous step, until we solve the problem for the original value of \( T \).
2.4 Network economics

Telecommunication network economics is not a new research topic, and a depth review of the state of the art was needed before starting our research. In this study of the state of the art, we focused on the application of microeconomics to solve several problems related with the next generation networks, as well as to study the economic viability of different network scenarios in the framework of the IoT and MTC. Firstly, the access network problems and the different approaches to solve were analyzed. And secondly, the economic viability is studied.

2.4.0.1 Access Networks

There are different approaches trying to solve the massive access of MTC devices. On the one hand there are local access technologies with a short or medium coverage range. These technologies including IEEE 802.11 and smallcells/picocells can be used in small areas with high density of devices, nevertheless this solution requires adapting the protocols and improving the security, furthermore their deployment is expensive due to the small coverage and in some areas may result impossible. On the other hand there are global access technologies like cellular networks adapted to IoT requirements like narrowband IoT, and unlicensed frequency technologies such as LoRa and Sigfox [16, 68–70]. Some of the advantages of these technologies are the higher coverage, the infrastructure is cheap to deploy, and the protocols are robust and safe[71, 72].

Cellular networks were not created to grant access to a huge number of devices. The recent LTE standards in mobile networks, which are implemented in the nowadays access networks, were not designed to provide service to MTCu but to HTCu, and the new 5G standards are still being developed. HTC traffic needs high bandwidth, mobility and small delays in order to carry services like voice and video streaming. By contrast, MTC traffic is mainly characterized by a huge number of devices, small packet sizes and low mobility. The development of new technologies can help to solve the problems that appear when MTC traffic coexists with HTC traffic in a common infrastructure[73]. There are a lot of proposals trying to manage the congestion in networks with MTCu based on protocol modifications [74–80], but also there are proposals using network pricing as a congestion control tool [81, 82] and as an efficient power control mechanism [83, 84], showing promising results in both, the distribution of the system load and in the control of the energy usage. For instance, the work in [85] proposes a pricing mechanism to prevent the congestion due to a large number of MTC devices trying to access to the channel within a short time interval. An adaptive price mechanism that increases the price when the network is overloaded is proposed and meets two objectives: congestion management and service differentiation mechanism. Reference [86] proposes a model where a centralized system is implemented to control the channel sharing in cognitive radio networks based on a credits system and how this mechanism allows to achieve different sharing objectives. Network pricing also has been used in combination with game theory and machine learning to study the competition in access networks [41], showing an improvement in the network usage and energy consumption.

Nevertheless, these works are focused in very specific aspects of access networks and WSN service provision, and they do not analyze an end-to-end business model, which provides a global point of view of all the system, from then sensors to the final users. For this reason, it is also necessary an analysis from an economic point of view.
2.4.0.2 Economic Viability

As we have shown in the previous point, new technologies are emerging to solve the IoT problems, but a study of the economic viability of these technologies is needed before its adoption by the operators and users. The main objective of the study of the economic models is to identify the viable scenarios where both, operators and users get positive payoffs.

Network pricing is a useful tool for congestion control, but also is a tool needed in the study of the economic viability of a scenario. It helps us to obtain the equilibrium prices where supply and demand are balanced and the operator profits are maximized [54]. To study the economic viability of a network scenario the users utility and the operators profits should be known, and consequently a pricing scheme is needed. The OP profit maximization problem has been addressed several times in the literature as a pricing problem [87–90].

There are works that model simplified cases with one network operator without competition, like [34], which proposes priority queuing to model service differentiation with different delay restrictions and analyzes the provider’s profit, or like [55], that uses game theory to find, select and improve the efficiency of equilibria in wireless networks. There are works that also analyze different types of services like [77], that develops a pricing scheme for a model with one operator providing macrocell and femtocell services.

On the other hand, there are models where two or more operators compete for serving users while they try to maximize their own profits. In these cases the study of competition is needed. Historically, the concepts of the economic analysis of transport systems field [57,91], based on game theory approaches, have been applied to the competition in the networks field [90]. Many works study the competition between operators with several considerations. Some studies analyze the competition in models with homogeneous traffic profiles [92], and also that analyze the competition between operators offering heterogeneous services like [93] or [46], which studies the feasibility of the competitive scenario making use of game theory and a two-queue model to model priority and non-priority services. However the work only considers one type of traffic and does not analyze different profiles, like the coexistence of HTCu and MTCu.

Typically, the economic viability was studied in network connectivity provision scenarios, however, due to the growing of the IoT and the WSN, the study of provision of different services over the network is becoming more relevant. The economic analysis of the services provision allows us to study how the IoT data could be monetized, and therefore study the economic viability from the point of view of actors other than the operators, such as the IoT devices owner. This is a very important point because it allows us to know which scenarios are most likely for IoT and how it will affect the HTCu perception of the network. Despite the small number of studies in this topic, there are some interesting contributions such as [94], which proposes a new business model for WSN-based services, where virtualization of WSN is studied. The virtualization allows the author to separate the WSN infrastructure from the services offered to final users, however the model is not studied from a mathematical perspective. Another business model is studied in [42,88], where a bundling platform acts as an intermediary, buying the data from WSN and selling data-based services to final users, however the model does not analyze the cost of collecting and transmitting the sensors’ data nor a competition scenario. The pricing mechanisms are studied in both articles using game theory and a solution maximizing the platforms’ profit is shown to exist. Another approach based on bundling is [87], where several business models are proposed, nevertheless the work is too general and does not analyze the models in depth. The work in [95] proposes several models, where users purchase providers’ IoT data through a marketplace and analyzes several economic concepts, such as value and pricing of information. In addition, it also analyzes the
competition between providers using a game theory approach, nevertheless the model does not evaluate the quality of the information and how the information is transported from providers to consumers.

The analysis is typically solved statically, and the results are obtained in the equilibrium, where the actors have no incentive to change their decisions [96, 97]. However, some works go one step further, analyzing dynamic problems, where the system parameters may vary over time and the optimization is done within a time interval [39, 49, 84, 98, 99].
CHAPTER 3
ANALYZED SCENARIOS

In this dissertation we propose different scenarios, in order to cover some of the different opened research topics related with the economic viability of wireless communications, in the framework of IoT and MTC. Our global objective in this dissertation is to analyze from an economic point of view the evolution of wireless networks, from the nowadays networks designed to serve HTC to end-to-end MTC solutions. In order to achieve that objective we have analyzed three scenarios. Scenario 1, studies the first stages of the transition, where HTC and MTC traffic must coexist on a common network infrastructure. The next logic step is to provide dedicated connectivity solutions to MTC users, which is analyzed in Scenario 2. Finally, once the MTC connectivity problem is analyzed, we can go one step further, and study the implications of providing services based on the data gathered by the MTC devices, such as sensors. One specific case of that is studied in Scenario 3.

All the scenarios are analyzed using the tools detailed in the previous chapter. In the following sections we describe with more detail the aim of each scenario before analyzing them in depth in the following chapters.

3.1 Scenario 1: HTC and MTC Service Provision on a Common Network Infrastructure

The deployment of networks to serve MTC implies huge investments and the lack of funding is a common problem [100, 101]. To minimize the costs of serving MTC users, we propose a model where the existing network infrastructures are reused in a shared manner between HTC and MTC in Chapter 4. Our objective is to study the economic viability of a transition scenario, where both, HTC and MTC, will be served simultaneously on the same network infrastructure [11], without the need of high investments in network improvements. The scenario is focused on the interaction between users and operators, and between the competing operators. We have modeled the interaction between the operators by applying the concepts of game theory and transport engineering [91] to networks economics. Our main
contributions are to prove that the duopoly provision is economically viable and allows the HTC and MTC to coexist.

3.2 Scenario 2: Dynamic Capacity Provision for Wireless Sensors Connectivity

In Chapter 5, we analyze a scenario where an operator provides a connectivity service to WSN. The OP chooses dynamically the amount of capacity provided in order to optimize its profits, given that the prices are fixed by a regulator. The scenario is analyzed using mathematical modeling and game theory.

Firstly, a static model is solved as a first approximation, then we propose a more realistic dynamic model, using evolutionary games and optimal control theory. The behavior of the sensors is modeled using a delay-sensitive utility function.

The aim of this work is to show the feasibility of the proposed IoT scenario. To achieve this objective we maximize the profits of the network operator in a given time interval. We provide detailed mathematical procedures, not only for optimization problems with fixed parameters, but also for problems where the parameters may vary over time.

3.3 Scenario 3: Wireless Sensor Network-Based Service Provision in a Duopoly Setting with a Monopolist Operator

In Chapter 6, we analyze a scenario where several service providers collect sensing-data and compete to provide a service based on the collected data. The scenario analyzes an end-to-end business model, which provides a global point of view of all the system, from then sensors to the final users.

In the scenario we propose a model where the IoT-SPs are the owners of the WSN. The scenario analyzes not only the competition between IoT-SPs, but also how the sensing data is obtained and the related costs. We study the feasibility of the model from a positive-profit point of view for all the actors. The model is analyzed as two games with two stages each one using game theory. Our model has the peculiarity that both games are connected, through the amount of data collected by the sinks and the price of transmit that data.

3.4 Scenario 4: Wireless Sensor Network-Based Service Provision in a Duopoly Setting with a Monopolist Operator: A Dynamic Approach

In Chapter 7, we go one step further, analyzing the scenario described in Chapter 6 dynamically, which allows us to obtain realistic conclusions, not only about static, but also about evolving scenarios. In the scenario we consider different time scales in the sinks-OP and users-IoT-SPs games. We consider that the changes in the first game are much less frequent, and therefore we can use the static equilibrium
values of the first game to solve the second game dynamically. The behavior of the users is modeled using a Logit dynamic, while the competitions between IoT-SPs is modeled using a differential game.
CHAPTER 4

SCENARIO 1: HTC AND MTC SERVICE PROVISION ON A COMMON NETWORK INFRASTRUCTURE

The objective of this dissertation is to analyze the economic viability of all the stages in the transition from a HTC to a MTC centered network. This scenario attempts to model the early stages of such transition, where HTC and MTC users are served in a common network infrastructure. This approach is a soft transition, where the initial investments in infrastructures are minimal, and allows us to analyze the different problems and implications that may appear in the dedicated MTC networks.

The chapter has the following structure: Section 4.1 describes the scenario, the queue model employed to model it and the pricing scheme, whereas Section 4.2 describes the strategic interactions in the model as well as the different game stages and the strategies used to solve it. Subsection 4.2.1 analyzes the baseline case with one operator serving HTC, while Subsection 4.2.2 studies the case with HTC and MTC in the duopoly case and Subsection 4.2.3 in the monopoly case. Finally, Section 4.3 discus the results.

4.1 General Model

The paper models the following cases:

- **Baseline case**: One operator serving one HTC flow.
- **Duopoly case**: One operator serving an HTC flow and the other operator serving an MTC flow.
- **Monopoly case**: One operator serving one HTC flow and one MTC flow.
Hereinafter the operator serving the HTC (resp. MTC) flow will be called Human Type Communications Operator (HTCo) (resp. Machine Type Communications Operator (MTCo)).

To model the service provided in the above scenarios, we use the two-priority queue model shown in Figure 4.1, where the HTC flow (Q1-priority) has priority over the MTC flow (Q2-ordinary), which is served only when the HTC queue is empty. The HTC flow may suffer a little degradation in its QoS due to the arrival of MTC packets. This is modeled with a non-preemptive model, that is, when one HTC packet tries to access the network and a MTC packet is being served, the HTC packet has to wait. The service discipline of each queue is FIFO (First In First Out). Each flow traffic follows a Poisson process with a mean rate $\lambda_i$, and the transmission time follows a exponential distribution with mean $\tau_i$, where $i = 1$ for the HTC and $i = 2$ for the MTC. It is assumed that the buffer space is unlimited.

The quality perceived by each flow is proposed to be given by a variation of the expression used in [34, 44–46]:

$$Q_i \equiv c \left( \frac{T_i}{\tau_i} \right)^{-\alpha},$$

(4.1)

where $T_i/\tau_i$ is the mean packet service time, normalized by the mean packet transmission time, that is, the minimum value of $T_i$, for each flow. The selected quality function is suitable for many MTC applications with delay requirements [47]. We define $0 < \alpha < 1$ as the user sensitivity to delay, which is assumed to be the same for the two flows; for a given normalized value of the delay, a greater $\alpha$ translates into a worse perceived quality. Note that $Q_i$ decreases with $T_i$, which means that the higher delay, the worse quality and that $Q_i \geq c$. Note that the quality function is different for each flow if the packet size (equivalently $\tau_i$) is different.

HTCo charges a price $p_1$ per packet to the HTC flow, while MTCo charges a price $p_2$ per packet to the MTC flow. Operators’ profits are given by

- $\Pi = \lambda p$ in the baseline case.
- $\Pi_i = \lambda_i p_i \quad i = 1, 2$ in the duopoly case.
- $\Pi_{12} = \lambda_1 p_1 + \lambda_2 p_2$ in the monopoly case.

Our model does not include marginal costs since they are negligible compared to the fixed costs that prevail in telecommunication network operation.
The arrival rate of each traffic flow is influenced by the serving operator through the price \( p_i \) it charges and through the quality \( Q_i \) it offers. The utility that each flow receives is proposed to be given by the difference between the quality perceived by users in monetary units minus the price charged by operators [35]:

\[
U_i = Q_i - p_i \quad i = 1, 2, \quad (4.2)
\]

The adjustment of \( \lambda_i \) (or equivalently \( \rho_i = \lambda_i \pi_i \)) hereinafter will be known as flow i’s subscription decision. The selection of the price \( p_i \) by each operator hereinafter will be known as pricing decision.

### 4.2 Game Analysis

From the model description above we can find the following strategic interactions:

- **Flow i’s subscription decision** depends on **Operator i’s pricing decision**.
- **Operator i’s profit** depends on **flow i’s subscription decision**.
- **Flow i’s subscription decision** depends on **flow j’s decision** with \((j \neq i)\), through the \( Q_i \) factor.
- **Operator i’s profit** depends on **Operator j’s pricing decision** indirectly trough **flow j’s subscription decision**.

The scenarios with such strategic interactions can be modeled and analyzed using game theory. The structure of the games is summarized in Figure 4.2

**Game I: Baseline Case**
- **Stage I: Operator Pricing Decision**
  - The operator chooses a price that maximizes its profit. The optimal price is obtained solving a profit maximization problem.
- **Stage II: HTC flow subscription decision**
  - Given the optimal price fixed by the operator, the value of \( \rho \) is obtained assuming that the population has reached the Wardrop equilibrium.

**Game II: Duopoly Case**
- **Stage I: Operators Pricing Decisions**
  - The HTCo and MTCo compete in a rational manner with the price \( p_i \) in order to maximize their own profits.
- **Stage II: HTC and MTC flows subscription decisions**
  - Given the prices fixed by the operators competition, the values of \( \rho_1 \) and \( \rho_2 \) are obtained.

**Game III: Monopoly Case**
- **Stage I: Operator Pricing Decisions**
  - The monopolistic operator chooses the prices \( p_1 \) and \( p_2 \) that maximizes its profit. The optimal prices are obtained solving a multi-variable optimization problem.
- **Stage II: HTC and MTC flows subscription decisions**
  - Given the prices fixed by the monopolistic operator, the values of \( \rho_1 \) and \( \rho_2 \) are obtained.

Figure 4.2: Description of the games

In all the cases we use two-stage games. In the first stage of Game I/Game III the operator chooses the price/s that maximizes its own profits. In the first stage of Game II each operator chooses its pricing strategy simultaneously and in an independent way, in order to maximize its profits, taking into account the behavior of the other operator. The equilibrium concept used here is the Nash equilibrium. In the second stage of all the games, each flow makes its subscription decision/s in order to reach the Wardrop equilibrium, based on the price published by the operator who serves the flow.
All the games were solved using backward induction. Backward induction consists in deducing backwards from the end of a problem to the beginning to infer a sequence of optimal actions. Extensive form games may have several Nash equilibria and backward induction helps us to pick out a good equilibrium. Any Nash equilibrium found using backward induction is also a Nash equilibrium for every subgame, or equivalently a Subgame Perfect Nash Equilibrium (SPNE) [26, 61].

In Stage II of each game, Assuming a number of packets high enough, the flow subscription decision of one user will not affect the utility perceived by the rest of the users. Under these conditions the equilibrium reached is that postulated by Wardrop [62], either \( \rho_i > 0 \) and \( U_i = 0 \); or \( \rho_i = 0 \) and \( U_i < 0 \). Under these circumstances a user equilibrium always exists and is unique. Four cases may be identified:

- **Case 1:** \( U_1 = 0 \) and \( U_2 = 0 \).
  \[ Q_i - p_i = 0 \quad i = 1, 2. \] (4.3)

- **Case 2:** \( U_1 = 0 \) and \( U_2 < 0 \), which means \( \rho_2 = 0 \).

- **Case 3:** \( U_1 < 0 \) and \( U_2 = 0 \), which means \( \rho_1 = 0 \).

- **Case 4:** \( U_1 < 0 \) and \( U_2 < 0 \), which means \( \rho_1 = \rho_2 = 0 \).

In Stage I of Games I and III, the prices selected by the operator are obtained solving maximization problems, given the flow subscription decision equilibriums as described above. However, in Game II, given that there is competition between the operators, the price selected by each operator will be such that:

\[ p_1^* = \arg\max_{p_1} \Pi_1(p_1, p_2^*), \] (4.4)

\[ p_2^* = \arg\max_{p_2} \Pi_2(p_1^*, p_2). \] (4.5)

### 4.2.1 Game I: Baseline Case - HTC service provision

In this section we study the baseline case: one operator providing service to an HTC flow. The first stage described in the previous section is reduced to a HTCo pricing decision with the objective of maximize its profits, while the second stage is reduced to the choice between the service provided by the HTCo or no service, where the utility is assumed to be zero.

#### 4.2.1.1 Stage II: HTC flow subscription decision

The mean service time for the M/M/1 queue is given by [23]:

\[ T = \frac{\rho \bar{x}}{1 - \rho} + \bar{x}, \] (4.6)
the quality perceived by the users is
\[ Q = c \left( \frac{1}{1 - \rho} \right)^{-\alpha}, \tag{4.7} \]

Analyzing the flow subscription decision we observe that given a price \( p \) announced by the HTCo, the Wardrop equilibrium is reached when:

- **Case 1:** The packet flow increases its rate until the utility is zero. The \( \rho \) value will be such that makes the utility zero
  \[ Q - p = 0. \tag{4.8} \]
- **Case 2:** The price is so high that makes the utility negative \( (u < 0) \), so no packets subscribe the service \( \rho = 0 \). This happen when \( Q(0) - p < 0 \), or in a similar way \( c^{-\alpha} < p \).

Assuming equilibrium is reached in Case 1 we can obtain \( \rho \) solving (4.8)
\[ \rho = 1 - \left( \frac{p}{c} \right)^{1/\alpha}, \tag{4.9} \]
and its derivative is
\[ \frac{\partial \rho}{\partial p} = -\frac{\left( \frac{p}{c} \right)^{1/\alpha}}{\alpha p}. \tag{4.10} \]

### 4.2.1.2 Stage I: Operator Pricing Decision

The HTCo’s profit is
\[ \Pi = \lambda p = \frac{\rho}{x} p. \tag{4.11} \]

We can maximize the profit of the monopolistic operator in (4.11) setting its derivative with respect to the price equal to zero and checking if the solution obtained is a maximum:
\[ \frac{\partial \Pi}{\partial p} = \frac{1}{x} \left( \rho + p \frac{\partial \rho}{\partial p} \right) = 0, \tag{4.12} \]
Replacing (4.9) and (4.10) in (4.12) and solving we can obtain the price candidate to maximize HTCo’s profit
\[ p^* = \left( \frac{\alpha}{\alpha + 1} \right)^{\alpha} c \tag{4.13} \]
and replacing the maximized price (4.13) in (4.9), we obtain the traffic in the optimal case
\[ \rho^* = \frac{1}{\alpha + 1}. \tag{4.14} \]

Finally we can obtain the maximum profit replacing (4.13) and (4.14) in (4.11)
\[ \Pi^* = \frac{\rho^*}{x} p^* = c \pi^{-1} \frac{\alpha^\alpha}{(1 + \alpha)^{1+\alpha}}. \tag{4.15} \]
4.2.2 Game II: Duopoly Case - HTC and MTC service provision by two competing operators

In this section, we analyze the duopoly case: two competing operators serving the HTC flow (HTCo) and MTC flow (MTCo).

We define the mean service time $T_i \equiv W_i + \pi_i$, where $W_i$ is the mean waiting time. The mean waiting time for each flow for the M/M/1 non-preemptive priority queue can be computed as [23]:

$$ W_1 = \frac{\rho_1 \pi_1 + \rho_2 \pi_2}{1 - \rho_1}, \quad (4.16) $$

$$ W_2 = \frac{\rho_1 \pi_1 + \rho_2 \pi_2}{(\rho_1 - 1)(\rho_1 + \rho_2 - 1)}, \quad (4.17) $$

so that

$$ T_1 = W_1 + \pi_1 = \frac{\rho_2 \pi_2 + \pi_1}{1 - \rho_1}, \quad (4.18) $$

$$ T_2 = W_2 + \pi_2 = \frac{\rho_1 \pi_1 + (\rho_1 + \rho_2 - 2) \pi_2}{(\rho_1 - 1)(\rho_1 + \rho_2 - 1)}, \quad (4.19) $$

Finally, the expressions for the qualities are

$$ Q_1(\rho_1, \rho_2) = c \left( \frac{\rho_2 \pi_2 + \pi_1}{\pi_1 - \rho_1 \pi_1} \right)^{-\alpha}, \quad (4.20) $$

$$ Q_2(\rho_1, \rho_2) = c \left( \frac{(\rho_1 + \rho_2 - 2) \rho_1 + \rho_1 \pi_1 + \rho_1 \pi_2}{(\rho_1 - 1)(\rho_1 + \rho_2 - 1) \pi_2} \right)^{-\alpha}. \quad (4.21) $$

4.2.2.1 Stage II: HTC and MTC flows subscription decisions

As discussed in Section 4.2, the Wardrop equilibria in this specific case are the following ones:

**Case I:** The conditions $U_1 = 0$ and $U_2 = 0$ give equilibrium values $\rho_1$ and $\rho_2$ such that $Q_1(\rho_1^*, \rho_2^*) - p_1 = 0$ and $Q_2(\rho_1^*, \rho_2^*) - p_2 = 0$, and using (4.20) and (4.21) we can get:

$$ \rho_1^* = \frac{\pi_2 \left( \frac{\rho_2}{\rho_2} \right)^{1/\alpha} - 1}{\pi_2 - \pi_1 \left( \frac{\rho_2}{\rho_2} \right)^{1/\alpha}} \left( \frac{c}{p_1} \right)^{1/\alpha} - 1, \quad (4.22) $$

$$ \rho_2^* = \frac{\pi_1 \left( \frac{c}{p_1} \right)^{1/\alpha} - 1}{\pi_2 \left( \frac{\rho_2}{\rho_2} \right)^{1/\alpha} - 1} \left( \frac{\rho_2}{\rho_2} \right)^{1/\alpha} \pi_2 - \pi_1 \left( \frac{\rho_2}{\rho_2} \right)^{1/\alpha} \left( \frac{c}{p_1} \right)^{1/\alpha} - 1, \quad (4.23) $$
Case 2: $\rho_2^* = 0$, so the equations (4.20) and (4.21) become

\[
Q_1(\rho_1, 0) = c \left( \frac{1}{1 - \rho_1} \right)^{-\alpha},
\]

(4.24)

\[
Q_2(\rho_1, 0) = c \left( \frac{(\rho_1 - 1)^2 \bar{x}_2 + \rho_1 \bar{x}_1}{(\rho_1 - 1)^2 \bar{x}_2} \right)^{-\alpha}.
\]

(4.25)

Solving $U_1 = Q_1(\rho_1^*, 0) - p_1 = 0$, we define obtain the value for $\rho_1^*$ when $\rho_2^* = 0$ as

\[
\rho_{1,0}^* = 1 - \left( \frac{p_1}{c} \right)^{1/\alpha}.
\]

(4.26)

Solving $U_2 = Q_2(\rho_1^*, 0) - p_2 < 0$ we obtain the condition

\[
p_2 > \hat{p}_{2L} \equiv Q_2(\rho_{1,0}^*, 0) = c \left( \frac{\bar{x}_2 - \bar{x}_1 \left( \frac{p_1}{c} \right)^{-2/\alpha} \left( \left( \frac{p_1}{c} \right)^{1/\alpha} - 1 \right)}{\bar{x}_2} \right)^{-\alpha}.
\]

(4.27)

Note that $\rho_2^* = 0$ means that MTCo has zero profit.

Case 3: $\rho_1^* = 0$, so the equations (4.20) and (4.21) become

\[
Q_1(0, \rho_2) = c \left( \frac{\rho_2 \bar{x}_2 + \bar{x}_1}{\bar{x}_1} \right)^{-\alpha},
\]

(4.28)

\[
Q_2(0, \rho_2) = c \left( -\frac{1}{\rho_2 - 1} \right)^{-\alpha}.
\]

(4.29)

Solving $U_2 = Q_2(0, \rho_2^*) - p_2 = 0$ for $\rho_2^*$ we obtain

\[
\rho_{2,0}^* = 1 - \left( \frac{p_2}{c} \right)^{1/\alpha}.
\]

(4.30)

Solving $U_1 = Q_1(0, \rho_2^*) - p_1 < 0$ we obtain the condition

\[
p_1 > \hat{p}_{1L} \equiv Q_1(0, \rho_{2,0}^*) = c \left( \frac{\bar{x}_1 - \bar{x}_2 \left( \left( \frac{p_2}{c} \right)^{1/\alpha} - 1 \right)}{\bar{x}_1} \right)^{-\alpha}.
\]

(4.31)

Note that $\rho_1^* = 0$ means that HTCo has zero profit.

Case 4: Finally, with $\rho_1^* = \rho_2^* = 0$ the equations (4.20) and (4.21) become

\[
Q_1(0, 0) = c,
\]

(4.32)

\[
Q_2(0, 0) = c,
\]

(4.33)

and the conditions $U_i = Q_i(0, 0) - p_i < 0$ ($i = 1, 2$) are held for

\[
p_i > \hat{p}_i \equiv Q_i(0, 0) = c.
\]

(4.34)

Note that $\rho_1^* = \rho_2^* = 0$ means that both operators have zero profits.
Table 4.1: HTC & MTC flows: Wardrop Equilibrium Conditions

<table>
<thead>
<tr>
<th>Case</th>
<th>$p_1$ conditions</th>
<th>$p_2$ conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$p_1 \leq \hat{p}<em>1 = \min(\hat{p}</em>{1L}, \hat{p}_{1U})$</td>
<td>$p_2 \leq \hat{p}<em>2 = \min(\hat{p}</em>{2L}, \hat{p}_{2U})$</td>
</tr>
<tr>
<td>2</td>
<td>$p_1 \leq \hat{p}_{1U}$</td>
<td>$p_2 &gt; \hat{p}_{2L}$</td>
</tr>
<tr>
<td>3</td>
<td>$p_1 &gt; \hat{p}_{1L}$</td>
<td>$p_2 \leq \hat{p}_{2U}$</td>
</tr>
<tr>
<td>4</td>
<td>$p_1 &gt; \hat{p}_{1U}$</td>
<td>$p_2 &gt; \hat{p}_{2U}$</td>
</tr>
</tbody>
</table>

As a corollary, the price values for Case 1 are defined by

$$p_1 \leq \hat{p}_1 \quad p_2 \leq \hat{p}_2,$$

where

$$\hat{p}_1 \equiv \min(\hat{p}_{1L}, \hat{p}_{1U}),$$

$$\hat{p}_2 \equiv \min(\hat{p}_{2L}, \hat{p}_{2U}).$$

Table 4.1 summarizes the restrictions on $p_1$ and $p_2$ that define the space of the problem and all the possible cases, while Fig. 4.3 shows a concrete graphical representation for a specific parameter configuration.

Figure 4.3: Wardrop equilibrium regions
4.2.2.2 Stage I: Operators Pricing Decisions

At this point we proceed to analyze the equilibrium prices $p_i^*$, given the values for $\rho_1$ and $\rho_2$ from the previous subsection.

Assuming Case 1 holds, we can compute the profit for each operator in the duopoly scenario, that is

$$\Pi_1 = \frac{p_1 \rho_1}{\bar{x}_1},$$

(4.38)

$$\Pi_2 = \frac{p_2 \rho_2}{\bar{x}_2}.$$  

(4.39)

Applying the Nash equilibrium concept, we obtain the best responses making the partial derivatives equal to zero to obtain the critical points, and checking if the solutions obtained are maximum of the functions

$$\frac{\partial \Pi_1}{\partial p_1} = 0,$$  

(4.40)

$$\frac{\partial \Pi_2}{\partial p_2} = 0.$$  

(4.41)

Proceeding as indicated we obtain the following system:

$$p_1 \frac{\partial \rho_1}{\partial p_1} + \rho_1 = 0,$$  

(4.42)

$$p_2 \frac{\partial \rho_2}{\partial p_2} + \rho_2 = 0,$$  

(4.43)

where

$$\frac{\partial \rho_1}{\partial p_1} = -\frac{\bar{x}_1 \left( \frac{c}{p_1} \right)^{1/\alpha} \left( \bar{x}_1 \left( \frac{c}{p_1} \right)^{1/\alpha} - \bar{x}_2 \right)}{\alpha p_1 \left( \left( \frac{c}{p_2} \right)^{1/\alpha} - 1 \right) \left( \bar{x}_2 - \bar{x}_1 \left( \frac{c}{p_1} \right)^{1/\alpha} \right)^2},$$  

(4.44)

$$\frac{\partial \rho_2}{\partial p_2} = -\frac{\bar{x}_1^2 \left( \left( \frac{c}{p_2} \right)^{1/\alpha} - 1 \right) \left( \frac{c}{p_1} \right)^{1/\alpha} \left( \frac{c}{p_2} \right)^{1/\alpha}}{\alpha p_2 \bar{x}_2 \left( \left( \frac{c}{p_2} \right)^{1/\alpha} - 1 \right)^2 \left( \bar{x}_1 \left( \frac{c}{p_1} \right)^{1/\alpha} - \bar{x}_2 \right)}.$$  

(4.45)

We can simplify the system and obtain

$$A_2 = \frac{\bar{x}_2 (\alpha \bar{x}_2 - (\alpha + 1)A_1 \bar{x}_1)}{A_1 \bar{x}_1^2 \left( \alpha (A_1 - 1) - 1 \right) + \alpha (1 - 2A_1) \bar{x}_2 \bar{x}_1 + \alpha \bar{x}_2^2},$$

(4.46)

where

$$A_1 = \left( \frac{c}{p_1} \right)^{\frac{1}{\alpha}},$$  

(4.47)

and an additional (and cumbersome) equation in $A_1$.

Finally, solving the latter equation numerically and replacing its value in (4.46), we can get the values for $p_1$ and $p_2$ using the inverse transformation of (4.47). Since we have not introduced the restrictions of the Case 1 (4.35) in the optimization problem for analytical tractability reasons, we must check that the solution obtained complies with these restrictions, which will be done in Section 4.3.
4.2.3 Game III: Monopoly Case - HTC and MTC service provision by one operator

In this section we analyze the monopoly case, where one operator is serving both the HTC and the MTC flows.

4.2.3.1 Stage II: HTC and MTC flows subscription decisions

The analysis for the flow subscription decision is the same as in Subsection 4.2.2.1.

4.2.3.2 Stage I: Operator Pricing Decisions

Monopolistic operator’s profit is now

\[ \Pi_{12} = \frac{p_1 \rho_1}{x_1} + \frac{p_2 \rho_2}{x_2}. \]  

(4.48)

The operator will choose \( p_1 \) and \( p_2 \) in order to maximize \( \Pi_{12} \), that is, the first stage is reduced again to an optimal decision problem.

We have solved the maximization problem across the four regions defined in subsection 4.2.2.1. If we assume that the mean MTC flow packet length is lower than the one for the HTC flow \((x_2 < x_1)\), the maximum profit is always reached in Case 3 region, which means \( \rho_1 = 0 \), or equivalently, the operator only serves MTCu given that they are more profitable. It can be easily proved using the equilibrium profits of the baseline case (4.15) and reducing the value of \( \bar{x} \). The operator’s profit is reduced to

\[ \Pi_{12} = \frac{p_2}{\bar{x}_2} p_2. \]  

(4.49)

It can be checked that the expression (4.49) and the restriction for the case 3 (4.29) are the same equations that appear in baseline case (4.11) and (4.7). The only one difference is that this time the service is provided to the MTC instead of HTC, so the results will be the same replacing \( \bar{x} \) by \( \bar{x}_2 \).

The above results imply that a profit maximizing operator that serves both the HTC and the MTC flows will evict the HTC flow. Since our objective is to study the coexistence of HTC and MTC, we will not discuss the monopoly case in the results section.

4.3 Results and discussion

In this section we compute numerically the equilibrium prices, traffic and profits and compare the duopoly case against the baseline case, where there is a monopolistic operator with only HTC.

We set parameters \( \bar{x}_1 = 1, \bar{x}_2 = 0.1, c = 1 \) and we vary \( \alpha \in [0.1, 0.9] \). Figures 4.4, 4.5 and 4.6 show the equilibrium traffic, prices and profits respectively. The values \( \rho^*, p^* \), \( \Pi^* \) refer to the baseline case, while the values \( \rho_1, \hat{\rho}_1, p_1, \Pi_1 \) refer to the HTC flow and \( \rho_2, \hat{\rho}_2, p_2, \Pi_2 \) refer to the MTC flow in the duopoly case. The monopoly case with HTC and MTC is not represented, given that it is equivalent to the baseline case changing the value of \( \bar{x} \), as demonstrated above.
Fig. 4.4 shows that the carried traffic by the HTCo ($\rho_1$) in the duopoly case is greater than the carried traffic in the baseline case ($\rho^*$); and that the carried traffic by the MTCo ($\rho_2$) is greater than zero. In other words, the coexistence is feasible. As $\alpha$ increases, $\rho_2$ increases while $\rho_1$ decreases, which means that HTCu are more affected by delay than MTCu. Furthermore, the overall traffic in the duopoly case is greater than the carried traffic in the baseline case. We can conclude that the efficiency in the usage of the network increases when HTC and MTC coexists.

Fig. 4.5 shows that the prices $p_1$ and $p_2$ are below $\hat{p}_1$ and $\hat{p}_2$ respectively, which confirms that Case 1 holds, as assumed in Section 4.2.2. As $\alpha$ increases all prices decrease, due to the smaller perceived utility, and the sensitivity $\alpha$ has a higher impact on the price in the duopoly case than in the baseline case. We then conclude that the entry of the MTCo causes the price charged both by the HTCo and by the MTCo to decrease, and it is therefore beneficial for the users, given that it allows to increase the number of users subscribed to the service.

Fig. 4.6 shows that the HTCo suffers a negligible decrease in its profit when the MTCo enters, which is caused by the non-preemptive behavior. In addition, the overall profits in the duopoly case are higher than in the baseline case thanks to the MTCo profit contribution. Given that, the operators can agree a payment from the MTCo to the HTCo that makes the MTCo’s entry incentive compatible to both, HTCo and MTCo. We can conclude then that HTC and MTC coexistence is desirable from the point of view of both operators.
Figure 4.5: Comparison between *equilibrium prices* in the duopoly case \((p_1^*, p_2^*)\) and baseline case \((p^*)\) when sensitivity to delay \(\alpha\) varies.

Figure 4.6: Comparison between *equilibrium profits* in the duopoly case \((\Pi_1^*, \Pi_2^*)\) and baseline case \((\Pi^*)\) when sensitivity to delay \(\alpha\) varies.
In the previous chapter we have observed that the service provision to HTC and MTC on a common infrastructure is feasible under certain circumstances. This chapter analyzes the next step, where the transition has been completed and the MTC service provision is done using a dedicated wireless access infrastructure. Specifically, we analyze the connectivity service provision to wireless sensors.

This chapter is organized as follows: in Section 5.1 we describe in detail the scenario and the behavior of the actors involved, the utility of the sinks and the operator profit. In Section 5.2 the scenario is analyzed using a static and a dynamic model. The sinks subscription problem as well as OP profit maximization problem are solved using game theory and optimization. Finally, Section 5.3 shows and discus the results.

5.1 General Model

We consider the IoT scenario which is depicted in Figure 5.1 with several clusters uploading their sensing data to the Internet through a OP. The sensor nodes are grouped into clusters. Each cluster has a large number of sensing nodes connected through a multi-hop wireless network [102]. Each cluster has a sink node, which transmits the data collected by all the nodes in the cluster to the Internet through the OP wireless network. Our scenario is based on [43], and analyzes the interaction between the sinks and the OP. The analyzed model has the following market actors:

- **Sinks.**
Network Operator (OP).

Sinks

Each sink belongs to only one cluster. Each sink is responsible of transmitting all the data collected by its sensors in a cluster to the Internet. They are the clients of the wireless connectivity service offered by the OP. The number of sinks is $N$, where $N \gg 1$.

In order to model the utility perceived by the sinks that subscribe to the OP we use a quality function $Q$ based on [34, 43–46, 103], which evaluates the service offered by the OP:

$$Q \equiv c (T)^{-1},$$  \hspace{1cm} (5.1)

where $c > 0$ is a conversion factor and $T$ is the mean sensing-data-unit (s.d.u) service time. Note that when the service time $T$ increases $Q$ decreases, or equivalently, the sinks perceive a worse quality when the delay of the network increases. This function has the ability to model the congestion in the wireless network, which is suitable for IoT scenarios with delay constraints [47]. We model the OP service as an M/M/1 system, and compute the mean service time $T$ [23] as

$$T = \frac{1}{\mu - \lambda},$$  \hspace{1cm} (5.2)

where $\mu$ is inverse of the mean sensing-data-unit transmission time $\tau = \frac{1}{\mu}$ or simply the system capacity and $\lambda$ is the arrival rate of the s.d.u. We propose a utility function, which models the perception of
the sinks about the service offered by the OP, as the difference between the quality perceived by the sinks and the price charged by the OP. This utility function is also known as compensated utility, and is commonly used in telecommunications [34, 35, 37, 38]

\[ U_s = Q - p = c (\mu - x_1 Nr) - p, \]  

where we have re-written the arrival rates as the traffic generated by all the sinks being served \( \lambda = x_1 r N \), \( \mu \) is inverse of the mean service time or simply the system capacity, \( r \) is the sensing data unit generation rate of one sink, \( p \) is the price in monetary units (m.u.) per s.d.u charged by the OP to each sink when it transmits one sensing data unit and \( x_1 \) is the fraction of sinks being served by the OP.

The utility must be non-negative \( U_s \geq 0 \) or the sink will not subscribe to the service. Note that all the sinks in the system perceive the same utility, and if we consider a number of sensors large enough, distributed randomly, in each WSN, the willingness to pay of each sink is also the same. The distribution of sinks in the system is described by the vector \( X_s = (x_0, x_1) \), where \( x_0 \) and \( x_1 \), are the fraction of sinks being served and not being served by the OP respectively and \( x_0 + x_1 = 1 \).

**Network Operator**

The OP offers a wireless connectivity service to the sinks, that allows them to transmit the data collected, and charges a price \( p \) to the corresponding sink per sensing data unit transmitted.

The objective of the OP is to maximize its own profit choosing the system capacity \( \mu \) given a price \( p > 0 \) fixed by a regulatory authority. The OP profit is:

\[ \Pi_{OP} = x_1 Nr p - k \mu^2, \]  

where \( Nprx_1 \) are the revenues obtained from sinks and we assume quadratic investment costs \( k \mu^2 \) [53] of leasing a system capacity \( \mu \), where \( k \) is a cost scale factor. The convex cost factor allows us to prevent an aggressive behavior of the OP [99, 104], opening the possibility to analyze competitive scenarios in future studies.

Figure 5.2 shows the payment flow described in this section, we observe that the amount of money perceived by the OP is proportional to the traffic generated by all the sinks multiplied by the price that each sink pays per data unit.

![Figure 5.2: Model payments flow and actors involved](image)

### 5.2 Game Analysis

The model described in the previous section can be analyzed as two games with two stages each one. The first game is a static analysis, while the second game is a dynamic analysis of the model. Both games have the following structure: firstly an optimization stage where the OP chooses the capacity that maximizes its profits and secondly a sink subscription stage. The games are summarized in Figure 5.3.
5.2.1 Game I: Static Analysis

This game analyzes our scenario using a static model, where all the parameters are fixed. In this game the actors act with perfect rationality and their decisions are instantaneous. The solution of this game is a Nash equilibrium where no actor has incentive to change its own decisions.

5.2.1.1 Stage II: Sinks Subscription Game

This stage is played once the OP has fixed its $\mu$. Sinks equilibrium was solved using the unified framework provided by Population Games described in [63]. This framework is useful for studying strategic interactions between agents with certain properties that our model satisfies.

Population Game

- **Strategies:** $S = \{0, 1\}$, where 0 means not to subscribe to the OP and 1 means to subscribe to the OP.
- **Social State:** $X_s = \{x_0, x_1\}$, $x_0 + x_1 = 1$. Sinks distribution between not being served and OP.
- **Payoffs:** $F_s(x_0, x_1) = \{F_{s_0}(X), F_{s_1}(X)\} = \{0, U_s\}$, where $U_s(F_{s_1}(X))$ is the utility of the sinks subscribing to the OP defined in (5.3) and $F_{s_0}(X)$ is the utility of the sinks not subscribing to the OP.
**Pure Best Response**  The Pure Best Response $b(X_s)$, is the best response where the actors can only choose a pure strategy [63]. In this case, a pure strategy means that all the population of sinks choose the same strategy. The first step for solving the population game is to obtain the pure strategies that are optimal at each social state $X_s$.

$$b(X_s) \equiv \arg\max_{i \in S} F_{si}(X_s) = \begin{cases} i = 0 & \text{if } \mu \leq \frac{p}{c} + x_1 N r \\ i = 1 & \text{if } \mu \geq \frac{p}{c} + x_1 N r \end{cases}, \quad (5.5)$$

where $i$ is the pure strategy chosen by all the population.

**Mixed Best Response**  The Mixed Best Response $B(X_s)$, is the best response where the actors can choose a mixed strategy [63]. In this case, a mixed strategy means that each sink in the population chooses its strategy based on probabilities, and therefore, the population could be split into several strategies. Once we have obtained the pure best responses we can extend the results to include the best mixed strategies.

$$B(X_s) \equiv \{[z_0 + z_1 = 1; z_i \in R_+] : z_i > 0 \Rightarrow i \in b(X_s)\} = \begin{cases} z_0 = 1, z_1 = 0 & \text{if } x_1 \geq \frac{\mu - p}{c N r} \\ z_0 \geq 0, z_1 \geq 0 & \text{if } x_1 = \frac{\mu - p}{c N r} \\ z_0 = 0, z_1 = 1 & \text{if } x_1 \leq \frac{\mu - p}{c N r} \end{cases}, \quad (5.6)$$

where $z_i$ is the fraction of the population choosing the strategy $i$.

**Nash Equilibrium**  At this point social state $x \in X_s$ is a Nash equilibrium of the game $F_s$ if all the agents choose a best response to $x \in X_s$:

$$NE(F_s) \equiv \{x \in X_s : x \in B(X_s)\} = \begin{cases} (1, 0) & \text{if } \mu \leq \frac{p}{c} \\ (1 - \frac{\mu - p}{c N r}, \frac{\mu - p}{c N r}) & \text{if } \frac{p}{c} \leq \mu \leq \frac{p}{c} + N r \\ (0, 1) & \text{if } \mu \geq \frac{p}{c} + N r \end{cases}. \quad (5.7)$$

### 5.2.1.2 Stage I: OP Capacity Optimization

In this stage the OP wants to maximize its profit given by (5.4) using $\mu$ as the optimization variable and considering the price $p$ fixed by a regulatory authority. Given the three cases obtained from (5.7) we analyze the case where the maximum profit is reached.

$$\Pi_{OP} = \begin{cases} -k \mu^2 & \text{if } \mu \leq \frac{p}{c} \\ \frac{\mu - p}{c} - k \mu^2 & \text{if } \frac{p}{c} \leq \mu \leq \frac{p}{c} + N r \\ N r p - k \mu^2 & \text{if } \mu \geq \frac{p}{c} + N r \end{cases}. \quad (5.8)$$
• Case 1: $\mu \leq \frac{p}{c}$:
  In this case the maximum profit is obtained solving the optimization problem
  \[
  \max_{\mu} \pi_{OP_{c1}} = -k\mu^2
  \]
  subject to $\mu \leq \frac{p}{c}, \quad (5.9)$
  where $\pi_{OP_{c1}}^*$ is the profit obtained in (5.8) for the Case $i$. The solution for the problem defined in (5.9) is
  \[
  \pi_{OP_{c1}}^* = 0 \quad \text{with} \quad \mu^* = 0. \quad (5.10)
  \]
  Note that in this case it is not possible to obtain positive profit.

• Case 2: $\frac{p}{c} \leq \mu \leq \frac{p}{c} + Nr$:
  In this case the maximum profit is obtained solving the optimization problem
  \[
  \max_{\mu} \pi_{OP_{c2}} = \frac{c\mu - p}{c} - k\mu^2
  \]
  subject to $\frac{p}{c} \leq \mu \leq \frac{p}{c} + Nr, \quad (5.11)$
  The problem in (5.11) is solved using KKT conditions and its solution is:
  \[
  \pi_{OP_{c2}}^* = \begin{cases}
  \frac{(c-4k)p^2}{4ck} & \text{if } k > \frac{cp}{2(p+cNr)} \quad \text{with} \quad \mu^* = \frac{p}{2k} \\
  \frac{c^2Npr-k(p+cNr)^2}{c^2} & \text{if } k \leq \frac{cp}{2(p+cNr)} \quad \text{with} \quad \mu^* = \frac{p}{c} + Nr. \quad (5.12)
  \end{cases}
  \]

• Case 3: $\mu \geq \frac{p}{c} + Nr$:
  In this case the maximum profit is obtained solving the optimization problem
  \[
  \max_{\mu} \pi_{OP_{c3}} = -k\mu^2
  \]
  subject to $\mu \geq \frac{p}{c} + Nr, \quad (5.13)$
  The problem in (5.13) is solved again using KKT conditions and its solution is:
  \[
  \pi_{OP_{c3}}^* = \frac{c^2Npr-k(cNr+p)^2}{c^2} \quad \text{with} \quad \mu^* = \frac{p}{c} + Nr. \quad (5.14)
  \]
  Given that the first part of (5.12) is always greater than (5.14) for the problem restrictions, the OP optimal profit can be summarized as:
  \[
  \pi_{OP}^* = \begin{cases}
  \frac{(c-4k)p^2}{4ck} & \text{if } k > \frac{cp}{2(p+cNr)} \quad \text{with} \quad \mu^* = \frac{p}{2k} \\
  \frac{c^2Npr-k(p+cNr)^2}{c^2} & \text{if } k \leq \frac{cp}{2(p+cNr)} \quad \text{with} \quad \mu^* = \frac{p}{c} + Nr. \quad (5.15)
  \end{cases}
  \]
  Analyzing the previous results we observe that $\pi_{OP}^* > 0$ if the following conditions are met:

• Case $k > \frac{cp}{2(p+cNr)}$
  \[
  k < \frac{c}{4} \quad (5.16)
  \]
In this case there are two possible interpretations depending on which is more restrictive (5.17) or 
k \leq \frac{cp}{2(p+cNr)}$. If $c > \frac{p}{N_r}$ then the case condition $k \leq \frac{cp}{2(p+cNr)}$ is more restrictive than (5.17) and therefore there are no additional conditions. However, if $c \leq \frac{p}{N_r}$ then (5.17) is more restrictive and it must be met in order to obtain positive profits.

As shown in the previous analysis the value of $k$ has a vital role in the feasibility of the system, and therefore, has to be bounded in order to obtain positive profits.

5.2.2 Game II: Dynamic Analysis

This game analyzes our scenario using a dynamic model, where the parameters and the decisions of the actors may change over the time. The dynamic analysis was conducted using evolutionary game theory for the Sink Subscription Game, while for the OP capacity optimization stage optimal control theory and PMP were used.

5.2.2.1 Stage II: Sink Evolutionary Subscription Game

In order to maximize the user utility described in Equation 5.3, we define the following evolutionary game:

- **Strategies:** $S = \{S_0, S_1\}$, where $S_0$ means not to subscribe to the OP and $S_1$ means to subscribe to the OP.
- **Social State:** $X_s(t) = \{x_0(t), x_1(t)\}$, $x_0 + x_1 = 1$. Sinks distribution between not being served and being served by the OP.
- **Payoffs:** $U_s(t) = \{u_0(t), u_1(t)\} = \{0, U_s(t)\}$, where $U_s(t)$ ($u_1(t)$) is the utility of the sinks subscribing to the OP as a function of time defined in (5.3) and $u_0(t)$ is the utility of the sinks not subscribing to the OP. Note than here the utility varies with the time due to the variation on the social state.

The sinks use a set of rules to update their strategies. This set of rules is known as revision protocol [63] and determines the evolutionary dynamic. There are several revision protocols, but we are interested in the imitative protocols and direct selection protocols. In the imitative protocols the users updates their strategies taking into account the strategies chosen by other users. But imitative protocols admit boundary rest points that are not Nash equilibria of the underlying game [64]. On the other hand direct selection protocols are not directly influenced by the choice of others and this characteristic prevents the boundary rest points. In this work we have chosen an imitative protocol, given that it is tractable analytically and widely used in the literature. However we need to be cautious about the boundary rest points.
The revision protocol used in this work can be described by the following action:

- At the time instant $t$ a user with strategy $S_i$ imitates the strategy $S_j (j \neq i)$ selected by other user if $U_i(t) > U_j(t)$ with probability:

$$\Gamma_{ij}^I(t, x_j, U_i, U_j) = x_j(t)[U_j(t) - U_i(t)]^+. \quad (5.18)$$

The revision protocol was introduced by Schlag in a population game context [105]. Under this protocol a user switches its strategy only if the other user has a better utility. The switching rate is proportional to the difference in the utility and the number of users in the destination strategy. The protocol has D2 data requirements [64].

The mean dynamic can be derived from the proposed revision protocol (5.18) as follows:

$$\dot{x}_i = \sum_{j \in S} x_j \Gamma_{ji} - x_i \sum_{j \in S} \Gamma_{ij} = \sum_{j \in S} x_i x_j [U_i - U_j]^+ - x_i \sum_{j \in S} x_j [U_j - U_i]^+ = x_i \sum_{j \in S} x_j (U_i - U_j) = x_i \left( \sum_{j \in S} x_j U_j \right) = x_i (U_i - U_{avg})$$

$$\dot{x}_i = \delta x_i (U_i - U_{avg}), \quad (5.19)$$

where $\delta$ is the learning rate and $U_{avg} = \sum_{j \in S} x_j U_j$ is the average utility of all the users in the model.

Following the mean dynamic described above users learn progressively the best choice until the market reaches a stationary point, where the action of one user has no impact on the utility of the other users and no user has an incentive to switch its strategy. When the equilibrium is reached the utility of all the users is the same $U_i = U_j \forall i, j \in \mathbb{N}$. This mean dynamic is also known as Replicator Dynamic. Adapting (5.19) to our model we obtain:

$$\dot{x}_0 = \delta x_0 (U_0 - x_0 U_0 - x_1 U_1) = \delta x_0 (-x_1 U_1),$$

$$\dot{x}_1 = \delta x_1 (U_1 - x_0 U_0 - x_1 U_1) = \delta x_1 (U_1 - x_1 U_1). \quad (5.20)$$

Given that $x_1 = 1 - x_0$ we can work only with one of the previous equations without loss of generality.

**Dynamic Stationary Points**

The dynamic reaches a stationary point when no user is willing to change its strategy, or equivalently when $\dot{x}_i = 0$.

$$\dot{x}_1 = \delta x_1 (U_1 - x_1 U_1) = 0,$$

$$\delta x_1 U_1 (1 - x_1) = 0$$

Solving the previous equation and assuming that $\delta > 0$ we get the following steady-states:

- Case 1:

$$x_1 = 0, \quad x_0 = 1. \quad (5.21)$$
• Case 2:
\[
1 - x_1 = 0 \\
x_1 = 1, \quad x_0 = 0.
\] (5.22)

• Case 3:
\[
U_1 = c(\mu - x_1 r N) - p = 0 \\
x_1 = \frac{c\mu - p}{cN r}, \quad x_0 = 1 - \frac{c\mu - p}{cN r}.
\] (5.23)

### Stability of Stationary Points

Once we have found the stationary points it is necessary to characterize their stability. Consider a steady-state \(x \in X_s\) where sinks perceive a utility \(U_s(x)\) and an invader state \(y \in X_s\) where some sinks move to a different strategy and they perceive a utility \(U_s(y)\). We can affirm that \(x \in X_s\) is a GESS \([63]\) if:
\[
U_s(y) - U_s(x) < 0 \quad \forall \quad y \in X - \{x\},
\] (5.24)
which means that the utility perceived by the sinks who do not switch their strategy from state \(x \in X_s\) is higher than the utility perceived by the sinks who switched it. An equivalent definition is that the utility of sinks who switch their strategy decreases or the utility of sinks who keeps their strategy increases, while the utility of sinks who switch remains constant \([65]\). We can apply this definition to the steady-states found in the previous point:

• Case 1: \(X = (x_0 = 1, \quad x_1 = 0)\)
  Consider that a mass of sinks \(\epsilon\) migrate from strategy \(S_0\) to \(S_1\), which leads us to the new social state
  \[
  X' = (x'_0 = 1 - \epsilon, \quad x'_1 = \epsilon).
  \]
The utility of sinks in both states is:
  \[
  U_s(x_0) = 0, \quad U_s(x_1) = c\mu - p, \\
  U_s(x'_0) = 0, \quad U_s(x'_1) = c(\mu - \epsilon N r) - p.
  \]
This steady-state is a GESS if
\[
U_s(x_0) > U_s(x'_1) \quad \text{or} \quad U_s(x'_0) > U_s(x'_1) \\
0 > c(\mu - \epsilon N r) - p.
\]
For all the possible values of \(\epsilon \in [0, 1]\) it is true if
\[
\mu \leq \frac{p}{c}.
\] (5.25)

• Case 2: \(X = (x_0 = 0, \quad x_1 = 1)\)
  Consider that a mass of sinks \(\epsilon\) migrate from strategy \(S_1\) to \(S_0\), which leads us to the new social state
  \[
  X' = (x'_0 = \epsilon, \quad x'_1 = 1 - \epsilon).
  \]
The utility of sinks in both states is:

\[ U_s(x_0) = 0, \quad U_s(x_1) = c(\mu - Nr) - p, \]
\[ U_s(x'_0) = 0, \quad U_s(x'_1) = c(\mu - \epsilon Nr) - p. \]

This steady-state is a GESS if

\[ U_s(x_1) > U_s(x'_0) \quad \text{or} \quad U_s(x'_1) > U_s(x'_0) \]
\[ c(\mu - Nr) - p > 0 \quad \text{or} \quad c(\mu - \epsilon Nr) - p > 0. \]

For all the possible values of \( \epsilon \in [0, 1] \) it is true if

\[ \mu \geq \frac{p}{c} + Nr. \quad (5.26) \]

Case 3: \( X = (x_0 = 1 - \frac{c\mu - p}{cN_r}, \quad x_1 = \frac{c\mu - p}{cN_r}) \)

Consider that a mass of sinks \( \epsilon \) migrate from strategy \( S_1 \) to \( S_0 \), which leads us to the new social state

\[ X = (x_0 = 1 - \epsilon - \frac{c\mu - p}{cN_r}, \quad x_1 = \frac{c\mu - p}{cN_r} - \epsilon). \]

The utility of sinks in both states is:

\[ U_s(x_0) = 0, \quad U_s(x_1) = c(\mu - \frac{c\mu - p}{cN_r} Nr) - p = 0, \]
\[ U_s(x'_0) = 0, \quad U_s(x'_1) = c(\mu - \left(\frac{c\mu - p}{cN_r} - \epsilon\right) Nr) - p. \]

The necessary conditions to be a GESS are

\[ U_s(x_1) > U_s(x'_0) \quad \text{or} \quad U_s(x'_1) > U_s(x'_0) \]
\[ 0 > 0 \quad \text{or} \quad c(\mu - \left(\frac{c\mu - p}{cN_r} - \epsilon\right) Nr) - p > 0. \]

For all the possible values of \( \epsilon \in \left[0, \frac{c\mu - p}{cN_r}\right] \) it is true if

\[ \mu > \frac{p}{c}. \quad (5.27) \]

On the other hand, if we analyze the case when a mass of sinks \( \epsilon \) migrate from strategy \( S_0 \) to \( S_1 \), we obtain the new social state

\[ X = (x_0 = 1 - \epsilon - \frac{c\mu - p}{cN_r}, \quad x_1 = \frac{c\mu - p}{cN_r} + \epsilon). \]

The utility of sinks in both states is:

\[ U_s(x_0) = 0, \quad U_s(x_1) = c(\mu - \frac{c\mu - p}{cN_r} Nr) - p = 0, \]
\[ U_s(x'_0) = 0, \quad U_s(x'_1) = c(\mu - \left(\frac{c\mu - p}{cN_r} + \epsilon\right) Nr) - p. \]

The necessary conditions to be a GESS are

\[ U_s(x_0) > U_s(x'_1) \quad \text{or} \quad U_s(x'_0) > U_s(x'_1) \]
\[ 0 > c\left(\mu - \left(\frac{c\mu - p}{cN_r} + \epsilon\right) Nr\right) - p. \]

For all the possible values of \( \epsilon \in \left[0, 1 - \frac{c\mu - p}{cN_r}\right] \) it is true if

\[ \mu < \frac{p}{c} + Nr. \quad (5.28) \]

With (5.27) and (5.28) we have the sufficient conditions where this state is a GESS:

\[ \frac{p}{c} < \mu < \frac{p}{c} + Nr. \quad (5.29) \]
In the previous analysis we have demonstrated that there is a GESS for all the possible values of the control variable $\mu$. Furthermore, in every single population game, like in our model, it can be demonstrated that every GESS is unique and it is also a Nash equilibrium [63]. In addition, every GESS is also an ESS and, as proven by Barron, it is also an asymptotically stable solution of the dynamic [61].

Figure 5.4 shows a particular case when the GESS is the mixed strategy equilibrium (5.23).

5.2.2.2 Stage I: OP Dynamic Capacity Optimization

The capacity optimization stage was solved using optimal control theory [30], which allows us to do a dynamic optimization within a time horizon and not only in the steady states. As a result of the dynamic optimization we obtained a control function in every instant of time $t$ that optimizes the objective function within a time horizon $t \in [0, T]$. The problem that we are going to solve is to obtain the optimal capacity that maximizes the profits of the OP, given that the behavior of sinks is modeled by the dynamic (5.19):

$$\max_{\mu} \Pi_{OP}(\mu) = \int_0^T e^{-\eta t} \Pi_{OP_{ins}}(\mu) dt,$$

s.t. $\dot{x}_i = \delta x_i (U_i - U_{avg})$, $X_s(0) = X_0$, and $\mu \in \mathbb{R}_{>0},$

where $\eta$ is a given discount rate, $\Pi_{OP_{ins}}(\mu)$ is the instantaneous profit of the OP defined in (5.4) and $X_0$ is the initial distribution of the population.

In order to solve the previous problem we used the PMP, which provides the necessary conditions to find the candidate optimal strategies for the open-loop case. The hamiltonian function of the OP is defined
as:

\[ H = \Pi_{OP_{ns}} + \sigma \dot{x}_1, \]

where \( \sigma \) is the adjoint variable of the OP. Rewriting the Hamiltonian in terms of our model we have:

\[ H = x_1 (\delta \sigma x_1 (-c(\mu + Nr) + cNr x_1 + p) + \delta \sigma (c\mu - p) + Npr) - k\mu^2. \quad (5.31) \]

Following the PMP, all candidate optimal strategies must satisfy the necessary conditions:

\[
\begin{align*}
\mu^*(t) &= \max_{\mu \in [0, R^+]} H, \quad (5.32) \\
\dot{x}_1 &= \delta x_1 (U_1 - U_{avg}), \quad (5.33) \\
\dot{\sigma} &= \sigma \eta - \frac{\partial H}{\partial x_1}, \quad (5.34) \\
\sigma(T) &= 0. \quad (5.35)
\end{align*}
\]

where (5.32) is the maximality condition, (5.33) is the replicator dynamic, which models the behavior of the sinks, (5.34) is the adjoint equation and (5.35) is the transversality condition. Solving (5.32) we obtain the candidate strategy to maximum in terms of the state \( x_1 \) and the adjoint variable \( \sigma \):

\[ \mu^*(t) = -\frac{c\delta \sigma (x_1 - 1) x_1}{2k}. \quad (5.36) \]

Replacing the optimal candidate strategy (5.36) in the remaining PMP conditions and with the initial state condition we have the following system of PDE:

\[
\begin{align*}
\dot{x}_1 &= \frac{\delta (x_1 - 1)x_1(c\delta \sigma + c\delta \sigma x_1 + 2kNr) + 2kp}{2k} \\
\dot{\sigma} &= \frac{2k(\sigma(\delta p + \Gamma) - Npr) - \delta x_1 (c^2 \delta \sigma + 4k(p - cNr) + cx_1 (-3c\delta \sigma + 2c\delta \sigma x_1 + 6kNr))}{2k} \\
x_1(0) &= x_0 \\
\sigma(T) &= 0.
\end{align*}
\quad (5.37)
\]

The previous system is a TPBVP and cannot be solved using traditional methods for PDEs, given that it has no initial conditions for all its variables. Instead of that, it has an initial condition and an end condition. This problem has been solved numerically using the shooting method [67]. Given that the shooting method requires a good initial estimation for the value of \( \sigma(0) \) otherwise it may be unstable, we have solved the problem in several steps, beginning with small values of \( T \) and increasing it in the following stages, using the solution of \( \sigma(0) \) of the previous stage as initial estimation for the present stage.

### 5.3 Results and discussion

In this section, we present the numerical results for the static and dynamic games analyzed in the previous section. The results were obtained for the case when the equilibrium is a mixed strategy. The figures were calculated for the values shown in Table 5.1 unless otherwise specified.
Table 5.1: Reference Case - Static Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality conversion factor ((c))</td>
<td>1</td>
<td>m.u s</td>
</tr>
<tr>
<td>Sensor data generation ratio ((r))</td>
<td>1</td>
<td>m.u s.d.u.</td>
</tr>
<tr>
<td>Operator price ((p))</td>
<td>0.2</td>
<td>m.u s.d.u.</td>
</tr>
<tr>
<td>Total Number of sensors ((N))</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Capacity cost scale parameter ((k))</td>
<td>(\frac{cp}{1.5(Nr+p)})</td>
<td>m.u s.d.u.</td>
</tr>
<tr>
<td>Dynamic’s learning rate ((\delta))</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>Initial social state ((X_s(0)))</td>
<td>({0.05, 0.95})</td>
<td></td>
</tr>
<tr>
<td>End time horizon ((T))</td>
<td>1</td>
<td>s</td>
</tr>
<tr>
<td>Discount rate ((\rho))</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.5: OP optimal capacity in the static and dynamic cases for different values of \(N\).

5.3.1 OP optimal control and sinks’ distribution with static parameters

In order to study the static and dynamic results, we show the optimal capacity \(\mu^*(t)\) and the fraction of sinks being served by the OP \(x_1(t)\) as a function of the time \(t\), for different values of the number of sinks \(N\).

Figure 5.5 shows the OP optimal capacity in the static case and in the dynamic case for different values of \(N\). In both the static and the dynamic analysis, when \(N\) increases, the optimal capacity increases in order to be able to serve the higher number of sinks. Comparing the static and the dynamic analysis we observe that the provider chooses a similar strategy for low values of \(t\). It is different due to the existence of the discount rate \(\eta\). Nevertheless, when \(t\) is close to \(T\) the provider decreases the reserved capacity and when \(t = T\), the total capacity reserved is zero. This behavior makes sense given that the OP optimizes its decision for a limited time interval, and it is not worthy to have costs when the OP has not to provide more services. Fig. 5.6 shows a similar behavior. For low values of \(t\) the population learns the optimal strategy by imitation moving from the initial state to the static Nash equilibrium. The population learns faster the optimal strategy when it has a higher amount of sinks. For values of \(t\) close to \(T\) the utility perceived by the sinks decreases due to the decrease in the capacity offered by the provider. The sinks start to leave the OP service but they are not able to learn fast enough and some sinks remain in the OP when \(t = T\) and it offers no service at all.
5.3.2 OP optimal control and sinks’ distribution with dynamic parameters

In this subsection we show the evolution of the optimal capacity \( \mu^*(t) \) and the fraction of sinks being served by the OP \( x_1(t) \), when the number of sinks in the system is also a function of the time \( N(t) \). The results for two different scenarios are shown. Figures 5.7, 5.8, 5.9 and 5.10 are related to Scenario 1, while figures 5.11, 5.12, 5.13 and 5.14 are related to Scenario 2. The figures for each scenario were calculated for the values shown in Table 5.2.

In both scenarios are shown three different cases.

- Case 1: In this case the values of \( \mu^*(t) \) and \( x_1(t) \) are obtained using the solutions for the static equilibrium obtained in (5.7) and (5.15) for each instant of time. The values of \( \mu^*(t) \) and \( x_1(t) \) are represented in the figures with the names "\( \mu^* \) Static" and "\( x_1^* \) Static" respectively.

- Case 2: In this case the value of \( \mu^*(t) \) is obtained using the solution for the static equilibrium obtained in (5.7) for each time instant. However, the value of \( x_1(t) \) is obtained from the replicator dynamic defined in (5.20). The values of \( \mu^*(t) \) and \( x_1(t) \) are represented in the figures with the names "\( \mu^* \) Static" and "\( x_1^* \) Replicator" respectively. Note that the value of \( \mu^*(t) \) is the same in Case 1 and Case 2. This case analyzes the scenario with a more realistic model, where the behavior of the sinks is not ideal and their reaction against a change in the market is not instantaneous.

- Case 3: In this case the values of \( \mu^*(t) \) and \( x_1(t) \) are obtained from the solution to the optimal control problem defined in (5.37). The values of \( \mu^*(t) \) and \( x_1(t) \) are represented in the figures with the names "\( \mu^* \) Optimal Control" and "\( x_1^* \) Optimal Control" respectively.

5.3.2.1 Scenario 2.1: Decreasing number of sensors

This scenario models a decreasing number of sensors over time due to failures in the sensors during its life as shown in Table 5.3 and Fig. 5.7. The figures were calculated for the values shown in Tables 5.2 and 5.3.

Due to the variation in the number of sensors \( N \), the optimal decision for the OP over time may vary. Fig. 5.8 shows how the system is able to adapt its decisions to variations not only in the distribution
Table 5.2: Reference Case - Dynamic Common Parameters 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenarios 1 and 2</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality conversion factor (c)</td>
<td>1</td>
<td>( \text{m.u s} )</td>
</tr>
<tr>
<td>Sensor data generation ratio (r)</td>
<td>1</td>
<td>( \text{s.d.u} )</td>
</tr>
<tr>
<td>Operator price (p)</td>
<td>0.2</td>
<td>( \text{m.u} )</td>
</tr>
<tr>
<td>Initial Number of sensors (N(0))</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>Dynamic’s learning rate (δ)</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>Initial social state (X_s(0))</td>
<td>{0.25, 0.75}</td>
<td></td>
</tr>
<tr>
<td>End time horizon (T)</td>
<td>0.5</td>
<td>( \text{s} )</td>
</tr>
<tr>
<td>Discount rate (ρ)</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Reference Case - Dynamic Parameters 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario 1 Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolution of Number of sensors (N(t))</td>
<td>( N(0) - \frac{0.7 N(0)}{T e^{0.8 T}} t e^{0.8 t} )</td>
</tr>
<tr>
<td>Capacity cost scale parameter (k)</td>
<td>( \frac{cp}{1.8(cN(0)r + p)} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario 2 Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evolution of Number of sensors (N(t))</td>
<td>( N(0) + \frac{0.7 N(0)}{T e^{0.8 T}} t e^{0.8 t} )</td>
</tr>
<tr>
<td>Capacity cost scale parameter (k)</td>
<td>( \frac{cp}{2.75(cN(0)r + p)} )</td>
</tr>
</tbody>
</table>

of the sinks, but also in the system parameters. The difference between Cases 1 and 2 and Case 3 is small for small values of \( t \), but it increases when \( t \) is close to \( T \). Fig. 5.9 shows the distribution of the sinks as a function of time, while Fig. 5.10 shows the instantaneous profit for all the cases, while the aggregated profits are 48.46 for Case 1, 44.36 for Case 2 and 45.56 for Case 3. We observe how the optimal control strategy, represented in Case 3, allows to increase the OP profits compared with Case 2 despite the lower number of sinks subscribed. This is possible thanks to the lower value of \( \mu^* \), and therefore, a reduction in the investment costs. We also observe how the non-optimal behavior of the sinks caused by the replicator dynamic decreases the OP profits with respect to Case 1, however, a scenario with instantaneous sink decisions is not realistic.

5.3.2.2 Scenario 2.2: Increasing number of sensors

This scenario models an increasing number of sensors over time due to a progressive deployment of new sensors as shown in Table 5.3 and Fig. 5.11. The figures were calculated for the values shown in Tables 5.2 and 5.3.

As in the previous scenario, the variation in the number of sensors changes the OP optimal static solution \( \mu^\text{static} \), as shown in Fig. 5.12. However, in this case the optimal control decision does not follow the static optimal solution. This is possible given that the OP knows in advance the evolution of \( N \) over time and can adapt its strategy to optimize not only the instantaneous profits, but the profits in all the time interval. This strategy allows the OP to maintain all the sensors subscribed during more time as shown in Fig. 5.13 and allows the OP to increase its profits with respect to the static optimization. Fig. 5.14
Figure 5.7: Scenario 2.1: Evolution of the number of sinks $N$ as a function of $t$.

Figure 5.8: Scenario 2.1: OP optimal capacity in the cases with static and dynamic optimization as a function of $t$.

Figure 5.9: Scenario 2.1: Social state in the three studied cases as a function of $t$. 
Figure 5.10: Scenario 2.1: Evolution of the OP profits for different strategies as a function of \( t \) and total profits.

shows the instantaneous profit for all the cases, while the aggregated profits are 81.06 for Case 1, 80.14 for Case 2 and 82.77 for Case 3.
Figure 5.11: Scenario 2.2: Evolution of the number of sinks $N$ as a function of $t$.

Figure 5.12: Scenario 2.2: OP optimal capacity in the cases with static and dynamic optimization as a function of $t$.

Figure 5.13: Scenario 2.2: Social state in the three studied cases as a function of $t$. 
Figure 5.14: Scenario 2.2: Evolution of the OP profits for different strategies as a function of $t$ and total profits.
Once we have demonstrated that it is possible to provide connectivity service to MTC users using a dedicated infrastructure, we can analyze the provision of services based on MTC data. In this scenario we analyze the provision of services based on WSN data using an end-to-end approach, from the gathering of the data by the sensors to the competition between the service providers in order to offer a sensor-data-based service to final users.

This chapter is organized as follows: in Section 6.1, we describe in detail the model with the actors, the utility of each actor and the pricing scheme. In Section 6.2, the two games of the model are described and the subscription and pricing strategies are solved. Finally, Section 6.3 shows and discuss the results.

6.1 General Model

We consider the IoT scenario that is depicted in Figure 6.1 with two IoT-SPs deploying their private WSN in order to provide sensor-data-based services to sensor-data users or simply final users, who pay to the IoT-SPs for this service. The sensor nodes are grouped into clusters. Each cluster has a large number of sensing nodes connected through a multi-hop wireless network [102], and belong to only one IoT-SP. Each cluster has a sink node, which transmits the data collected by all the nodes in the cluster to their IoT-SP server (IoT-SP, srv) through a OP and Internet. In the IoT-SP servers the data is aggregated in order to provide a service to final users. Our scenario has the following market actors:
Figure 6.1: Analyzed scenario with all the actors of the market. Each IoT-SP collects its sensing data through an OP and transmits it to a server (srv) where it is processed in order to offer a service to the Sensor Data Users.

- **Sinks.**
- **Network Operator (OP).**
- **Users.**
- **Internet of Things-Service Providers (IoT-SPs).**

### 6.1.1 Sinks

Each sink belongs to only one IoT-SP. They are responsible of transmitting all the data collected by sensors in a WSN to their IoT-SP server. They are the clients of the wireless connectivity service offered by the OP. The number of IoT-SP \( i \) sinks is \( N_j \), where \( N_j \gg 1 \) (\( j = 1, 2 \)), and \( N_1 + N_2 = N \).

In order to model the utility perceived by the sinks that subscribe to the OP we use a quality function \( Q \) based in [34,44–46,103], which evaluates the service offered by the OP as a latency based service [106]:

\[
Q \equiv c \left( \frac{T}{\tau} \right)^{-1},
\]

where \( c > 0 \) is a conversion factor and \( T/\tau \) is the mean sensing-data-unit service time normalized by the mean sensing-data-unit transmission time \( \tau = \frac{1}{P} \), that is the minimum possible value of \( T \). Note that \( Q \) decreases when the service time \( T \) increases, which means that the users perceive a worse quality when the delay of the network increases. We have chosen this function due to its ability to model
the congestion in the wireless network, which makes it suitable for many IoT scenarios with delay constraints [47]. This quality function also has the ability to model different kinds of users through the value of $\tau$ and different queueing systems through $T$, however, in this model we consider homogeneous sinks, given that we study the competition in a single service provision. We model the OP service as a M/M/1 system, and compute the mean service time $T$ [23] as

$$T = \frac{\tau}{1 - \tau \lambda}.$$  

(6.2)

The utility function models the perception that sinks have about the OP connectivity service. We propose a utility function for the sinks that subscribe to the OP as the difference between the quality perceived by the sinks and the price charged by the operator, also called compensated utility, which is a function widely used in economics and telecommunications [34–38]

$$U_s \equiv Q - p = c(1 - x_1 rN\tau) - p,$$  

(6.3)

where we have re-written the arrival rates as the traffic generated by all the sinks being served $\lambda = x_1 rN$, $r$ is the sensing-data-unit generation rate of one sink, $p$ is the price charged by the OP to each IoT-SP $j$ ($j = 1,2$) when its sinks transmit one sensing-data-unit and $x_1$ is the fraction of sinks being served by the OP. The utility must be positive $U_s \geq 0$, or equivalently, the price charged by the OP should not be higher than the service value perceived by the sink, otherwise the sink will not subscribe to the service. Note that all the sinks, whichever IoT-SP they belong to, perceive the same utility, which means that the fraction of sinks served by the OP is the same for all the IoT-SPs. The distribution of sinks in the system is described by the vector $X_s = (x_0, x_1)$, where $x_0$ and $x_1$, are the fraction of sinks not being served and being served by the OP respectively and $x_0 + x_1 = 1$.

### 6.1.2 Network Operator

The OP offers a wireless connectivity service to the sinks, that allows them to transmit the data collected to their IoT-SP, and charges a price $p$ to the corresponding IoT-SP per sensing-data-unit transmitted.

The objective of the OP is to maximize its own profit announcing a price $p > 0$. The OP profit is:

$$\Pi_{OP} = px_1 rN.$$  

(6.4)

### 6.1.3 Users

Users want to subscribe to a sensor-data service offered by the IoT-SPs. The number of users is $M$, where $M \gg 1$. The utility of a user making the choice $j$ is based on [28, 42]

$$U_{u_j} = \varphi \log \left( \frac{\beta R_j}{f_j} \right) + \kappa_{u_j},$$  

(6.5)

where the first part of the expression is deterministic and is related with the market parameters while the second part $\kappa_{u_j}$ is treated as a random variable that models the unobserved user-specific part of the utility. The random variable $\kappa_{u_j}$ follows a Gumbel distribution of mean 0. The human behavior is hard to predict and usually users within the same population do not have the same preferences. For instance, while some users always prefer the cheapest option others only will change their decision if the difference in
Figure 6.2: Model payments flow and actors involved.

The perceived utility is high enough. All these unknown effects are aggregated in the random variable $\kappa_{u_j}$. In the deterministic part $R_j$ is the quality of the data provided by the WSN to the IoT-SP$_j$, $f_j$ is the price per time unit that users pay to the IoT-SP$_j$ for its service, $\beta$ is a conversion factor and $\varphi > 0$ is a sensitivity parameter that models the relative importance of the rate $R/f$. Larger values of $\varphi$ increase the impact of the rate $R/f$ in users’ choices, while lower values of $\varphi$ reduce the impact. In our model we set the conversion factor $\beta = 1$. We obtain the expression for $R_j$ assuming that the quality of the information is proportional to the number of sinks sending data to the IoT-SP$_j$

$$R_j = x_1 r N_j.$$  \hspace{1cm} (6.6)

The logarithmic relation between physical magnitudes and the human perception observed in (6.5) has been justified in telecommunications through the Weber-Fechner Law \cite{33, 51, 52}.

The users will choose the IoT-SP$_j$ that maximizes their utility $U_{u_j} \geq U_{u_k}$ for $k \neq j$. The distribution of users in the system is described by the vector $X_u = (y_0, y_1, y_2)$, where $y_0$ is the fraction of users not subscribing to any IoT-SP and $y_j$, $j = 1, 2$ is the fraction of users subscribing to IoT-SP$_j$. Note that $y_0 + y_1 + y_2 = 1$.

6.1.4 IoT-Service Providers

The IoT-SPs are the owners of the sensors. IoT-SP$_j$ pays a price $p$ for each sensing-data-unit transmitted by its sinks through the OP and announces a price $f_j$ per time unit that will be charged to its users. According to the previous information, we can compute the IoT-SP$_j$ profit as:

$$\Pi_{IoT-SP_j} = y_j M f_j - x_1 r N_j p = y_j M f_j - R_j p,$$  \hspace{1cm} (6.7)

where $y_j M$ is the number of users subscribed to the IoT-SP$_j$ service and $x_1 r N_j$ is the number of sensing-data-units transmitted by the sinks per time unit through the OP. The first part of the expression is the revenues obtained from the users, while the second part is the cost of transmitting the sensors data through the OP network.

Figure 6.2 shows the pricing scheme of the model described in this section, where WSN$_j$ are all the sinks of the IoT-SP$_j$ ($j = 1, 2$).

6.2 Game Analysis

Optimal profits could be obtained if the IoT-SPs were able to change their sinks’ decisions, however, in most real scenarios it is not possible due to energy limitations. Changing sinks’ decisions implies a constant communication between the IoT-SPs and sinks, which requires a lot of energy, which
typically is a limited resource in WSN [14, 107], although there are cases where the sensors could be wireless-powered [108]. In this paper, we consider the case where the energy is a limited resource, in order to be as general as possible. Assuming that the IoT-SPs cannot influence in the decisions of their sinks, the model can be analyzed as two games of two stages. The model has the characteristic that both games are connected through the value of \( R_j \) in (6.6). Both games have a similar structure: firstly a pricing stage and secondly a subscription stage. The game model is summarized in Figure 6.3.

![Game I: OP and Sinks](image)

**Figure 6.3:** Description of the games stages.

Both games were solved using backward induction. The correct way forward is to solve first Game I and then solve Game II replacing the variables with the equilibrium values obtained in the solution of Game I. In Game I, the second stage is solved using Population Games described in [63], while the pricing stage is solved using optimization methods. In Game II, the second stage is solved using the probability of choice for the Logit model [48], while the first stage is solved using game theory and the concept of Nash Equilibrium.

### 6.2.1 Game I: OP and Sinks

In the first stage, hereinafter **OP pricing stage**, the OP chooses the price \( p \) in order to maximize its profit. The optimal price \( p^* \) is given by the problem

\[
p^* = \arg\max_p \Pi_{OP}(p, X_s).
\]

(6.8)

In the second stage, called **WSNs subscription game**, sinks decide to subscribe or not to the OP connectivity service based on the perceived utility. Sinks have limited information due to the restrictions in power, processing capabilities and memory [14] and their subscription decisions may not be optimal for their IoT-SP.

### 6.2.1.1 WSN Subscription Game

This stage is played once the OP has fixed its price \( p \). Sinks equilibrium is solved using the unified framework provided by Population Games described in [63]. This framework is useful for studying strategic interactions between agents with certain properties that our model satisfies. Furthermore, the analysis is easily extensible from static to dynamic games, which will allow us to obtain more realistic conclusions in future studies. The equilibrium reached is a Nash equilibrium.
Population Game

- **Strategies:** \( S = \{0, 1\} \), where 0 means not to subscribe to the OP and 1 means to subscribe to the OP.
- **Social State:** \( X_s = \{x_0, x_1\} \), \( x_0 + x_1 = 1 \). Sinks distribution between not being served and OP.
- **Payoffs:** \( F_s(x_0, x_1) = \{F_{s0}(X), F_{s1}(X)\} = \{0, U_s(6.3)\} \), where \( F_{s0}(X) \) is the utility of the users choosing the strategy of not to subscribe to the OP and \( F_{s1}(X) \) is the utility of the users choosing the strategy of subscribe to the OP.

**Pure Best Response** The first step for solving the population game is to obtain the pure strategies that are optimal at each social state \( X_s \).

\[
b(X_s) \equiv \arg \max_{i \in S} F_{si}(X_s) = \begin{cases} 
  i = 1 & \text{if } F_{s1}(X_s) \geq F_{s0}(X_s) \Leftrightarrow x_1 \leq \frac{c-p}{\tau Nr} \\
  i = 0 & \text{if } F_{s0}(X_s) \geq F_{s1}(X_s) \Leftrightarrow x_1 \geq \frac{c-p}{\tau Nr}
\end{cases}.
\] (6.9)

**Mixed Best Response** Once we have obtained the pure best responses, we can extend the results to include the optimal mixed strategies.

\[
B(X_s) \equiv \{[z_0 + z_1 = 1; z_i \in R_+] : z_i > 0 \Rightarrow i \in b(X_s)\} = \begin{cases} 
  z_0 = 0, z_1 = 1 & \text{if } x_1 \leq \frac{c-p}{\tau Nr} \\
  z_0 \geq 0, z_1 \geq 0 & \text{if } x_1 = \frac{c-p}{\tau Nr} \\
  z_0 = 1, z_1 = 0 & \text{if } x_1 \geq \frac{c-p}{\tau Nr}
\end{cases}.
\] (6.10)

**Nash Equilibrium** At this point social state \( x \in X_s \) is a Nash equilibrium of the game \( F_s \) if all the agents chooses a best response to \( x \in X_s \):

\[
NE(F_s) \equiv \{x \in X_s : x \in B(X_s)\} = \begin{cases} 
  (0, 1) & \text{if } p \leq c (1 - \tau Nr) \\
  (1 - \frac{c-p}{\tau Nr}, \frac{c-p}{\tau Nr}) & \text{if } c (1 - \tau Nr) \leq p \leq c \\
  (1, 0) & \text{if } p \geq c
\end{cases}.
\] (6.11)

**6.2.1.2 OP Pricing Stage**

In this stage, the OP wants to maximize its profit given by (6.4). Given the three cases obtained from (6.11) we analyze the case where the maximum profit is reached.

\[
\Pi_{OP} = \begin{cases} 
  pNr & \text{if } 0 < p \leq c (1 - \tau Nr) \\
  \frac{c-p}{\tau} & \text{if } c (1 - \tau Nr) \leq p \leq c \\
  0 & \text{if } p \geq c
\end{cases}.
\] (6.12)

- Case \( 0 < p \leq c (1 - \tau Nr) \):
  
  In this case, the maximum profit is obtained solving the optimization problem

\[
\max_p \quad \Pi_{OP}^* = pNr \\
\text{subject to } \quad 0 < p \leq c (1 - \tau Nr).
\] (6.13)
The solution for the previous problem is
\[ \Pi^*_\text{OP}_1 = c (1 - \tau Nr) Nr \quad \text{if} \quad 0 \leq \tau < \frac{1}{Nr} \quad \text{with} \quad p^* = c (1 - \tau Nr). \quad (6.14) \]

Note that if \( \tau Nr > 1 \) the upper limit \( c (1 - \tau Nr) \) is negative and, therefore, there is no possible solution for \( p^* \) in this case.

- **Case** \( c (1 - \tau Nr) \leq p \leq c \):
  
  In this case, the maximum profit is obtained solving the optimization problem
  
  \[ \max_p \quad \Pi^*_\text{OP}_2 = \left( \frac{p^* - p}{\tau} \right) \]
  
  subject to \( c (1 - \tau Nr) \leq p \leq c \) \hspace{1cm} (6.15)

  The problem in (6.15) is solved using KKT conditions in Appendix A, and its solution is:
  
  \[ \Pi^*_\text{OP}_2 = \begin{cases} 
  c (1 - \tau Nr) Nr & \text{if} \quad 0 \leq \tau < \frac{1}{2Nr} \\
  \frac{c}{4\tau} & \text{if} \quad \tau \geq \frac{1}{2Nr} 
  \end{cases} \quad (6.16) \]

  - **Case** \( p \geq c \):
    
    In this case, for any value of \( p \) the maximum profit is
    
    \[ \Pi_{\text{OP}_3} = 0. \quad (6.17) \]

Combining (6.14) and (6.16) the OP optimal profit can be summarized as:

\[ \Pi^*_\text{OP} = \begin{cases} 
  c (1 - \tau Nr) Nr & \text{if} \quad \tau < \frac{1}{2Nr} \quad \text{with} \quad p^* = c (1 - \tau Nr) \\
  \max \left( \{ cNr (1 - \tau Nr) \}, \frac{c}{4\tau} \right) & \text{if} \quad \frac{1}{2Nr} \leq \tau \leq \frac{1}{Nr} \quad \text{with} \quad p^* = \left[ c (1 - \tau Nr), \frac{c}{2} \right] \\
  \frac{c}{4\tau} & \text{if} \quad \frac{1}{Nr} < \tau \quad \text{with} \quad p^* = \frac{c}{2} 
  \end{cases} \quad (6.18) \]

The expression for the profit in (6.18) can be simplified given that \( \frac{c}{4\tau} \geq cNr (1 - \tau Nr) \) for any value of \( c, Nr, r \) and \( \tau \). To prove this we analyze the expressions for any value of \( \tau \)

\[ \frac{c}{4\tau} \geq cNr (1 - \tau Nr), \quad (6.19) \]

re-writing with \( A = \tau Nr \), the previous expression is simplified to:

\[ 1 \geq 4A - 4A^2. \]

We can demonstrate that

\[ \max_A (4A - 4A^2) \leq 1 \quad (6.20) \]

\[ \frac{\partial}{\partial A} (4A - 4A^2) = 0 \]

\[ \frac{\partial}{\partial A} (4A - 4A^2) = 4 - 8A \]

\[ 4 - 8A^* = 0 \]

\[ A^* = \frac{1}{2} \]

\[ 4A^* - 4A^{*2} = 1, \]
which proves (6.19). Figure 6.4 shows a particular case of the demonstration, where we can see how 
\[ \frac{c}{\tau} \geq cN_r (1 - \tau N_r) \] for the range of interest \( \tau \geq \frac{1}{2N_r} \). With the previous demonstration, OP optimal profit can be simplified to:

\[ \Pi_{OP}^* = \begin{cases} 
c(1 - \tau N_r) N_r & \text{if } \tau < \frac{1}{2N_r} \\
c \frac{4}{\tau} & \text{if } \frac{1}{2N_r} \leq \tau 
\end{cases} \] \hspace{1cm} (6.21)

\[ p^* = \begin{cases} 
c(1 - \tau N_r) & \text{if } \tau < \frac{1}{2N_r} \\
c \frac{2}{\tau} & \text{if } \frac{1}{2N_r} \leq \tau 
\end{cases} \] \hspace{1cm} (6.22)

\[ x_{1}^* = \begin{cases} 
1 & \text{if } \tau < \frac{1}{2N_r} \\
\frac{1}{2\tau N_r} & \text{if } \frac{1}{2N_r} \leq \tau 
\end{cases} \] \hspace{1cm} (6.23)

In order to understand better the behavior of the first game we can re-write the equations in terms of the maximum amount of data generated by sensors normalized by the system capacity, which we define as maximum system load \( L \):

\[ L = \tau N_r = \frac{N_r}{\mu} \]

obtaining

\[ \Pi_{OP}^* = \begin{cases} 
c(1 - L) L \mu & \text{if } L < \frac{1}{2} \\
c \frac{4}{L} & \text{if } L \geq \frac{1}{2} 
\end{cases} \] \hspace{1cm} (6.24)

\[ p^* = \begin{cases} 
c(1 - L) & \text{if } L < \frac{1}{2} \\
c \frac{2}{L} & \text{if } L \geq \frac{1}{2} 
\end{cases} \] \hspace{1cm} (6.25)

\[ x_{1}^* = \begin{cases} 
1 & \text{if } L < \frac{1}{2} \\
\frac{1}{2L} & \text{if } L \geq \frac{1}{2} 
\end{cases} \] \hspace{1cm} (6.26)

### 6.2.2 Game II: Internet of Things-Service Providers (IoT-SPs) and Users

The scenario analyzed in this section is a model with two IoT-SPs and M users. In the first stage, also known as *IoT-SPs Pricing stage*, the IoT-SPs compete with the pricing strategies in order to maximize their profits given by (6.7). This game is solved assuming the solution for Game I obtained above.

#### 6.2.2.1 Users Subscription Game

This stage is played when the IoT-SPs have decided their prices \( f_j^* \). The concept of equilibrium used for users is Nash equilibrium.
The utility of the users described in (6.5) is a Logit discrete choice model. In such a model, if the number of users $M$ is large enough, it can be proved that the portion of user choosing the IoT-SP$_j$ equals the probability of a user choosing that option [28, 50]:

$$\omega_j = \frac{\left(\frac{R_j}{f_j}\right)^\varphi}{\sum_{k=0}^{n} \left(\frac{R_k}{f_k}\right)^\varphi} = y_j,$$

where $n$ is the number of IoT-SPs and $\varphi$ is the sensitivity parameter described in (6.5). Given that the utility of the users that do not subscribe is zero $U_{u_i} = 0$, the "no-operator" option is characterized by the ratio $\left(\frac{R_0}{f_0}\right) = 1$. The distribution of users choosing each strategy can be expressed as:

$$y_0 = \frac{1}{\left(\frac{R_1}{f_1}\right)^\varphi + \left(\frac{R_2}{f_2}\right)^\varphi + 1},$$

$$y_1 = \frac{\left(\frac{R_1}{f_1}\right)^\varphi}{\left(\frac{R_1}{f_1}\right)^\varphi + \left(\frac{R_2}{f_2}\right)^\varphi + 1},$$

$$y_2 = \frac{\left(\frac{R_2}{f_2}\right)^\varphi}{\left(\frac{R_1}{f_1}\right)^\varphi + \left(\frac{R_2}{f_2}\right)^\varphi + 1}.$$ 

where $y_0$ is the fraction of users not subscribing and $y_1, y_2$ are the portion of users subscribing to IoT-SP$_1$ and IoT-SP$_2$ respectively.

### 6.2.2.2 IoT-SPs Pricing Stage

In this stage, each IoT-SP wants to maximize its own profit given by (6.7). Given the solution of the previous stage (6.28) and the solution for Game I in (6.21) the providers’ profits in the Nash equilibrium are going to be analyzed.
With the solution of OP-Sinks game and users subscription game we can re-write the profit for the IoT-SPi as:

$$\Pi_{\text{IoT-SPi}}(f_1, f_2) = \frac{f_i M \left( \frac{R_i}{f_i} \right)^\varphi}{\left( \frac{R_i}{f_1} \right)^\varphi + \left( \frac{R_i}{f_2} \right)^\varphi + 1} - pR_i, \quad i = 1, 2. \quad (6.29)$$

In order to find the Nash equilibrium we use the best response functions for both operators defined as follows:

$$BR_1(f_2) = f^*_1(f_2) = \arg \max_{f_1 > 0} \Pi_{\text{IoT-SP1}}(f_1, f_2),$$

$$BR_2(f_1) = f^*_2(f_1) = \arg \max_{f_2 > 0} \Pi_{\text{IoT-SP2}}(f_1, f_2).$$

The Nash equilibrium is obtained from the equation system

$$f^*_1 = \arg \max_{f_1 > 0} \Pi_{\text{IoT-SP1}}(f_1, f^*_2) \quad \text{s.t.} \quad f_1 > 0,$$

$$f^*_2 = \arg \max_{f_2 > 0} \Pi_{\text{IoT-SP2}}(f^*_1, f_2) \quad \text{s.t.} \quad f_2 > 0. \quad (6.30)$$

In order to obtain the optimum prices we equal the partial derivatives to zero

$$\frac{\partial \Pi_{\text{IoT-SP1}}(f_1, f_2)}{\partial f_1} = \frac{M \left( \frac{R_1}{f_1} \right)^\varphi \left( -\varphi + \left( \frac{R_1}{f_1} \right)^\varphi - \left( \frac{R_2}{f_1} \right)^\varphi + 1 + 1 \right)}{\left( \left( \frac{R_1}{f_1} \right)^\varphi + \left( \frac{R_2}{f_1} \right)^\varphi + 1 \right)^2} = 0,$$

$$\frac{\partial \Pi_{\text{IoT-SP2}}(f_1, f_2)}{\partial f_2} = \frac{M \left( \frac{R_2}{f_2} \right)^\varphi \left( \left( \frac{R_2}{f_2} \right)^\varphi - \left( \frac{R_1}{f_2} \right)^\varphi + 1 \right)}{\left( \left( \frac{R_1}{f_1} \right)^\varphi + \left( \frac{R_2}{f_1} \right)^\varphi + 1 \right)^2} = 0.$$

With the change $Ai = \left( \frac{R_i}{f_i} \right)^\varphi$ and simplifying the system we obtain

$$A1 = (\varphi - 1)(A2 + 1), \quad (6.31)$$

$$A2 = (\varphi - 1)(A1 + 1). \quad (6.32)$$

Solving the previous equation system we obtain

$$A1^* = A2^* = \frac{1 - \varphi}{\varphi - 2}. \quad (6.33)$$

Given that $R_i$ and $f_i$ are positive, $Ai$ has to be positive. From Equation (6.33) we can infer that $1 < \varphi \leq 2$, otherwise $Ai$ would be negative, and there would be no real solutions for $f_i$. In addition, we see that there is only one pricing equilibrium different than $(f^*_1 = 0, f^*_2 = 0)$ if and only if $1 < \varphi < 2$. Figure 6.5a shows a particular solution when $1 < \varphi < 2$, where we observe that there is a Nash equilibrium where the best response functions intersect. On the other hand, Figure 6.5b shows how the best response functions of both operators only intersect in $(0, 0)$, and therefore the only possible solution when $\varphi = 2$ is $(f^*_1 = 0, f^*_2 = 0)$.
Reverting the change $f_i = R_i Ai^{-\frac{1}{\varphi}}$, we get the pricing strategies for both IoT-SPs in the equilibrium

$$
\begin{align*}
    f_1^* &= \left( \frac{1}{2-\varphi} - 1 \right)^{-1/\varphi} R_1 \quad \text{s.t.} \quad 1 < \varphi < 2, \\
    f_2^* &= \left( \frac{1}{2-\varphi} - 1 \right)^{-1/\varphi} R_2 \quad \text{s.t.} \quad 1 < \varphi < 2.
\end{align*}
$$

(6.34)

Replacing (6.34) in (6.28) we obtain the users distribution in the equilibrium, that depends only on $\varphi$.

$$
\begin{align*}
    y_0^* &= \frac{2}{\varphi} - 1, \quad y_1^* = \frac{\varphi - 1}{\varphi}, \quad y_2^* = \frac{\varphi - 1}{\varphi}.
\end{align*}
$$

(6.35)

Figure 6.6 shows how when the value of $\varphi$ is close to 1 the percentage of users that subscribe is very small and the prices of the providers are very high. This could be counter-intuitive, but it explains cases where some users are willing to pay huge amounts of money even without clear evidence of a good quality of service. On the other hand, when $\varphi$ is close to 2, almost all the users decide to subscribe. This is caused because all the users act in a more rational behavior, and the providers adjust its prices in order to attract the largest possible number of them.

Finally, replacing the values obtained in (6.22), (6.23) and (6.34) in (6.29) we obtain the profits in the equilibrium for both operators

$$
\begin{align*}
    \Pi_{*\text{IoT-SP1}} &= \begin{cases} 
    N_1 r \left( c (\tau N r - 1) + \frac{(\varphi - 1)(\frac{1}{\varphi} - 1)^{-1/\varphi} M}{\varphi} \right) & \text{if } \tau < \frac{1}{2N r} \\
    \frac{N_1}{4N r} \left( 2 \left( \frac{1}{\varphi} - 1 \right)^{-1/\varphi} (\varphi - 1) M - c \right) & \text{if } \tau \geq \frac{1}{2N r}
    \end{cases} \\
    \Pi_{*\text{IoT-SP2}} &= \begin{cases} 
    N_2 r \left( c (\tau N r - 1) + \frac{(\varphi - 1)(\frac{1}{\varphi} - 1)^{-1/\varphi} M}{\varphi} \right) & \text{if } \tau < \frac{1}{2N r} \\
    \frac{N_2}{4N r} \left( 2 \left( \frac{1}{\varphi} - 1 \right)^{-1/\varphi} (\varphi - 1) M - c \right) & \text{if } \tau \geq \frac{1}{2N r}
    \end{cases}
\end{align*}
$$

(6.36) (6.37)
Analyzing the previous results we observe that $\Pi_{IoT-SP}^* > 0$ if the following conditions are met:

Case $\tau < \frac{1}{2N_r}$:

$$M > \frac{\left(\frac{1}{2-\varphi} - 1\right)^{1/\varphi}}{\varphi - 1} \frac{\varphi c (1 - \tau N_r)}{\varphi - 1}. \quad (6.38)$$

Case $\tau \geq \frac{1}{2N_r}$:

$$M > \frac{\left(\frac{1}{2-\varphi} - 1\right)^{1/\varphi}}{2(\varphi - 1)} \frac{\varphi c}{\varphi - 1}. \quad (6.39)$$

Restrictions (6.38) and (6.39) are represented in Figure 6.7 as $\min M_1$ and $\min M_2$ respectively. When the value of $\varphi$ is near to 1, the impact of the ratio $R/f$ in users’ utility is low, and the providers can increase their prices, obtaining, as shown, positive profits with a very small pool of users. However, when the value of $\varphi$ increases the providers have to decrease their prices in order to attract users and the revenue per user decreases drastically, while the cost of sensor data collection remains constant. In order to obtain positive profits an increasing number of users $M$ is needed as $\varphi$ increases, with an asymptotic behavior in $\varphi = 2$.

### 6.3 Results and Discussion

In this section, we present the numerical results for the games analyzed in the previous section. The results were obtained for the reference case shown in Table 6.1 unless otherwise specified. The figures are structured as follows: Figures 6.8–6.10 are related to Game I, while Figures 6.11–6.16 are related to Game II-IoT-SP$_1$ and Figures 6.17–6.22 are related to Game II-IoT-SP$_2$. 

![Figure 6.6: Distribution of the users between the strategies in the equilibrium.](image-url)
Figure 6.7: Minimum value of $M$ to obtain positive profits with $c = 1$ and $\tau N r = 1/3$.

Table 6.1: Reference Case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality conversion factor ($c$)</td>
<td>1</td>
</tr>
<tr>
<td>Sensor data generation ratio ($r$)</td>
<td>1</td>
</tr>
<tr>
<td>Mean sensing-data-unit transmission time ($1/\mu$)</td>
<td>1/800</td>
</tr>
<tr>
<td>Total Number of sensors ($N$)</td>
<td>200</td>
</tr>
<tr>
<td>Number of IoT-SP$_1$ sensors ($N1$)</td>
<td>$\frac{1}{3}N$</td>
</tr>
<tr>
<td>Number of IoT-SP$_2$ sensors ($N2$)</td>
<td>$\frac{3}{4}N$</td>
</tr>
<tr>
<td>Number of users ($M$)</td>
<td>1000</td>
</tr>
<tr>
<td>Sensitivity ($\varphi$)</td>
<td>1.5</td>
</tr>
</tbody>
</table>
6.3.1 OP Pricing and Profit

In order to study the Game I results we show the optimal price \( p^* \) and the OP profit \( \Pi_{OP} \), varying the maximum system load \( L \) and the parameter \( c \).

Figure 6.8 shows the OP optimal price as a function of \( L \) for different values of \( c \). When \( c \) increases, the optimal price increases as expected, given that \( c \) acts as a conversion factor. More interesting is the behavior of the price when it is analyzed in terms of the maximum system load \( L \). When the maximum system load (eq. L) increases the utility of the sinks decreases given the growing mean transmission time. When \( L < \frac{1}{2} \) the OP decreases its price and thanks to it all the sinks decide to subscribe. Nevertheless, when the generated traffic is more than the half of the network capacity, it is more profitable for the OP to keep constant the price and decrease the percentage of subscribed sinks as shown in Figures 6.8 and 6.9. In terms of real system load \( L_R = x_1 L \), it is equivalent to the maximum system load while \( L \leq 1/2 \), but when \( L > 1/2 \) the real load remains constant in \( L_R = 1/2 \), which means that real system load never exceeds the 50% of the capacity. Another approximation studied in [103] where different priorities were used in the OP wireless network obtained a better efficiency. In order to implement this improvement in our model a sensing data differentiation in delay requirements is needed, where priority traffic has a more restrictive utility function, while non-priority traffic utility function is more relaxed. This would allow us to obtain a better efficiency in the OP network and, in addition, allows the IoT-SPs to offer new services using the sensing data with lower requirements.

Figure 6.10 shows the OP profit as a function of \( L \) for different values of \( c \). The figure shows how the OP profit increases when the system load increases until \( L = \frac{1}{2} \). After this point, the profit remains constant with the system load. In addition OP profit also experiments an increase with \( c \) for any value of \( L \).

6.3.2 IoT-SP\(_1\) and IoT-SP\(_2\) Pricing and Profits

In order to study the Game II results we show the equilibrium price \( f_1^* \) and the IoT-SP\(_1\) profit \( \Pi_{IoT-SP_1} \), varying the sensitivity of the users to the providers’ price \( \varphi \) and the parameters \( c, N \) and \( M \).

Figures 6.11 and 6.12 show the IoT-SP\(_1\) equilibrium price \( f_1^* \) as a function of \( \varphi \) for different values of \( c \).
Figure 6.9: Social state as a function of $L$ for different values of $c$.

Figure 6.10: OP optimal profit as a function of $L$ for different values of $c$. 
and $N$ respectively. The equilibrium price does not depend on the value of $c$ and increases with $N$, due to the higher utility perceived by users. Note that it only happens if the maximum system load $L < 1/2$. We also observe that when users’ sensitivity to $R_1/f_1$ increases, the IoT-SP$_1$ optimal price decreases very fast.

Figure 6.13 shows the IoT-SP$_1$ profit as a function of $\varphi$ for different values of $c$. Similarly to the price the equilibrium profit does not depend on the value of $c$ and decreases with the sensitivity of the users to the price. On the other hand Figure 6.14 shows that the IoT-SP$_1$ profit increases with the value of $N$. This means that final users are willing to pay a higher price if the amount of data collected by the IoT-SP is higher. This will drive to a competition between the IoT-SPs to increase the number of sensors that is not studied in this paper. As shown in Figure 6.15 the profit also increases with $M$ due to the higher pool of users subscribed to the IoT-SP$_1$. Note that here there is not a congestion effect when $M$ increases. With low values of the sensitivity parameter users choices have a very weak dependence on the prices $f_i$ and the IoT-SPs increase hugely its prices. When it occurs the rate of users subscribed to the IoT-SPs is very low, but the higher prices offset it.

Figure 6.16 shows the IoT-SP$_1$ profit as a function of $N$. The figure shows how the profit increases with $N$ until the OP network is congested. After that, the profit remains constant.

The conclusions for the IoT-SP$_2$ obtained from Figures 6.17–6.22 are the same as those obtained for the IoT-SP$_1$ taking into account that the values of $N1$ and $N2$ are different.

From previous results we observe that users’ sensitivity parameter $\varphi$ is critical in the second game. For values of $\varphi < 1$ and $\varphi > 2$ it is not possible to reach an equilibrium with positive profits, as deduced from the analysis. In addition, if the value is in the range $1 < \varphi < 2$ it still has a huge relevance in the IoT-SPs equilibrium decisions and profits, not only in the value of them, but also in the feasibility of the whole model as deduced in (6.38) and (6.39).
Figure 6.12: IoT-SP\(_1\) equilibrium price as a function of \(\varphi\) for different values of \(N\).

Figure 6.13: Scenario 3: IoT-SP\(_1\) equilibrium profit as a function of \(\varphi\) for different values of \(c\).

Figure 6.14: IoT-SP\(_1\) equilibrium profit as a function of \(\varphi\) for different values of \(N\).
Figure 6.15: IoT-SP$_1$ equilibrium profit as a function of $\varphi$ for different values of $M$.

Figure 6.16: IoT-SP$_1$ equilibrium profit as a function of $N$.

Figure 6.17: IoT-SP$_2$ equilibrium price as a function of $\varphi$ for different values of $c$. 
Figure 6.18: IoT-SP2 equilibrium price as a function of $\varphi$ for different values of $N$.

Figure 6.19: IoT-SP2 equilibrium profit as a function of $\varphi$ for different values of $c$.

Figure 6.20: IoT-SP2 equilibrium profit as a function of $\varphi$ for different values of $N$. 
Figure 6.21: IoT-SP\textsubscript{2} equilibrium profit as a function of $\varphi$ for different values of $M$.

Figure 6.22: IoT-SP\textsubscript{2} equilibrium profit as a function of $N$. 

$\varphi$

$\Pi_{\text{IOT-SP2}}$

$N$

$\Pi_{\text{IOT-SP2}}$

$N$

$\varphi$

$M=500$

$M=1000$

$M=1500$

$M=2000$
In Chapter 5 we have analyzed statically and dynamically the provision of connectivity service to MTC users, while in Chapter 6 we have studied statically a more complex scenario, where two IoT-SPs compete to provide WSN-data-based services to final users. In this chapter we go one step further, analyzing the model shown in Chapter 6 dynamically, which allows us to obtain realistic conclusions, not only in static, but also in end-to-end evolving scenarios.

This chapter is organized as follows: in Section 7.1, we describe the model with the actors, the utility of each actor and the pricing scheme. In Section 7.2, the two games of the model are described and the subscription and pricing strategies are solved. Finally, Section 7.3 shows and discusses the results.

7.1 General Model

We consider the IoT scenario that is depicted Chapter 6, Figure 6.1. The scenario in this chapter is the same with small variations in the behavior of the final users and IoT-SPs, which are specified in the following subsection. These differences are due to the differences in the static and dynamic analysis.

In order to improve the readability of Chapter, we are going to explain briefly all the elements of the model. For a detailed explanation, the reader can go to Section 6.1. Our scenario has the following
actors:

- **Sinks**.
- **Network Operator (OP)**.
- **Users**.
- **Internet of Things-Service Providers (IoT-SPs)**.

### 7.1.1 Sinks

The behavior of the sinks is exactly the same shown in Chapter 6. The sinks subscribe to the connectivity service of the OP in order to transmit the gathered sensing data. The behavior of the sinks is modeled through the utility function

\[
U_s(t) \equiv Q(t) - p(t) = c(1 - x_1(t)rN\tau) - p(t),
\]

where \(x_1\) is the fraction of sinks subscribed to the OP, \(r\) is the sensing-data-unit generation rate of one sink, \(N = N_1 + N_2\) is the number of sinks, where \(N_j\) is the number of sinks of the IoT-SP \(j\), \(\tau\) is the mean sensing-data-unit transmission time and \(p\) is the price charged by the OP to each IoT-SP \(j (j = 1, 2)\) when its sinks transmit one sensing-data-unit.

The distribution of sinks in the system is described by the vector \(X_s(t) = (x_0(t), x_1(t))\), where \(x_0\) and \(x_1\), are the fraction of sinks not being served and being served by the OP respectively and \(x_0(t) + x_1(t) = 1\).

### 7.1.2 Network Operator

The profit of the OP is exactly the same shown in Chapter 6. The OP offers a wireless connectivity service to the sinks and charges a price \(p\) to the corresponding IoT-SP per sensing-data-unit transmitted. The OP instantaneous profit function is:

\[
\Pi_{OP_{ins}}(t) = p(t)x_1(t)rN.
\]

### 7.1.3 Users

The behavior of the users has been slightly modified due to mathematical limitations. The users subscribe to the sensing-data-based service of the IoT-SPs. The preferences a user \(j\) are modeled using the Logit discrete choice model and a utility function based on [49]:

\[
U_{u_j}(t) = \varphi \log (\beta R_j(t) - f_j(t)) + \kappa_{u_j},
\]

where \(\varphi \log (\beta R_j(t) - f_j(t))\) is a deterministic part, related with the market parameters, while \(\kappa_{u_j}\) is treated as a random variable that models the unobserved user-specific part of the utility. The random variable \(\kappa_{u_j}\) follows a Gumbel distribution of mean 0. The parameter \(R_j = x_1rN_j\) is the amount of data transmitted to the to the IoT-SP \(j\), \(f_j\) is the price per time unit that users pay to the IoT-SP \(j\) for its service, \(\beta = 1\) is a conversion factor and \(\varphi > 0\) is a sensitivity parameter that models the relative importance of the difference \(R_j - f_j\).
The users will choose the IoT-SP \( j \) that maximizes their utility \( U_{u_j} \geq U_{u_k} \) \( \forall k \neq j \). The distribution of users in the system is described by the vector \( X_u(t) = (y_1(t), y_2(t)) \), where \( y_j \) is the fraction of users subscribed to IoT-SP \( j \). Note that \( y_1(t) + y_2(t) = 1 \), and that all the users subscribe to one of the IoT-SPs.

### 7.1.4 IoT-Service Providers

The profit of the OP is exactly the same shown in Chapter 6. The IoT-SPs provide a sensing-data-based service to the users. Each IoT-SP pays a price \( p \) to the OP for each sensing-data-unit transmitted by its sinks, and charges a price \( f_j \) per time unit to the users subscribed to its service. The IoT-SP \( j \) instantaneous profit is:

\[
\Pi_{IoT-SP_{nx_j}}(t) = y_j(t)M f_j(t) - R_j(t)p(t).
\]  

(7.4)

### 7.2 Game Analysis

The scenario defined in the previous section is analyzed dynamically. The model is analyzed as two different games using game theory. Both games have a similar structure: firstly a pricing stage and secondly a subscription stage. The game model is summarized in Figure 7.1.

In this scenario we can assume that the changes in the Game I, for instance in the deployment of WSNs, are much less frequent than the changes in the Game II, such as the variation in the number of users in the market. In this case, we can consider that most of the time, the outcome of the Game I is constant while the Game II is being played. In addition, the outcome of the Game I is a stationary point that matches with the Nash equilibrium of the static analysis, which could be obtained using backward induction. On the other hand, the changes in the Game II are relatively frequent, and therefore, the equilibrium solution is a function of time, which is obtained using differential games [30].

#### 7.2.1 Game I: OP and Sinks

In this game, the sinks decide to subscribe or not to the OP service in order to transmit the gathered data to their IoT-SP, while the objective of the OP is to maximize its profit. Given that we can consider the Game I static from the point of view of the Game II the solution of this game is the same that we obtained in Chapter 6:
\[ \Pi_{OP}^* = \begin{cases} c(1 - \tau N_r) N_r & \text{if } \tau < \frac{1}{2 N_r} \\ \frac{c}{2} & \text{if } \frac{1}{2 N_r} \leq \tau \end{cases} \quad (7.5) \]

\[ p^* = \begin{cases} c(1 - \tau N_r) & \text{if } \tau < \frac{1}{2 N_r} \\ \frac{c}{2} & \text{if } \frac{1}{2 N_r} \leq \tau \end{cases} \quad (7.6) \]

\[ x_1^* = \begin{cases} 1 & \text{if } \tau < \frac{1}{2 N_r} \\ \frac{1}{2} \tau N_r & \text{if } \frac{1}{2 N_r} \leq \tau \end{cases} \quad (7.7) \]

### 7.2.2 Game II: Internet of Things-Service Providers (IoT-SPs) and Users

In this game we analyze the competition in prices between the two IoT-SPs in order to maximize their profits and the subscription of the users to the IoT-SPs service. This game is analyzed dynamically assuming the static solution for Game I obtained above. The dynamic analysis was conducted using evolutionary game theory for the Users Subscription Game and differential games for the IoT-SPs pricing stage.

#### 7.2.2.1 Users Subscription Stage

In this stage the users decide with which IoT-SP subscribe. The behavior of the users is modeled by the utility function described in (7.3), which is a Logit discrete choice model. In the logit model, we can obtain the probability of a user choosing the IoT-SP \( j \) as :

\[ \omega_j = \frac{e^{\log(R_j - f_j)}}{\sum_{k=1}^{2} e^{\log(R_k - f_k)}}, \quad (7.8) \]

and therefore, the probability of subscribing with each IoT-SP is:

\[ \omega_1 = \frac{R_1 - f_1}{(R_1 - f_1) + (R_2 - f_2)}, \quad (7.9) \]

\[ \omega_2 = \frac{R_2 - f_2}{(R_1 - f_1) + (R_2 - f_2)}. \quad (7.10) \]

In order to maximize dynamically the user’s utility described in (7.3), we define the following evolutionary game:

- **Strategies**: \( S = \{S_1, S_2\} \), where \( S_j \) means to subscribe to the IoT-SP \( j \) service.
- **Social State**: \( X_u(t) = \{y_1(t), y_2(t)\}, \quad y_1 + y_2 = 1 \). Users’ distribution between IoT-SPs.
- **Payoffs**: \( F_a(t) = \{U_{u_1}(t), U_{u_2}(t)\} \). Note than here the utility varies with the time due to the variation on the social state and the prices.
The previous evolutionary game is modeled by the mean dynamic:

\[ \dot{y}_i = \delta \left( \omega_i - y_i \right), \quad (7.11) \]

where \( \dot{y}_i \) represents the instantaneous variation of the population of users subscribed to the IoT-SP \( i \) and \( \delta \) is the learning rate. This mean dynamic is known as the Logit dynamic [49,63]. Substituting the values of our model in (7.11), we obtain the instantaneous variation of the population of users subscribed to each IoT-SP:

\[ \dot{y}_1 = \delta \left( \frac{R_1 - f_1}{(R_1 - f_1) + (R_2 - f_2)} - y_1 \right), \]
\[ \dot{y}_2 = \delta \left( \frac{R_2 - f_2}{(R_1 - f_1) + (R_2 - f_2)} - y_2 \right). \quad (7.12) \]

Figure 7.2 shows the evolution of the population of users as a function of time, when their behavior is modeled by the equations (7.12).

The Logit dynamic, unlike the replicator dynamic, has the property that the pure strategies are not necessarily steady states, as shown in Figure 7.3, where we observe the evolution of the Logit dynamic in a particular case with four different initial states, where the GESS is the mixed strategy equilibrium.

### 7.2.2.2 IoT-SPs Pricing Stage

In Chapter 5 we used an optimal control problem to optimize the profit of the operator, however, in this stage we have two competing operators. In this case, we solve the stage using a differential game, which allows us to analyze the competition dynamically within a time horizon, and not only in the steady states. As a result of the differential game we obtain an optimal path for the price of each provider, given the optimal path of the other provider. In order to solve the optimal path of the dynamic competition, each IoT-SP solves an optimal control problem that maximizes its profits given the strategies of the other IoT-SP and the behavior of the users, or equivalently, a differential game. The dynamic competitions is solved within a time horizon \( t \in [0, T] \). The objective function the IoT-SP \( j \) is:

\[ \max_{f_j} \Pi_{IoT-SP_j} = \int_0^T e^{-\eta t} \Pi_{IoT-SP_{msj}} \, dt, \quad (7.13) \]
where \( \eta \) is a given discount rate, \( \Pi_{IoT-SP_{insj}} \) is the instantaneous profit of the IoT-SP \( j \) defined in (7.4) and \( X_u(0) \) is the initial distribution of the population of users.

In order to solve the previous problem we used the PMP, which provides the necessary conditions to find the candidate optimal strategies for the open-loop case. The Hamiltonian function of the IoT-SPs is defined as:

\[
H_1(f_1, f_2) = \Pi_{IoT-SP_{ins1}} + \sigma_{11}\dot{y}_1 + \sigma_{12}\dot{y}_2
\]
\[
H_2(f_1, f_2) = \Pi_{IoT-SP_{ins2}} + \sigma_{22}\dot{y}_2 + \sigma_{21}\dot{y}_1.
\]

Following the PMP, all candidate optimal strategies of the IoT-SP \( j \) must satisfy the necessary conditions:

\[
f_j^*(t) = \max_{f_j \in [0, \mathbb{R}^+] \cap \mathbb{Z}} H_j,
\]
\[
\dot{y}_j = \delta (\omega_j - y_j),
\]
\[
\dot{\sigma}_{ji} = \sigma_{ji}\eta - \frac{\partial H_j}{\partial y_i},
\]
\[
\sigma_{ji}(T) = 0.
\]

where (7.14) is the maximality condition, (7.15) is the Logit dynamic, which models the behavior of the users, (7.16) are the adjoint equations and (7.17) are the transversality conditions. Solving (7.14) we obtain the candidate strategies \( f_1^*, f_2^* \) to optimal paths in terms of the state \( y_1, y_2 \) and the adjoint variables \( \sigma_{11}, \sigma_{12}, \sigma_{22}, \sigma_{21} \). Replacing the optimal candidate strategies \( f_1^*, f_2^* \) in the remaining PMP
Table 7.1: Game I parameters

<table>
<thead>
<tr>
<th>Game I parameter</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality conversion factor ((c))</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sensor data generation ratio ((r))</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mean sensing-data-unit transmission time ((\tau))</td>
<td>(\frac{1}{600})</td>
<td>(\frac{1}{600})</td>
</tr>
<tr>
<td>Number of IoT-SP(_1) sensors ((N_1))</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>Number of IoT-SP(_2) sensors ((N_2))</td>
<td>100</td>
<td>150</td>
</tr>
</tbody>
</table>

conditions and with the initial state conditions we have the following system of PDE:

\[
\begin{align*}
\dot{y}_1 &= \delta (\omega^*_1 - y_1), \\
\dot{y}_2 &= \delta (\omega^*_2 - y_2), \\
\dot{\sigma}_{11} &= \sigma_{11} \eta - \partial H_{1}^* \partial y_1, \\
\dot{\sigma}_{12} &= \sigma_{12} \eta - \partial H_{1}^* \partial y_2, \\
\dot{\sigma}_{22} &= \sigma_{22} \eta - \partial H_{2}^* \partial y_2, \\
\dot{\sigma}_{21} &= \sigma_{21} \eta - \partial H_{2}^* \partial y_1, \\
\end{align*}
\]

\((7.18)\)

where \(\omega^*_i\) and \(H_i^*\) are the probability of choice the strategy \(i\) and the Hamiltonian function of the operator \(i\) when prices \(f_1, f_2\) are replaced by the candidates to optimal paths \(f^*_1, f^*_2\).

The system \((7.18)\) is a TPBVP that models the solution to the differential game, and cannot be solved using traditional methods for PDEs. This problem has been solved numerically using the shooting method \([67]\)], ans the results are shown in Section 7.3.

### 7.3 Results and Discussion

In this section, we present the numerical results for the Game II given the solutions for the Game I.

The value of the Game I parameters for the two scenarios that we analyzed are shown in Table 7.1, and the solutions of the Game I given that parameters are shown in Table 7.2. On the other hand, the Game II is solved using the reference case parameters shown in Table 7.3 for both scenarios.

#### 7.3.1 Scenario 1: IoT-SPs dynamic competitions with different number of sinks

In this subsection we show the IoT-SPs dynamic competition in the prices \(f^*_1(t)\) and \(f^*_2(t)\) when both providers have a different amount of sinks deployed, given the solution for the Game I shown in Table 7.2, column "Scenario 1". We also show the fraction of users subscribed to each IoT-SP \((y_1(t), y_2(t))\), and the profit of each IoT-SP.
Table 7.2: Game I solutions

<table>
<thead>
<tr>
<th>Game I Variable</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>OP price ($p$)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Fraction os sinks subscribed ($x_1$)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Amount of IoT-SP$_1$ data ($R_1$)</td>
<td>200</td>
<td>150</td>
</tr>
<tr>
<td>Amount of IoT-SP$_2$ data ($R_2$)</td>
<td>100</td>
<td>150</td>
</tr>
</tbody>
</table>

Table 7.3: Reference Case - Game II Differential Game parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scenario 1 and 2 Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of users subscribed to the IoT-SPs ($M$)</td>
<td>1000</td>
</tr>
<tr>
<td>Dynamic’s learning rate ($\delta$)</td>
<td>0.7</td>
</tr>
<tr>
<td>Initial social state ($X_u(0)$)</td>
<td>${0.2, 0.8}$</td>
</tr>
<tr>
<td>Final time horizon ($T$)</td>
<td>10</td>
</tr>
<tr>
<td>Discount rate ($\eta$)</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig. 7.4 shows the variation of the prices as a function of time, while Figure 7.5 shows the evolution of the users’ social state as a function of the time. For small values of $t$ the providers change their prices in order to compensate the changes in the population of users, due to the differences between the static equilibrium values and the initial populations. Once the static equilibrium is reached the behavior of the users and providers is stationary, until $t$ is close to $T$, where the providers increase their prices. We do not observe changes in the users’ social state when the prices increase, given that we have not included the option of not subscribing. It is also interesting to note that the provider with more sinks gathering and transmitting data ($R_i$), is able to fix a higher price and still maintain a higher number of users subscribed to it. We observe a similar behavior in the instantaneous profits of the providers in Figure 7.6, where there is a transition stage until the stationarity is reached, and the profits only deviates from the equilibrium when $t$ is close to $T$. The aggregated profits for the IoT-SP$_1$ are 839325 while for the IoT-SP$_2$ are 298900. We observe that the profits for the IoT-SP$_1$ are higher than the profits of the IoT-SP$_2$ as expected, given that $R_1 > R_2$.

### 7.3.2 Scenario 2: IoT-SPs dynamic competitions with same number of sinks

In this subsection we analyze the same variables when both providers have the same amount of sinks deployed.

In this scenario we observe a similar behavior than in the scenario 1, with the difference that given that the two provider have the same amount of sinks, the equilibrium values for the prices, population and profits are the same for both providers, as shown in Figures 7.7, 7.8 and 7.9. In this case, the aggregated profits are 492837 for IoT-SP$_1$ and 558575 for IoT-SP$_2$. They are not the same due to the differences between the initial state of the population of users and the equilibrium.
Figure 7.4: Scenario 1: IoT-SPs equilibrium prices as a function of $t$.

Figure 7.5: Scenario 1: Social state as a function of $t$.

Figure 7.6: Scenario 1: Evolution of the IoT-SPs profits as a function of $t$. 
Figure 7.7: Scenario 2: IoT-SPs equilibrium prices as a function of $t$.

Figure 7.8: Scenario 2: Social state as a function of $t$.

Figure 7.9: Scenario 2: Evolution of the IoT-SPs profits as a function of $t$. 
CHAPTER 8

CONCLUSIONS AND FUTURE WORK

8.1 Conclusions (English)

Wireless communications are evolving continuously. In the recent years they have experienced an enormous growth thanks to the HTC users. In addition, the introduction of the IoT paradigm and MTC, that are still being studied, are increasing the growth even more. These new paradigms require the solution of new technical problems, but it is also needed to prove that it will be economically feasible for the main network actors, the network operators.

In this dissertation we have analyzed the transition from HTC to MTC centered networks, trying to prove that wireless sensor-based service provision scenarios are feasible from an economic point of view. The analysis was conducted using concepts of microeconomics, game theory, optimal control and optimization. The different stages of the transition are analyzed in three different scenarios. The first scenario is focused on the coexistence of HTC and MTC, where both traffic profiles are served on a common network infrastructure. The second scenario analyzes the connectivity provision service to MTC users using a dedicated network infrastructure. Finally, given that the connectivity of MTC users is solved in the previous scenario, the third scenario is focused on the study of the provision of MTC-data-based services to final users from an end-to-end point of view.

Thanks to the analysis in the previous chapters we have observed that, in the analyzed scenarios, the transition from HTC to MTC centered networks is feasible and that the provision of services is viable with certain restrictions. The first restriction is related with the users of the services. It has been observed that the behavior of the users is vital in order to determine the optimal decisions of the operators and the economic feasibility of the scenarios. Specifically, the most relevant factors are the tolerance of the users to the delay and to the quality of the data gathered by the sensors. The second restriction is related with the investment costs of the service. It has been shown that the variable costs of the providers have to be upper bounded, otherwise the feasibility of the service provision could not be guaranteed for all the actors. We also have observed that separating the different traffic profiles in different categories,
with different priorities in the service, improves the utilization of the networks. In addition, if each traffic profile is served by a different operator in a competitive scenario, it also allows to increase the revenues for all the actors in the market, improving the monetization of the investments and opening the possibility to create new services based on data, that otherwise, would not be used due to its low quality. It also has been demonstrated the possibility of creating a dedicated role in the telecommunications market to offer IoT services, the IoT-SPs, which provide sensor-data services to final users using their own sensors and a third party access infrastructure. Furthermore, we have shown that the pricing is not the only one tool that can be used to optimize the profits of the operators, but also the capacity provision, which could be used in scenarios where the price if fixed by external factors.

In the following sections we describe in detail the results obtained in each scenario.

8.1.1 Main Contributions

8.1.1.1 Scenario 1:

In this scenario we modeled a transition scenario, where human and machine users shared the wireless access infrastructure. The human traffic (HTC) and the machine traffic (MTC) differentiation was modeled using a two-priority queue. The economic viability of the coexistence of both traffic profiles was studied under both, monopolistic and duopolistic scenarios and these scenarios are compared with the baseline case, where only one operator serves HTCu.

We have proved that the HTC-MTC coexistence is only feasible in a duopolistic scenario, given that in monopolistic scenarios, the operator decides to serve only the more profitable users. The entry of a second operator to serve MTC users is desirable not only from the point of view of resource usage efficiency and from the point of view of the users, but also from the point of view of both operators, if a payment mechanism between MTCo and HTCo is agreed. Furthermore, the HTCo profit is hardly affected by the entry of the second operator, given that we consider opportunistic access, however, the quality of the service is also lower, and it has to be compensated with a lower price charged for the MTCo service.

8.1.1.2 Scenario 2:

A capacity provision scenario for wireless sensors connectivity is proposed. The scenario was studied using both, a static model and a more complex, but also more realistic, dynamic model.

The behavior of the sensors was modeled through a utility function based on a congestion model, while the subscription decision was modeled using both: static equilibrium and the replicator dynamic. The OP profit was modeled using the revenues obtained from the sensors and a quadratic investment costs function. The optimal profit in a defined time interval was obtained solving an optimal control problem, using the network capacity as a control variable, and compared against the static optimization.

It has been shown that the optimization using optimal control, when the users are modeled using the replicator dynamic, allows the OP to obtain higher profits than the optimization using the equilibrium solution. In addition, the dynamic optimization allowed the operator to optimize its profits not only in a scenario with fixed parameters, but also in a scenario where the system parameters, like the number of sensors, changed over time. Given the obtained results, we can conclude that the proposed scenario is
feasible from an economic point of view for all the actors if the investment costs scale factor is bounded. Therefore, it is important to invest in the development of the technology, in order to reduce as much as possible these costs. In addition, we show that the optimal control theory is a profitable and a powerful tool for the maximization of the network operator profits in dynamic IoT scenarios and that capacity provision is an alternative to pricing in the profit optimization.

8.1.1.3 Scenario 3:

A novel network model for providing IoT-based services with private sensor networks, using third party access infrastructure, has been studied. The model was analyzed as two games using game theory, population games, Logit discrete choice model, optimization and Nash equilibrium concepts.

Firstly, a congestion model was proposed for the utility of the sensors and it was shown economically viable for the network operator to offer connectivity service, however, the system load never exceeded half of the maximum possible load.

Secondly, a Logit discrete choice model was chosen to model users’ decisions with two IoT-SPs competing for serving them maximizing their own profits. It has been shown that, in the equilibrium, both IoT-SPs obtain the same profits multiplied by the portion of the total sensors that each one has. We observed that the value of users’ sensitivity to the data/price ratio had to be $1 < \alpha < 2$, in order to obtain a providers’ pricing equilibrium other than $(0,0)$. In addition, the number of potential clients had to be high enough to guarantee the feasibility for the IoT-SPs.

Given that both stages have been shown feasible under specific conditions, we can conclude that the whole network model is conditionally feasible from an economic point of view, and therefore, the provision of services based on MTC data is possible, from the deployment of the WSNs that gather the sensing data, to the data-based service provision to the final users.

8.1.1.4 Scenario 4:

The dynamic provision of IoT-based services with private sensor networks, using third party access infrastructure, has been studied. The model was analyzed as two games using concepts of game theory, population games, Logit dynamic, optimization and differential games.

Firstly, the model was analyzed as two games. The first game analyzed the interaction between the OP and the sinks, while the second game analyzed the competition between IoT-SPs to provide services to final users and the behavior of the users. The first game was considered static from the point of view of the users and IoT-SPs, given that the frequency of the changes in that game is much lower than in the second game.

Secondly, a Logit dynamic was chosen to model users’ behavior, while the competition between IoT-SPs in order to maximize their profits was analyzed using a differential game.

It has been shown how the dynamic analysis of the scenario is economically viable in the analyzed cases. In addition, we observe that the competition between providers keeps the prices bounded until the time is close to the end of the optimization interval, therefore, it is recommended to introduce a reservation value or an infinite interval in order to observe the long term behavior. We also observe how the provider with more sinks is able to fix a higher price, keep a higher portion of users subscribed and therefore,
obtain higher profits. Finally, it has been shown how the initial distribution of users alters the profits of two competing providers with the same number of sensors in short term optimizations.

8.1.2 Future Work

Future work will mainly involve the dynamic profit optimization, given that the results are better in the dynamic than in the static approaches. In addition, it is necessary to study more complex scenarios with several competing operators using differential games, which will allow us to study a broader range of scenarios with a higher dynamism in parameters and to identify additional business opportunities for the network actors. This tool will allow us to study scenarios with variable number of clients or the entry of new operators in a market that has already reached the equilibrium, among others.

In our work we have been focused in machine communications with delay-based utilities, however, there are other types of services with a higher dependency in other parameters, such as bandwidth and coverage. In future studies we recommend to analyze these models using a similar methodology, in order to validate our results for a wider range of scenarios.

Given that outcomes may change with the behavior of the users it is necessary to conduct rigorous studies, which allow us to obtain reliable data about users’ behaviors and preferences, in order to be able to tune the parameters of our models and analyze more realistic and complex situations in a precise way.

The analytical methods employed in our analysis gave us a wide perspective of the problem. Nevertheless, this analysis is limited to relatively simple models, and it is not possible to use it in more realistic and complex models due to mathematical limitations. One of the main future research lines is to develop numerical methods that will allow us to study the competition in complex scenarios, using mechanisms such as genetic algorithms.

8.2 Conclusiones (Spanish)

Las comunicaciones inalámbricas están en constante evolución. En los últimos años han experimentado un enorme crecimiento debido al tráfico HTC, sin embargo todavía están en constante evolución. Actualmente, el internet de las cosas y las comunicaciones MTC son paradigmas que todavía están en desarrollo, sin embargo, están aumentando el crecimiento de las redes de una forma más pronunciada. Estos nuevos paradigmas requieren la solución de nuevos problemas técnicos, pero también es necesario para demostrar que será económicamente viable para los principales actores de la red, los operadores de red.

En esta tesis hemos analizado la transición de las redes centradas en usuarios y servicios HTC a MTC, tratando de demostrar que los escenarios de provisión de servicios basados en sensores inalámbricos son factibles desde el punto de vista económico. El análisis se realizó utilizando conceptos de microeconomía, teoría de juegos, control óptimo y optimización. Las diferentes etapas de la transición se han analizado en tres escenarios diferentes. El primer escenario se centra en la coexistencia de HTC y MTC, donde ambos perfiles de tráfico se emplean una infraestructura de red común. El segundo escenario analiza el servicio de provisión de conectividad para usuarios MTC mediante una infraestructura de red dedicada. Finalmente, dado que la conectividad de los usuarios MTC queda resuelta en el escenario anterior, el tercer escenario se centra en el estudio de la provisión de servicios
basados en datos MTC a usuarios finales con un enfoque end-to-end.

Gracias al análisis de los capítulos anteriores, hemos observado que, en dichos escenarios, la transición de las redes centradas en usuarios HTC a MTC es factible y que la provisión de servicios es viable bajo determinadas condiciones. La primera condición está relacionada con los usuarios de los servicios. Se ha observado que el comportamiento de los usuarios es vital para determinar las decisiones óptimas de los operadores y la viabilidad económica de los escenarios. Específicamente, los factores más relevantes son la tolerancia de los usuarios al retardo de los datos y la calidad de los datos recopilados por los sensores. La segunda condición hace referencia a los costes de inversión del servicio. Se ha demostrado que los costes variables de los proveedores deben estar acotados, de lo contrario, la viabilidad de la provisión del servicio no se puede garantizar para todos los actores. También hemos observado que separar los diferentes perfiles de tráfico en diferentes categorías con diferentes prioridades en el servicio mejora la utilización de las redes. Además, si cada perfil de tráfico es atendido por un operador diferente y el escenario es competitivo, también permite aumentar los ingresos para todos los actores del mercado, mejorando la monetización de las inversiones y permitiendo crear nuevos servicios basados en datos, que de lo contrario, serían deshechos. También se ha demostrado la posibilidad de crear un rol específico en el mercado de las telecomunicaciones para ofrecer servicios IoT, los IoT-SPs, que proporcionan servicios basados en datos de sensores a usuarios finales, empleando sus propias redes de sensores y la infraestructura de acceso de un tercero. Además, hemos podido mostrar como la elección de precios no es la única herramienta disponible para la optimización de beneficios, sino también la provisión de capacidad, que resulta eficaz en escenarios donde el precio es fijado por factores externos. En las siguientes secciones se describe en detalle los resultados obtenidos en cada escenario.

8.2.1 Principales contribuciones

8.2.1.1 Escenario 1:

El escenario modelado está planteado como un escenario de transición entre las redes clásicas y redes dedicadas exclusivamente al servicio de MTCu. El escenario modela una etapa temprana en la transición, en la que usuarios HTCu y MTCu comparten una infraestructura de acceso inalámbrico. Los diferentes perfiles de tráfico se diferencian mediante el uso de una cola con dos prioridades. En el escenario se analiza la viabilidad económica de la coexistencia de ambos perfiles de tráfico cuando son servidos por un único operador monopolista o por dos operadores en competencia. Los resultados se comparan con un caso base, en el que un solo operador ofrece servicio a HTCu.

En el trabajo demostramos que la coexistencia HTC-MTC es únicamente factible en un escenario donde los operadores compiten, ya que en el caso de monopolio el operador decide servir únicamente a los clientes que le proporcionan un mayor beneficio. La entrada de un segundo operador para servir a los HTCu es deseable desde un punto de vista de eficiencia de la red, así como desde el punto de vista de los usuarios. Por otro lado, si consideramos un mecanismo de pago entre el operador entrante y el operado existente, el escenario también es factible desde el punto de vista de los operadores. Finalmente, debido a la disciplina de servicio empleada en la cola, se observa que el operador HTC existente apenas sufre variación con la entrada del nuevo operador MTC, sin embargo, la calidad del servicio también es menor, y debe compensarse con un precio más bajo por el servicio del MTCo.
8.2.1.2 Escenario 2:

En este capítulo se propone un escenario de provisión de capacidad para la conectividad de sensores inalámbricos. El escenario es analizado usando dos modelos: un modelo estático y un modelo dinámico, más complejo pero también más realista.

El comportamiento de los sensores se modela empleando una función de utilidad basada en un modelo de congestión, mientras que las decisiones de los usuarios se estudian empleando los conceptos de equilibrio estático y la dinámica del replicador. Los beneficios del OP se modelan utilizando los ingresos percibidos por el pago de los sensores y una función de costes cuadrática con la capacidad de la red reservada. Los beneficios se obtienen para un intervalo de tiempo. En el caso dinámico se resuelve un problema de control óptimo, en el que la capacidad de la red se utiliza como variable de control, y se comparan con los beneficios de la optimización estática.

En los resultados observamos que la optimización mediante control óptimo, cuando los usuarios se comportan según la dinámica del replicador, permite al OP obtener mayores beneficios que la optimización empleando la solución del equilibrio estático. Otra de las ventajas de la optimización dinámica es que permite a los operadores optimizar sus beneficios en escenarios tanto estáticos, con parámetros fijos, como con dinámicos, donde los parámetros, como el número de sensores, varían con el tiempo.

Con los resultados obtenidos podemos concluir que el escenario planteado es factible desde un punto de vista económico para todos los actores. Además, hemos mostrado como la teoría de control óptimo es una herramienta que permite mejorar la maximización de los beneficios de los operadores de red en escenarios IoT dinámicos y que la provisión de capacidad es una alternativa a la fijación de precios para la optimización de beneficios.

8.2.1.3 Escenario 3:

En este escenario presentamos un novedoso modelo para la provisión de servicios de IoT basados en datos de redes de sensores privadas que usan una infraestructura de acceso inalámbrico de un tercero. El modelo se analiza como dos juegos utilizando los conceptos de teoría de juegos, juegos poblacionales, modelo Logit de elección discreta, equilibrio de Nash y optimización.

En el primer juego, se propone un modelo de congestión para la utilidad de los sensores que recopilan los datos y se demuestra que resulta económicamente viable para el operador de red ofrecer un servicio de conectividad a los mismos. Sin embargo, las decisiones óptimas para el operador y el modelo de congestión para los sensores hacen que en el equilibrio la carga del sistema nunca supere la mitad de la capacidad máxima.

En el segundo juego, se propone un modelo Logit de elección discreta para modelar las decisiones de los usuarios, mientras que dos proveedores de servicios IoT compiten para servir a los usuarios buscando maximizar sus beneficios. Se observa que en el equilibrio ambos operadores obtienen los mismos beneficios, multiplicados por la porción total de sensores que cada uno posee. También se observa que la sensibilidad de los usuarios a la relación calidad del servicio/precio debe estar comprendida entre $1 < \alpha < 2$, de modo que se pueda obtener un equilibrio diferente del $(0, 0)$ en los precios de los proveedores de servicio. Finalmente, también se observa que el número de potenciales usuarios debe ser suficiente grande para garantizar la viabilidad del modelo para los IoT-SPs.
Debido a que ambos juegos se demuestran factibles bajo ciertas circunstancias, podemos concluir que el modelo completo de red es condicionalmente factible desde un punto de vista económico y por lo tanto, la provisión de servicios basados en datos MTC es posible, desde el despliegue de los WSNs que se encargan de obtener los datos, hasta la provisión de servicios basados dichos datos a los usuarios finales.

8.2.1.4 Escenario 4:

Se ha estudiado la provisión dinámica de servicios basados en el IoT con redes de sensores privadas, utilizando la infraestructura de acceso de un tercero. El modelo fue analizado como dos juegos usando conceptos de teoría de juegos, juegos de población, la dinámica Logit, optimización y juegos diferenciales.

El modelo se analizó como dos juegos: en el primer juego se analiza la interacción entre el OP y los sensores, mientras que el segundo juego se analizó la competencia entre IoT-SPs y el comportamiento de los usuarios. Dado que los cambios en el primer juego es mucho menor que en el segundo, se pueden considerar escalas de tiempo diferentes, lo que nos permite considerar el primer juego estático desde el punto de vista del segundo juego.

En segundo lugar, se eligió una dinámica Logit para modelar el comportamiento de los usuarios, mientras que la competencia entre IoT-SPs para maximizar sus ganancias se analizó utilizando un juego diferencial.

Se ha demostrado cómo el escenario analizado mediante modelos dinámicos es económicamente viable en los casos analizados. Además, observamos que la competencia entre proveedores mantiene los precios acotados hasta que el tiempo está cerca del final del intervalo de optimización, por lo que se recomienda introducir un valor de reserva o un intervalo de optimización infinito para observar la evolución a largo plazo. También observamos cómo el proveedor con más sensores puede fijar un precio más alto, mantener una mayor proporción de usuarios suscritos y, por lo tanto, obtener mayores ganancias. Finalmente, se ha observado que la distribución inicial de usuarios es capaz de influir en las ganancias de proveedores con las mismas características en optimizaciones a corto plazo.

8.2.2 Trabajos futuros

En los trabajos futuros se recomienda principalmente utilizar métodos de optimización dinámica, debido a que se ha probado que permite obtener mejores resultados que las aproximaciones estáticas. Por otro lado, resulta necesario estudiar mediante juegos diferenciales escenarios más complejos, con un mayor número de operadores y proveedores de servicio compitiendo, lo que nos permitirá estudiar una mayor abanico de escenarios con un alto dinamismo e identificar nuevas oportunidades de negocio para los diferentes actores. Esta herramienta nos permitirá estudiar escenarios con un número variable de clientes o la entrada de nuevos operadores en un mercado que ya ha alcanzado el equilibrio, entre otros.

En nuestro trabajo principalmente nos hemos centrado en modelar comunicaciones MTC con funciones de utilidad basadas en el retardo, sin embargo, hay otros tipos de servicios con una mayor dependencia de otros parámetros, como el ancho de banda y la cobertura. En futuros estudios se recomienda estudiar dichos escenarios empleando una metodología similar a la actual, a fin de revalidar los resultados ya obtenidos en un mayor abanico de escenarios.
En los diferentes escenarios se ha observado que la viabilidad de los mismos está estrechamente ligada con el comportamiento de los usuarios. Resulta por tanto necesario realizar estudios rigurosos que nos permitan obtener datos fiables sobre el comportamiento y las preferencias de los usuarios, para poder ajustar los parámetros de nuestros modelos y analizar de manera más realista y precisa situaciones complejas.

Finalmente, los métodos analíticos empleados en nuestro análisis nos han dado una amplia perspectiva del problema. Sin embargo, este tipo de análisis está limitado a modelos relativamente simples, y no es posible emplearlos en modelos más complejos, donde las expresiones analíticas se vuelven intratables. En futuros estudios se recomienda emplear herramientas como algoritmos genéticos y métodos numéricos para estudiar la competencia en escenarios complejos.


APPENDIX A

SCENARIO 3: OP PRICING STAGE - SOLUTION OF KKT PROBLEM

The candidates to local maximum of the optimization problem with restrictions

\[
\max_p \quad \Pi_{OP_2} = \left( p \frac{c-p}{ct} \right)
\]

subject to \( c(1 - \tau N r) \leq p \leq c \) \hspace{1cm} (A.1)

must hold KKT conditions

\[
\nabla f_0(p^*) + \lambda_1 \nabla g_1(p^*) + \lambda_2 \nabla g_2(p^*) = 0
\]

\( \lambda_1, \lambda_2 \geq 0 \)

\( \lambda_1 g_1(p^*) = 0 \)

\( \lambda_2 g_2(p^*) = 0 \), \hspace{1cm} (A.2)

where

\[
\begin{align*}
f_0(p) &= p \frac{c-p}{ct} \quad & \nabla f_0(p) &= \frac{c-2p}{ct} \\
g_1(p) &= p - c(1 - \tau N r) \quad & \nabla g_1(p) &= 1 \\
g_2(p) &= c - p \quad & \nabla g_2(p) &= -1
\end{align*}
\]

The solution to the KKT conditions is a global maximum if \( f_0(p) \), \( g_1(p) \) and \( g_2(p) \) are concave \( C^1 \) functions. We observe that \( g_1(p) \) and \( g_2(p) \) functions are both convex and concave since they are linear and \( f_0(p) \) is concave since its second derivative \( f''_0(p) = -\frac{2}{ct} \) is negative for any value of \( p \).

Solution of the problem with the KKT conditions:

- \( \lambda_1 > 0 \):

\[
\begin{align*}
g_1(p^*) &= 0 \iff p^* - c(1 - \tau N r) = 0 \\
p^* &= c(1 - \tau N r)
\end{align*}
\]
\[ \lambda_2 > 0: \]
\[ g_2(p^*) = 0 \iff c - p^* = 0 \]
\[ p^* = c. \quad (A.4) \]

Given that \( p^* \) must meet (A.3) and (A.4) simultaneously we have
\[ \tau N_r = 0, \quad (A.5) \]

which is impossible.

\(- \lambda_2 = 0:\)
From this restriction we can obtain
\[
\begin{align*}
\frac{c - 2p^*}{c\tau} + \lambda_1 &= 0 \\
\frac{c - 2c(1 - \tau N_r)}{c\tau} + \lambda_1 &= 0 \\
\frac{2\tau N_r - 1}{\tau} &< 0 \\
2\tau N_r - 1 &< 0 \\
\tau &< \frac{1}{2N_r}.
\end{align*}
\]

This case is valid if (A.6) holds. And the profit is
\[ \Pi_{OP_2}(c(1 - \tau N_r)) = c(1 - \tau N_r) \left( \frac{c - (1 - \tau N_r)}{c\tau} \right) = c(1 - \tau N_r) N_r \quad (A.7) \]

Note that the profit is always positive and
\[ \Pi_{OP_2}(p^*) > \frac{cN_r}{2} \quad \text{s.t.} \quad \tau < \frac{1}{2N_r} \quad (A.8) \]

\(- \lambda_1 = 0:\)

\(- \lambda_2 > 0:\)
\[ g_2(p^*) = 0 \iff p^* = c \]
\[ \frac{c - 2p^*}{c\tau} - \lambda_2 = 0 \]
\[ \lambda_2 = -\frac{1}{\tau} \iff \lambda_2 < 0, \]

which is impossible.

\(- \lambda_2 = 0:\)
\[ \frac{c - 2p^*}{c\tau} = 0 \]
\[ p^* = \frac{c}{2}. \quad (A.9) \]

This case is valid only if
\[ c \geq \frac{c}{2} \quad \text{and} \quad c(1 - \tau N_r) \leq \frac{c}{2} \iff \tau \geq \frac{1}{2N_r}, \]

and the optimal profit is
\[ \Pi_{OP_2}(p^*) = \frac{c}{4\tau} \quad \text{s.t.} \quad \tau \geq \frac{1}{2N_r}. \quad (A.10) \]

From previous analysis we conclude that the solution for the KKT problem defined in (A.2), and therefore the solution for the optimization problem (6.15) is:
\[ \Pi^*_x_{OP_2} = \begin{cases} 
  c(1 - \tau N_r) N_r & \text{if} \quad 0 \leq \tau < \frac{1}{2N_r} \\
  \frac{c}{4\tau} & \text{if} \quad \tau \geq \frac{1}{2N_r}
\end{cases}. \quad (A.11) \]
APPENDIX ABBREVIATIONS

**CAPEX** Capital Expenditures. 10

**ESS** Evolutionary Stable Strategy. 16, 49

**GESS** Globally Evolutionary Stable Strategy. 15, 16, 47–49, 83

**HTC** Human Type Communications. i–iv, vi, vii, 1, 2, 5, 20, 23, 24, 27, 28, 30, 32, 36, 37, 39, 89, 90, 92, 93

**HTCo** Human Type Communications Operator. 28, 30–33, 37, 90

**HTCu** Human Type Communications users. i, 20, 21, 27, 37, 90, 93

**IoT** Internet of Things. i–vii, 1–3, 5, 8, 20, 21, 23, 24, 39, 40, 59, 61, 79, 89–91, 93–95

**IoT-SP** Internet of Things Service Provider. ii, iv, vii, 7, 9, 24, 25, 59–63, 66–69, 71, 72, 74, 79–86, 90, 91, 93–95

**KKT** Karush-Kuhn-Tucker. 3, 17, 44, 65, 105, 106

**M2M** Machine to Machine communications. 1–3

**MINECO** Spanish Ministry of Economy and Competitiveness. 3, 4

**MPNE** Markov-Perfect Nash Equilibrium. 16

**MTC** Machine Type Communications. i–vii, 1–3, 5, 20, 23, 24, 27, 28, 32, 36, 37, 39, 59, 79, 89–93, 95

**MTCo** Machine Type Communications Operator. 28, 32, 33, 37, 90, 93

**MTCu** Machine Type Communications users. i, 20, 21, 23, 27, 36, 37, 93

**OLNE** Open-Loop Nash Equilibrium. 16
OP  Network Operator. i–iv, vi, vii, 7–9, 18, 21, 24, 39–45, 49–53, 59–66, 68, 72, 74, 80, 81, 84, 86, 90, 91, 94, 95

PDE  Partial Differential Equation. 19, 50, 85

PMP  Pontryagin’s Maximum Principle. 18, 19, 45, 49, 50, 84, 108

SPNE  Subgame Perfect Nash Equilibrium. 30

TPBVP  Two Point Boundary Value Problem. 19, 50, 85

WSN  Wireless Sensor Networks. i–iv, vi, vii, 2, 3, 20, 21, 24, 41, 59, 60, 62, 63, 79, 81, 91, 95
NOMENCLATURE

Common Terms

* All the symbols with a * represent the value of such symbols in the optimal/equilibrium case

α Customer sensitivity to the delay in the packets

λ_i Arrival rate of the queue i

Π Operators or service providers profit

c Scale factor of the quality Q

c_v(n) Variable costs

K Fixed costs

n Number of users subscribed to the operator service

p_i Price charged by the network operator i to the users of its connectivity service

Q_i Quality perceived by the users choosing the queue i

S Set of all possible strategies in a game

T_i Mean packet service time in the queue i

U_i Utility of the users choosing the strategy i

W_i Mean waiting time in the queue i

X Social state of a system

Scenario 1

μ_i Mean packet transmission time in the queue i

Π Operator profit in the baseline case

Π_i Operator i (i = 1, 2) profit in the duopoly case
Operator profit in the monopoly case

Total server load

Server load due to the queue $i$

Upper threshold to the price $p_i$ in order to warranty the coexistence of two operators

Operator price in the baseline case

Price charged to the users of the queue $i$

**Scenario 2 and 4**

Learning rate of the evolutionary dynamic

Instantaneous variation of the adjoint variable (see PMP)

Instantaneous variation of the social state $i$

Discount rate of the instantaneous profits

Revision protocol. Switching rate from strategy $i$ to strategy $j$

Aleatory part of the Logit utility $U_{i,n}$. The variable $\kappa_{i,n}$ follows a Gumbel distribution of mean 0

Service rate of a server, or equivalently, system capacity

Probability that a user following the Logit utility chooses the strategy $i$

Instantaneous profit of the operator $i$ at time $t$

Adjoint variable (see PMP)

Sensitivity parameter in the User Logit utility $U_{i,n}$ to the deterministic part

Hamiltonian function

Scale factor of the investment costs

Total number of sinks in the scenario

Price charged by the OP to the sinks

Sensing data unit generation rate of one sink

Final time horizon of the optimization

Average utility of all the players in the system

Logit utility $U_{i,n}$

Deterministic part of the Logit utility $U_{i,n}$

Fraction of sinks not being served

Fraction of sinks being served by the OP

**Scenario 3 and 4**
\(\mu\)  Service rate of a server, or equivalently, system capacity

\(f_i\)  Price charged by the IoT-SP\(_i\) to the users subscribed to its service

\(k\)  Scale factor of the investment costs

\(L\)  Maximum theoretical system load

\(M\)  Total number of users that can subscribe to the IoT-SPs services

\(N\)  Total number of sinks in the scenario

\(p\)  Price charged by the OP to the sinks

\(r\)  Sensing data unit generation rate of one sink

\(R_i\)  Quality of the data provided by the IoT-SP\(_i\)

\(x_0\)  Fraction of sinks not being served

\(x_1\)  Fraction of sinks being served by the OP

\(y_0\)  Fraction of users not subscribed to any service

\(y_1\)  Fraction of users subscribed to the service of the IoT-SP\(_1\)

\(y_2\)  Fraction of users subscribed to the service of the IoT-SP\(_2\)