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Capacitance Evaluation on Non-parallel Thick-Plate Capacitors by Means of Finite Element Analysis

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Abstract: In this work we show the influence of the edge-effect on the electric field distribution and, hence, on the inner and outer capacitance in an inclined-plate capacitor system surrounded by an insulating medium taking into account the thickness of the conducting plates for a complete set of dimensions and insulating characteristics. Where available, we compare our results with previously published works. Finally, using statistical tools, we obtain approximate expression for computing the relationship between capacitance and insulation material characteristics, insulation gap, plate dimensions and angle.

Key words: Capacitance, edge-effect, electric field, finite elements method, modeling.

1. Introduction

In recent years the development of the hybrid microelectronics technology [1], has opened new possibilities for the resolution of the technological problems that typically arise in electrical engineering. The development of etched fuses, for instance, has been a field of particular interest for the last three decades [2, 3]: One of the main advantages of this technology is the miniaturization of the electric circuits. But, for the same reason, small lengths and high voltages combined together imply non-negligible capacitances and very high electric fields.

While exact analytical solutions are appropriate only for simplified geometries, approximate numerical solutions can be obtained for any real situation.

Several iterative algorithms have been used to solve the equation system representing this behavior [4-8], these algorithms are designed to obtain surface charge distribution, electric field and capacitance. The disadvantage of these algorithms is that they are designed for a few specific cases, so that, if the geometry changes, the algorithm should be reconstructed completely.

Nowadays, owe to the development of the finite elements method (FEM), commercial software exists that integrate these methods to solve accurately the quasi-electrostatics generic problem, independently of the geometry. These tools can increase accuracy, flexibility and minimize cost and time of development; however, in general, the modeling effort for each tiny fuse is usually not justified.

Moreover, even though these software tools provide interfaces increasingly friendly, high skilled and experienced personnel is still needed to avoid common errors and completely wrong solutions.

The aim of this work is to obtain approximate analytical solutions for the distribution of the electric field and the capacitance of the system defined by two conductors of rectangular section valid for a wide range of material, sizes and geometries.

In this work, FEM was implemented using the electrostatics module included in the commercial software ANSYS [9].

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2. Inclined Plate Capacitor

The most common capacitor system found in hybrid microelectronics is formed by two conductors.

Due to manufacturing and application constraints, one conductor plate is often inclined, is not parallel, to the other [10-12]. Recently, mathematical expressions based on hyperbolic functions were presented to calculate the theoretical capacitance of this type of capacitors, bearing in mind the increased importance of the "edge-effect".

In order to compare this real capacitor system to the ideal one, the commercial software has been used to calculate capacitance [10] and electric field distribution between the plates [13]. For this purpose, a two-plane inclined plate capacitor has been parametrically modeled. Relative permittivity and geometry are the parameters.

The geometry used in this model is shown in Fig. 1, where e is the conductors thickness, l is the width of both plates, d is the radius of the plate point nearest to the coordinates origin, α is the angle, ε_1 is the relative permittivity of the dielectric volume between plates and ε_2 is the relative permittivity of the dielectric volume surrounding the system. The length of both plates is assumed to be large enough.

Table 1 shows the variables of the model and its corresponding values used in the simulation. Merging the values in this table, 3125 simulations have been carried out.

Table 2 shows the results of the simulation for angle α = 15°, length l = 0.5 m, thickness e = 0.01 m and ε_2 = 1.

To evaluate the capacitance of the ideal capacitor, the equation proposed by Y. Xiang can be directly used [11]. However, assuming zero thickness and neglecting the edge-effect, ideal analytical expressions are easily obtained [14]:

Considering both plates in electrostatic balance, with a constant charge q uniformly distributed over the plates surface and a voltage ΔV. Provided that the electric field E is normal to the plates, the field lines

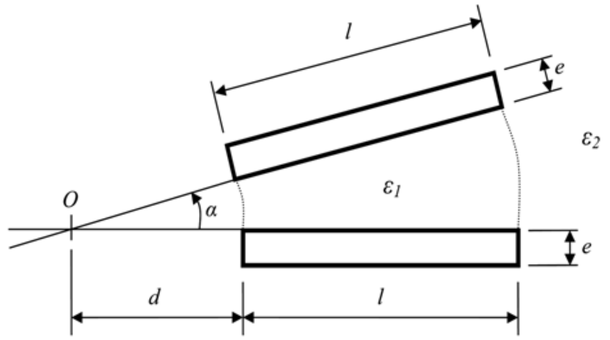

Fig. 1 Two-dimensional model and inclined plate capacitor.

Table 1 Variables of the model and values used for the simulation.

α (°)	l (m)	e (m)	d (m)	ε_1	ε_2
15	0.5	0.01	0.1	1	1
30	0.875	0.0325	0.325	251	1
45	1.25	0.055	0.55	500	1
60	1.62	0.0775	0.775	750	1
75	2	0.1	1	1000	1

are circumference arcs (see Fig. 2) whose axis is the point O (see Fig. 1). The electric field intensity has constant module along each field line and is only a function of the radius r. So the potential difference ΔV between plates is equal to the circulation of the electric field along a field line:

$$\Delta V = \int_L \vec{E}\cdot\vec{dl} = \int_{\theta=0}^{\theta=\alpha} E(r)\cdot r\cdot d\theta = E(r)\cdot r\cdot\alpha \qquad (1)$$

The surface charge density σ on the plates, evaluated at a radius r, is:

$$\sigma(r) = \varepsilon_0\cdot\varepsilon_1\cdot E(r) \qquad (2)$$

From Eqs. (1) and (2):

$$\sigma(r) = \frac{\varepsilon_0\cdot\varepsilon_1}{r\cdot\alpha}\cdot\Delta V \qquad (3)$$

At any surface element dS the electric charge is:

$$dq = \sigma(r)\cdot dS = \sigma(r)\cdot 1\cdot dr \qquad (4)$$

Integrating, the total charge is:

$$q = \frac{\varepsilon_0\cdot\varepsilon_1}{\alpha}\cdot\Delta V\cdot\int_d^{d+l}\frac{dr}{r} = \frac{\varepsilon_0\cdot\varepsilon_1}{\alpha}\cdot\Delta V\cdot\ln\left[1+\frac{l}{d}\right] \qquad (5)$$

Taking into account the definition of capacitance: $q=C\cdot\Delta V$, the capacitance value is:

$$C = \frac{\varepsilon_0\cdot\varepsilon_1}{\alpha}\cdot\ln\left[1+\frac{l}{d}\right] \qquad (6)$$

Applying the Taylor series transformation to the logarithmic term of the Eq. (6), the following

Table 2 Simulation results with an angle α = 15°, a length l = 0.5 m, a thickness e = 0.01 m and ε_2 = 1.

l/d	ε_1	C_{IDEAL}	$C_{SIMULATION}$
	1	6.06E-11	1.51E-10
	251	1.11E-08	1.39E-08
5.000	500	2.22E-08	2.77E-08
	750	3.32E-08	4.15E-08
	1000	4.43E-08	5.53E-08
	1	1.36E-11	2.28E-11
	251	3.42E-09	5.73E-09
1.540	500	6.82E-09	1.14E-08
	750	1.02E-08	1.71E-08
	1000	1.36E-08	2.28E-08
	1	8.05E-12	1.65E-11
	251	2.02E-09	4.15E-09
0.909	500	4.03E-09	8.26E-09
	750	6.04E-09	1.24E-08
	1000	8.05E-09	1.65E-08
	1	5.71E-12	1.38E-11
	251	1.43E-09	3.45E-09
0.645	500	2.86E-09	6.88E-09
	750	4.29E-09	1.03E-08
	1000	5.71E-09	1.38E-08

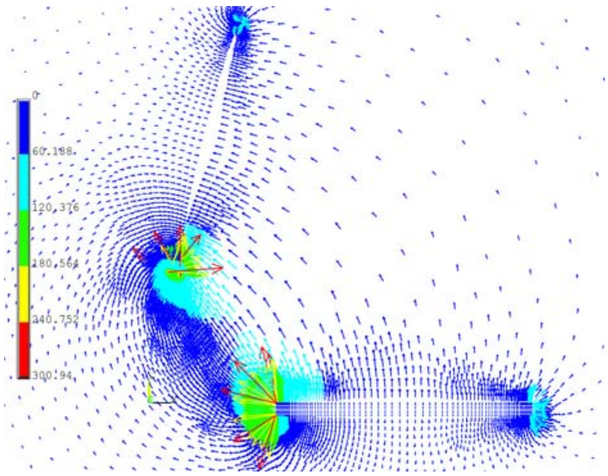

Fig. 2 Electric Field on an inclined thick plate capacitor. α = 75°, d = 1 m, e = 0.1 m, l = 2 m, ε_1 = 1000, ε_2 = 1.

expression is obtained:

$$C = \frac{\varepsilon_0 \cdot \varepsilon_1}{\alpha} \cdot \sum_{n=1}^{m} (-1)^{n-1} \frac{\left(l/d\right)^n}{n} \qquad (7)$$

Fig. 3 and Table 2 show the results obtained modeling the exact geometry compared to the ideal behavior value. It can be seen that the difference between the behavior of a real capacitor and an ideal one increases as the ratio length/distance diminishes.

This difference between the behaviour of a real capacitor and an ideal one is obviously due to the increasing influence of the edge-effect.

The simulation also shows that the maximum charge density, and hence the maximum electric field, increases as the upper plate moves away from the lower plate.

From the obtained results, the capacitance of a real capacitor according to 6 basic parameters was calculated. These are: angle of inclination (α), length (l), radius (d), thickness (e), dielectric relative permittivity of the dielectric volume between the plates (ε_1), and dielectric relative permittivity of the dielectric volume that surrounds the plates (ε_2).

By means of an n-dimensional analysis [15], the polynomial function (8) and the coefficients (9) has been obtained to define the behavior of a real capacitor.

The obtained R^2 error in the interpolation is 0.9996, which indicates that the equation fits well to the interpolated points.

3. Model Validation

The validation of the model has been conducted comparing the values of the capacitance obtained in the simulation, with those calculated using Eq. (10) proposed in Ref. [12], which is valid for relative permittivity of the dielectric volume between plates ε_1

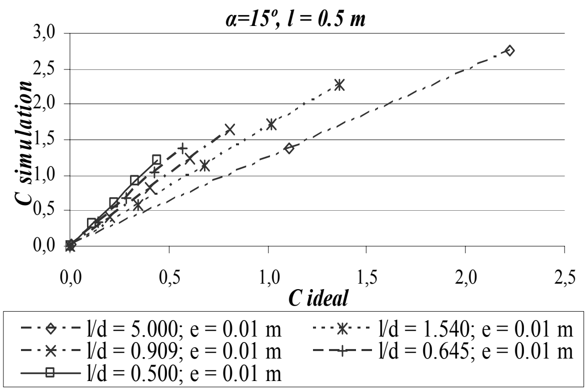

Fig. 3 Simulation results varying inter plates relative permittivity and separation.

= 1, equal length of both plates, zero plate thickness and neglecting the edge-effect. The coefficients k_R and k_L in Eq. (11) are valid for the case of the perpendicular electrode plates (see Fig. 4).

Fig. 5 shows the calculated capacitance based on FEM with the assumption of Xiang. It can be seen that it fits well to the capacitance calculated by Xiang in Ref. [12], using the expression (10).

Fig. 6 shows the electric field distribution in the model with the simplifications of Xiang.

Fig. 7 shows the electric field distribution on the real geometry considering the edge-effect at the plates and the surrounding volume.

Fig. 8 shows the relation between the real capacitance and the geometric model variables.

$$C_{regression} = C_{ideal} \cdot A \cdot e + B \cdot d + C + D \cdot \alpha + E \cdot \frac{l}{d} + F \cdot l + G \cdot \varepsilon_0 \cdot \varepsilon_1 + H \cdot \varepsilon_0 \cdot \varepsilon_2 \quad (8)$$

Where, the obtained coefficients are:

$$A = 1.2154;\ B = 0.2184;\ C = 2.2061;\ D = 0.0077;\ E = 0.0018;\ F = -0.1124;\ G = 1.5127;\ H = -0.2799; \quad (9)$$

$$C_{Xiang} = \varepsilon_0 \cdot \left(\frac{K'(k_{R\ in})}{K(k_{R\ in})} + \frac{K'(k_{R\ out})}{K(k_{R\ out})} + \frac{K'(k_{L\ in})}{K(k_{L\ in})} + \frac{K'(k_{L\ out})}{K(k_{L\ out})} \right) \quad (10)$$

Where the coefficients k_R and k_L are calculated by the following expressions:

$$k_{R\ in} = \frac{r_1}{r_1 + l_1} ? \sqrt{\frac{(r_1 + l_1)^2 + l_{2R}^2}{r_1^2 + l_{2R}^2}}$$

$$k_{L\ in} = \frac{r_1}{r_1 + l_1} ? \sqrt{\frac{(r_1 + l_1)^2 + l_{2L}^2}{r_1^2 + l_{2L}^2}}$$

$$k_{R\ out} = \sqrt{\frac{r_1^{(2/3)} \cdot \left((r_1 + l_1)^{(2/3)} + l_{2R}^{(2/3)}\right)}{(r_1 + l_1)^{(2/3)} \cdot \left(r_1^{(2/3)} + l_{2R}^{(2/3)}\right)}}; \quad (11)$$

$$k_{L\ out} = \sqrt{\frac{r_1^{(2/3)} \cdot \left((r_1 + l_1)^{(2/3)} + l_{2L}^{(2/3)}\right)}{(r_1 + l_1)^{(2/3)} \cdot \left(r_1^{(2/3)} + l_{2L}^{(2/3)}\right)}};$$

The capacitance values taking into account the "edge-effect" ($C_{Sim\_Real}/\varepsilon_0$) (see Fig. 8) are significantly higher than those obtained using Xiang assumptions (C_{Xiang}/ε_0) [12] (see Fig. 5). Table 3 show

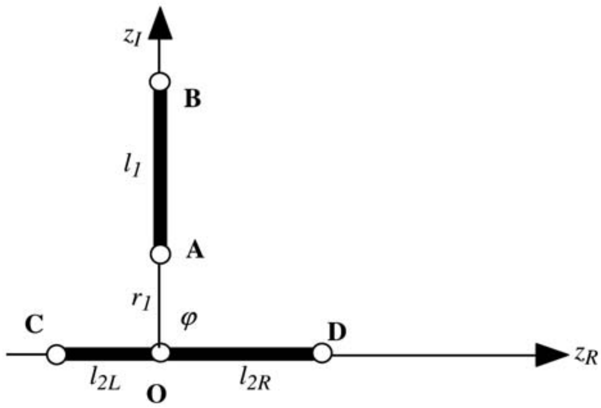

Fig. 4 Two perpendicular electrode plates [12].

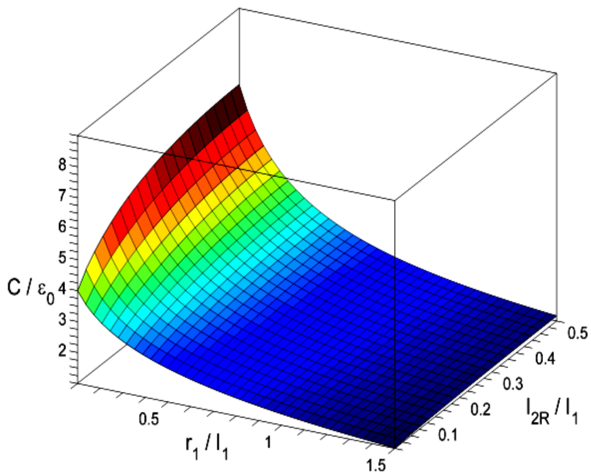

Fig. 5 Capacitances with Y. Xiang assumptions.

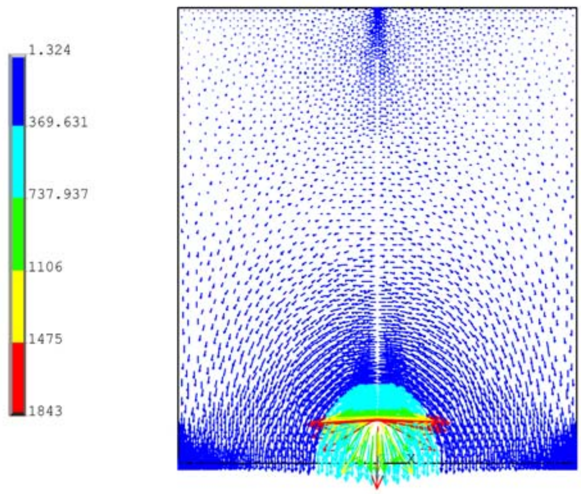

Fig. 6 Electric field distribution with Y. Xiang assumptions.

the specific capacitance values for l_{2R}/l_1 = 0.5 where the largest capacitance value changes have been obtained for the same variation of r_1/l_1. This fact

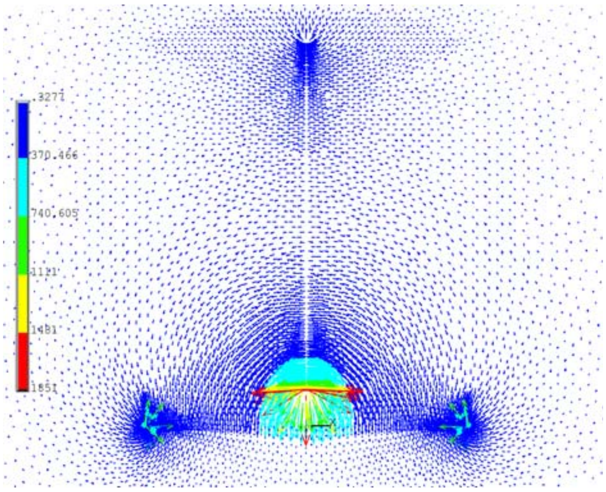

Fig. 7 Real geometry electric field distribution.

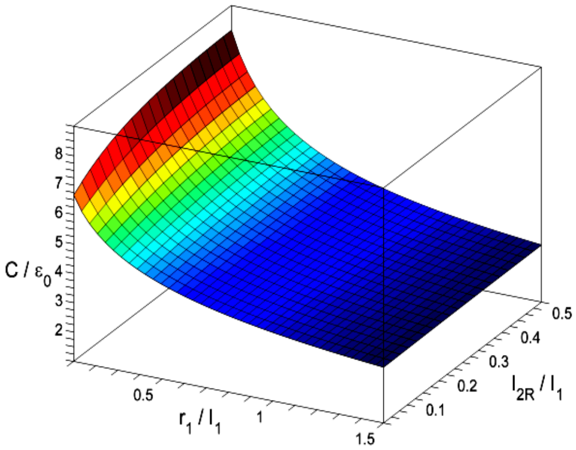

Fig. 8 Real geometry capacitances.

Table 3 Capacitances of the models for l_{2R}/l_1 = 0.5.

r_1/l_1	C_{Xiang}/ε_0	$C_{Sim\_Real}/\varepsilon_0$
0.10	6.54	8.15
0.17	5.18	6.85
0.24	4.34	6.04
0.32	3.74	5.48
0.39	3.30	5.06
0.46	2.95	4.72
0.53	2.67	4.45
0.60	2.43	4.22
0.75	2.07	3.87
0.89	1.80	3.61
1.18	1.43	3.23
1.50	1.16	2.94

highlights the need for taking into account the "edge-effect" when estimating the capacitance value in real applications.

4. Conclusions

In this paper it has been shown the effectiveness of the Finite Element Method based tools to calculate the capacitance of real geometries system. The developed model considers both the volume that surrounds the conductors and the edge-effect that has been shown to be non-negligible. For a wide range of angles, lengths, radius, thicknesses and dielectric permittivities, a polynomial approximate analytical expression has been obtained for the capacitance of a two-plane inclined plate capacitor.

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