Degree of Static Indeterminacy

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1 Summary of key ideas

Before solving a framed structure statically, it is necessary to identify the static unknowns. In this article we will explain the concept of degree of static indeterminacy and how to obtain it in the case of a framed structure.

2 Introduction

A framed structure has to be calculated, both statically and kinematically.

The static unknowns are the reactions and the internal forces. The kinematic unknowns are the joints movements and the member-end movements.

Depending on the method of analysis, static or kinematic variables are taken as fundamental and will be obtained first. If we choose the static variables as fundamental it is necessary to determine the number of independent static unknows (degree of static indeterminacy) to choose an adequate method according to the type of structure.

Structures can be classified, according to the degree of static indeterminacy, in statically determinate or statically indeterminate. In the first case they can be solved statically only with the equilibrium equations. In the second case the compatibility and the constitutive equations must be included in the analysis.

In this document we will explain the concept of degree of static indeterminacy and how to obtain it, providing some examples.

3 Objectives

After reading this document, the student will be able to:

- Determine the number of static unknowns
- Determine the number of equilibrium equations
- Determine the degree of static indeterminacy

4 Degree of static indeterminacy

4.1 Definition

The DEGREE OF STATIC INDETERMINACY (DSI) is the number of redundant forces in the structure.

Redundant forces are the forces that cannot be found by writing and solving only the equations of equilibrium. They must be independent.

Therefore, the degree of static indeterminacy (DSI) represents the difference between the number of static unknowns (reactions and internal forces) and the number of static equations (equilibrium equations).

For a given structure the degree of static indeterminacy is unique but different sets of static unknowns can be selected as redundant forces, as we will see later on in the document.

4.2 Obtaining the degree of static indeterminacy (DSI)

To obtain the degree of static indeterminacy we compare the number of static unknowns with the number of equilibrium equations.

It is important to point out that the DSI includes, not only the external redundancy, which will be obtained comparing the number of external reactions with the number of equilibrium equations of the overall structure (3 in planar structures), but also the internal redundancy. This is the reason why we have to consider all the static unknowns (in members and joints) and all the equilibrium equations (in members and joints).

a) STATIC UNKNOWNS

The static unknowns are the external reactions (or support reactions) and the member-end internal forces.

In a planar framed structure, the support reactions will be a maximum of 3, if the support joint is fixed and a minimum of 1, if two movements are permitted (2 full releases in the support). With 3 movements permitted the joint wouldn't be a support but a free joint with no reactions.

In a planar framed structure, the member-end internal forces are 6, 3 in each member end.

Number of static unknowns: $\sum Rext+6M$ (1)

Being:

$\sum Rext$: total number of external reactions
M: number of members

b) EQUILIBRIUM EQUATIONS

In a planar framed structure, we can formulate 3 global equilibrium equations (in X', Y', Z' axes) in each joint (free joints and support joints), 3 equilibrium equations in each member and an equation corresponding to each full release in a member-end (the internal member-end force corresponding to the member-end release is zero).

Number of equilibrium equations: $3FJ+3SJ+3M+\sum FRm$ (2)

Being:

FJ: number of free joints
SJ: number of support joints
M: number of members
$\sum FRm$: total number of full releases in member-ends

c) DEGREE OF STATIC INDETERMINACY

The degree of static indeterminacy is the difference between the static unknowns and the equilibrium equations, thus:

$$DSI = (\sum Rext + 6M) - (3FJ + 3SJ + 3M + \sum FRm) \quad (3)$$

Considering that in each member there are 6 static unknowns (3 internal forces in each member-end) and 3 equilibrium equations, there is a balance of 3 static unknowns in each member, therefore we can simplify the expression (3) into the following:

$$DSI = (\sum Rext + 3M) - (3FJ + 3SJ + \sum FRm \quad (4)$$

Being:

$\sum Rext$: total number of external reactions
M: number of members
FJ: number of free joints
SJ: number of support joints
$\sum FRm$: total number of full releases in member-ends

This expression can be simplified again if we consider that, in a support, the number of reactions depends on the restrained conditions, that is, in a fixed support of a planar framed structure the number of reactions will be 3, decreasing by one unit for each full release, for example, a pinned support will have 2 reactions (3-1) and a roller support will have 1 reaction (3-2).

Hence, the total number of reactions can be expressed as follows:

$$\sum Rext = 3SJ - \sum FRs \quad (5)$$

Being:

$\sum Rext$: total number of external reactions
SJ: number of support joints
$\sum FRS$: number of full releases in the supports.

Substituting in (4)

$$DSI = (3SJ - \sum FRs + 3M) - (3FJ + 3SJ + \sum FRm)$$

Now simplifying we obtain another expression of the Degree of Static Indeterminacy (DSI):

$$DSI = 3M - (3FJ + \sum FRm + \sum FRs) \quad (6)$$

Being:

M: number of members
FJ: number of free joints
$\sum FRm$: total number of full releases in member-ends
$\sum FRS$: number of full releases in the supports

Both expressions, (4) and (6), can be used to determine the Degree of static indeterminacy. The first expression needs, usually, to define previously the model of the structure.

4.3 Examples

We will obtain, as a practical application, the degree of static indeterminacy of some structures.

EXAMPLE 1 (figure 1)

This structure consists of 2 members and 3 joints. Joint C is free and joints A and B are supports, fixed and pinned, respectively.

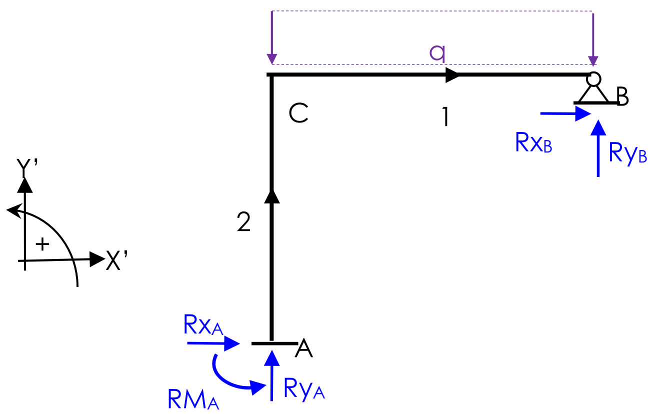

Figure 1. Example1

Making use of the expression (4) for the structure under study:

DSI= (∑Rext+3M) - (3FJ+3SJ+∑FRm) = (5+3·2) - (3·1+3·2+0) = 11-9 = 2

(∑Rext=5, M=2, FJ=1, SJ=2, ∑FRm=0)

Making use of the expression (6):

DSI= 3M - (3FJ+∑FRm+ ∑FRs) = 3·2 - (3·1+0+1) = 6-4 = 2

(M=2, FJ=1, ∑FRm=0, ∑FRs=1)

The structure is hyperstatic (or statically indeterminate) to the 2nd degree

Therefore, the number of redundant forces is 2. There are a lot of pairs of static unknowns that can be selected as redundants. For example: Rx_B and Ry_B, Rx_A and RM_A, Rx_B and RM_A or Mj_2 and RM_A.

There are more possibilities but be careful because not all the pairs are possible. For instance, RxA and RxB cannot be selected together as redundant because they are dependent one to each other.

EXAMPLE 2 (figure 2 and 3)

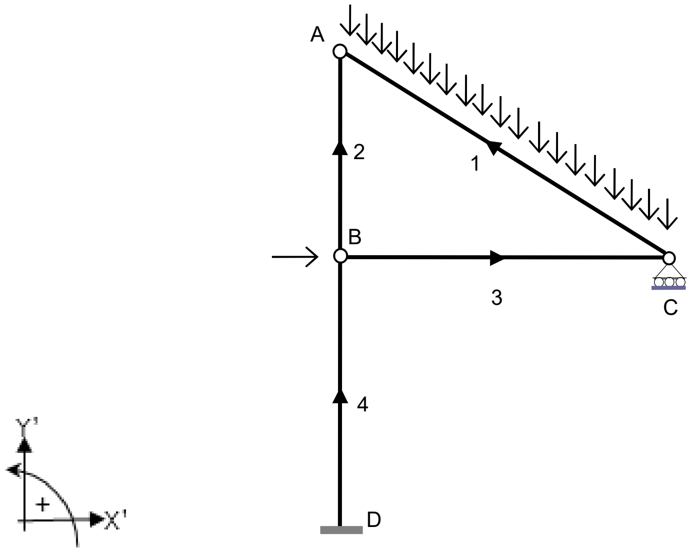

Figure 2. Example2

This structure has 4 joints (2 free joints and 2 supports, a roller and a fixed support) and 4 members. All the members coming to joints A and B are pinned.

To define the releases in members and/or in the supports and consequently the number of reactions we will, previously, define the model. There are different possibilities, one of them is drawn in figure 3.

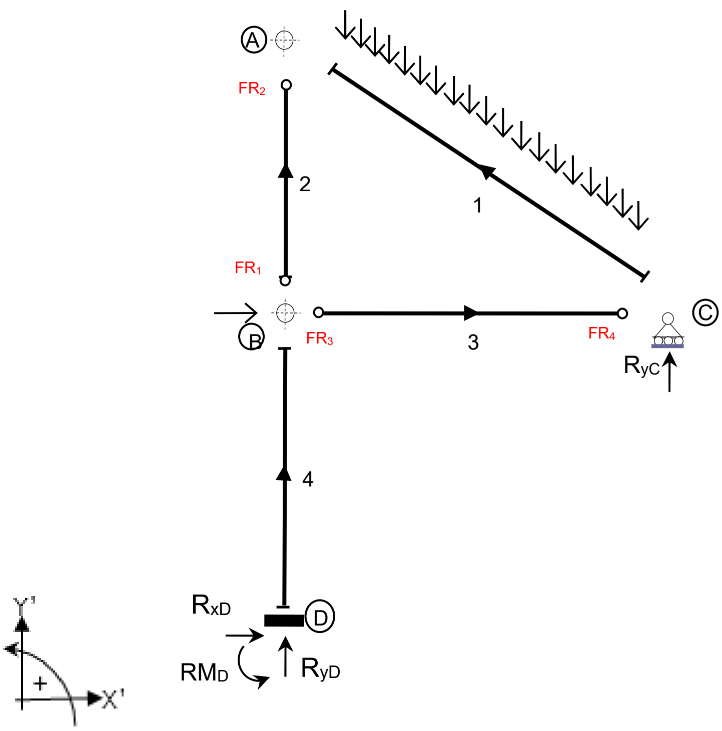

Figure 3. Example2- Model

According to the model there are 4 releases in members (2 hinges in each member-end of 1 and 3). Support C has only 1 reaction because it has 2 releases.

Making use of the expression (4) for the structure under study:

DSI= (∑Rext+3M) - (3FJ+3SJ+∑FRm) = (4+3·4) - (3·2+3·2+4) = 16-16 = 0

(∑Rext=4, M=4, FJ=2, SJ=2, ∑FRm=4)

Making use of the expression (6):

DSI= 3M - (3FJ+∑FRm+ ∑FRs) = 3·4 - (3·2+4+2) = 12-12 = 0

(M=4, FJ=2, ∑FRm=4, ∑FRs=2)

The structure is isostatic (or statically determinate)

5 Closing

In this document we have obtained the degree of static indeterminacy (number of redundant forces of a structure) of a planar framed structure, helping us to classify the structure as statically determinate or statically indeterminate.

As a practical application and self-training, we propose the student to obtain the degree of static indeterminacy of the structure in figure 4.

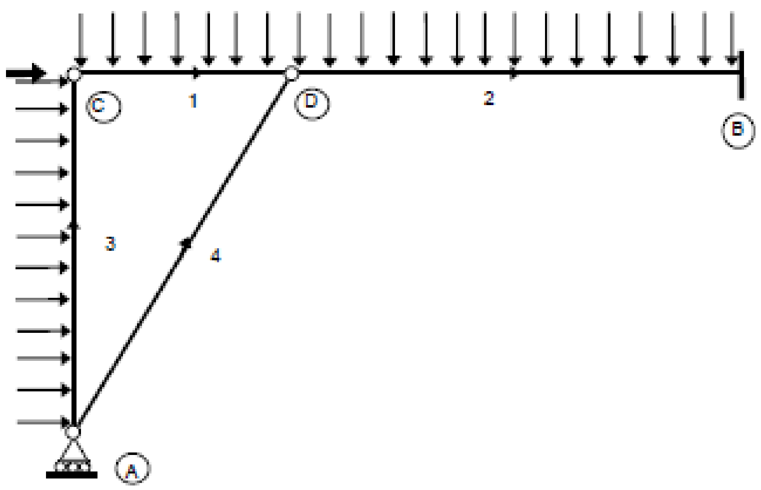

Figure 4. Self-training example

(Results:
The degree of static indeterminacy is 0. The structure is statically determinate)

6 Bibliography

6.1 Books:

[1] Basset, L.; Cálculo matricial de estructuras. Desconexiones y vínculos

[2] Basset, L. "Clasificación estática de las estructuras", Artículo Docente ETSA, 2012. Disponible en Riunet: http://hdl.handle.net/10251/16447

[3] Basset, L. "Cálculo estático de una estructura isostática", Artículo Docente ETSA, 2012. Disponible en Riunet: http://hdl.handle.net/10251/16480

6.2 Figures:

Figure 1. Example 1.
Figure 2. Example 2
Figure 3. Example 2-Model
Figure 4. Self-training example

Author of the figures: Luisa Basset