Degree of Kinematic Indeterminacy

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1 Summary of key ideas

Calculating a structure kinematically consists of obtaining the kinematic unknowns, therefore the first step is to know their number and to identify them. In this document we will explain the concept of degree of kinematic indeterminacy and how to obtain it in the case of a framed structure.

2 Introduction

Framed structures can be solved using different methods. Depending on the method of analysis, kinematic variables are taken as fundamental (i.e. stiffness method) and will be obtained first. Then, using the relations between kinematic and static variables, the structure will be solved statically too.

If we choose the kinematic variables as fundamental (the movements in the key points which are the joints and the member-ends) it is necessary to determine the number of independent kinematic unknowns. This number is the degree of kinematic indeterminacy.

In this document we will explain the concept of degree of kinematic indeterminacy and how to obtain it, giving some examples to illustrate the process.

3 Objectives

After reading this document, the student will be able to:

- Determine the number of kinematic unknowns in joints and members
- Determine the number of rigid modes, if any and the conditions of dependence
- Determine the degree of kinematic indeterminacy
- Identify the independent movements

4 Degree of kinematic indeterminacy

4.1 Definition

The degree of kinematic indeterminacy (DKI) is the minimum number of movements (degrees of freedom, DOF) with which the kinematic configuration of the overall structure can be defined, that is, the number of unknown independent movements of the structure.

The kinematic unknowns are the joint movements and the member-end movements. Formulating the compatibility equations in the members and between member-ends and joints, the number of kinematic unknowns can be reduced to the number of joint movements and the member-end movements which are different to the corresponding adjacent joints (total or partial member-end releases). In a planar framed structure there are 3 movements for each free joint (two displacements and a rotation), 1 or 2 for the supports if the boundary conditions allow 1 or 2 free movements, respectively (1 or 2 releases in the support) and 1, 2 or 3 movements in the case of an elastic support.

Therefore, the minimum number of movements with which the kinematic configuration of the overall structure can be defined, DKI, is the number of joint movements and released member-end movements, if they are independents.

For a given structure the degree of kinematic indeterminacy is unique, although, depending on the model, a movement can be associated to a joint or to a member-end.

4.2 Obtaining the degree of static indeterminacy (DSI)

The degree of kinematic indeterminacy, DKI, can be obtained deducting from the total number of unknown movements the number of dependent movements.

a) KINEMATIC CAPACITY OF THE STRUCTURE (total number of unknown movements):

The kinematic capacity (KC) of the structure is the total number of unknown movements, that is, all the non-zero joint movements and released member-end movements. We obtain the kinematic capacity deducting the zero movements (in supports) from the number of kinematic coordinates.

The number of kinematic coordinates (NKC) is the number of components of the joints movements (taking as origin their original position) and of the member-ends movements (if different from the corresponding component of the associated joint movement).

$$NKC = 3(FJ+SJ) + \sum FR_m + \sum PR_m \qquad (1)$$

Being:

FJ: number of free joints
SJ: number of support joints
ΣFR_m: total number of full releases in member-ends
ΣPR_m: total number of partial releases in member-ends (springs)

Therefore, being MZ the number of zero movements at the supports, the expression of the kinematic capacity, KC, is the following:

$$KC = NKC - MZ = 3(FJ+SJ) + \sum FR_m + \sum PR_m - MZ \qquad (2)$$

Being:

NKC: number of kinematic coordinates
MZ: number of zero movements at the supports
FJ: number of free joints
SJ: number of support joints
ΣFR_m: total number of full releases in member-ends
ΣPR_m: total number of partial releases in member-ends (springs)

This expression can be simplified again if we consider that, in a support, the number of zero movements depends on the restrained conditions, that is, in a fixed support of a planar framed structure the number of zero movements will be 3, decreasing by one unit for each permitted movement (full or partial release), or in other words, if we deduct from the total number of supports multiplied by 3 (planar framed structure) the number of movements which are zero, we will have the total number of permitted movements in the supports (full or partial release).

$$(3SJ-MZ= \sum FR_S +\sum PR_S) \quad (3)$$

Being:

SJ: number of support joints
MZ: number of zero movements at the supports
ΣFR_S: total number of full releases in supports
ΣPR_S: total number of partial releases in supports (springs)

Hence, the kinematic capacity can also be expressed as:

$$KC = 3FJ + \sum FR_m + \sum PR_m + \sum FR_S + \sum PR_S \quad (4)$$

Being:

FJ: number of free joints
ΣFR_m: total number of full releases in member-ends
ΣPR_m: total number of partial releases in member-ends (springs)
ΣFR_S: total number of full releases in supports
ΣPR_S: total number of partial releases in supports (springs)

b) NUMBER OF DEPENDENT MOVEMENTS

The number of dependent movements is the same as the number of conditions of dependence ($\sum CD$) or formulated relations between movements, given that each condition of dependence expresses a dependent movement as a function of one or more independent movements.

These conditions of dependence must be formulated if there are rigid modes in the members (one condition if there is an axial rigid mode, ARM, and two conditions for bending/shear rigid mode, B/S RM) or inclined supports (one condition of dependence).

When a member has an axial rigid mode (ARM) the axial deformation will be zero, hence the displacement in both member-ends in the direction of x axis shall be the same. The displacement in the j-end is dependent (equal) to the displacement in i-end or vice versa. There is a condition of dependence.

When there is a bending/shear rigid mode, B/S RM, the deformation of the member in the direction of axes Y and z will be zero and the member will remain straight. The rotation at both ends will be the same and will be related to the displacement along Y axis. There will be two conditions of dependence, being the rotations at both member-ends dependent movements. This is the case, for example, of a double pinned member with no transverse load.

In inclined supports (support with a displacement along a surface forming an angle γ with the global X' axis), the components of the displacements referred to the global axes are related being one of them dependent of the other one.

c) DEGREE OF KINEMATIC INDETERMINACY

We obtain the degree of kinematic indeterminacy (DKI) by deducting from the kinematic capacity of the structure (KC), the number of conditions of dependence (∑CD).

$$DKI = KC - \sum CD \qquad (5)$$

Substituting KC from (2) the expression of the DKI is:

$$DKI = [3(FJ+SJ) + \sum FR_m + \sum PR_m - MZ] - \sum CD \qquad (6)$$

Being:

FJ: number of free joints
SJ: number of support joints
ΣFR_m: total number of full releases in member-ends
ΣPR_m: total number of partial releases in member-ends (springs
MZ: number of zero movements at the supports
∑CD: number of conditions of dependence (dependent movements)

Substituting KC from (4) the expression of the DKI is:

$$DKI = [3FJ + \sum FR_m + \sum PR_m + \sum FR_S + \sum PR_S] - \sum CD \qquad (7)$$

Being:

FJ: number of free joints
ΣFR_m: total number of full releases in member-ends
ΣPR_m: total number of partial releases in member-ends (springs)
ΣFR_S: total number of full releases in supports
ΣPR_S: total number of partial releases in supports (springs)
∑CD: number of conditions of dependence (dependent movements)

Both expressions, (6) and (7), can be used to determine the degree of kinematic indeterminacy. The first expression needs, usually, to define previously the model of the structure.

4.3 Examples

Let's obtain the degree of kinematic indeterminacy of some structures.

EXAMPLE 1 (figure 1)

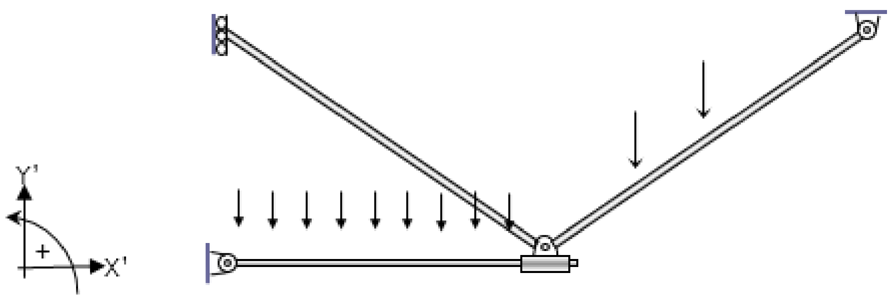

Figure 1. Example1

This structure consists of 3 members and 4 joints. To obtain the degree of kinematic indeterminacy we will start by drawing the model (figure 2)

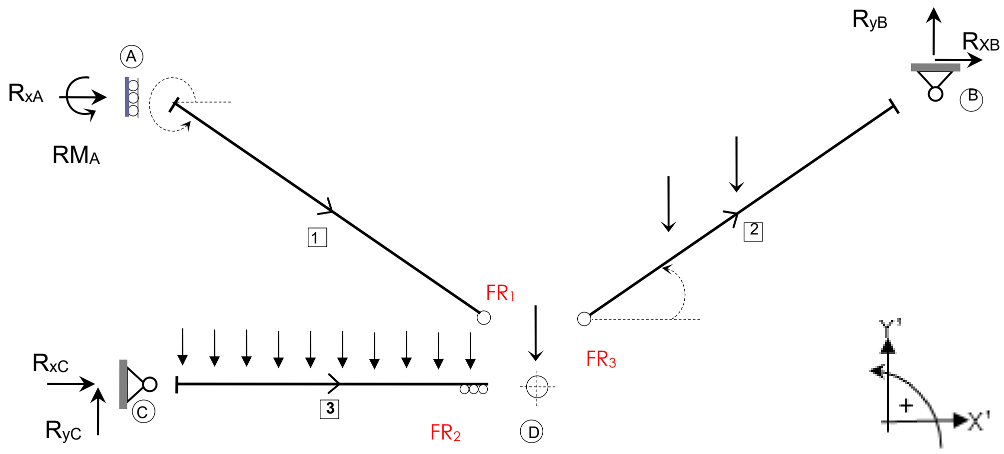

Figure 2. Example1-Model

Joint D is free. The members that are coming to this joint have different rotations, therefore two of them (members 1 and 2) have a hinge in the corresponding member-end (FR_1 and FR_3). On the other hand, member 3 has a different displacement in X' direction, which is represented in the model with a release to x-displacement (FR2)

Support A has a vertical displacement (2 zero-movements) and supports B and C are hinges, thus the rotation is permitted but not the displacements (2 zero-movements). In the model these three movements have been assigned to the supports, but we could have made another choice assigning, for example, the rotations of joints B and C to the j-end of member 2 and i-end of member 3, respectively.

There are no conditions of dependence, thus no dependent movements.

Making use of the expression (6) for the structure under study, the degree of kinematic indeterminacy is:

$DKI = [3(FJ+SJ) + \sum FR_m + \sum PR_m - MZ] - \sum CD = [3 \cdot (1+3) + 3 + 0 - 6] - 0 = 9$

($FJ=1$, $SJ=3$, $\sum FR_m = 3$, $\sum PR_m = 0$, $MZ=6$, $\sum CD=0$)

Making use of the expression (7), the degree of kinematic indeterminacy is:

$DKI = [3FJ + \sum FR_m + \sum PR_m + \sum FR_S + \sum PR_S] - \sum CD = [3 \cdot 1 + 3 + 0 + 3 + 0] - 0 = 9$

($FJ=1$, $\sum FR_m = 3$, $\sum PR_m = 0$, $\sum FR_S = 3$, $\sum PR_S = 0$, $\sum CD=0$)

The independent movements are: dy_A, θ_B, θ_C, dx_D, dy_D, θ_D, θ_{j1}, θ_{i2}, dx_{j3}.

EXAMPLE 2 (figure 3)

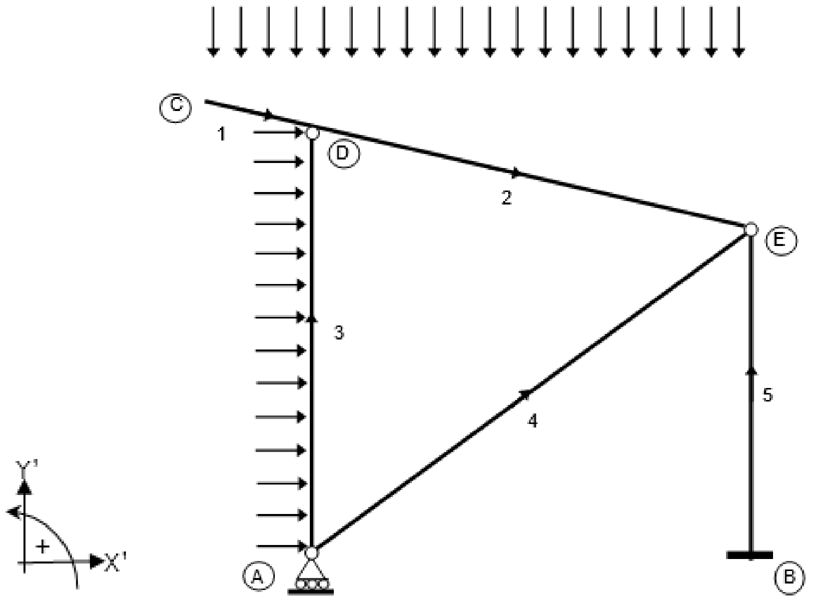

Figure 3. Example2

This structure has 5 joints (3 free joints and 2 supports, a roller and a fixed support) and 5 members. We are going to use the expression 7 to obtain the degree of kinematic indeterminacy. With this expression we don't need to draw the model.

All the members coming to joint E are pinned. That means that there are 2 full releases, because at least one member-end must be fixed to the joint.

Members 1 and 2 are continuous, sharing their j-end and i-end movements, respectively, which are the movements of joint D. Member 3, although having the same displacements has a different rotation, so there will be a release (a hinge in this case) in its j-end.

Finally, let's analyse support A together with i-ends of members 3 and 4. There is an horizontal displacement and 2 different rotations, so, in total, 3 different movements. The horizontal displacement is a full release of the support, but the rotations can be either both full releases in the members or a full release in one member and another full release in the support.

Member 4 is double pinned and there isn't any load acting on it. Consequently, this member has a bending/shear rigid mode (B/S RM), so there will be 2 conditions of dependence, being dependent the rotations of its ends.

Making use of the expression (7), the degree of kinematic indeterminacy is:

$DKI = [3FJ + \sum FR_m + \sum PR_m + \sum FR_S + \sum PR_S] - \sum CD = [3FJ + \sum(FR_m + FR_S) + \sum(PR_m + PR_S)] - \sum CD = [3 \cdot 3 + 6 + 0] - 2 = 13$

$(FJ=3, \sum(FR_m + FR_S) = 6, \sum(PR_m + PR_S) = 0, \sum CD = 2)$

The independent movements are: dx_A, θ_{i3}, dx_C, dy_C, θ_C, θ_{j1}, dx_D, dy_D, θ_D, dx_E, dy_E, θ_E ($=\theta_{j2}$ assigning the joint rotation to j-end of member 2), θ_{j5}.

The rotations θ_{i4} and θ_{j4} are dependent.

5 Closing

In this document we have explained and formulated the expressions to determine the degree of kinematic indeterminacy (number of independent movements or degrees of freedom) of a framed structure. We have also obtained the degree of kinematic indeterminacy of two examples, indicating which are the independent movements.

As a practical application and self-training, we propose the student to obtain the degree of kinematic indeterminacy of the structure in figure 4, indicating which are the independent movements

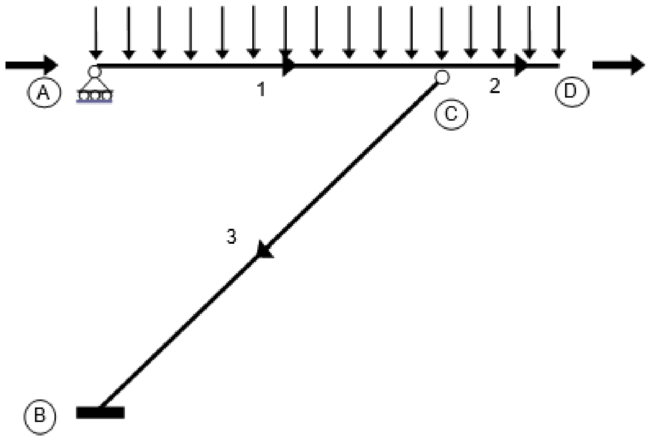

Figure 4. Self-training example

(Results:

The degree of kinematic indeterminacy is 9. The independent movements are dx_A, $\theta_A(=\theta_{i1})$, dx_C, dy_C, θ_C, θ_{i3}, dx_D, dy_D, θ_D. There are no dependent movements)

6 Bibliography

6.1 Books:

[1] Basset, L.; Cálculo matricial de estructuras. Desconexiones y vínculos

[2] Basset, L. "Clasificación cinemática de las estructuras", Artículo Docente ETSA, 2012. Disponible en Riunet: http://hdl.handle.net/10251/12713

[3] Basset, L. "Cálculo cinemático de una estructura isostática", Artículo Docente ETSA, 2012. Disponible en Riunet: http://hdl.handle.net/10251/16497

6.2 Figures:

Figure 1. Example 1.
Figure 2. Example 1-Model
Figure 3. Example 2
Figure 4. Self-training example
Author of the figures: Luisa Basset