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Analytical model for predicting thermal track buckling in dual gauge rail tracks

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Abstract

In rail transport, track gauge is one of the principal factors that condition the passage of trains. For technical and economic reasons, in some circumstances it is necessary to build and operate a so-called dual gauged track, in which a third rail is added to allow circulation of trains of two separate gauges. Although the problem of lateral buckling of rail tracks under thermal loading has been well researched, the addition of the third rail increases the steel area subjected to thermal loads, thus requiring a more accurate analysis. The objective of this paper is to develop an analytical model to analyze the lateral buckling under thermal loads on dual gauge rail tracks. A deep analysis of the effects on the thermal track buckling response produced by each fundamental parameter is presented and discussed. It is found that the risk of buckling is far greater for dual gauge tracks in comparison with conventional tracks. Finally, this model establishes a mechanism that can be used to perform a more effective infrastructure management policy.

Keywords

Track buckling; continuous welded rail; dual gauge; analytical model; rail temperature; ballast resistance.

Introduction

In some particular conditions, it is necessary to build and operate a so called dual gauge track. Dual gauge track is formed by three rails, on which vehicles of two different gauges are able to travel. Some examples of this particular track configuration can be found in USA, Japan, Australia, Europe, Vietnam and Russia. The principal characteristic of these stretches is their short length and reduced traffic. However, a particular situation is presented in the actual Spanish rail network, which is characterized by the coexistence of two rail networks with different gauges. First, the Iberian gauge (1668 mm) was adopted in 1955 and is the most extensively gauge installed with more than 11.000 km. Secondly, in 1992 the standard track gauge (1435 mm) was introduced with the construction of new high-speed rail lines. Actually, more than 3,000 km have been constructed, providing direct connections without break-of-gauge with the rest of the Europe.

Under these particular conditions and considering that, in principle, trains cannot pass from a line with Iberian gauge to standard gauge and vice-versa, some interoperability issues have been recently addressed by considering the concept of dual gauge tracks in order to avoid transfer or changing train operations, which are related with an inconvenient and time-consuming process. This rail track is formed by special concrete sleepers that allow installing three rails: two adjacent outer rails for each gauge, while the single outer rail is common to trains of both gauges, as shown in Figure 1.

“[insert Figure 1.]"

Figure 1. Dual gauge sleeper

With the implementation of dual gauge track, several problems caused by the break of gauge have been solved, but the use of Continuous Welded Rails (CWR) together with the addition of the third rails increases the steel section and therefore the axial compression, which may increase the risk of buckling. In CWR, rails are constrained by sleepers, fasteners and ballast. Under these conditions and considering that rail longitudinal movements are restricted, if the rail temperature becomes substantially higher than the Rail Neutral Temperature (RNT is the temperature of rails, at which there is no axial force), longitudinal forces can build up and accentuate the risk of misalignment. Moreover, these longitudinal forces contribute to many problems such as rail joint failure, rail break or failure of turnouts, but the most important problem in terms of cost and safety is track buckling. Accordingly, infrastructure managers must know the allowable track shifting forces in order to limit vehicle loads and operating speeds while achieving the best performance through applying a cost-effective maintenance strategy, according to the UIC Leaflet 720R safety fundamentals.

Literature Review

To control and prevent buckling events, researches and experimental tests were developed during the latest 40 years. The mechanical behaviour of CWR is a function of the lateral track resistance, climatic conditions, and misalignment as well as other secondary variables. As a result, the mechanism of the buckling phenomenon is extremely complex, especially due to the large number of interrelated variables that influence the problem resolution. Current methodologies use rail track models to predict buckling temperatures from the lateral response of the superstructure, considering different buckling factors. As a result, an important number of approaches have been proposed, based on different assumptions and numerical methods.

There are several investigations of thermal buckling to be found in the literature, with different degrees of complexity. Usually, track buckling theories are based on a mechanistic model using analytical or numerical solutions. Although numerical models provide much more detailed results than analytical ones, their solution is more complex due to numerous, usually unknown factors describing track structure behaviour. On the other hand, analytical models have advantages of speed and simplicity, which can be efficiently implemented and used.

Currently, the solution of the thermal instability problem of the railway track is based on calculation schemes, where the rails and the sleepers are modelled with beam elements, while the fastenings and the ballast bed are idealized with spring elements. These models generally are divided into two groups: beam and rail–sleeper models. The first beam model was developed in the 1950s and 1960s by Bijl[1], with a simple beam formulation of track using energy methods. In order to introduce more realistic solutions, a beam model solution has been developed by Kerr[2] on the basis of the principle of virtual displacements, introducing the concept of the buckling region. Nevertheless, aspects related with maintenance operations and vertical loads were not considered, and the solution was obtained with an iterative method that complicates its resolution. In a series of papers, Kish et al.[3,4,5] redefined the previous models introducing the aspect of "safe temperature", below which it is expected that buckling will not occur. Factors such as track misalignments and non-linearity of axial, lateral and torsional resistances of the track were considered, but only standard track gauge was investigated. El-Ghazaly et al.[6] established a three-dimensional finite element model of a single beam. The model was used to analyze the track behaviour under static, deterministic loads, considering ballast resistance and fastener stiffness constant.

Previous track buckling models have rarely considered lateral deflections. More recently, Samavedam[7], Van[8] and Esveld[9] have conducted different studies in which the lateral deflection due to a certain compressive strength could be calculated. The major limitation of these models is that they are only solved for small displacements and the lateral resistance is proportional to the displacement of the track panel. In 1993, the European Railway Research Institute Committee D-202 and Delft University of Technology started a four-year research program entitled "Improved knowledge of CWR, including switches" to develop a uniform safety philosophy for CWR track analysis and to serve as a basis for updating UIC Leaflet 720 R. As a result, a computer model called CWERRI was developed, which analyses track stability in combination with longitudinal and vertical loads. This model consists of a Timoshenko beam supported on elastic bearings and elements to represent the lateral resistance with a Mohr–Coulomb failure criterion. The major application of CWERRI is its use as a tool for safety analyses. However, the software does not consider the addition of a third rail in the track panel; hence it is not applicable to dual gauge tracks. Recently, Grissom[10] used a new frame-type equations formulation that more accurately represents the lateral behavior of the track structure. Results concluded that the rotational fastener stiffness and the lateral ballast stiffness have a profound effect on track buckling and should be maintained as high as possible to reduce the possibility of lateral track buckling. In the same year, López Pita[11] presented a model based on the formulation of the equilibrium position of the deformed track. However the resolution required many simplifications, limiting its applicability. Recently, Navarro et al.[12] put forward a new buckling model, in which sleeper type, the passing of running vehicles, maintenance operations, type of misalignment and the variation of the ballast height were considered.

Although dual gauge track modifies the classical track configuration, none of the previous studies addresses their influence. Given that the addition of the third rail increases the steel section, considering dual gauge track system in the buckling analysis is needed in order to ensure safety. Accordingly, a numerical finite element CWR model was developed by Cuadrado et al.[13] based upon the latest reports of the European Railway Research Institute (ERRI[14]). There were a few important limitations to this study. The model comprises two sub-models, one for simulating the behaviour in the vertical plane and other for the horizontal plane, where the only action considered is the increase of temperature. As a result, the resolution must be done in two steps and only curvature influence was compared and evaluated. More recently, Villalba et al.[15] applied a 3-D finite element model to the analysis of dual gauge track stability and a numerical model was developed. The overall conclusion is that the risk of buckling is far greater for dual-gauge tracks.

As pointed out above, the existing simple beam and numerical models have their inherent shortcomings for the CWR dual gauge track buckling analysis, and most studies about track buckling restrict themselves to the classical two-rail layout. Additionally, analytical models developed until now do not consider the presence of the third rail. Addressing these limitations, a new analytical rail track model which takes dual gauge track into consideration is developed in this paper, accounting for both simple and dual gauge track buckling and in the context of railway buckling. The model is used to analyze its sensitivity to the variations of the main parameters characterizing the railway track behaviour. The results show the applicability of the proposed model for maintenance planning, specifically for preventive maintenance.

Fundamental assumptions

In order to obtain the buckling temperature, the following assumptions were made to establish the model of dual gauge track, in which the rails were represented as a monorail equivalent model. The following assumptions were made:

1. The buckling load can be obtained from the system's total potential V, applying the principle of stationary potential energy, with respect to any admissible infinitesimal variation of the deformations.
2. The three rail-sleeper structure is replaced by an equivalent beam, where the axial and the bending stiffness in the lateral plane of the two or three rails are assigned to a monorail having the mechanical and inertial properties of the three rails. Therefore, the beam is elastically supported by lateral and vertical resistance.
3. Only longitudinal loads due to trains passage and thermal variations have been considered in the model.

4. Three different spring elements have been used to represent the torsional stiffness between rail and sleeper and the longitudinal and vertical resistance of the ballast, connected to the ground.
5. The hypothesis of an elastoplastic constitutive model for the ballast lateral resistance has been implemented in the model.
6. An initial sinusoidal misalignment with a length L and amplitude f_o has been taken into account.

Proposed methodology

In order to analyze the problem of thermal buckling in dual gauge tracks, a new analytical model is proposed. The superstructure is represented as a straight, two-dimensional bending beam loaded by axial compressive forces. The model's mechanical features are the cross-section and the horizontal moment of inertia corresponding to the type and number of rails used. Due to the fact that rail buckling often occurs on the horizontal plane in presence of small alignment defects (real track is never perfectly straight, as it shows some form of geometrical misalignments), the horizontal deformation of the rails under the action of the compressive temperature force can vary due to different loading scenarios. For conciseness, it is assumed that the horizontal misalignment can be represented by a symmetrical sinusoidal-shaped wave, according with Esveld[9]. As a result, during the buckling phenomena the track will deform by amplifying the initial misalignment proposed. Hence, the lateral displacement can be defined as follows:

$$f(x) = f_0 \, sin\left({}^{n\pi x}/_{L}\right) \tag{1}$$

where $f(x)$ = track lateral deflection; f_0 = amplitude of misalignment; n = defines the form of the initial misalignment, with a half-wave ($n = 1$) or a full-wave ($n = 2$); and L = total length of the misalignment.

While rail temperatures due to the sun irradiation cause steel tracks to expand and buckle, the ballast lateral resistance plays an essential role on the track stability against thermal buckling. However, ballast lateral resistance values strongly depend on construction and maintenance factors that define the local and instantaneous response. That circumstance justifies the limited and restricted values available in the literature, despite the great number of research activities carried out. Taking into account these limitations, the ballast lateral resistance is modelled as an elastoplastic behaviour, representing the interaction forces through the contact surfaces between the sleepers and the ballast layer, as represented in Figure 2.

“[insert Figure 2.]"

Figure 2. Ballast lateral resistance

Model resolution

With the previous considerations and the initial misalignment, a compressive load P is applied in both ends of the beam, causing additional bending moment. The total potential energy of the model V can be obtained by considering the work done by an internal force, which is the sum of the deformation energy associated with the bending energy of the beam $U_{bending}$ and the energy associated to the ballast lateral resistance $U_{ballast}$, as well as the work done by the compressive forces W_P. Accordingly, the total potential energy of the rail track based on the principle of minimum stationary potential energy in the buckled zone is expressed as follows:

$$V = U_{bending} + U_{ballast} - W_P \quad (2)$$

Once the model is subject to axial compression (representing the effects of thermal increase) and assuming a buckling shape of sinusoidal wave over length L in the horizontal plane, the bending energy stored in the beam can be expressed as:

$$U_{bending} = \frac{1}{2}\int_0^L EI_h y''^2 dx \quad (3)$$

where E = modulus of elasticity; I_h = moment of inertia in the horizontal plane; y'' = second derivative of the lateral deflection in the horizontal plane.

Assuming that lateral resistance is proportional to the lateral deflection, the energy exerted by the ballast layer can be formulated as follows:

$$U_{ballast} = \int_0^L \varphi \cdot y \, dx \quad (4)$$

where φ = track ballast resistance per unit of length

Finally, the work done by the compressive forces W_P can be expressed as:

$$W_p = \frac{1}{2}\int_0^L P \left[(y' + f')^2 - f^2)\right] dx \quad (5)$$

where f represents the initial defect in the differential element of the beam.

Taking into account the expression defined by Eq. (2), the potential energy is a function of the variables L, y, and the external applied compressive force P. Assuming that the length of misalignment and the buckling force remain constant during the buckling process, the system will be in equilibrium when its total potential energy V is stationary. That means that their first derivative with respect to the unknown parameters must be equal to zero.

$$\frac{\partial V(y,L,P)}{\partial y} \qquad \frac{\partial V(y,L,P)}{\partial L} \tag{6}$$

Finally, applying the buckling criteria the second variation of the total potential energy $\bar{\delta}^2 V$ evaluated at an equilibrium configuration must be zero, which represents the state in which the equilibrium changes from stable to unstable. Although the total potential V depends on three variables $V(y, L, P)$, only the lateral deflection y and the length of the misalignment or buckling region are considered as generalized coordinates. Therefore, the second variation can be expressed as follows:

$$\bar{\delta}^2 V = \begin{vmatrix} \frac{\partial^2 V(y,L,P)}{\partial y^2} & \frac{\partial^2 V(y,L,P)}{\partial y \partial L} \\ \frac{\partial^2 V(y,L,P)}{\partial L \partial y} & \frac{\partial^2 V(y,L,P)}{\partial L^2} \end{vmatrix} = 0 \tag{7}$$

With the previous equations, the buckling load is the smallest load of compressive force P which causes buckling. The stability criterion provides not only the buckling load, but also the amplitude of the deflection y and the length of the misalignment L when buckling begins to develop. One of the important advantages is related with the buckling load, which is independent of the initially assumed length of the existing track misalignment.

Finally, the required temperature rise to buckle the track with respect to the neutral temperature is derived by using continuity requirements on the longitudinal displacement between the buckled and adjoining zones. This is expressed as follows:

$$\Delta T = {}^{P}/_{E\ A\ \alpha} \tag{8}$$

where A = coss section of the rail; E = Young's modulus of steel; α = thermal expansion coefficient (˚C^{-1}).

Parametric study

The presented analytical model was developed to perform a parametric study of dual gauge track stability and the parameters involved, which represent a deterministic method of predicting the buckling behaviour. Considering that the objective of this investigation is to quantify the stability of dual gauge tracks, different levels for each parameter were evaluated. From a practical point of view, it is desirable to reduce the number of parameters to the primary or principal group. Consequently, the study analyses the ballast lateral resistance, the amplitude and form of the initial misalignment, the type of rail and sleeper. To accomplish this task, each parameter was varied within a practical range, based on the latest reports of the European Railway Research Institute[16]. Table 1 summarizes the values considered for the fixed parameters and levels considered for each factor in the analysis.

Table 1. Track analytical model values

Parameter	Unit	Reference value	Varied range
Rail size	-	UIC 60	UIC 54 / UIC 60 / UIC 71
Sleeper spacing	m	0.6	-
Sleeper type	-	Monoblock	Monoblock / Bi-block
Lateral ballast resistance	(kN/m)	11	7 - 15
Stiffness of rail supporting point	kNm/rad	100	-
Stiffness of ballast longitudinal resistance	kN/m/m	70000	-
Stiffness of ballast vertical resistance	kN/mm	100	-
Initial misalignment amplitude	mm	0.5	0.5 - 5
Misalignment form	-	Half-wave	Half-wave / Full-wave

With the above values, the track response was analyzed, obtaining the buckling temperature, the lateral deflection and the longitude of the misalignment in the buckled zone. For a better understanding and interpretation of the results, the buckling loads were expressed in terms of rail temperature variations with respect to the neutral temperature.

Effects of rail size

Figure 3 shows the effect of rail size on buckling temperatures, with the corresponding length of the track misalignment before the occurrence of buckling. As can be seen, the buckling temperature decreases as the rail size increases because the bending moment of inertia and area increase, thus reducing the lateral stability. This is in accordance with other previous studies such as Tew et al.[16], Kish[17] and Choi[18]. Conversely, for increasing values of the rail size, the length of buckling is progressively greater. Although smaller rail sections provide higher buckling temperatures, a significant decrease in the bending stiffness reduces the maximum axle load that can be supported. Hence an optimization between buckling temperature and fatigue is necessary to select an appropriate rail size.

“[insert Figure 3.]"

Figure 3. Buckling temperatures for different rail sizes

Effects of sleeper type

In order to analyze the influence of the sleeper type, buckling temperatures are obtained by type of sleeper. According to their shape, dual gauge sleepers can be monoblock or bi-block. As showed in Figure 4, the buckling temperature for a track with bi-block sleepers is approximately 20% higher than the buckling load for a track with monoblock concrete sleepers due to their different properties in terms of lateral resistance and interaction with ballast material. With bi-block sleepers, ballast grains penetrate and increase lateral stiffness developed by double number of surfaces.

“[insert Figure 4.]"

Figure 4. Buckling temperatures for different sleeper type

Effects of lateral ballast resistance

In principle, the influence of lateral ballast resistance is evident from the point of view of buckling stability because it provides the ability to resist lateral forces. Figure 5 represents the influence of the ballast lateral resistance on the buckling temperature which was varied over a range of 65% to 135% of the reference value (11000 kN/m), according with the reference values provided by ERRI D-202. The results obtained show that the increasing of lateral resistance appears to have a greater effect on the buckling temperature of the track than other parameters studied, varying misalignment wavelengths from 8 to 12 m. As such, lateral ballast resistance becomes a fundamental and yet highly variable parameter, but its value is affected by several factors such as ballast type and condition, weight, shape, and spacing of sleepers, type of rails and fasteners, maintenance operations, climate conditions, etc.

“[insert Figure 5.]"

Figure 5. Buckling temperatures for different lateral resistances

Effects of initial misalignment

As showed previously, track buckling takes place at a certain temperature under predefined conditions. If such tracks have an initial geometric imperfection, deformations will increase from the initial deformation, thereby having a lower buckling temperature. As a result, rail tracks with smaller lateral imperfections buckled at much higher temperature increases than those with important imperfections (Pandit[19]). In order to examine their influence, a range from 0.5 to 5 cm was analyzed, considering that the shape of the buckled zone remains constant during buckling.

Figure 6 presents the effects of misalignment amplitude on buckling temperature. As can be seen, buckling starts at lower temperatures (< 50ºC) when the amplitude of misalignment reaches values over 1 cm. Furthermore, the buckling length increase as the misalignment amplitude increases, reaching values of 12 m for misalignment amplitudes of 50 mm.

“[insert Figure 6.]"

Figure 6. Buckling temperatures for different misalignment amplitudes

In order to carry out an in-depth study on the influence of misalignment, Figure 7 represents the influence of the misalignment form on the buckling temperature. Calculations were performed using an initial misalignment with a half-wave and full-wave sinusoidal curve form. It can be seen that for a half-wave form, the buckling temperature is, approximately, 16% lower than that obtained for full-wave sinusoidal form. This behaviour is confirmed in the literature, since the tendency to obtain safer results is to consider the initial misalignments as a half sine wave (Kish et al.[3], Choi et al.[18], Lim et al.[20]).

“[insert Figure 7.]"

Figure 7. Buckling temperatures for different misalignment forms

Comparative study between conventional and dual gauge track

A comparative study was made between conventional and dual gauge tracks, using the reference values summarized in Table 1. Accordingly, the differences between the two models consisted of the presence of the third rail. With the previous considerations, the results obtained for these track factors are represented in Figure 8 and Figure 9.

“[insert Figure 8.]"

Figure 8. Buckling temperatures comparative for different lateral resistances

“[insert Figure 9.]"

Figure 9. Buckling temperatures comparative for different misalignment amplitudes

As can be seen, buckling temperatures in dual gauge tracks are around 5ºC to 15ºC lower than those obtained for conventional tracks under the same track conditions. Therefore, the risk of buckling is far greater for dual gauge tracks, requiring special attention and more accurate maintenance operations. However, track parameters are based on the analysis of existing field test and databases and their reliability de-pends on data quantity and quality. In addition, the buckling temperatures follow the same trends with increasing lateral resistance or initial misalignments.

Therefore, the decrease in dual gauge track buckling temperatures may reduce performance and require more maintenance with respect to conventional tracks.

Conclusions

In the last decades, many special models were developed for the lateral stability of rail tracks. However, dual gauge tracks imply a substantial modification of the conventional track panel, requiring a new approach. Previous models analyzed do not consider the presence of the third rail in the track section. Therefore, the main contribution of this paper is related to the possibility of calculating the buckling temperatures in dual gauge tracks, given a specific track conditions. The analytical treatment applied introduces the rail-sleeper structure with an equivalent beam, using the energy method. The results of sensitivity analyses quantify the influence in buckling temperatures of factors such as ballast lateral resistance, amplitude and form of the initial misalignment, type of rail and sleeper.

Additionally, a comparison between conventional and dual gauge tracks was performed. Results shows that buckling temperatures in dual gauge tracks are around 5ºC to 15ºC lower than those obtained for conventional tracks. Consequently and considering the limitations on the accuracy of the proposed method, the risk of buckling is far greater for dual gauge tracks and the decrease in safety using dual gauge tracks require optimized maintenance operations.

Finally, the model in this paper can be further developed and incorporated into railways’ specific safety management systems so as to efficiently manage dual gauge tracks.

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