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Medina, JR.; Sánchez Carratalá, CR. (1991). Robust AR representations of ocean spectra. Journal of Engineering Mechanics. 117(12):2926-2930. doi:10.1061/(ASCE)0733-9399(1991)117:12(2926)

The final publication is available at

http://doi.org/10.1061/(ASCE)0733-9399(1991)117:12(2926)

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Additional Information

ROBUST AR REPRESENTATIONS OF OCEAN SPECTRA

By

Josep R. Medina[1], Member ASCE, and Carlos R. Sánchez-Carratalá[2]

INTRODUCTION

Numerical simulation techniques of stochastic processes defined by a continuous variance spectrum are used to solve problems in the area of engineering mechanics, as well as structural and ocean engineering. Borgman(1969) was the first to introduce the two basic linear simulation techniques in maritime engineering: wave superposition and filtering of white noise. Recently, Funke and Mansard(1987) have presented a complete review of the wave generation techniques used in physical modelling of maritime structures; they pointed out the relationship between modern wave generators and the corresponding numerical simulation techniques. Although simulations by wave superposition are extensively used, the design of linear filters for efficient simulation have been receiving increasing interest. Spanos and Hansen(1981) introduced a method for determining the AR parameters corresponding to a given ocean spectrum. Samii and Vandiver(1984) presented a method based on ARMA filters for simulating forces on offshore structures. Some other authors have proposed different methods for estimating the parameters of ARMA models associated with ocean spectra which can be also extended to multi-variate and multi-dimensional stochastic processes.

The first objective of this note is to justify a reasonable criterion for qualifying the goodness of fitting the transfer

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[1]Profesor Titular, Dept. of Transportation, ETSI Caminos, Univ. Politécnica de Valencia,Camino de Vera s/n,46071-Valencia, SPAIN.
[2]Research Asst., Dept. of Transportation, ETSI Caminos, Univ. Politécnica de Valencia,Camino de Vera s/n,46071-Valencia, SPAIN.
FAX:34-6-3604208 ;TELEX:62808 UPVA-E ;TELP:34-6-3615051-EXT-372

function of a proposed ARMA model to a target ocean spectrum. Once the criterion to qualifying simulators is stablished, the existing techniques to define ARMA models are analyzed. As a result, a new robust AR representation arise as the natural basic element for ARMA representations of ocean spectra.

FILTERING OF WHITE NOISE

Numerical simulations of a stochastic process described by its variance spectrum, $S_\eta(f)$, can be obtained by filtering white noise through a linear filter. The technique consists of calculating the parameters of an ARMA filter whose transfer function corresponds to the given target spectrum of the stochastic process. The ARMA model is described by

$$\eta(n\Delta t) \approx z_n = - \sum_{k=1}^{p} a_k z_{n-k} + w_n + \sum_{m=1}^{q} b_m w_{n-m} \qquad (1)$$

in which z_n is the generated time series; w_n is a white noise with variance σ_w^2; $\{a_k\}$ are the p autoregressive parameters; and $\{b_m\}$ are the q moving average parameters. The spectrum of the simulations generated with the ARMA model given by Eq. 1 is

$$S(f) = 2\sigma_w^2\Delta t|H(f)|^2 = 2\sigma_w^2\Delta t \frac{|1+ \sum_{m=1}^{q} b_m \exp(-j2\pi fm\Delta t)|^2}{|1+ \sum_{k=1}^{p} a_k \exp(-j2\pi fk\Delta t)|^2} \; ; \quad 0\leq f\leq 1/(2\Delta t) \qquad (2)$$

in which H(f) is the frequency response function of the filter. Simulations by filtering white noise require a reliable methodology for determining the ARMA parameters that fit the target spectrum. A process considering only the autoregressive parameters and referred to as AR(p) is described by

$$\eta(n\Delta t) \approx z_n = - \sum_{k=1}^{p} a_k z_{n-k} + w_n \qquad (3)$$

Spanos and Hansen(1981), Spanos(1983), and Spanos and Mignolet(1986) presented succesive methods for calculating the parameters of the AR model corresponding to a target ocean spectrum, $S_\eta(f)$. Specifically an eight-term Taylor expansion of the exponential term of the P-M spectrum was used while trying to avoid the stability problems.

The Yule-Walker equations applied to the theoretical autocorrelation function are the basis for estimating classic AR representations of ocean spectra. Fig. 1 shows a typical spectrum corresponding to an AR representation (dotted line) of a target JONSWAP spectrum (solid line). In spite of the sharp peaks observed, the classic method accurately fits the low order spectral moments (m_0, m_1, m_2,...) where

$$m_n = \int_0^\infty f^n S_\eta(f)\, df \quad (4)$$

However, the precise fitting of spectral moments shown by the classic AR representations does not guarantee an acceptable transfer function for practical applications in maritime engineering. The sharp peaks of the transfer function will significantly modify the mean group lengths, variability of variance, and other important aspects of the sea wave simulations.

[Insert Fig.1]

CRITERIA TO QUALIFY ARMA REPRESENTATIONS

A variety of methods to determine ARMA representations have been proposed. Sometimes it is easy to discriminate between acceptable and unacceptable ARMA representations, such as those shown in Fig.1 . However, when the quality of the representation increases, it is not easy to decide which is the best model. Medina and Sánchez-Carratalá(1988) introduced a hierarchic order of spectral parameters for qualifying the goodness of fitting a model to target ocean spectra. This hierarchic order appears to be a reasonable basis to establish a useful criterion to qualify ARMA representations. A reasonable qualification criterion is not only convenient to select the best ARMA model among alternatives, but also necessary to reject poor ARMA models for ocean engineering applications.

For most common applications of simulation techniques, the variance of the process, $\sigma_w^2=m_0$, is the most important sea wave characteristic. The variance is related to the mean value of the squared wave height, the mean value of the flux of energy, and other basic characteristics of the irregular waves. Therefore, it is reasonable to affirm that any ARMA representation of ocean spectra must fit exactly the variance of the process to be modelled. From a practical point of view, this condition may be achieved by selecting σ_w^2 in Eq.2 to fit the target variance, and also by checking the whiteness of the source of white noise used in the simulation.

Once the variance of the target spectrum has been fitted, the mean value of wave periods appears to be the second most important characteristic of sea waves for most applications. According to Hudspeth and Medina(1988), the mean period of the orbital motion of a point floating in the sea surface is given by $T_{01}=m_0/m_1$. Therefore, one should keep in mind that the relative error of the first spectral moment of the ARMA spectrum in Eq.2 will be the same as the relative error of the mean wave period of the corresponding simulations.

Once the target variance is exactly fitted (m_0), and the mean wave period is approximately fitted (m_1), attention should be paid to the spectral peakedness. According to Medina and Hudspeth(1987), the mean wave group length and the variability of variance are highly sensitive to the spectral peakedness characterized by

$$Q_e=\frac{2m_1}{m_0^3}\int_0^\infty S_\eta^2(f)\ df \tag{5}$$

in which Q_e is a dimensionless spectral peakedness parameter. Many ocean engineering applications require an appropriate description of wave group lengths, correlation between consecutive wave heights, and variability of variance. For these cases, one should take into consideration that the relative error of the coefficient of variation of ζ_{rms} of the simulations is approximately one half the addition of the relative errors of Q_e

and m_1 of the ARMA spectrum. Additionally, the relative errors of the mean wave group lengths and the correlation between consecutive wave heights are approximately the same as the relative error of Q_e of the ARMA spectrum.

Finally, it seems reasonable to try to fit the second spectral moment, m_2. For applications in which the joint distribution of wave heights and periods were important, one should take into account that the model proposed by Longuet-Higgins(1975) depends on the dimensionless spectral parameter $\nu^2=(m_2 m_0/m_1{}^2-1)$. Therefore, the errors of m_2 and m_1 will alter the p.d.f. of wave periods of the simulations.

A reasonable general criterion to qualify ARMA representations of ocean spectra is to demand first an exact fitting of the m_0 ARMA spectrum to the target variance, and then to calculate the relative errors of m_1, Q_e, and m_2. If the three errors show values below the acceptable levels for the particular application, the ARMA representation can be accepted. Otherwise, it should be rejected.

ROBUST AR REPRESENTATIONS

Most proposed ARMA representations of ocean spectra are based on a classic AR representation, using the Yule-Walker equations on the theoretical values of the autocorrelation function. A typical AR representation of ocean spectra shows the sharp peaks noted in Fig.1. One could expect that by increasing the order of the AR model, better representations would be obtained. However, by selecting larger p, the corresponding Toeplitz matrix gradually becomes ill-conditioned, generating spectral shapes with sharp deviations around the target function. The classic method accurately fits the lower order spectral moments when possible, but it does not fit the spectral peakedness.

The unacceptable sharp peaks for most common applications and the instability problems shown by the classic method are related to the fact that theoretical ocean spectra show very low values in the low frequency range. This problem may be avoided by considering that real ocean spectra are the common theoretical spectra with a modicum level of white noise. This reasonable consideration generates robust AR representations of ocean spectra; the Yule-Walker equations can then provide stable solutions with insignificant errors for practical applications. The addition of white noise with a variance about $0.0025 \cdot m_0$ allows for an acceptable fitting of the AR transfer function to JONSWAP type spectra (γ=1 to 10) with only 15 to 30 parameters. Using robust AR representations as a basis, the authors have found that most proposed techniques to design MA and ARMA models for ocean spectra are significantly improved.

Although not published in current literature, it has come to the authors attention that some researchers have introduced in their oral presentations the idea that the addition of white noise to P-M spectra is a computational trick or a numerical manoeuver to avoid instability and not a break through in the mathematical basis for the simulation procedure. On the contrary, the proposed technique is firmly based from a mathematical point of view. It must be considered that the following inequality

$$\int_0^\infty \log[S_\eta(f)]\, df > -\infty \qquad (6)$$

is a necessary condition for the existence of a finite AR representation of a stochastic process and has also been associated wih the existence of sharp peaks of the transfer function of an AR representation as well as with the ill conditioning of the autocovariance matrix used in fitting an autoregressive model.

The proposed addition of a modicum level of white noise solves the failure to fullfil Eq.6 by the ocean spectra commonly used, and is acceptable for all practical applications in engineering design. Real wave records from the ocean and hydraulic laboratories in fact contain large levels of uncontrolled noise.

CONCLUSIONS

A general criterion to qualify ARMA representations of ocean spectra is presented. An objective criterion to qualify simulators becomes the key element to justify the selection and rejection of ARMA representations for practical applications. A new technique to calculate robust AR representations appears to be the natural engineering solution to avoid the instability and poor results provided by classic methods. The addition of a modicum of white noise ($0.0025 \cdot m_0$) in the theoretical JONSWAP type spectra has proved to be sufficient enough to generate robust AR representations. Said representations are free of instabilities and show an excellent qualification in accordance with the general criterion justified for most ocean engineering applications.

APPENDIX I. REFERENCES

Borgman, L.E.(1969). "Ocean Wave Simulation for Engineering Design." Journal of the Waterways and Harbors Division, ASCE, 95(WW4), 557-583.

Funke, E.R., and Mansard, E.P.D.(1987). "A Rationale for the Use of Deterministic Approach to Laboratory Wave Generation." Proc. IAHR Seminar on Wave Analysis and Generation in Lab. Basins, 22nd IAHR Congress, Lausanne, Switzerland, 153-195.

Hudspeth, R.T., and Medina, J.R.(1988). "Wave Groups from Analyses of Wave Height Functions." Proc. 21st International Conference on Coastal Engineering, ASCE, 884-898.

Longuet-Higgins, M.S.(1975). "On the Joint Distributions of the Periods and Amplitudes of Sea Waves." Journal of Geophysical Research, 80(18), 2688-2694.

Medina, J.R., and Hudspeth, R.T.(1987). "Sea States Defined by Wave Height and Period Functions." Proc. IAHR Seminar on Wave Analysis and Generation in Lab. Basins, 22nd IAHR Congress, Lausanne, Switzerland, 249-259.

Medina, J.R., and Sánchez-Carratalá, C.R.(1988). "Comparisons of Numerical Random Wave Simulators." Proc. 21st International Conference on Coastal Engineering, ASCE, 941-955.

Samii, K., and Vandiver, J.K.(1984). "A Numerical Efficient Technique for the Simulation of Random Wave Forces on Offshore Structures." Proc. 16th Offshore Technology Conference, Houston, TX, 301-308.

Spanos, P-T.D.(1983). "ARMA Algorithms for Ocean Wave Modelling." Journal of Energy Resources Technology, ASME, 105, 300-309.

Spanos, P-T.D., and Hansen, J.E.(1981). "Linear Prediction Theory for Digital Simulation of Sea Waves." Journal of Energy Resources Technology, ASME, 103, 243-249.

Spanos, P-T.D., and Mignolet, M.P.(1986). "Z-Transform Modelling of P-M Wave Spectrum." Journal of Engineering Mechanics, 112(8), 745-759.

LIST OF CAPTIONS

FIG. 1.- Classic and Robust AR(30) Spectra Compared to the Target JONSWAP Spectrum (γ=3.3; f_p=0.1 Hz; Δt=1 sec; H_s=4 m).

GAMMA

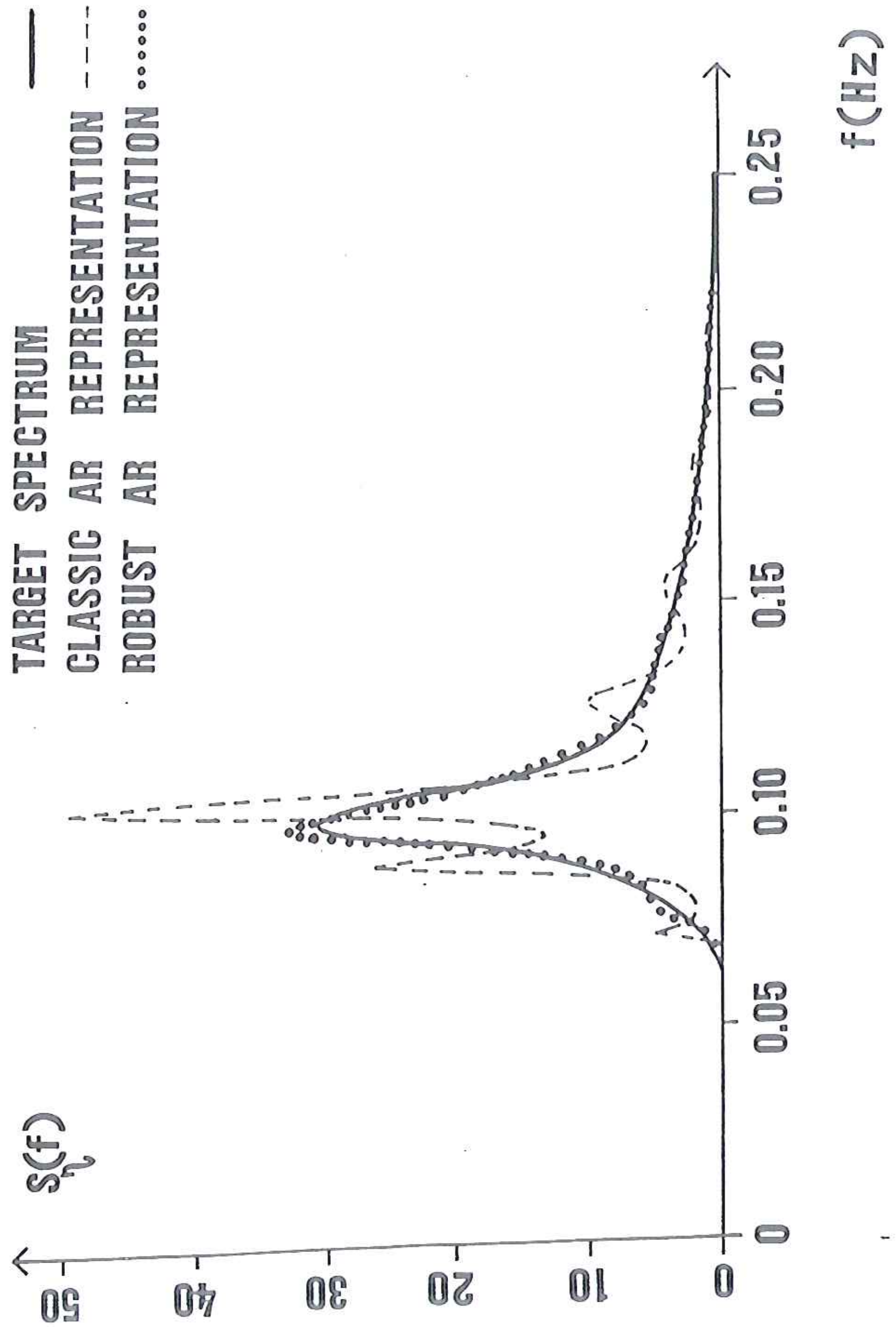

FIG.1 / MLDINA

KEYWORDS

Numerical Filters
Ocean Waves
Computer Simulation
Stochastic Processes

ABSTRACT

An objective general criterion is given to qualify ARMA representations of ocean spectra. A new technique to calculate robust AR models appears to be the natural solution to avoid the instability and poor results provided by classic methods. An addition of modicum white noise to JONSWAP spectra has shown excellent results.