

## Research Article

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# Application of the parametric proper generalized decomposition to the frequency-dependent calculation of the impedance of an AC line with rectangular conductors

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**Abstract:** AC lines of industrial busbar systems are usually built using conductors with rectangular cross sections, where each phase can have several parallel conductors to carry high currents. The current density in a rectangular conductor, under sinusoidal conditions, is not uniform. It depends on the frequency, on the conductor shape, and on the distance between conductors, due to the skin effect and to proximity effects. Contrary to circular conductors, there are not closed analytical formulas for obtaining the frequency-dependent impedance of conductors with rectangular cross-section. It is necessary to resort to numerical simulations to obtain the resistance and the inductance of the phases, one for each desired frequency and also for each distance between the phases' conductors. On the contrary, the use of the parametric proper generalized decomposition (PGD) allows to obtain the frequency-dependent impedance of an AC line for a wide range of frequencies and distances between the phases' conductors by solving a single simulation in a 4D domain (spatial coordinates  $x$  and  $y$ , the frequency and the separation between conductors). In this way, a general "virtual chart" solution is obtained, which contains the solution for any frequency and for any separation of the conductors, and stores it in a compact separated representations form, which can be easily embedded on a more general software for the design of electrical installations. The approach presented in this work for rectangular conductors can be easily extended to conductors with an arbitrary shape.

**Keywords:** proper generalized decomposition, skin effect, parametric simulation, rectangular conductor, virtual chart

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## 1 Introduction

The impedance per unit length of AC lines with rectangular conductors is a key parameter for the calculation of voltage drops and short-circuit currents. Calculating this parameter is a key issue for optimizing industrial busbar systems, in order to reduce the power losses and the drop of voltage in the phases' conductors. But this impedance is highly dependent on the frequency, in case of permanent regime, or on the transient impulses which appear in case of a short-circuit. Both the skin effect and the proximity effects must be taken into account to obtain this impedance, so not only the frequency, but also the geometrical layout of the conductors determine the impedance of the line. No closed analytical formulas exist in the case of rectangular conduc-

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tors [1, 2], so a numerical approach must be used. In this work, a parametric formulation using the PGD [3, 4] is presented to calculate the AC resistance and AC inductance of a single-phase and of a three-phase electric power systems [2, 5], for a wide range of frequencies and distances between the conductors. The use of traditional numerical methods, such as the finite elements method (FEM), requires running one simulation for each desired value of the frequency and for each value of the distance between the phases' conductors, which may require heavy computational resources and long simulation times. On the contrary, only one simulation is needed with the parametric PGD [6–9]. To achieve this goal, the formulation is established using a four-dimensional domain: two spatial dimensions,  $x$ ,  $y$ , and two parametric dimensions, frequency and distance between conductors. The addition of these two parametric dimensions has a negligible computational impact on the time needed to simulate the power line [10]. Besides, the results obtained in this 4D domain include all the individual solutions for each frequency and distance in the selected ranges of the domain. This single 4D solution can be considered as a "virtual chart", which provides the solution for a particular value of frequency and distance just by particularizing this 4D solution, without requiring any further numerical simulation.

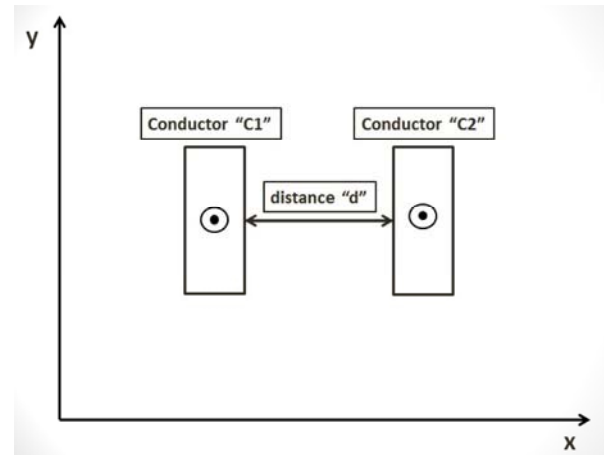
The structure of this paper is as follows. In Section 2, the problem is formulated in a 4D domain, and the parametric formulation of the PGD for this problem is presented. In Section 3, the results obtained for different values of frequency and distance between conductors are shown, and compared with FEM simulations. In Section 4, the frequency is replaced by the time as a parametric dimension, in order to simulate the short-circuit currents and forces in a busbar system. Finally, in Section 5, the conclusions of this work are presented.

## 2 Formulation of the problem in a 4D domain

A phase with two rectangular conductors, separated a distance  $d$  (see Figure 1), and connected in parallel, is the setup simulated in this work for computing the phase's impedance. The conductors are considered to be parallel to the  $z$  axis, so that the magnetic vector potential (MVP) generated by the current has a single component  $A$  directed along the  $z$  axis. Following the PGD procedure, as presented in full detail in [6], the MVP is expressed as a sum of products, termed modes, in a multidimensional domain: the spatial dimensions,  $x$ ,  $y$ , the frequency dimen-

sion,  $f$ , and the distance between conductors dimension,  $d$ , as

$$A(x, y, f, d) = \sum_{i=1}^n (X_i(x) \cdot Y_i(y) \cdot F_i(f) \cdot D_i(d)). \quad (1)$$



**Figure 1:** Phase with two rectangular conductors, connected in parallel, and separated a distance  $d$ , whose impedance as a function of the frequency  $f$  and the distance  $d$  is computed in this work using the PGD

The diffusion equation under sinusoidal regime, which includes as the unknown quantity the MVP, and as the source term the current density  $J_0$ , is established in this 4D domain as

$$\begin{aligned} \frac{\partial^2 A(x, y, f, d)}{\partial x^2} + \frac{\partial^2 A(x, y, f, d)}{\partial y^2} = \\ = -\mu_0 J_0(x, y, f, d) + j2\pi f \mu_0 \sigma(x, y, f, d) \cdot A(x, y, f, d), \end{aligned} \quad (2)$$

with boundary conditions

$$A(x, y, f, d) \Big|_{x=\infty} = A(x, y, f, d) \Big|_{y=\infty} = 0, \quad (3)$$

where  $\mu_0$  is the permeability of free space,  $\sigma$  is the conductivity of the conductor (copper or aluminum),  $j$  is the imaginary unit, and  $J_0$  is the imposed current. The imposed current density and the conductivity in (2),

$J_0(x, y, d)$  and  $\sigma(x, y, d)$ , are represented also as a sum of products,

$$J_0(x, y, f, d) = \sum_{i=1}^{i=m} Jx_i(x) \cdot Jy_i(y) \cdot Jf_i(f) \cdot Jd_i(d) \quad (4)$$

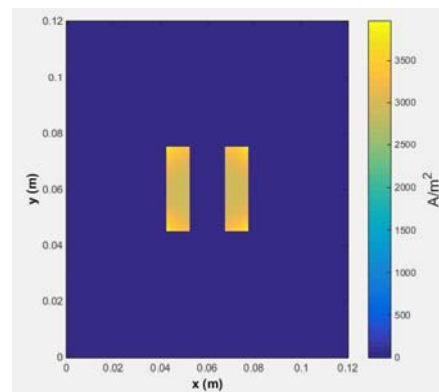
$$\sigma(x, y, f, d) = \sum_{i=1}^{i=l} \sigma x_i(x) \cdot \sigma y_i(y) \cdot \sigma d_i(f) \cdot \sigma d_i(d).$$

### 3 Resolution of the problem in a 4D domain

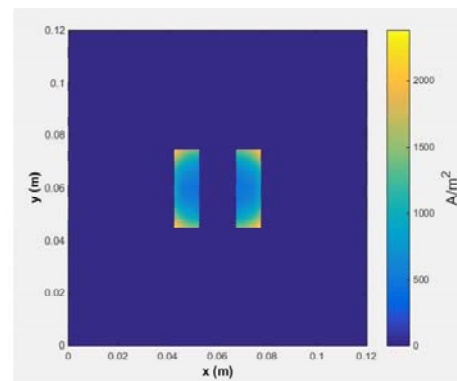
Solving (2) provides the value of the MVP, and from this value the phase impedance, for every frequency and distance between conductors, is easily obtained with the total current density in the conductor, the right side term in (2) [11, 12]. The numerical solution of equations (2), (3), and (4) using traditional methods such as FEM is intractable, because a 4D mesh of the domain has a huge number of degrees of freedom (DOFs). For example, using a coarse mesh of just 100 nodes per dimension would require a total number of DOFs proportional to  $(10^2)^4 = 10^8$ , which difficult this approach using standard software and computer resources. Unlike these traditional methods, the parametric PGD procedure, as presented in detail in [6], uses just four single dimensional meshes, one for each of the four dimensions of the problem. This approach allows to obtain the solution of (2) even with large and dense meshes. In this work, the line presented in Figure 1, has been solved with two parallel, rectangular conductors (10×30 mm), for a range of frequencies [0 – 550 Hz], and for a range of distances [10 – 25 mm].

As an example, three particular values of the full 4D solution of (2) have been selected by assigning particular values to the frequency and distance dimensions, and plotting the spatial values of the total current density in the  $x$  and  $y$  plane, for this particular values. Figure 2 shows the current density for  $f = 50$  Hz and  $d = 15$  mm, Figure 3 for  $f = 200$  Hz and  $d = 15$  mm, and Figure 4 for  $f = 500$  Hz and  $d = 15$  mm.

With the values of the total current density, the AC impedance of the line is computed as a function of both the frequency and the distance between the conductors of Figure 1. It is advisable, as is usually done in the technical literature, to represent the AC impedance not as a function of the frequency, but of a parameter  $k \cdot w$ , where  $k = \sqrt{2\pi f \mu_0 \sigma}$  and  $w$  is the width of the conductor. So, Figure 5 represents the AC resistance of the line (left) and the



**Figure 2:** Current density for a frequency  $f = 50$  Hz computed using the solution of (2), obtained with the PGD, particularized for distance  $d = 15$  mm



**Figure 3:** Current density for a frequency  $f = 200$  Hz computed using the solution of (2), obtained with the PGD, particularized for distance  $d = 15$  mm

AC reactance of the line (right), as a function of the parameter  $k \cdot w$ , for a distance between conductors  $d = 15$  mm. Figure 6 presents the same data but for a distance between conductors  $d = 20$  mm. To assess the validity of the solution obtained with the parametric PGD, the same values of AC impedance have been computed with a standard FEM solution, and they have been represented superimposed to the PGD solutions (in Figures 5 and 6, green lines). It is worth to mention that, for plotting the FEM solution, multiple FEM simulations have been performed, one for each frequency and distance value in the plots, while, in the case of the PGD, just one simulation has been performed for getting the solutions for all the frequencies and distances in the domain range. An excellent agreement has been found between the FEM results and the PGD results.

The results presented in the previous figures are simply particularizations of the most general 4D solution obtained with the PGD, applied to solve (2). With the full 4D solution obtained with the PGD, it is possible to represent simultaneously the AC impedance of the line for a given

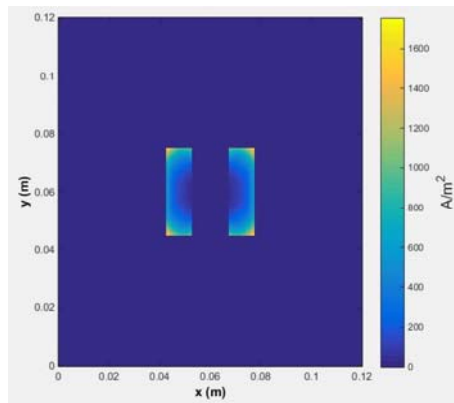


Figure 4: Current density for a frequency  $f = 500$  Hz computed using the solution of (2), obtained with the PGD, particularized for distance  $d = 15$  mm

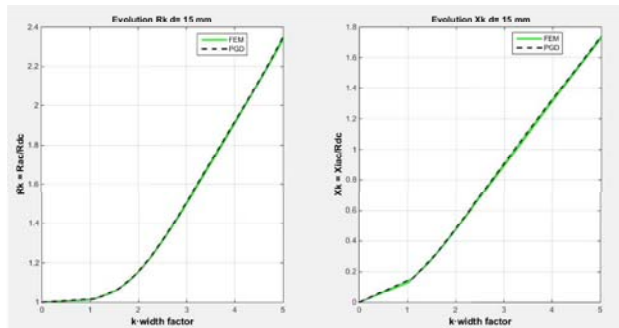


Figure 5: AC resistance (left) and reactance (right) obtained from the PGD solution of (2), as a function of the parameter  $k \cdot w$ , particularized for a distance  $d = 15$  mm, and compared with the solutions obtained with multiple FEM simulations (one for each frequency)

range of frequencies and distances. In this way, a virtual chart is built, like the one used for plotting the AC resistance in Figure 7, and the AC reactance in Figure 8. Besides, the storage of this full 4D solution is very compact, because the number of modes in (1) is small (only 45 modes have been used in this work).

The procedure presented in this work can be extended to the simulation of more complex, industrial busbar systems, such as a three-phase busbar system, whose MVP is presented in Figure 9, and its current density in Figure 10, for a frequency  $f = 500$  Hz and a distance between conductors  $d = 10$  mm. The 4D solution obtained with the PGD includes the solution for the full range of frequencies ([0 – 550 Hz]) and separation between conductors ([10 – 25 mm]). In fact, Figures 9 and 10 are just a 2D solution extracted from this full 4D solution for a particular point in the frequency dimension,  $f = 500$  Hz, and in the distance dimension,  $d = 10$  mm.

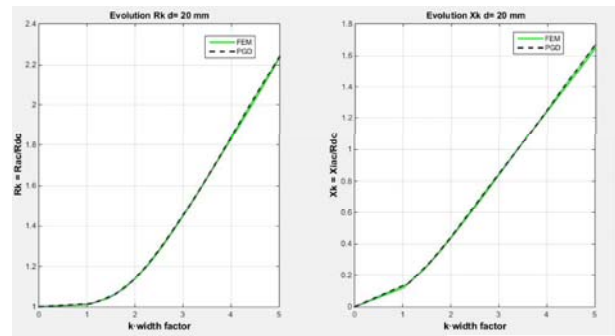


Figure 6: AC resistance obtained from the PGD solution of (2), as a function of the parameter  $k \cdot w$ , particularized for a distance  $d = 20$  mm, and compared with the solutions obtained with multiple FEM simulations (one for each frequency)

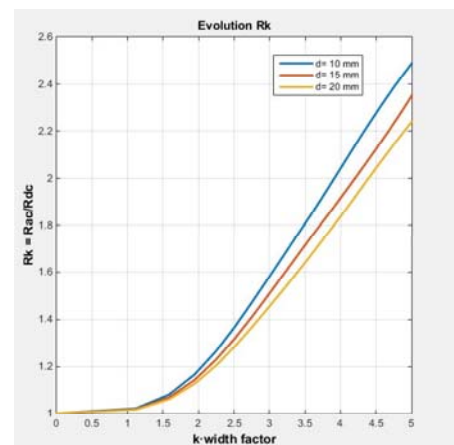


Figure 7: AC resistance of the phase shown in Figure 1, as a function of the parameter  $k \cdot w$ , and the distance  $d$  between the conductors, obtained with the parametric PGD proposed in this work

#### 4 Formulation of the problem in a 4D domain including time instead of frequency

The procedure presented in the previous sections is able to obtain the solution to (2) using a parametric formulation of the PGD, in a 4D domain (spatial dimensions  $x, y$ , frequency  $f$ , and distance  $d$  between conductors). The results generated by the PGD allows to obtain the AC resistance and AC reactance of parallel rectangular conductors, for a range of frequencies and distances, which can be used for the harmonic analysis of the AC lines. Nevertheless, from an industrial point of view, it is also necessary to compute the force between conductors, which can damage them in case of very high short-circuit currents. In this case, it is necessary to replace the frequency with the time in the 4D

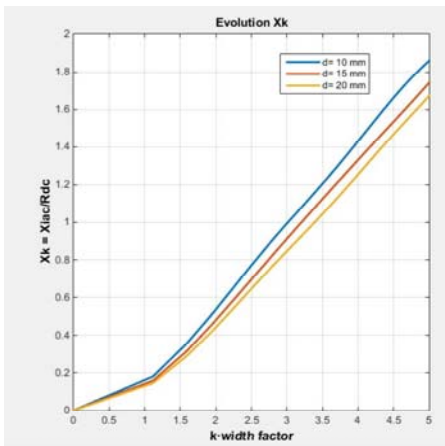


Figure 8: AC reactance of the phase shown in Figure 1, as a function of the parameter  $k \cdot w$ , and the distance  $d$  between the conductors, obtained with the parametric PGD proposed in this work

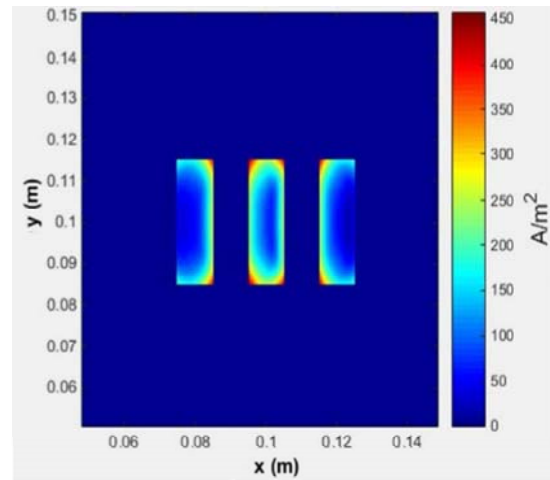


Figure 10: MVP( left) and current density of a three-phase busbar at 500 Hz, for a separation between conductors of 10 mm

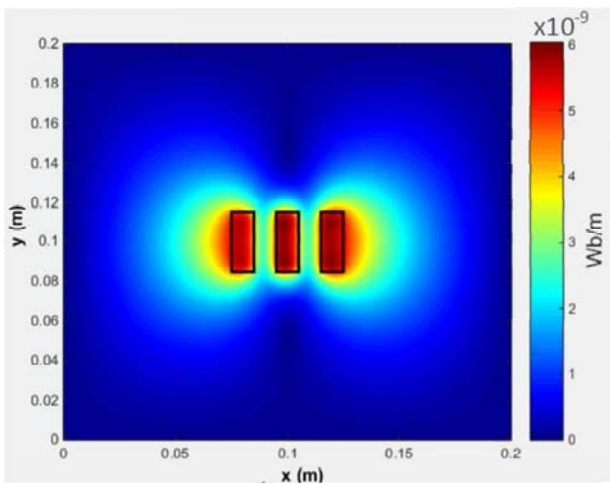


Figure 9: MVP( left) and current density of a three-phase busbar at 500 Hz, for a separation between conductors of 10 mm

and assuming that the conductivity  $\sigma$  does not vary with time.

The configuration analyzed in this work is shown in Figure 11. It represents a single phase line, with two conductors carrying the current in one direction (I-A, I-B), and two conductors carrying the same current in the opposite direction (V-A, V-B).

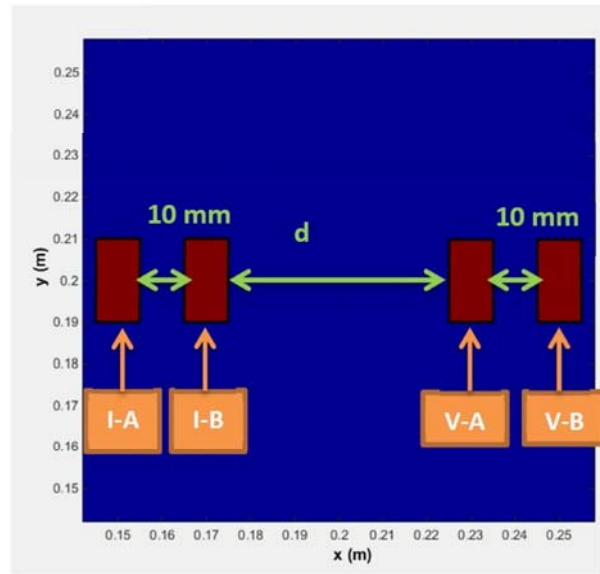


Figure 11: Single phase line, with two conductors carrying the current in one direction (I-A, I-B), and two conductors carrying the same current in the opposite direction (V-A, V-B)

formulation of the PGD, that is,

$$A(x, y, t, d) = \sum_{i=1}^n X_i(x) \cdot Y_i(y) \cdot T_i(t) \cdot D_i(d). \quad (5)$$

The problem to solve in the 4D domain  $(x, y, t, d)$  is now

$$\begin{aligned} \frac{\partial^2 A(x, y, t, d)}{\partial x^2} + \frac{\partial^2 A(x, y, t, d)}{\partial y^2} &= \\ &= -\mu_0 J_0(x, y, t, d) + \mu_0 \sigma(x, y, t, d) \cdot \frac{\partial A(x, y, t, d)}{\partial t}, \end{aligned} \quad (6)$$

with boundary conditions

$$A(x, y, t, d) \Big|_{x=0} = A(x, y, t, d) \Big|_{y=\infty} = 0, \quad (7)$$

One of the strengths of the PGD is that post processing the solution is very fast, because the solution is obtained

directly in separated form. In this section the force per-unit length that appears on a single conductor is computed as

$$F = \iint_S \vec{j} \times \vec{B} \, dS = \iint_S \vec{j} \times (\nabla \times A) \, dS, \quad (8)$$

where  $S$  is the cross-section of the conductor. Taking advantage that the MVP is obtained in separated form by the PGD after solving (6), the components of  $\vec{B}$  can be obtained without computing the MVP in the full 4D domain, which is impractical. Instead, these components can be obtained directly from the separated representation of the MVP, as

$$B_x = \frac{\partial A}{\partial y} = \sum_{i=1}^n (X_i(x) \cdot \frac{dY_i(y)}{dy} \cdot T_i(t) \cdot D_i(d)), \quad (9)$$

$$B_y = -\frac{\partial A}{\partial x} = -\sum_{i=1}^n \left( \frac{dX_i(x)}{dx} \cdot Y_i(x) \cdot T_i(t) \cdot D_i(d) \right). \quad (10)$$

The computation of the force and the corresponding short-circuit current, for conductors I-A and I-B of Figure 11, are represented respectively in Figures 12 and 13, for the particular case of  $d = 50$  mm.

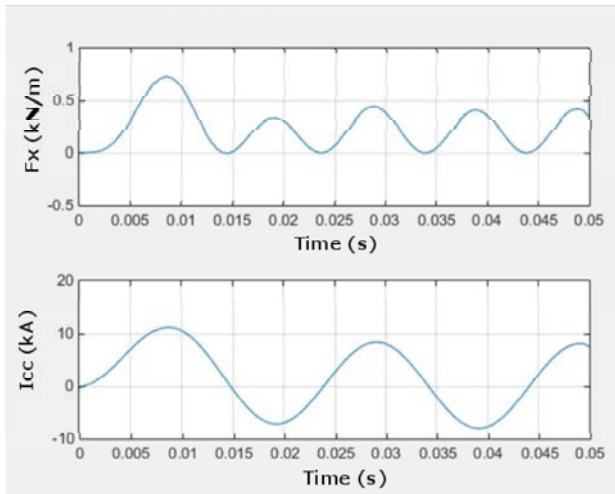


Figure 12: Force (top) and short-circuit current (bottom) of conductor I-A of Figure 11, for  $d = 50$  mm

## 5 Conclusions

In this paper, the AC impedance of a power line with rectangular conductors has been obtained for a given range of

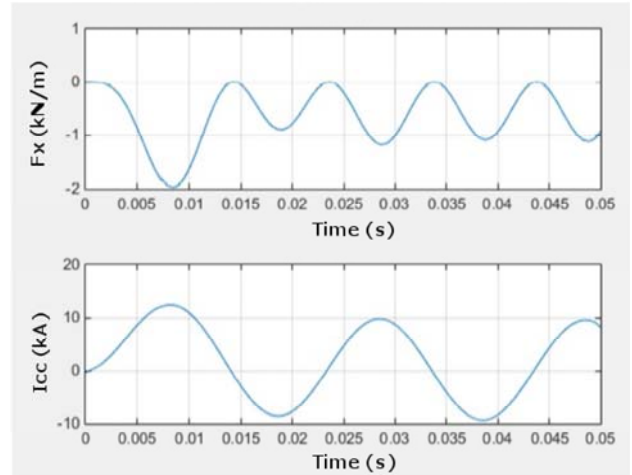


Figure 13: Force (top) and short-circuit current (bottom) of conductor I-B of Figure 11, for  $d = 50$  mm

frequencies and distances between the conductors, using a parametric PGD approach. Under this approach, both the frequency and the distance are considered as additional, parametric, dimensions, so a full solution in a 4D domain (spatial dimensions  $x$ ,  $y$ , frequency  $f$ , and distance  $d$  between conductors) is obtained, in a single execution of the PGD algorithm. This 4D solution has been used for obtaining a "virtual chart" that gives the impedance of the AC line for any value of the frequency and distance, in the range specified as the solution's domain. This approach has been applied to a single-phase and a three-phase line. The results have been validated with multiple 2D FEM simulations, one for each value of the frequency and for each value of the separation between conductors, with an excellent agreement. Besides, the proposed approach can be easily applied to conductors with complex, non-rectangular cross-sections.

The solutions obtained with proposed method, using the frequency as one of the parametric dimensions, are the AC resistance and the AC reactance of the line for any desired frequency, and is useful for the harmonic analysis of the AC line with non-linear loads. If the frequency is replaced with the time as a parametric dimension, then it is possible to simulate short-circuit transients, and to compute the short-circuit forces between the phase conductors as a function of the time and the distance between them. The combination of both multidimensional, parametric approaches allows the simulation and the optimization of the busbar systems both in permanent, sinusoidal regime, including the effect of non-linear loads, and in transient regime, including the effect of the short-circuit forces.

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