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Modeling Corneal Oxygen with Scleral GP Lens Wear

Vicente Compañ,[+] Marcel Aguilella-Arzo,^ Timothy B. Edrington,* Barry A. Weissman*[#]

From:
[+]Departamento de Termodin´amica Aplicada, ETSII, Universidad Polit´ecnica de Valencia, Valencia, Spain
^Departamento de F´ısica, Universitat Jaume I, Castell´on, Spain
*The Southern California College of Optometry at Marshall B Ketchum University, Fullerton CA, 92831 USA
[#]Stein Eye Institute, Department of Ophthalmology, David Geffen School of Medicine at UCLA, Los Angeles CA 90095 USA\

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Abstract

Purpose: The main goal of this current work is to use an updated calculation paradigm, and updated boundary conditions, to provide theoretical guidelines to assist the clinician whose goal is to improve his or her scleral GP contact lens wearing patients' anterior corneal oxygen supply.

Methods: Our model uses a variable value of corneal oxygen consumption developed through Monod equations that disallows negative oxygen tensions within the stroma to predict oxygen tension at the anterior corneal surface of scleral GP contact lens wearing eyes, and to describe oxygen tension and flux profiles, for various boundary conditions, through the lens, tears and cornea. We use several updated tissue and boundary parameters in our model. Tear exchange with GP scleral lenses is considered non-existent in this model.

Conclusion: Our results suggest the majority of current scleral GP contact lenses should produce some levels of corneal hypoxia under open eye conditions regardless of use of updated calculation methods and values for corneal oxygen consumption, permeability, and boundary conditions. Only lenses producing the thinnest of tear vaults should result in anterior corneal surface oxygen tensions greater than a presumed critical oxygen tension of 100 mmHg. We also find that corneal oxygen tension and flux are each more sensitive to modification in tear vault than to changes in lens oxygen permeability, within the ranges of current clinical manipulation. Clinicians would therefore be prudent to prescribe scleral GP lenses manufactured from higher oxygen permeability materials and especially to fit without excessive corneal clearance.

Key words: Scleral lenses, oxygen tension, oxygen consumption rate, tear film, corneal hypoxia.

Introduction

Maintenance of ongoing mammalian physiology demands adequate oxygen supply. The human cornea is known to receive its oxygen supply, under normal conditions, anteriorly through the tears (from the atmosphere when the eye is open and from the tarsal conjunctiva when the eye is closed, e.g. during sleep) and posteriorly from the eye's anterior chamber.[1,2]

Smelser & Ozanics[1] were the first to demonstrate that contact lenses (CLs), then large scleral lenses made of oxygen impermeable glass, would interfere with the cornea's anterior oxygen supply leading to a number of complications (well defined over the past half century) including: acute stromal swelling (and secondary disruption of optical clarity), chronic stromal thinning, epithelial microcysts and fragility, loss of corneal sensitivity, and eventual corneal vascularization.[3-7]

CL designs have evolved from large scleral CLs to both rigid CLs smaller in overall diameter than the cornea and flexible (soft) CLs, larger than the cornea but usually smaller than most scleral lenses. CL materials evolved over the same time period from oxygen impermeable glass and polymethylmethacrylate to various oxygen permeable plastics, both rigid and flexible, to address the oxygen supply restriction discussed above.

The term "Dk" is used to define oxygen permeability in the CL literature, whether considering a cornea or a gas permeable (GP) plastic CL; D is the oxygen diffusion coefficient (in cm^2/sec) and k represents the Henry's law oxygen solubility of a given material (in cm^3 O_2 /cm^3 mm Hg).[8] Dk, however, incompletely describes the oxygen performance of CLs, which have various thickness values (t, in cm). Oxygen transmissibility (Dk/t) has therefore been used to describe the oxygen performance of a CL.[8,9]* Variability in central CL thickness from one lens design to another is compounded, however, by considering that most optically powered lenses are individually not uniform in thickness (it is only reasonable to consider low minus powered lenses - approximately -0.75 DS - as uniformly thick[10]).

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*To help simplify the units of oxygen permeability or Dk ([cm^2/sec][ml O_2/ml mmHg]) and oxygen transmissibility or Dk/t ([cm/sec] [mlO_2/ml mmHg]), "Fatt Dk units" and "Fatt Dk/t units" have been respectively proposed[11] and will be used in our manuscript, where 1 Fatt Dk unit is equivalent to 1 Barrer = 10^{-11} (cm^2/sec)(ml O_2[STP]/ml mmHg).

Uniformity in t is certainly not the case in modern scleral GP lenses of complex designs. (While several studies suggested various ways to derive clinical average thickness values for optically powered CLs, it should be remembered that Fatt and Neumann[12] proposed a conservative approach; they suggested that the site of most oxygen resistance (greatest t, or lowest Dk/t) is the parameter of concern when considering hypoxic stress to the cornea.)

Our quantitative understanding of how much oxygen is required to maintain normal corneal metabolism has similarly evolved over the years, with various proposed metrics: corneal oxygen consumption (termed Q, in ml O_2 cm^{-3} s^{-1}), flux (termed j, in ul O_2/cm^2 hr), or tear layer oxygen partial pressure (pO_2, in mmHg) which maintains adequate j and Q.

As direct measurement of oxygen under in vivo CLs is technically quite challenging,[13] several attempts have been made over the years to model the physical situation of CLs on eyes and through these models predict the likely oxygen metrics available at the anterior corneal surface: j, Q, and/or tear pO_2.[9,14-19]

These models all depend on boundary conditions (e.g. oxygen partial pressures or tensions in the atmosphere, the anterior chamber, and in the tarsal conjunctivae) and the above defined physical intrinsic parameters of the cornea, CLs, and tears: oxygen consumptions (Q), oxygen diffusion coefficients (D), solubilities (k), and thicknesses (t), both of whole cornea and/or the various layers encountered.[20]

As our knowledge of the cornea has evolved over the decades, our ability to provide improved values for boundary conditions and well as improved values for Q, D, k, and t for use in these models, as well as the models themselves, continue to improve. Hence it is valuable to revisit new models over time. The last fifteen years have seen the publication of several serious efforts to develop improved quantification of Q for both whole cornea and the various layers.[15,16,18,19,21,22] Compan et al,[19] in particular, advanced the Monad equation model of Chhabra et al[18] which develops a variable Q that disallows negative oxygen tensions in the corneal stroma (an acknowledged flaw of previous constant Q models).

Scleral CLs, now made of GP plastics, have made a resurgence, being particularly useful in the treatment of eye diseases such as advanced keratoconus and severe dry eye. Modern scleral GP lenses are fitted to vault the cornea, creating a layer of tears underneath. Tear exchange is believed reduced or disallowed with scleral CLs[23] and therefore any oxygen reaching the anterior corneal surface must diffuse through two layers (similar to some hybrid and piggyback lens systems). This creates a situation of “resistance-in-series.”[24] Most research in this area, not surprisingly, suggests that oxygen supply may be somewhat more questionable with CLs that create resistance-in-series than that found with solo modern high Dk rigid or soft CLs.[17,19,25,26]

The main goal of this current work is to use an updated paradigm to provide theoretical guidelines to assist the clinician to improve his or her patients' corneal oxygen supply. Clinicians cannot change the patient's corneal thickness or physiology but can practically modify only one or more of three parameters: lens Dk, lens thickness, or tear layer thickness (called "vault" with scleral lenses). Lens thickness is the least clinically usable of these three parameters, as both required optical power and the necessity to minimize on-eye lens flexure limit clinical manipulation of this parameter.

This study updates the model of Jaynes et al[26] who modeled scleral GP CL-tear systems with a constant cornea Q; we used a Monad equation model with a variable Q. As two groups have used differing Dk values for the whole cornea, we will also consider both here: Dk_c = 86,[19] and Dk_c = 24.7[9] Fatt Dk units respectively to gain perspective on the difference each will make in modelling pO_2 at the corneal surface. We also use a higher value for the Dk of tears (93 Fatt Dk units, after Compañ et al[19]) rather than the value of 80 Fatt Dk units previously used.[9,20]

Literature Review

Several models estimating the in-vivo corneal oxygen supply associated with CL wear are in the literature. Each model has advantages and disadvantages.

Both Holden & Mertz,[5] and later Harvitt & Bonanno,[15] theorized that adverse corneal physiological changes would occur below a "critical" lens Dk/t value. Fatt, however, stated that Dk/t is an in-vitro measurement and therefore is a "disappointment" as a metric of in-vivo performance.[27]

Historically, in-vivo oxygen flux (j) was considered a potential alternative useful metric, but Brennan showed that j changes minimally when considering contemporary higher Dk lenses (>/= 50 Fatt Dk/t units).[16] Brennan also proposed a percent of normal corneal oxygen consumption ($\%Q_c$) as a potential metric, but $\%Q_c$ has the same problem as j and has gained minimal acceptance.

Polse & Mandell proposed a "critical" tear layer oxygen tension value (COT) at the anterior corneal surface below which corneal physiology would be compromised and corneal swelling induced.[3] Presumably, this is that tear layer oxygen tension below which both j and Q suffer. From their somewhat limited data, Mandell & Polse suggested a COT of 11-19 mmHg[3] but later researchers estimated COT at 70-100 mmHg or greater.[28,29] In our opinion, tear pO_2 is as good a metric as any proposed and will be used in this analysis. Weissman and Ye[17] suggested that a reasonable COT would be a tear pO_2 of 100 mmHg from a review of the previous literature (wherein COT ranges from the oxygen tension

found in palpebral conjunctiva during closed eye conditions (~50-60 mmHg) to 120 mmHg or greater.[29]

Weissman and Ye[17] adapted Huang et al's[30] interpretation of the Fatt model (but utilized most of Brennan's[16] boundary condition modifications), used tear pO_2 as their outcome metric, and considered the situation of piggyback CLs as a resistance-in-series[24,31] to oxygen diffusion. This same model was later applied to scleral GP lenses by Jaynes et al.[26]

Michaud et al calculated scleral GP lens oxygen resistance-in-series Dk/t at both the lens center and periphery.[25] Michaud et al[25] used the Holden & Mertz[5] criterion (Dk/t of 24 Fatt units) for the central cornea and the Harvitt & Bonanno[15] criterion (Dk/t of 35 Fatt units) for the peripheral cornea. As noted above, however, we believe that the metric used in this study (CL oxygen transmissibility or Dk/t), may not be the best possible metric to evaluate scleral GP lens in vivo oxygen effectivity and that tear pO_2 at the corneal surface could be a better metric.

In summary, our current model, discussed in more detail below, uses a variable value of Q developed through Monod equations (that disallows negative oxygen tensions within the stroma associated with the constant Q models) similar to the work of Chhabra et al.[32] We also use a tear layer Dk of 93 Fatt Dk units. And, as mentioned above, as whole corneal Dk has been previously proposed at both 86 and 24.7 Fatt units; we will below consider both.

Methods

Although each layer of the cornea has individual values for oxygen consumption (due to the characteristic metabolism and cells in each living layer: epithelium, stroma, endothelium), we here consider the cornea as a single layer and we assume that Q_c only depends on oxygen tension; therefore the complete cornea has the value of $Q_{c,max}$. This value was calculated from the *in vivo* human data of Bonanno et al[13] regarding measurements of oxygen tension in the post-lens tear film as a function of time. Chhabra et al[18] developed a maximum corneal oxygen consumption rate $Q_{c,max}$, and then calculated the spatial-averaged *in vivo* human maximum corneal oxygen consumption rate of 1.05×10^{-4} $ml \cdot cm^{-3} \cdot s^{-1}$. This value corresponds to the average of the obtained values ($Q_{c,max(ave)}$), and it is practically the same as the value found by del Castillo et al.[33] (On the other hand this new value is 2.34 times higher than the one given by Brennan:[16] $Q_{c,max} = 4.48 \times 10^{-5}$ $ml \cdot cm^{-3} s^{-1)}$; and it is 1.8 times higher than that found by Larrea et al:[22] $Q_{c,max} = 5.75 \times 10^{-5}$ $ml \cdot cm^{-3} \cdot s^{-1}$.)

To obtain oxygen tension and flux profiles through the total system cornea-post-lens tear film-scleral GP lens, we used the above Chhabra et al[18] value of $Q_{c,max}$ (1.05×10^{-4} $ml \cdot cm^{-3} \cdot s^{-1}$ and K=2.2 mmHg for the Monod kinetics value). For comparison, however, we will also

consider a $Q_{c,max}$ = 5.0 x10^{-5} ml O_2 cm^{-3} s^{-1} (equivalent to 9.7 ml O_2 /cm^2 h used previously by Jaynes et al[26] after the work of both Freeman[34] and Weissman[35]).

When trying to solve these analyses, there are some difficulties in selecting the "best" boundary conditions to use, such as: corneal and tear permeabilities, or the partial pressure of oxygen at the different interfaces. During the last decades there has been agreement in the partial pressure of oxygen at sea level for the open eye (155 mmHg). Other boundary conditions, however, have been re-evaluated and updated, such as the partial pressure of oxygen at the posterior (or endothelial) corneal surface. This was first considered to be 55 mmHg[14] but a more recent value is 24 mmHg.[13,16] We also used the palpebral conjunctiva pO_2 value of 60 mm Hg[15,18] for closed eye conditions rather than the previously used value of 55 mmHg.[14]

Thickness of endothelium, epithelium and stroma, have been taken at different values by different authors. We here used values of 2, 58 and 480 μm for endothelium, ephitelium and stroma thickness, respectively, from the average central cornea thickness of 540 μm provided by the meta-analysis of Doughty & Zaman[36] also bearing in mind other data.[37]

Tear film oxygen permeability should be identical to water permeability, taking into account the values of the oxygen diffusion and solubility coefficients in water solution at 25ºC of D= 3 x10^{-5} cm^2/sec and k=3.1 x10^{-6} cm^3 O_2/cm^3 mmHg, respectively; a tear permeability of 93 Fatt units[38] is therefore determined. Also, noting that the cornea is 78% water, we feel it is reasonable to use a value for oxygen permeability through the cornea tissue of Dk_c=86 Fatt Dk units (the product between the oxygen diffusivity D = 2.8 x10^{-5} cm^2/sec and k = 3.05 x10^{-6} cm^3 O_2/cm^3 mmHg measured for oxygen in water at 35ºC). Because several groups have directly measured Dk_c, however, we will also use a value of 24.7 Fatt Dk units and compare results of both values.

Importantly, it should also be noted that tear exchange with scleral GP lenses was not considered in our following analysis.

All the parameters considered in the calculations of oxygen tension at the interface cornea-post-lens scleral GP tear film, and both oxygen tension and flux profiles in the cornea, discussed below, are given in Table 1.

Insert Table 1 approximately here.

We used the same technical procedures from previous studies[19,33] employing FiPy 3.1, a finite volume PDE solver written in Python. We used a spatial grid with 10^3 points in all computations and time steps of 10^{-1}s for the time-dependent equations. All the computations were performed by a personal computer with an Intel Core i7-3770K under Debian Linux.

Our model is a 1D construct which assumes that there are three layers (cornea, post-lens tear film layer, and scleral GP lens) between the atmospheric air and the aqueous humour; resistance to oxygen flux in the very thin pre-lens tear film on the front surface of contact lenses is considered negligible (in comparison with the other 3 resistances). Only mono-dimensional oxygen flux is considered; diffusion parallel to the cornea is neglected, as it has been previously, because the cornea is very thin compared with its width.[9]

The non-steady state diffusion equation that describes oxygen tension as a function of time and position for the cornea supposing 1D model is given:

$$\frac{\partial^2 p_c}{\partial x^2} - \left(\frac{Q}{Dk}\right)_c = \frac{1}{D_c}\frac{\partial p_c}{\partial t} \qquad (1)$$

where, p_c is the partial pressure of oxygen in the cornea. As discussed above, D_c is the diffusion coefficient of oxygen in the corneal tissue, k_c is the oxygen solubility coefficient, and x is the distance perpendicular to the surface (cm); Q_c is the oxygen consumption rate into the cornea (ml O_2/cm^3 sec) and here t is time (sec). For the steady-state conditions, eq. (1) becomes:

$$\frac{\partial^2 p_c}{\partial x^2} = \left(\frac{Q}{Dk}\right)_c \qquad 0 \le x \le x_c \qquad (2)$$

At steady-state conditions, the following expression holds at the tear film and GP scleral lens, respectively:

$$\frac{\partial^2 p_{tear}}{\partial x^2} = 0 \qquad x_c \le x \le x_c + x_{tear} \qquad (3)$$

And

$$\frac{\partial^2 p_{lens}}{\partial x^2} = 0 \qquad x_c + x_{tear} \le x \le x_c + x_{tear} + x_{lens} \qquad (4)$$

where x_c, x_{tear} and x_{lens} are the thicknesses of the cornea, tear film and lens, respectively.

The solutions of eq. (2) for the cornea are functions of $Q_c(p_c)$ which consider that corneal oxygen consumption is function of oxygen partial pressure into the cornea as consequence of the aerobic metabolism.[18,20,39]

It is clear that aerobic metabolism does not occur at zero oxygen tension and therefore Q is zero at 0 pO_2. At high oxygen pressures, the reaction is limited by the equilibrium concentration of activated complexes formed by reactions between oxygen and the enzymes which act as catalysts; the reaction is then saturated and Q is independent of the oxygen partial pressure. In these cases, aerobic metabolism is quantified by Monod kinetics

model (also known as Michaelis Menton Model[21,40] and Q is related with oxygen tension by the expression:

$$Q_c(p_c) = \frac{Q_{c,\max} \cdot p_c(x)}{(K + p_c(x))} \qquad (5)$$

Here *K* is the Monod dissociation equilibrium constant, which represents the oxygen tension when corneal aerobic metabolism reaches maximum Q; that is, the oxygen pressure required for the cornea to be in equilibrium where the reaction is saturated and the system starts to have consumption of oxygen independent of partial pressure; $Q_{c,max}$ is the maximum corneal consumption of oxygen and p_c is the partial pressure of oxygen at the cornea-tear interface. As noted above, from Bonanno et al's[13] in vivo estimation of oxygen consumption Q_c, Chhabra et al[18] obtained, using a Monod kinetics constant with K=2.2 mmHg, $Q_{c,max}$=1.05 10^{-4} mL O_2 cm^{-3} s^{-1}. The inclusion of this Nonlinear Monod oxygen consumption for the cornea, described by equation (5), avoids aphysical oxygen partial pressures in the cornea (such can happen when models use a constant Q). In our study, the solution of equation (2), taking into account the equation (5) was obtained following our previously described procedure.[19,33]

A more detailed description of our procedure is given in the appendix of Compan et al[19] and discussed in Appendix I below. Table I shows the different values of parameters used in the numerical solution of Eqs.(2-4) taking into account eq.(5).

Results

Several following figures plot results of using our model.

Oxygen tension isolated to the cornea-post-lens tears film interface (that is, at the anterior surface of the cornea just in contact with the tears, or $x=x_c$) versus tear vault thickness for several specific scleral GP lens Dk and thickness (L here) values is shown in Figure 1. The reader should note that 2 of three scleral GP lens situations in our Figure 1 can be directly compared to Figure 1 of Jaynes et al.[26]

Insert Figure 1 approximately here.

If a COT of 100 mmHg is indeed a reasonable criteria, similar to the results of Jaynes et al[26] and Michaud et al[25], our results again suggest that scleral GP lenses for most tear film thickness combinations considered (except for perhaps the thinnest of tear vaults) are unlikely to provide a $pO2 \geq 100$ mmHg, e.g. enough oxygen to fully avoid hypoxic effects.

Electing a lower COT (e.g. 60 mmHg, such as provided by the inside of the lids during sleep), or a higher value such as 125 mmHg, would, of course, lead to a different prediction of oxygen-related CL physiological tolerability.

Oxygen tension (pO_2) and flux (j) profiles across the system (cornea – tears - scleral GP lens) for various tear vaults are shown in Figures 2 through 5.

The simulated scleral GP lens of Figure 2 and 3 has a Dk of 140 Fatt Dk units and is 250 μm thick. The left-hand panels of Figure 1 present oxygen tension profiles through the cornea, tear vault and lens, with different profiles for tear vault thickness varying from 50 to 300 μm. (Similar results could be obtained for other combinations of lenses with different oxygen transmissibilities and ranges of tear thicknesses). In the top panels, corneal Dk_c=24.7 while in the bottom panels, Dk_c= 86 Fatt units. Q is a variable function of oxygen tension, obtained following the Monod kinetics model with a maximum corneal oxygen consumption rate $Q_{c,max}$ of 1.05 $x10^{-4}$ ml O_2 cm^{-3} s^{-1} in Figure 2.

From the calculated oxygen tensions (pO_2) through cornea-tears-lens, we also calculated oxygen flux (j) using a combination of eqs (2), (5) and Fick's law of diffusion across each layer. The right hand panels in Figure 2 and 3 show oxygen flux profiles for the same scleral GP lens-tear vault systems shown in the left panels. Because there can be no oxygen partial pressure discontinuity at the interfaces between layers, through the boundary layers: cornea-tears, tears-lens and lens-atmosphere, the continuity between fluxes must be satisfied, such as has been observed in all Figures.

Figure 3 displays oxygen tension and flux profiles with maximum oxygen consumption $Q_{c,max}$ of $5x10^{-5}$ ml O_2 cm^{-3} s^{-1}.

As can be seen in Figures 2 and 3, distribution of oxygen tension and flux through the cornea, as well as pO_2 at the corneal surface, are all functions of the thickness of the tear layer trapped between the cornea and the scleral lens.

Similar results could be shown for other scenarios (other GP scleral lens oxygen permeabilities, lens thickness and tears layer thickness). (All results for all the systems cornea-tears-GP scleral lens analyzed here can be seen in Table 2 in Appendix II).

Insert Figures 2 & 3 approximately here.

Figures 4 and 5 similarly show results obtained from our simulations for oxygen tension (left panels) and flux (right panels) profiles where the maximum corneal oxygen consumption

rate, $Q_{c,max}$, is 1.05x10^{-4} ml O_2 cm^{-3} s^{-1} (Figure 4) and 5x10^{-5} ml O_2 cm^{-3} s^{-1} (Figure 5), respectively, for scleral GP lenses with tear vaults held constant at 250 μm. In Figures 4 and 5, however, lens Dk is allowed to vary (3 current clinically used values of 100, 140 and 170 Fatt units). Again corneal oxygen permeability (Dk_c) is 24.7 Fatt units in the top panels and 86 Fatt units in the bottom panels.

Insert Figures 4 & 5 approximately here.

Comparing Figures 2 and 3, where tear vault thickness is allowed to vary between clinically viable values (50 to 300 um), to Figures 4 and 5, where lens Dk is allowed to vary between clinically viable values (100, 140 and 170 Fatt Dk units), it is clear that both oxygen tension and oxygen flux profiles are far more clinically sensitive to changes in tear vault thicknesses than changes in these lens Dks. In Figure 2, with $Q_{c,max}$ of 1.05x10^{-4} ml O_2 cm^{-3}s^{-1}, and changes in tear vault thickness of 50 to 300 μm, anterior corneal surface tear pO_2 ranges 36-38 mmHg (if Dk_c is 24.7 Fatt units, pO_2 ranges from 67 to 103 mmHg, and if Dk_c is 86 Fatt units, pO_2 ranges from 38 to 76 mmHg). Similarly, in Figure 3 with $Q_{c,max}$ of 5x10^{-5} ml O_2 cm^{-3}s^{-1}, and changes in tear vault thickness of 50 to 300 μm, anterior corneal surface tear pO_2 ranges 31-36 mmHg (if Dk_c is 24.7 Fatt units, pO_2 ranges from 67 to 103 mmHg, and if Dk_c is 86 Fatt units, pO_2 ranges from 62 to 98 mmHg). On the other hand, in Figure 4 with lens Dk ranging 100 to 170 Fatt units, anterior corneal surface pO_2 only ranges 10-12 mmHg (if Dk_c is 24.7 Fatt units, pO_2 is 65, 73, and 77 mmHg respectively and if Dk_c is 86 Fatt units, pO2 is 36, 43, and 46 mmHg respectively). Similarly, in Figure 5, with lens Dk ranging 100 to 170 Fatt units, anterior corneal surface pO_2 only ranges 11 mmHg (if Dk_c is 24.7 Fatt units, pO2 is 84, 91, and 95 mmHg respectively and if Dk_c is 86 Fatt units, pO_2 is 60, 67, and 71 mmHg respectively).

Discussion

First of all, despite much improved mathematics (although maintaining a single chamber corneal model), anterior corneal oxygen tension values during GP scleral lens open eye wear predicted by this enhanced analysis differ little qualitatively (i.e. clinically) from that found by the simpler analysis of Jaynes et al.[26] If one accepts a COT of 100 mmHg, the majority of current scleral GP lenses should produce some levels of corneal hypoxia under open eye conditions regardless of use of enhanced paradigms for corneal oxygen consumption modeling, or corneal oxygen permeability, or updated boundary conditions.

Our results displayed in Figure 1 suggest the same conclusion as that of both Michaud et al[25] and Jaynes et al,[26] namely that: *"…clinicians would be prudent to prescribe scleral GP lenses manufactured from what we consider… the highest Dk materials and to fit without*

excessive corneal clearance, in our efforts to provide GP scleral lenses that minimize potential anterior segment hypoxia while providing the other optical and physiological benefits of these devices…".

Additionally, and importantly, we find that both anterior corneal surface oxygen tension and flux are each more sensitive to modification in tear vault (Figures 2 and 3) than they are to changes in lens material Dk (seen in Figures 4 and 5), within the ranges of current clinical manipulation.

This observation suggests that clinicians can exercise more control over anterior corneal oxygen supply by varying tear vault thickness than by changing scleral GP contact lens Dk, using the values for each of these clinically available at this time.

The question of tear exchange of course continues to lead to concerns however, as there are groups who believe that scleral contact lenses (and hybrid CLs as well) preclude any effective tear exchange while other groups believe tear exchange occurs with the wear of such lenses. As noted above, this study did not consider any tear exchange. Hopefully upcoming research will provide data supporting one or the other of these arguments -- and if there is tear exchange, allow some quantization of same. Any significant tear exchange during scleral GP lens wear, would substantially affect the values we report.

In conclusion, we sincerely hope our study above will assist clinicians engaged in the care of scleral GP contact lens wearing patients.

APPENDIX I

The general equation describing oxygen transport through the lens-corneal system, in one dimension, is Fick's second law with reaction {del Castillo et al 2015}

$$k(x)\frac{\partial p(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(x)D(x)\frac{\partial p(x,t)}{\partial x}\right) - Q(p(x,t)) \qquad \text{Ap.(1)}$$

where *p* is the oxygen partial pressure in the lens-corneal system, *t* is time and *x* is the coordinate for normal cornea, with *x*=0 in the interface between the anterior chamber and the cornea.

The second term on the right-hand side in Eq.(Ap.1) is the oxygen consumption as a function of the partial pressure, which is absent in the contact lens and tears film regions and follows a Monod kinetics form in the corneal system:

$$Q_c(p_c) = \frac{Q_{c,\max}\cdot p_c(x)}{(K_m + p_c(x))} \qquad \text{Ap(2)}$$

In Eq.(Ap.1), solubility (*k*) and the diffusion coefficient (*D*) are considered as a function of the position, taking constant values across each of the two regions (CL and cornea) in the system. By using the above approach, we could obtain the complete pressure profile, provided that the continuity of the pressure is satisfied in the lens-corneal interface. This is automatically satisfied within our numerical scheme.

We chose the standard Dirichlet boundary conditions in the spatial coordinate:

$$P(t,0) = P_{ac} \quad and \quad P(t, x = L_c + L_{tears} + L_{lens}) = P_{air} \qquad \text{(Ap. 3)}$$

where P_{air} is the open-eye pressure, corresponding to the atmospheric pressure, and P_{ac} is the oxygen pressure in the anterior chamber.

As for the initial condition, we need to feed the stationary pressure profile in Eq.(1) in order to reproduce the evolution of the pressure profile from the closed-eye condition. This stationary closed-eye profile can be obtained by solving the steady-state equation:

$$\frac{\partial}{\partial x}\left(k(x)D(x)\frac{\partial P_{est}(x)}{\partial x}\right) - Q(P_{est}(x)) = 0 \qquad \text{(Ap.4)}$$

which is obtained from Eq.(Ap.1) by removing the temporal evolution. Eq.(Ap.4) is subjected to the boundary conditions:

$$P_{est}(0) = P_{ac} \text{ and } P_{est}(x = L_c + L) = P_{PC}\,, \qquad \text{(Ap. 5)}$$

where P_{PC} is the contact-lens/palpebral conjunctiva oxygen pressure (P_{PC}*=61.4 mmHg)*.

We then used the solution to Eq.(Ap.4-5) to define:

$$P(0, x) = P_{est}(x) \qquad \text{(Ap. 6)}$$

as the last boundary condition for Eq.(Ap.1).

The system of Eq.(Ap. 4-5) and Eq.(Ap.1-3) and (Ap.6) are solved using FiPy,[41] a finite volume PDE solver written in Python. Table I shows the different values for the parameters used in the numerical solution of the equations. We used a spatial grid with 10^3 points in all computations and time steps of 10^{-1}s for the time-dependent equations.

First, Eq.(Ap. 4-5) are numerically solved and the resulting profile is used as initial condition for Eq.(1-3) and (6). An iterative procedure was used due to the nonlinear nature of the transport equations Ap.1 to Ap.6, by “sweeping” the solutions over few iterations (see FiPy manual for details http://www.ctcms.nist.gov/fipy). Convergence was reached after the residual was below a predefined value (10^{-11} in our case). We checked both grid size and time step parameters so that further decrease in size would not result in any improvement. All the computations were performed in a personal computer with an Intel Core i7-3770K under Debian Linux. FiPy version 3.0 was used in all computations.

Multidimensional parameter optimization subject to bounds was done through the “fmin_tnc” function in the Scipy package (http://www.scipy.org/), which uses a Newton Conjugate-Gradient method. We used this optimization procedure to determine the optimized values of $Q_{c,max}$ and K_m parameters in the Monod kinetics model, Q*, D_c and k_c in the Larrea et al {2009} model, and Q’ and k’ in the MMM, for a predefined set of remaining parameters in the model.

Appendix II

Insert Table 2 approximately here.

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Tables.

Table 1. Parameters considered in this study to obtain the oxygen tension and flux profiles across the cornea-tear vault-contact lens.

Parameter	Symbol	Value	Units
Atmospheric Partial Pressure of Oxygen under Open Eye Conditions	p_{tc}	155	mmHg
Aqueous Humor Oxygen Tension	P_{aq}	24	mmHg
Maximum Corneal Oxygen Consumption	$Q_{c,max}$	$5.0E^{-05}$ (Weissman) $1.05E^{-04}$(Chhabra et al.)	$cm^3(O_2)\cdot cm^{-3}\cdot s^{-1}$
Corneal permeability	Dk_c	24.7 (Weissman) 86 (Compañ et al.)	Fatt Dk unit
Central Corneal Thickness	**CCT**	540	μm
Epithelium	T_{ep}	58	μm
Stroma	T_{sp}	480	μm
Endothelium	T_{en}	2	μm
Water permeability	Dk_w	93	Fatt Dk unit
Post-lens tear film thickness	T_{post}	50 to 300	μm
Lens permeability	Dk_{lens}	100, 140, 170	Fatt Dk unit
Lens thickness (central)	**L**	300, 350	μm

Fatt Dk units = 10^{-11} (cm^2 /sec)[ml $O_2\cdot$ $cm^{-3}\cdot mmHg^{-1}$)])
Fatt Dk/t_{av} units = 10^{-9} (cm ml O_2)/(cm^{-3} sec mmHg)

Table 2. Oxygen tension (in mmHg) calculated by the method described, at the cornea-tears interface [column 3 $p(x=x_c)$] (i.e. at the anterior corneal surface), at the tears-lens interface [column 4 $p(x=x_c+x_{tears})$] (i.e., at back surface of the lens), and at the position of minimum oxygen tension in the stroma [column 6 ($p(x=x_{min})$], found for all the systems cornea-tears-GP scleral lens analyzed in this study, developed from the values of the parameters in Table1. Column 1 provides the lens thicknesses, column 2 the tear vault thicknesses, and column 5 the position of minimal oxygen tension in the cornea.

Dlkl=100 Fatt units, $Q_{c,max}=5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 24.7 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	107.55	116.07	176.82	2.64
	100	100.96	117.36	182.01	2.18
	150	94.99	118.6	186.59	1.83
	200	89.32	119.7	191.57	1.54
	250	84.06	120.79	197.08	1.31
	300	79.15	121.87	201.11	1.12
300	50	101.41	109.6	182.01	2.21
	100	95.4	111.08	186.59	1.86
	150	89.7	112.47	191.57	1.56
	200	84.42	113.87	197.08	1.32
	250	79.48	115.14	201.11	1.13
	300	74.75	116.35	205.77	0.97
350	50	95.82	103.67	186.59	1.88
	100	90.09	105.35	191.57	1.58
	150	84.78	106.92	196.04	1.34
	200	79.82	108.37	201.11	1.15
	250	75.06	109.79	205.77	0.98
	300	70.79	111.2	210.04	0.85
Dlkl=140 Fatt units, $Q_{c,max}=5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 24.7 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	117.09	125.97	168.42	3.46
	100	109.83	126.93	174.89	2.82
	150	103.23	127.85	180.01	2.34
	200	96.98	128.68	185.63	1.95
	250	91.15	129.49	190.84	1.63
	300	85.73	130.3	195.66	1.38
300	50	112.2	120.83	172.22	3.01
	100	105.43	121.94	178.13	2.50
	150	99.01	122.99	183.65	2.07
	200	93.05	124.06	188.76	1.72
	250	87.5	125.02	193.48	1.46
	300	82.18	125.94	198.93	1.23

350	50	107.69	116.04	176.25	2.67
	100	101.14	117.32	181.67	2.20
	150	95.04	118.53	186.68	1.83
	200	89.32	119.63	192.39	1.54
	250	83.87	120.72	196.65	1.30
	300	78.96	121.81	201.71	1.11
Dlkl=170 Fatt units, $Q_{c,max}=5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 24.7 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	121.6	130.64	165.06	3.90
	100	114.02	131.44	171.33	3.17
	150	107.13	132.22	176.25	2.62
	200	100.61	132.91	182.66	2.17
	250	94.5	133.58	187.72	1.80
	300	88.84	134.27	192.39	1.52
300	50	117.37	126.2	168.66	3.48
	100	110.24	127.14	174.37	2.87
	150	103.48	128.02	179.69	2.36
	200	97.2	128.92	185.64	1.95
	250	91.35	129.74	190.21	1.64
	300	85.75	130.52	195.51	1.37
350	50	113.46	122.04	171.55	3.14
	100	106.52	123.13	176.72	2.58
	150	100.04	124.15	182.52	2.13
	200	93.95	125.08	188.03	1.78
	250	88.16	126.01	193.23	1.48
	300	82.94	126.94	198.14	1.26

Table 2b

Dlkl=100 Fatt units, $Q_{c,max}=5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 86 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	85.84	98.18	58.38	23.08
	100	78.2	101.42	84.11	22.12
	150	71.53	104.36	106.69	21.00
	200	65.45	106.87	127.22	19.77
	250	60.01	109.19	145.08	18.49
	300	55.09	111.36	161.87	17.22
300	50	78.71	90.27	82.33	22.19
	100	71.98	93.81	104.81	21.08
	150	65.85	96.93	125.24	19.85
	200	60.37	99.85	144.04	18.58
	250	55.42	102.43	160.78	17.31

	300	50.79	104.75	176.13	16.02
350	50	72.43	83.39	103.87	21.16
	100	66.26	87.12	124.25	19.94
	150	60.75	90.45	143.00	18.67
	200	55.75	93.44	159.69	17.40
	250	51.09	96.17	174.99	16.11
	300	47.03	98.78	188.62	14.89
Dlkl=140 Fatt units, $Q_{c,max}=5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 86 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	97.59	110.94	19.74	23.90
	100	88.58	113.65	49.40	23.34
	150	80.75	116.06	75.67	22.47
	200	73.71	118.12	99.50	21.39
	250	67.39	119.99	121.16	20.17
	300	61.72	121.73	138.98	18.91
300	50	91.47	104.18	40.50	23.57
	100	83.31	107.19	67.21	22.79
	150	75.97	109.81	91.58	21.76
	200	69.43	112.25	113.88	20.58
	250	63.56	114.38	133.53	19.34
	300	58.13	116.29	151.05	18.04
350	50	85.97	98.15	58.75	23.09
	100	78.36	101.39	83.66	22.13
	150	71.59	104.25	106.60	21.00
	200	65.46	106.78	126.99	19.77
	250	59.84	109.08	145.35	18.47
	300	54.91	111.26	162.44	17.19
Dlkl=170 Fatt units, $Q_{c,max}=5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 86 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	103.45	117.3	0.42	24.00
	100	93.73	119.69	32.49	23.72
	150	85.31	121.82	60.63	23.02
	200	77.77	123.63	85.64	22.04
	250	71	125.25	108.68	20.89
	300	64.96	126.77	128.08	19.66
300	50	97.96	111.24	18.25	23.91
	100	89.03	113.93	48.41	23.38
	150	81.05	116.27	74.75	22.51
	200	73.95	118.43	99.32	21.42
	250	67.61	120.31	119.36	20.23
	300	61.77	121.99	138.51	18.94
350	50	92.98	105.78	35.25	23.67

	100	84.59	108.71	62.87	22.94
	150	77.13	111.28	87.88	21.94
	200	70.4	113.54	110.64	20.78
	250	64.28	115.59	130.53	19.52
	300	58.9	117.52	148.16	18.24

Table 2c

Dlkl=100 Fatt units, $Q_{c,max}$=10.5x10^{-5} mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 24.7 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**p(x=x_c) (mmHg)**	**p(x=x_c+x_{tears}) (mmHg)**	**X_{min} (μm)**	**P(x=x_{min}) (mmHg)**
250	50	91.78	103.13	214.62	0.037
	100	84.04	105.57	219.385	0.030
	150	77.3	107.86	223.25	0.025
	200	71.07	109.89	228.195	0.021
	250	65.48	111.83	231.4	0.017
	300	60.42	113.68	235.985	0.015
300	50	84.56	95.32	219.385	0.030
	100	77.76	98.08	223.25	0.025
	150	71.48	100.59	227.205	0.021
	200	65.85	103.05	231.4	0.018
	250	60.75	105.25	234.895	0.015
	300	55.95	107.29	238.83	0.013
350	50	78.22	88.42	223.25	0.025
	100	71.9	91.43	227.205	0.021
	150	66.23	94.21	231.4	0.018
	200	61.09	96.76	234.895	0.015
	250	56.25	99.15	238.83	0.013
	300	52.1	101.48	242.165	0.011
Dlkl=140 Fatt units, $Q_{c,max}$=10.5x10^{-5} mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 24.7 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**p(x=x_c) (mmHg)**	**p(x=x_c+x_{tears}) (mmHg)**	**X_{min} (μm)**	**P(x=x_{min}) (mmHg)**
250	50	103.44	115.52	207.9	0.049
	100	94.51	117.41	213.155	0.039
	150	86.71	119.18	217.61	0.032
	200	79.53	120.76	222.255	0.026
	250	73.05	122.25	226.2	0.022
	300	67.22	123.7	230.535	0.018
300	50	97.39	109	211.375	0.042
	100	89.29	111.18	215.73	0.035
	150	81.84	113.17	220.275	0.028
	200	75.14	115.11	225.16	0.023

	250	69.11	116.85	229.445	0.019
	300	63.46	118.47	233.13	0.016
350	50	91.97	103.09	214.79	0.037
	100	84.28	105.53	219.285	0.030
	150	77.36	107.76	224.12	0.025
	200	71.06	109.8	228.355	0.021
	250	65.21	111.73	231.99	0.017
	300	60.15	113.6	236.215	0.015
Dlkl=170 Fatt units, $Q_{c,max}=10.5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 24.7 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	109.14	121.55	204.54	0.057
	100	99.63	123.16	210.49	0.045
	150	91.30	124.68	214.79	0.036
	200	83.67	126.02	219.29	0.030
	250	76.76	127.29	224.12	0.024
	300	70.25	128.53	228.36	0.020
300	50	103.8	115.81	207.82	0.050
	100	95.04	117.68	212.91	0.040
	150	87.00	119.39	217.31	0.032
	200	79.78	128.06	222.04	0.026
	250	73.28	122.56	226.18	0.022
	300	67.20	123.96	230.85	0.018
350	50	98.97	110.53	210.09	0.044
	100	90.58	112.65	215.33	0.036
	150	83.02	114.59	219.96	0.029
	200	76.14	116.35	224.00	0.024
	250	69.77	118.03	228.57	0.020
	300	64.25	119.65	232.65	0.017

Table 2d

Dlkl=100 Fatt units, $Q_{c,max}=10.5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 86 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	62.05	78.62	200.34	5.23
	100	53.51	84.2	214.05	3.79
	150	46.71	89.29	226.07	2.83
	200	40.95	93.7	236.12	2.17
	250	36.19	97.7	244.92	1.71
	300	32.19	101.35	252.34	1.40
300	50	54.06	69.35	213.16	3.87
	100	47.15	75.52	225.13	2.89
	150	41.32	80.95	235.13	2.21

	200	36.5	85.94	243.88	1.74
	250	32.45	90.3	251.25	1.42
	300	28.85	94.17	259.35	1.17
350	50	47.6	61.86	224.19	2.94
	100	41.69	68.32	234.14	2.25
	150	36.81	74.06	242.84	1.77
	200	32.71	79.15	251.25	1.44
	250	29.07	83.7	258.21	1.18
	300	26.16	87.91	264.78	1.00
Dlkl=140 Fatt units, $Q_{c,max}=10.5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 86 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	76.36	94.64	178.50	8.13
	100	65.26	99.12	195.36	5.83
	150	56.34	103.25	210.09	4.22
	200	48.89	106.85	222.26	3.11
	250	42.74	110.13	232.44	2.36
	300	37.65	113.14	241.44	1.85
300	50	68.74	85.99	190.02	6.52
	100	59.19	91.10	205.39	4.70
	150	51.21	95.65	218.30	3.44
	200	44.67	99.88	229.32	2.57
	250	39.25	103.58	239.26	2.00
	300	34.54	106.87	247.95	1.59
350	50	62.24	78.60	200.69	5.25
	100	53.75	84.16	214.34	3.81
	150	46.77	89.15	226.20	2.83
	200	40.96	93.58	235.99	2.17
	250	35.96	97.56	244.53	1.71
	300	31.97	101.23	252.88	1.39
Dlkl=170 Fatt units, $Q_{c,max}=10.5x10^{-5}$ mL O_2 cm^{-3} s^{-1} , $(Dk)_c$= 86 Fatt units					
T_{lens} (μm)	**T_{tears} (μm)**	**$p(x=x_c)$ (mmHg)**	**$p(x=x_c+x_{tears})$ (mmHg)**	**X_{min} (μm)**	**$P(x=x_{min})$ (mmHg)**
250	50	83.91	103.01	166.74	9.79
	100	71.51	106.89	186.46	7.09
	150	61.47	110.47	201.63	5.11
	200	53.11	113.60	215.33	3.71
	250	46.19	116.46	226.20	2.76
	300	40.51	119.10	235.99	2.13
300	50	76.82	95.02	177.56	8.22
	100	65.85	99.50	195.05	5.93
	150	53.57	103.50	209.39	4.28
	200	49.14	107.23	222.04	3.14
	250	42.95	110.52	231.63	2.38

	300	37.61	113.43	241.11	1.85
350	50	70.62	88.03	187.53	6.89
	100	60.65	92.97	203.45	4.96
	150	52.43	97.41	215.80	3.60
	200	45.61	101.38	227.27	2.69
	250	39.82	104.96	237.69	2.06
	300	35.18	108.27	245.74	1.64

Figures:

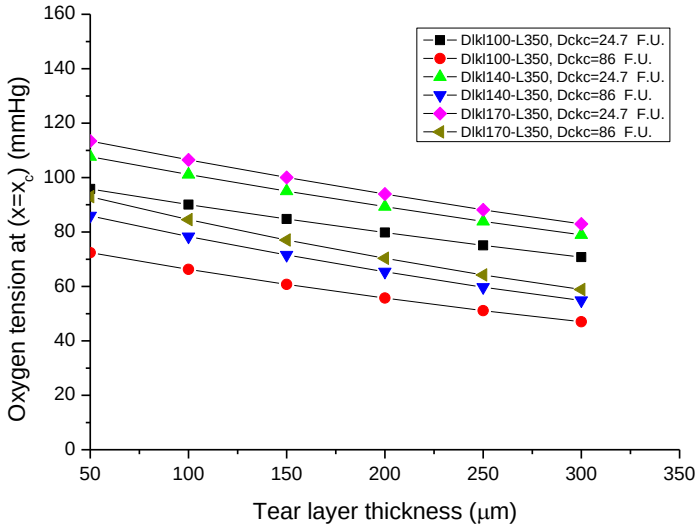

Figure 1. Predicted oxygen tension in the tear film under a scleral GP lens, just in the point in contact with the cornea (position $x=x_c$), as a function of the tear layer thickness for three simulated scleral GP lenses of different oxygen permeabilities (100, 140 and 170 Fatt Dk units) but one lens thickness (L=350 μm). Data has been obtained following the Monod kinetics model with a maximum corneal oxygen consumption rate Q_c,max = $5x10^{-5}$ ml O_2 cm-3 s-1. Both corneal oxygen permeabilities, Dk_c=24.7 or 86 Fatt units (barrer) are considered here, with corneal thickness t_c=540 um. Tear permeability $P_{tears}=P_{water}= D_wk_w$=93 Fatt units.

The reader should note that this figure may be directly compared to Figure 1 of Jaynes et al[16] but those authors used Dk_c of 24.7 Fatt units, and plotted a lens of Dk=140 Fatt units, 300 μm thick in place of the lens of Dk=170 Fatt units, 350 μm thick plotted here.

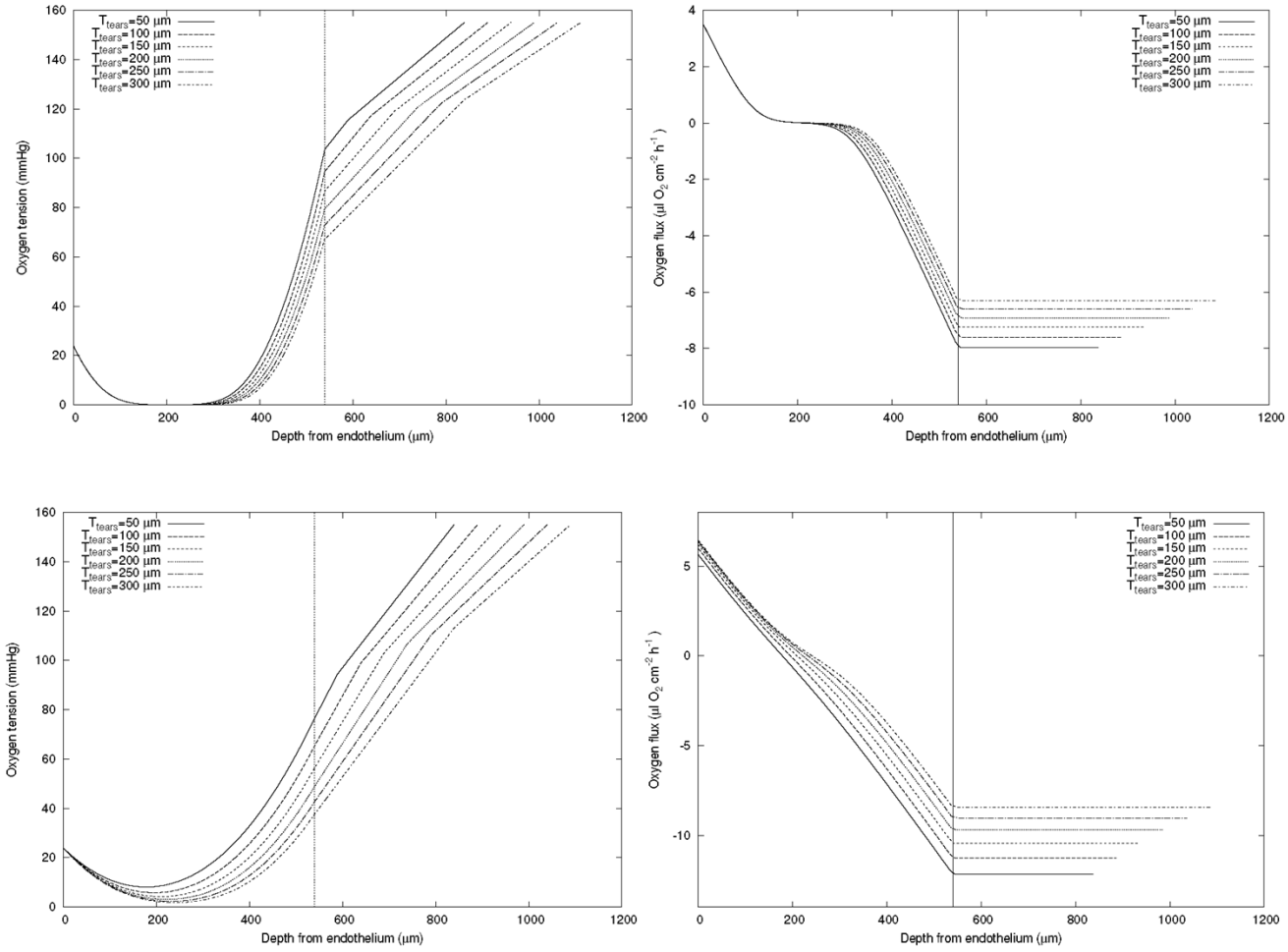

Figure 2. Oxygen tension (partial pressure) (**left panels**) and flux (**right panels**) profiles versus depth from endothelium for the system cornea-tears-lens at different tear vault thicknesses 50 to 300 um obtained following the Monod kinetics model with a maximum corneal oxygen consumption rate: $Q_{c,max} = 1.05x10^{-4}$ ml O_2 cm^{-3} s^{-1}. A vertical line marks the anterior corneal surface. Scleral GP lens thickness is 250 μm and its permeability 140 Fatt Dk units. **Top panels:** corneal oxygen permeability Dk_c is 24.7 Fatt units and **bottom panels**: Dk_c is 86 Fatt units. In both cases the cornea thickness is t_c=540 um and tear permeability $P_{tears}=P_{water}= D_w k_w$=93 Fatt units.

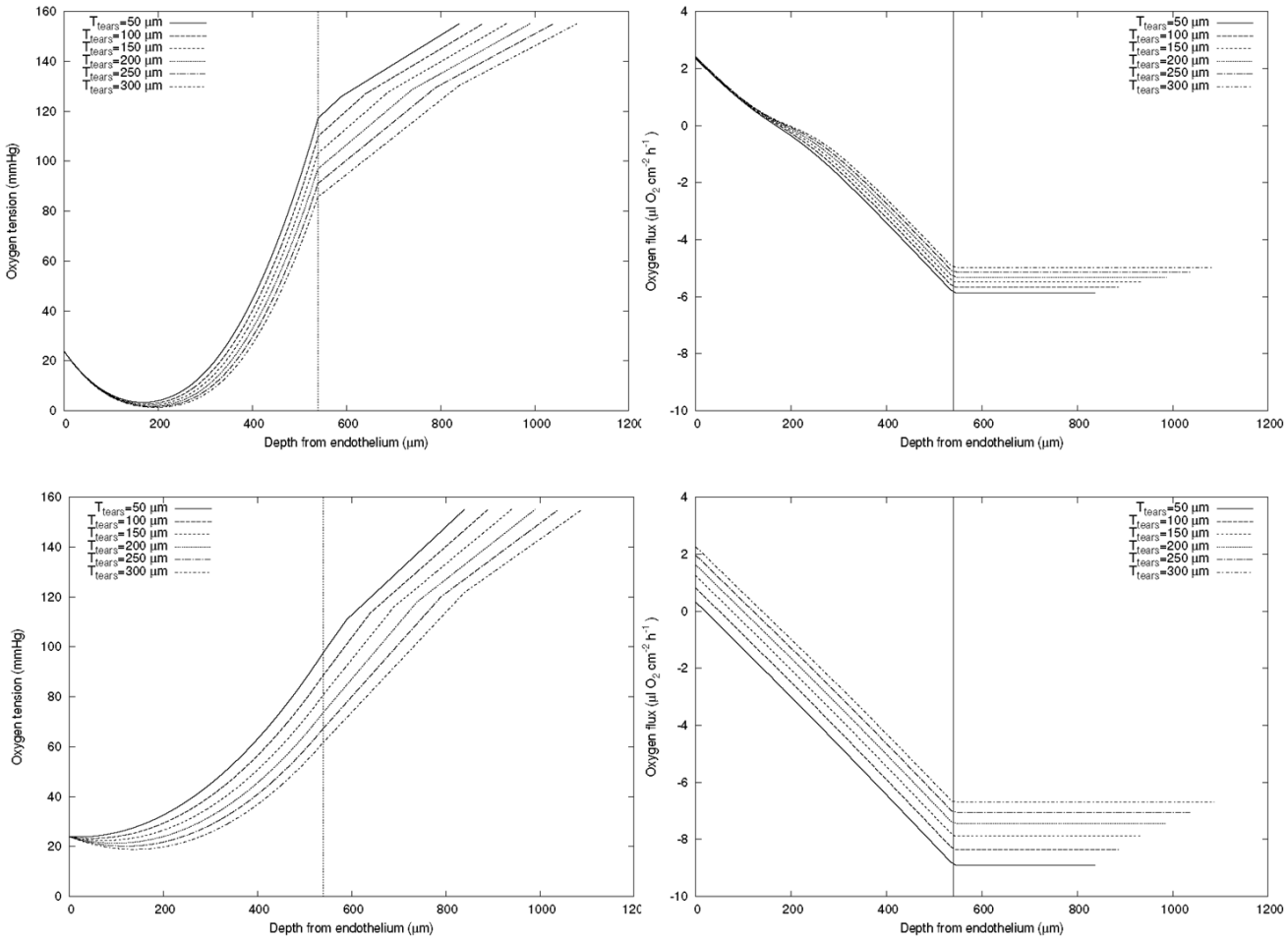

Figure 3. Oxygen tension (**left panels**) and flux (**right panels**) profiles versus depth from endothelium for the system cornea-tears-lens at different tear vault thicknesses 50 to 300 um, obtained following the Monod kinetics model with a maximum corneal oxygen consumption rate $Q_{c,max}$ = 5x10^{-5} ml O_2/cm^3 s. A vertical line marks the anterior corneal surface. Scleral lens thickness is 250 μm and its permeability 140 Fatt Dk units. **Top panels**: corneal oxygen permeability Dk_c=24.7 Fatt units and **bottom panels**: Dk_c=86 Fatt units. In both cases the cornea thickness is t_c=540 um and tear permeability $P_{tears}=P_{water}= D_w k_w$=93 Fatt units.

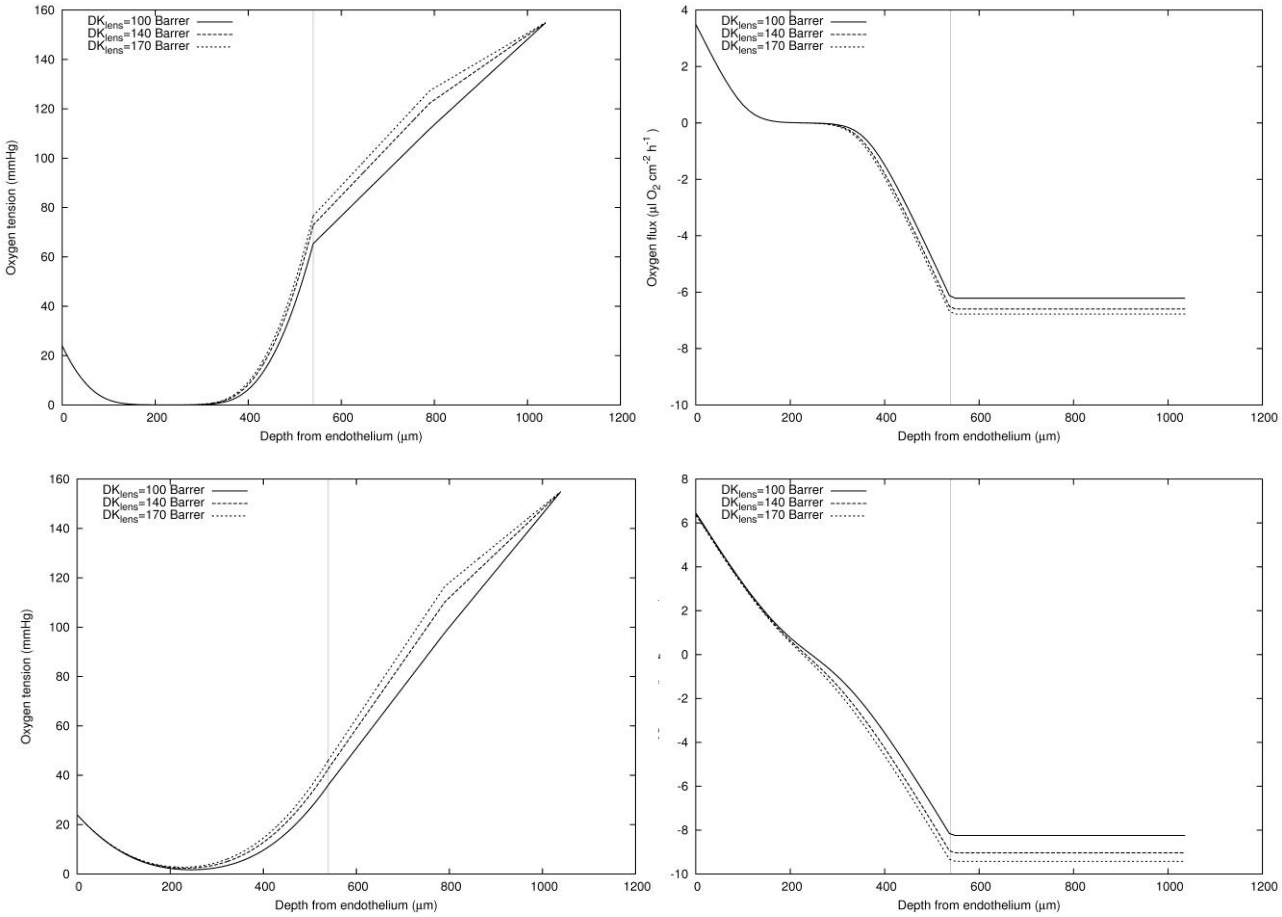

Figure 4. Oxygen tension (**left panels**) and flux (**right panels**) profiles versus depth from endothelium for the system cornea-tears-lens at different tear vault thicknesses 50 to 300 μm obtained following the Monod kinetic model with a maximum oxygen consumption rate $Q_{c,max}$ = 1.05x10^{-4} ml O_2 cm^{-3} s^{-1}. A vertical line marks the anterior corneal surface. Scleral GP lens and tear film thickness are both held constant at 250 μm but scleral Dk varies from 100, 140, 170 Fatt Dk units. **Top panels**: Corneal oxygen permeability Dk_c is 24.7 Fatt units and **bottom panels**: Dk_c is 86 Fatt units. Corneal thickness t_c is 540 μm and tear permeability, P_{tears}=P_{water}= D_wk_w=93 Fatt units.

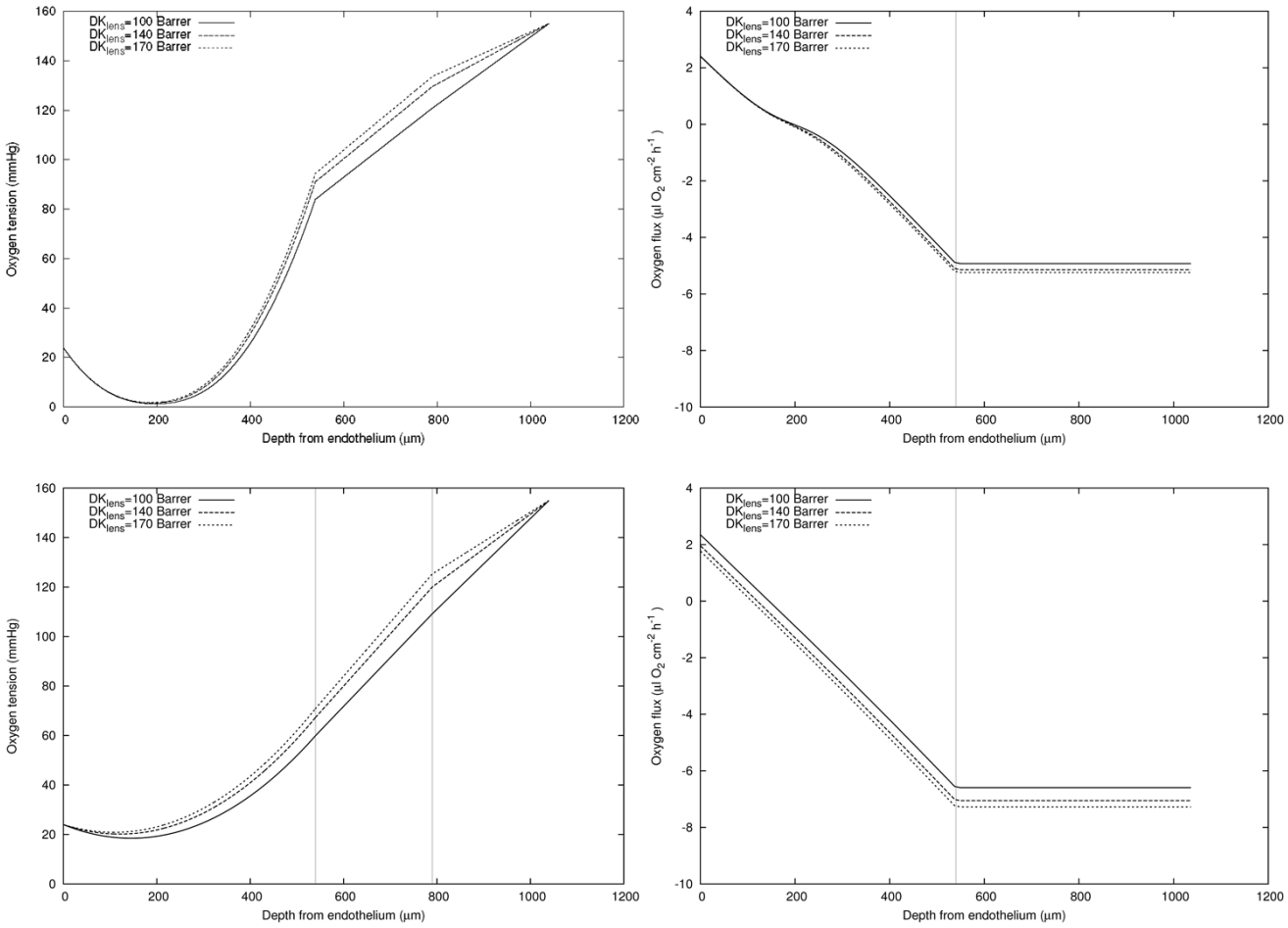

Figure 5. Oxygen tension (**left panels**) and flux **(right panels**) profiles versus depth from endothelium for the system cornea-tears-lens at different thicknesses of tears 50 to 300 μm obtained following the Monod kinetic model with a maximum oxygen consumption rate $Q_{c,max}$ = $5x10^{-5}$ mL O_2 cm^{-3} s^{-1}. A vertical line marks the anterior corneal surface. Scleral lens thickness and tear film thickness are both held constant both at 250 μm but scleral GP lens Dk varies (100, 140 and 170 Fatt units as shown). **Top panels**: corneal oxygen permeability Dk_c is 24.7 Fatt units, and **bottom panels**: Dk_c is 86 Fatt units. Corneal thickness t_c=540 μm and tear permeability is P_{tears}=P_{water}= D_wk_w=93 Fatt units.