MSEL
Modelling in Science Education and Learning

Modelling in Science Education and Learning
Volume 1, No. 1, 2008.
Instituto Universitario de Matemática Pura y Aplicada

The Engineering as Scenario and the Mathematical Models as Actors

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Abstract

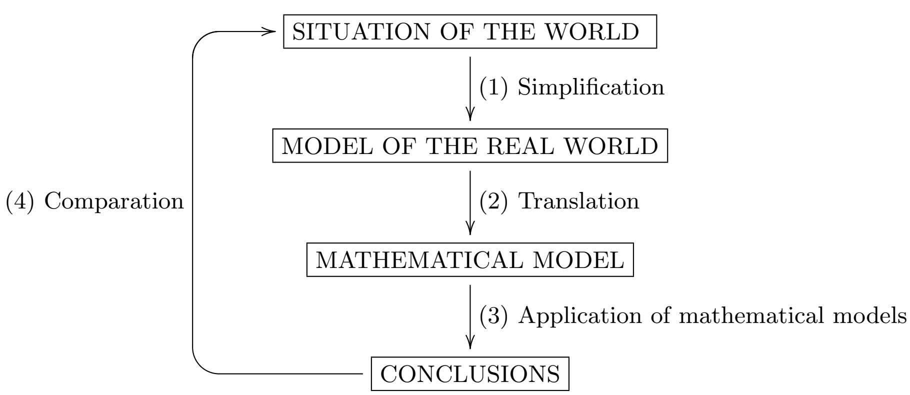
An experience of innovative teaching in the engineering curriculum is presented. It includes work in projects as a component of teaching-learning, analysing the validity and viability of the methodology of mathematical modelling processes. Some of the analysed models, among others, are based on the study of electrocardiograms, emphasizing the importance of Fourier series as a model to interpret this situation. The methodological change versus traditional teaching stands out, and it acquires a heuristic and cognitive trend emphasising the epistemology of mathematics and re-examining the evaluation processes.

1 Antecedents: mathematics and engineering, a necessary relationship

In the technologically advanced society of the XXI century it is necessary to use a pedagogy of the XXI century instead of a pedagogy, perhaps obsolete, of the XIX century; for this reason it is necessary to redefine the contents and methodologies of the formation of the users of mathematics. The step of the tradition to the innovation is not a change of support; it is to verify and to analyze new forms of teaching / learning which provide optimum cognitive results; for this reason not only is it required a good formation in mathematics, but also in didactics and interdisciplinary. I will describe experiences that have obtained results of improvement in the educational quality and which, in turn , highlight the mathematical and technological productions shown by the students. Puig Adam pointed. “One of the fundamental defects that the mathematical teaching for technicians had at the beginning of the century was its excess of abstraction, its unconscious estrangement of all immediate application to the real world..... the blame of its inability was not in the same mathematics, but in the way it had been taught”. The comfortable pretext: “You will see how this is applied in....” rarely had confirmation

2 Educational methodology

The methodological proposal is based on the mathematical modelling as teaching-learning tool. The mathematical modelling consists on formulating a problem of the daily life or technical situation, in mathematical terms, to solve it if it is possible and to interpret the results in terms of the problem and of the outlined situation.



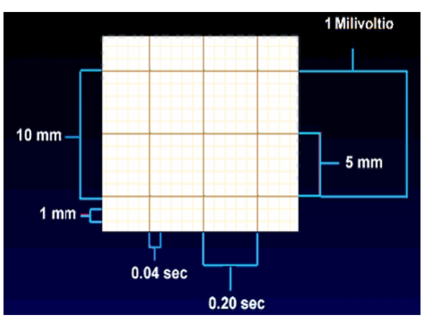
These methodological aspects are structured in what I call projects, in which the student learns the concepts and is not a mere spectator in the acquisition of knowledge. The objective of the projects is to discover if the students are able to develop a real problem starting from the information given by the professor and to build a theory which explains the studied phenomenon. The project is defended in public in the last working week of the course and it contemplates the interventions of the other students; in the exhibition the professor questions the students and the interventions are registered in videotape. The projects are concerted (in groups of between 3 and 4 people) in a previous interview with the professor. There is an investigation component: the student must pick up information with the purpose of developing the proposed activities, this way it is sought that the student gets in contact with the extra-academic world. I would like to highlight that there are other areas of knowledge involved in the elaboration of projects, which provides a bigger connection between the binomial mathematics-reality. We will understand the realization of a project like a form of interdisciplinary learning based on the experience and able to cohabit with other traditional forms of learning. The inclusion of

projects is justified by the following aspects: The acquisition of concepts. An active learning. Development of abilities. Work in group. Assumption of responsibilities.
In each project an evaluation test filled in by the professor is included and it is used to obtain information of the process of the students' learning. This grill contemplates aspects of global design, mathematical content, clarity, mathematical attitude, conclusions and comments and final punctuation. In the didactic investigation an articulation of the mathematics? content that favours the interdisciplinary perspective is outlined using and discovering mathematical concepts by means of outlining real situations. The methodological change versus the traditional teaching is emphasized acquiring a heuristic side that highlights the epistemology of the mathematics and it rethinks the evaluation processes.

3 Description of some models

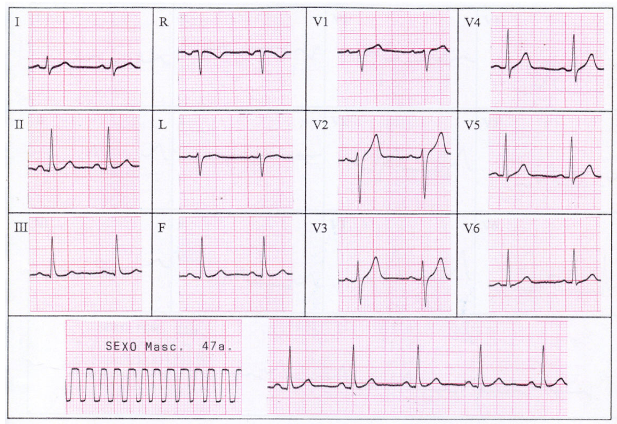
3.1 Modelling an Electrocardiogram

The objective is to build a mathematical model that gives us information about the condition of the heart. For this purpose real samples of electrocardiograms of different kinds (healthy heart, sick heart, tests carried out in adults, in children...)are picked up. In the work, the simulation and computer approach of the graphs that shows us the machine is explicit; this way Fourier?s series necessarily appear. By means of the simulation in MAPLE it is obtained that the values of the Fourier coefficients are not the same ones if it is a healthy or sick heart, neither necessarily the same ones if the range of the patient's age is different. The graph of the electrocardiogram is presented in a millimetre paper that measures the voltage vertically and the time horizontally, determined by the movement of the paper.



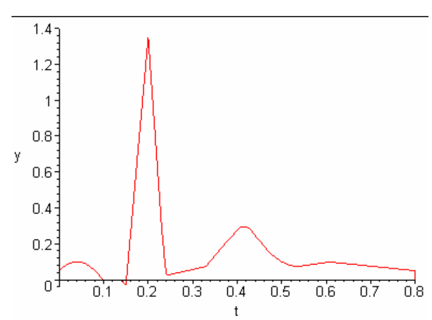
The data to be taken into account are:
- Speed of the electrocardiogram similar to 25mm/seg.
- 1mm = 0.04 seg.
- 5mm = 0.20 seg.
- 1mm vertical = 0.01 mvolt.

Therefore, by means of the visual analysis of a great number of electrocardiograms, it is possible to generalize that with something as simple as to look at the value of the width of wave of the electrocardiogram we can guess if it belongs to a healthy or a sick heart. With the work we can establish the conclusion that only checking that those values are between 0.15mvolt and 0.08mvotls, the corresponding electrocardiogram belongs to a healthy person. In the illustration a normal electrocardiogram corresponding to the heart of a 40 year-old male is shown



With the help of MAPLE7 we can simulate analytically and visualize the function. Let us notice that the feigned graph is defined in the interval $(0, 0.8)$. This function is extended periodically with a period of 0.8 seconds.

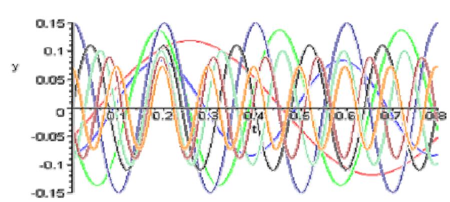
$$\left.\begin{array}{cc}
.2\sin(5.714285714\pi t + .2702702703\pi) - .1 & -t \le 0 \text{ and } t < .08 \\
-2.5t + .25.08 & -t \le 0 \text{ and } t < .1 \\
0 & .1 - t \le 0 \text{ and } t < .14 \\
-3t + .42 & .14 - t \le 0 \text{ and } t < .15 \\
7.6t - 4.17 & .15 - t \le 0 \text{ and } t < .2 \\
-35t + 8.35 & .2 - t \le 0 \text{ and } t < .23 \\
-27.5t + 6.625 & .023 - t \le 0 \text{ and } t < .24 \\
.55t - .108 & .24 - t \le 0 \text{ and } t < .33 \\
2.91t - .8850 & .33 - t \le 0 \text{ and } t < .39 \\
.2\sin(9.280742459\pi t + .6493506494\pi) + .1 & .39 - t \le 0 \text{ and } t < .44 \\
.2\sin(1.428571429\pi t + .7369196758\pi) + 2.075 & .44 - t \le 0 \text{ and } t < .53 \\
.35t - .114 & .53 - t \le 0 \text{ and } t < .6 \\
-.25t + .25 & .6 - t \le 0 \text{ and } t < .8
\end{array}\right\}$$



Fourier's series that will let us calculate the coefficients are used and, in turn, we compare different electrocardiograms. Carrying out calculations for the expression:

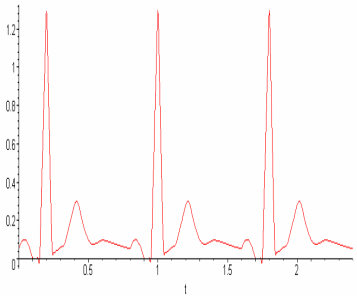
$$Sf(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \big(a_n \cos(nwt) + b_n \sin(nwt)\big).$$

with $T = 0.8$ and $w = \frac{2\pi}{T} = \frac{2\pi}{0.8}$. Superimposing 8 harmonics it is obtained:



The sum of the same ones is close to the initial function but using MAPLE it let us obtain a more precise information, for this we consider 50 harmonics and we will show the graphic visualization represented in three consecutive intervals of period 0.8 seconds:

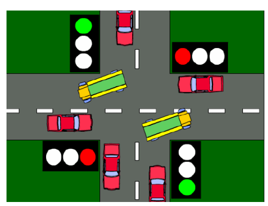
$$\mathtt{plot}\left(\frac{.3195816819}{8}+\sum_{n=1}^{50}\left(\cos\left(\frac{n2\pi t}{.8}\right)a_n+\sin\left(\frac{n2\pi t}{.8}\right)b_n\right), t=0..2.4, \mathtt{colour}=\mathtt{red}\right)$$



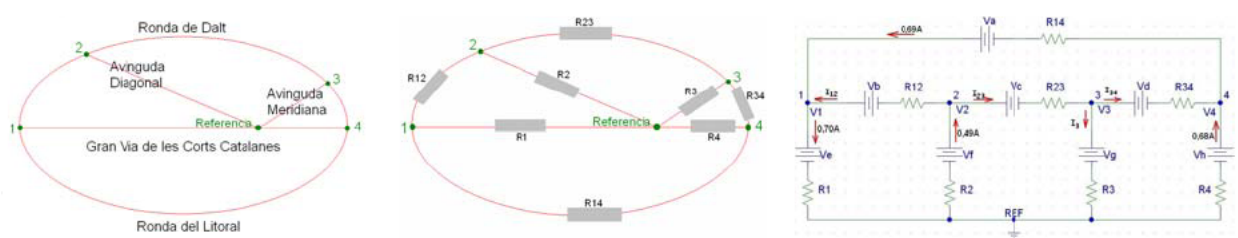
Let's observe that we have achieved a better model of the electrocardiogram.

3.2 Intersection regulated by traffic lights

In this case the operation of a net of traffic is analyzed. Specifically an intersection regulated by traffic lights is chosen to study the evolution of the traffic.
In the project the analogy between the classical elements of the theory of circuits and the characteristics of the traffic is established: Laws of Kirchoff, graphs theory ... and it is tried to show results as a first approach to the problem of the administration of the traffic. A real intersection was chosen and it was illustrated using flash of Macromedia and a scale model was even built to simulate the traffic net just as the following illustrations show

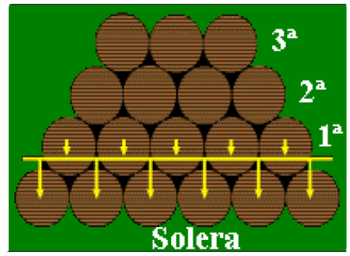


Next the sequence-diagram followed for modelling is reflected:



3.3 Method of the vintage

The oldest wine is in the inferior line of barrels and the newest in the line above. Every year, half of the content of the barrels of the inferior line is bottled as sherry and it is filled with wine of the barrels of the immediately superior line. The process is completed adding new wine to the barrels from the line above. The problem is to find a mathematical model that determines the quantity of wine of n years that is extracted from the k lines of barrels (Larson, 2003. Calculation I).



The students deduced, after considering numeric series, that the expression that determines the quantity of wine of n years that it is extracted of k lines of the casks is:

$$f(n,k) = \binom{n}{k}\frac{1}{2^n} \text{ for } k < n.$$

This is another modelling example that shows its epistemological component.

4 Conclusions: From the tradition to the innovation

In the epistemological environment a bigger connection with the engineering curriculum is achieved and the interdisciplinarity stands out, showing the mathematics not detached from the reality and from the professional environment. It highlights the formative aspect of the mathematics, in the sense that the modelling techniques allow to stimulate the interest for discovery and creativity. In the heuristic environment the work group is favoured, as a present characteristic in an advanced society of the XXI century, promoting debate and critical sense at the same time. The traditional teaching maintains excessive formalisms that often are far from the future engineer's reality. In the modelling the load of formalisms is avoided betting for a more intuitive and closer learning to the technical situations. In the cognitive environment we can establish that the mathematical modelling is a good tool for the teaching / learning mathematical process. In this type of experiences the prominent role of the mathematical modelling in the curricular scene of engineering is outlined, acquiring a protagonism the learning produced by the students that in turn assume the role of actors in their own learning.

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