

Estudio analítico y por elementos finitos de postpandeo de paneles aeronáuticos

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Grado en Ingeniería Aeroespacial
Universidad Politécnica de Valencia
Escuela Técnica Superior de Ingeniería del Diseño

17 de julio de 2018

1 Introducción

- Introducción al postpandeo en estructuras aeronáuticas
- Objetivos

2 Inicio pandeo

- Metodología
- Panel articulado

3 Postpandeo placas

- Metodología
- Resultados

4 Postpandeo paneles

- Panel articulado
- Deformada
- Tensión placa
- Larguerillo

5 Conclusiones

El pandeo en paneles aeronáuticos

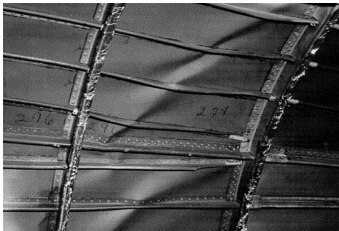
El pandeo en paneles aeronáuticos

- Estructura semimonocasco:
pequeño espesor



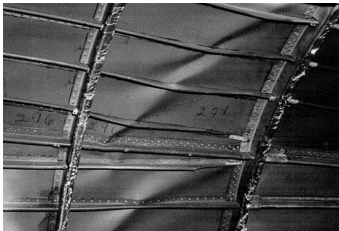
El pandeo en paneles aeronáuticos

- Estructura semimonocasco: pequeño espesor
- Pandeo: inestabilidad elástica



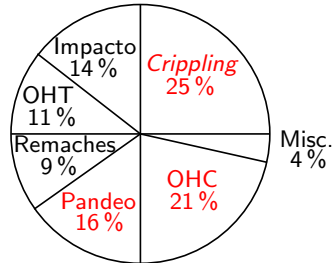
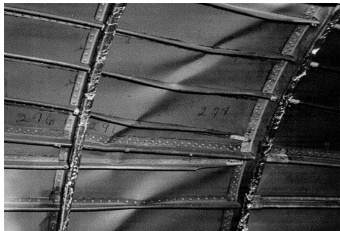
El pandeo en paneles aeronáuticos

- Estructura semimonocasco: pequeño espesor
- Pandeo: inestabilidad elástica
- Priorización pandeo de piel



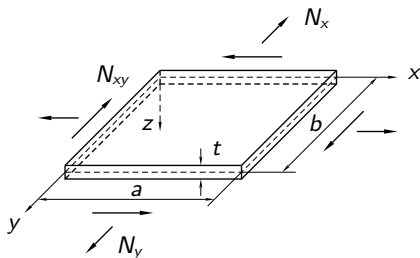
El pandeo en paneles aeronáuticos

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Objetivos

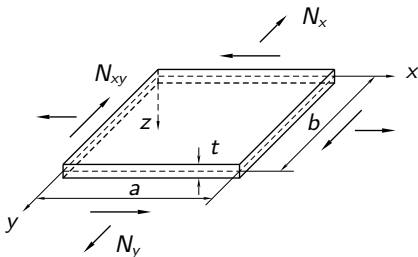
Objetivos del trabajo:



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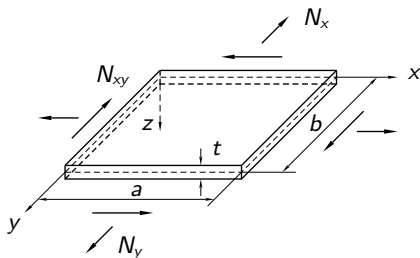
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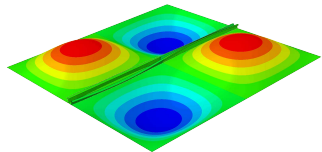
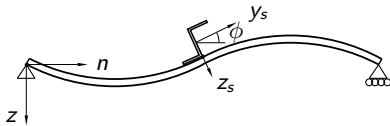
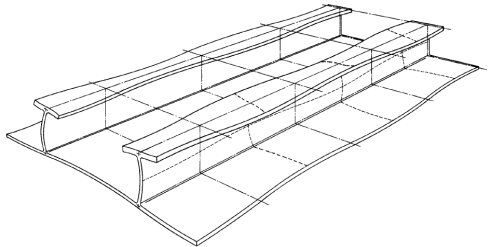
- Inicio pandeo
- Postpandeo placas



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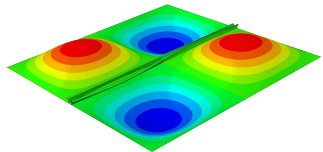
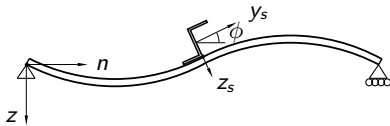
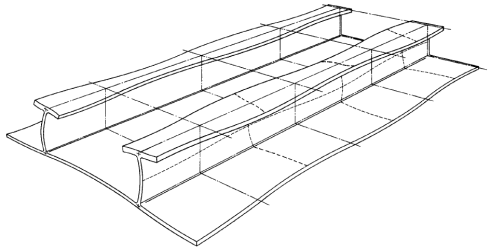
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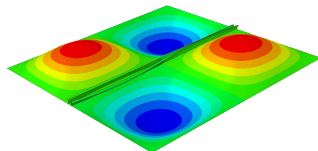
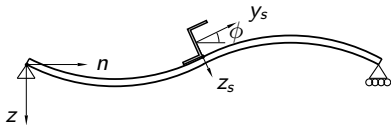
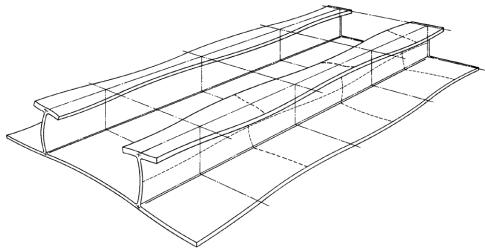
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- Validaciones:



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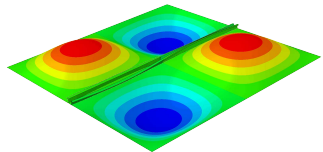
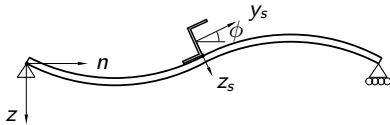
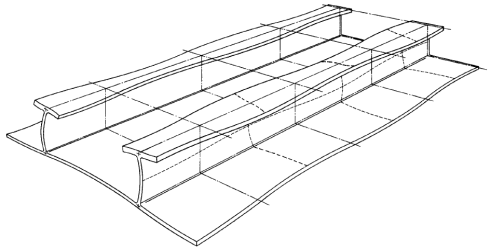
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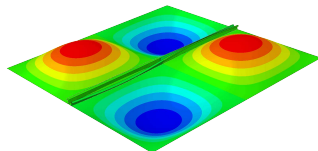
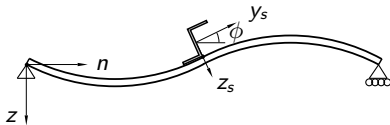
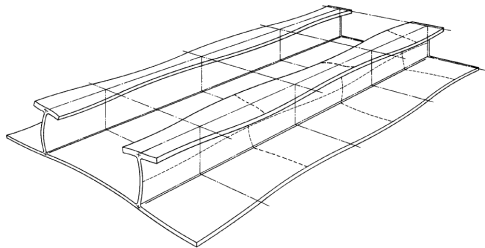
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Metodología - Analítica

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- Ecuación de gobierno:

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}$$

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$$\frac{\partial \Pi}{\partial w_{mn}} = 0$$

Metodología - Analítica

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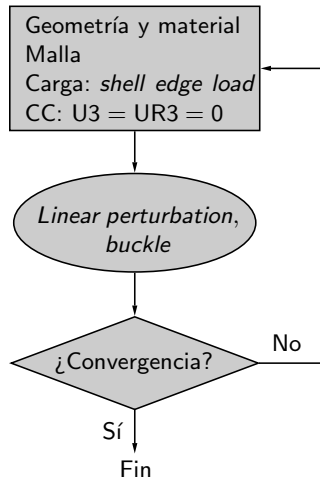
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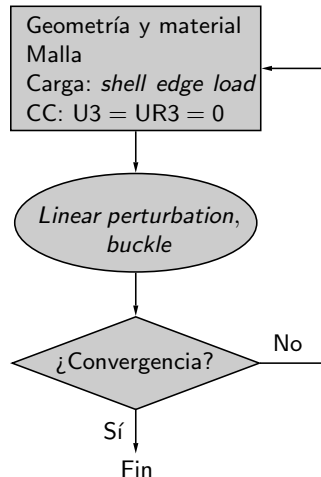
$$\frac{\partial \Pi}{\partial w_{mn}} = 0 \rightarrow \begin{cases} \text{Carga crítica, } N_{cr} \\ \text{Modos} \end{cases}$$

Metodología - Abaqus[®]



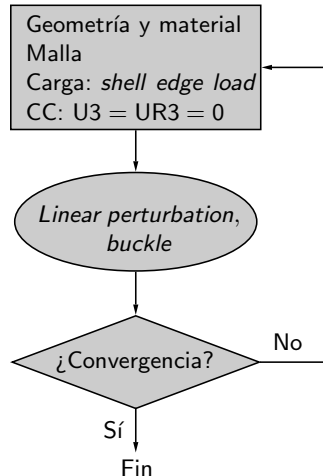
Metodología - Abaqus[®]

- Geometría *shell*



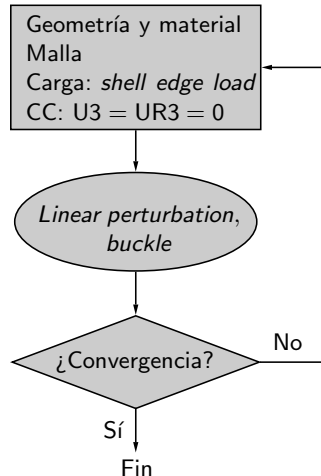
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- Geometría *shell*
- Elástico lineal



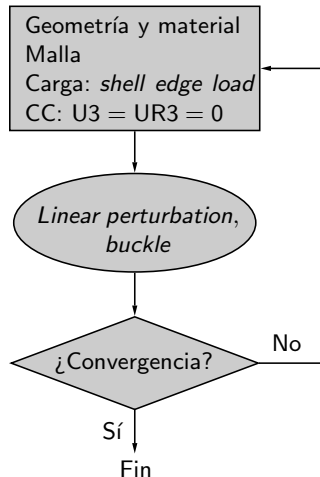
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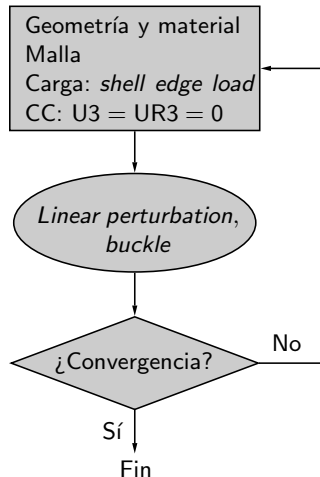
Metodología - Abaqus[®]

- Geometría *shell*
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- Interpolación lineal

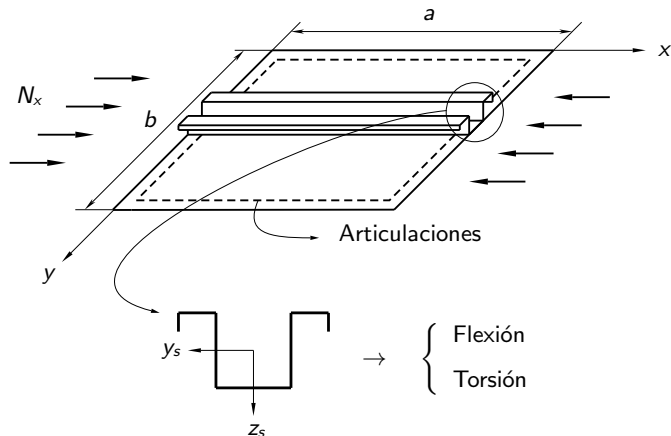


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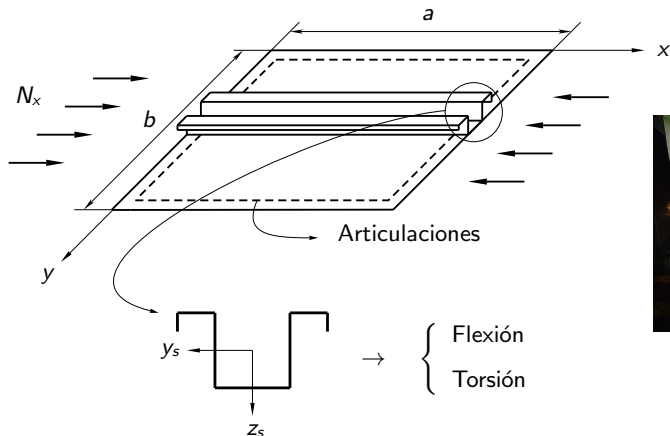
- Geometría *shell*
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- Interpolación lineal
- Contacto nodo (larguerrillo) - superficie (placa)



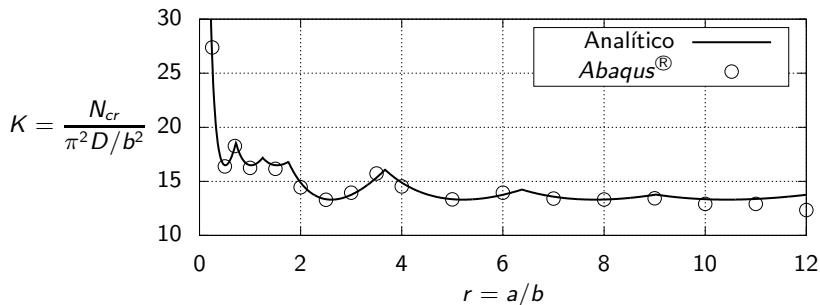
Panel articulado - Boeing 747



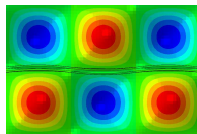
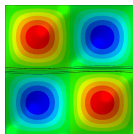
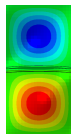
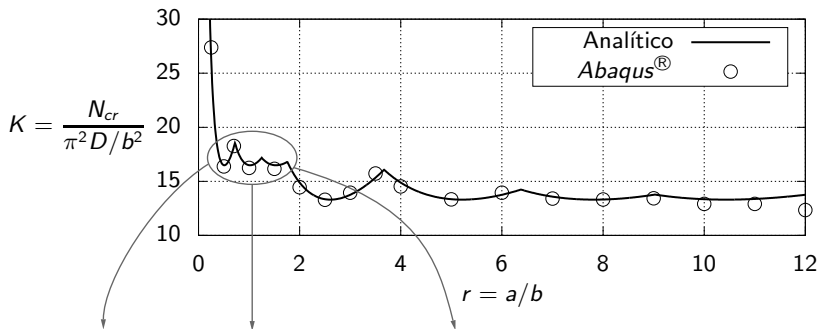
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Panel articulado - Carga crítica y modos de pandeo



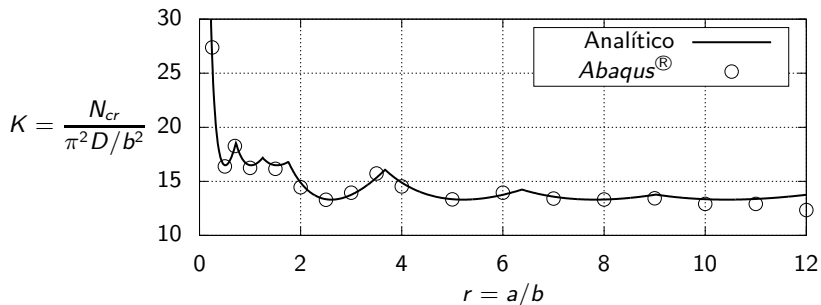
Panel articulado - Carga crítica y modos de pandeo



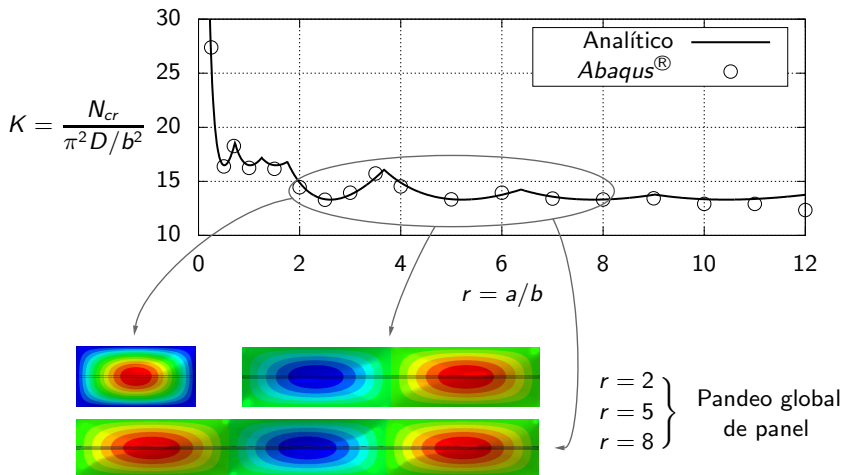
$r = 0.5$
 $r = 1$
 $r = 1.5$

} Pandeo local de piel

Panel articulado - Carga crítica y modos de pandeo

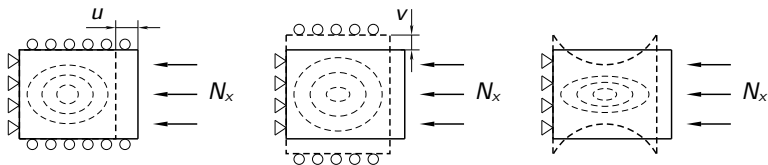


Panel articulado - Carga crítica y modos de pandeo

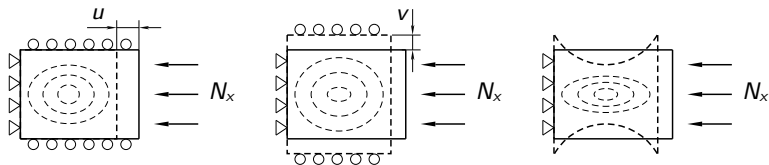


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Postpandeo de placas a compresión



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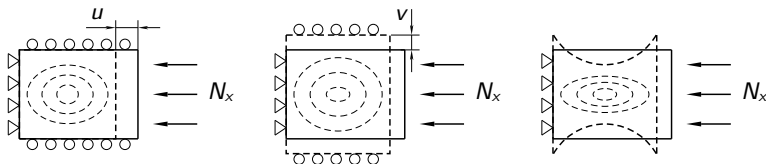


Ecuaciones de Von Karman; incógnitas w , Φ (función de Airy):

$$\frac{D}{t} \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) = \frac{\partial^2 \Phi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 \Phi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 \Phi}{\partial x^2} \frac{\partial^2 w}{\partial y^2}$$

$$\frac{1}{E} \left(\frac{\partial^4 \Phi}{\partial x^4} + 2 \frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} \right) = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}$$

Postpandeo de placas a compresión

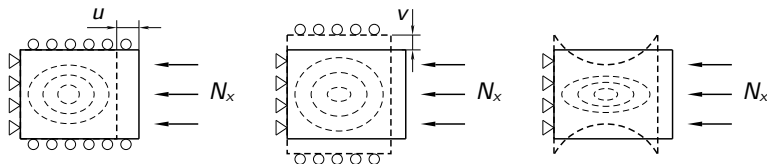


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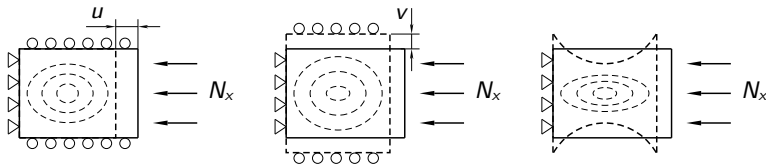


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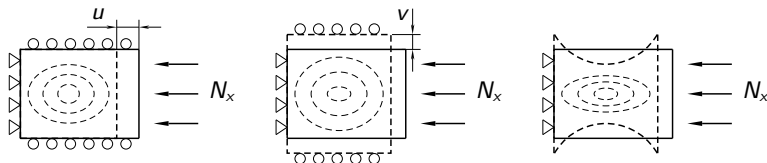


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EDPs no lineales

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- Solución prueba, 2 modos:

$$w(x, y) = w_{11} \sin\left(\frac{\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) + w_{21} \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right)$$

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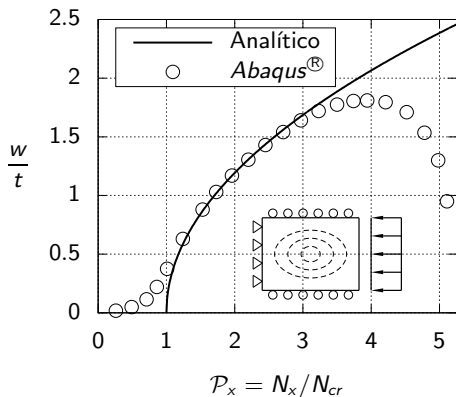
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- Función de Airy 2ª ecuación: $\Phi(x, y) = \Phi_H(x, y) + \Phi_P(x, y)$
- Coeficientes w_{11} , w_{21} 1ª ecuación, método de Galerkin:

$$\int_0^a \int_0^b \left[\frac{D}{t} \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) - \frac{\partial^2 \Phi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} \right. \\ \left. - 2 \frac{\partial^2 \Phi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 \Phi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \sin\left(\frac{p\pi x}{a}\right) \sin\left(\frac{q\pi y}{b}\right) dx dy = 0$$

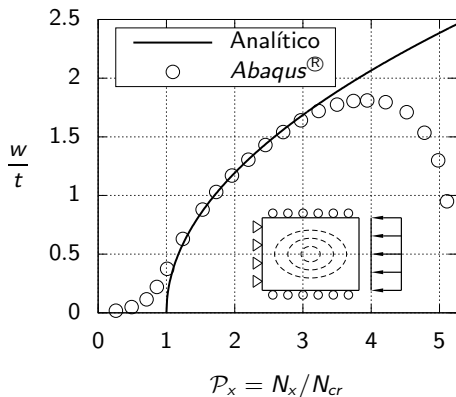
Postpandeo por compresión

- Grandes deflexiones (Riks):

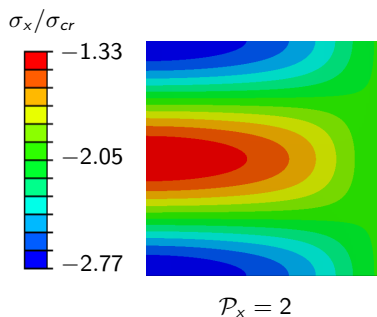


Postpandeo por compresión

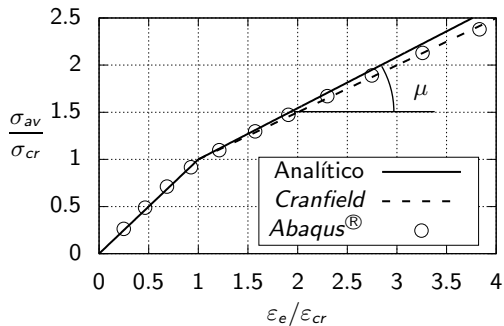
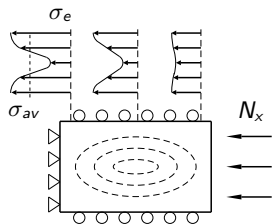
- Grandes deflexiones (Riks):



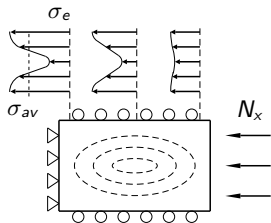
- Redistribución de tensión:



Pérdida de rigidez compresiva

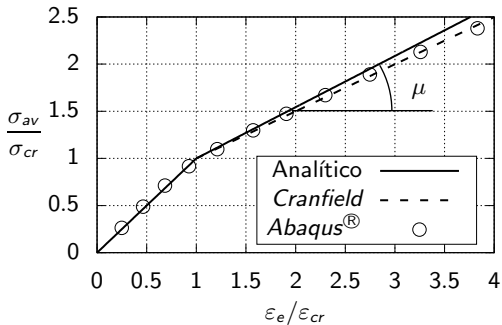


Pérdida de rigidez compressiva

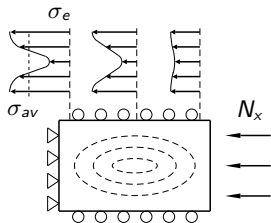


- Módulo Young aparente:

$$\mu = \frac{\partial E^*}{\partial E}$$



Pérdida de rigidez compresiva

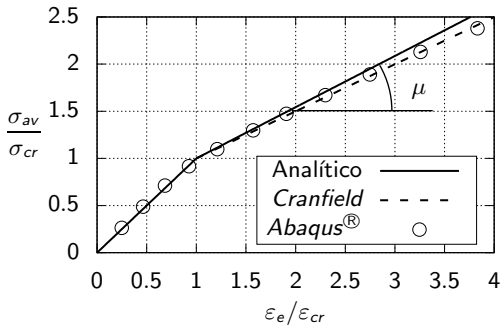


- Módulo Young aparente:

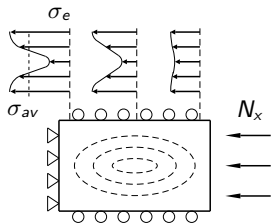
$$\mu = \frac{\partial E^*}{\partial E}$$

- Postpandeo:

$$\mu \cong 0.5 \Rightarrow E^* \cong \frac{E}{2}$$



Pérdida de rigidez compresiva



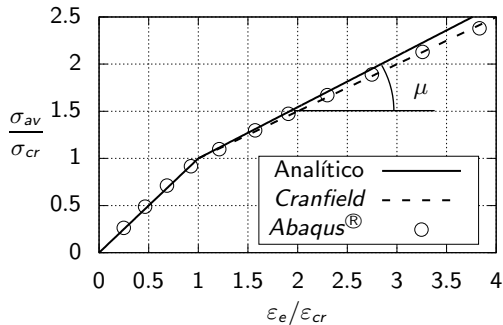
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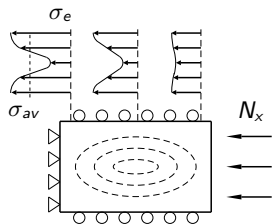
- Postpandeo:

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- Centro pierde rigidez



Pérdida de rigidez compressiva



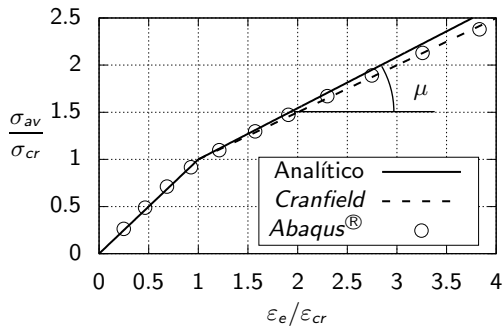
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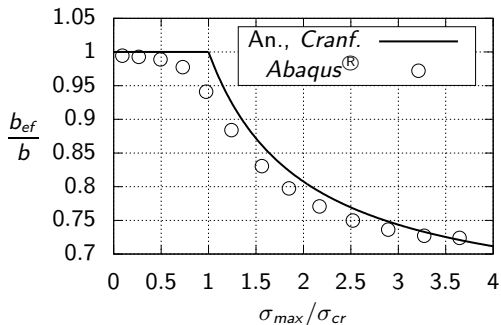
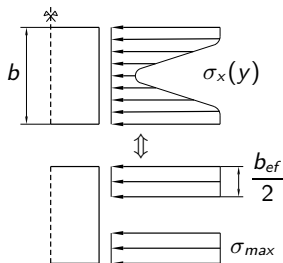
- Postpandeo:

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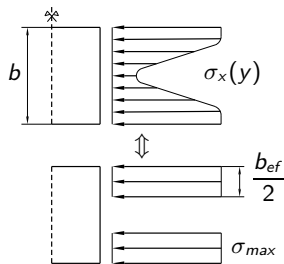
- Centro pierde rigidez
- Bordes descargados: σ_{max}



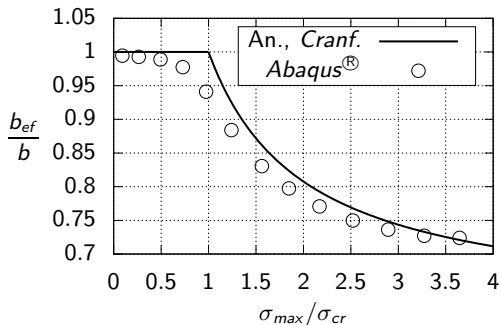
Ancho efectivo



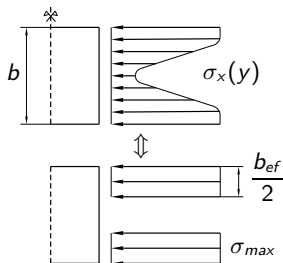
Ancho efectivo



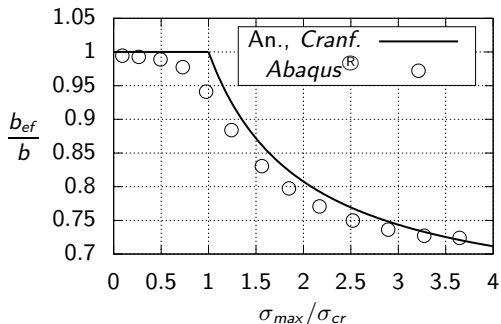
• Si $b_{ef} \downarrow \Rightarrow \sigma_{max} \gg \sigma_{av}$



Ancho efectivo



- Si $b_{ef} \downarrow \Rightarrow \sigma_{max} \gg \sigma_{av}$
- Dimensionado de estructuras aeronáuticas



1 Introducción

- Introducción al postpandeo en estructuras aeronáuticas
- Objetivos

2 Inicio pandeo

- Metodología
- Panel articulado

3 Postpandeo placas

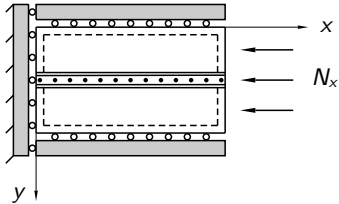
- Metodología
- Resultados

4 Postpandeo paneles

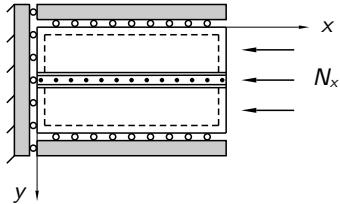
- Panel articulado
- Deformada
- Tensión placa
- Larguerillo

5 Conclusiones

Panel articulado

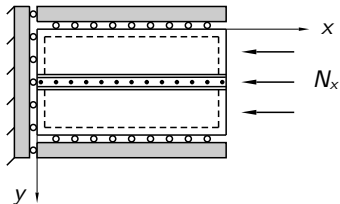


Panel articulado



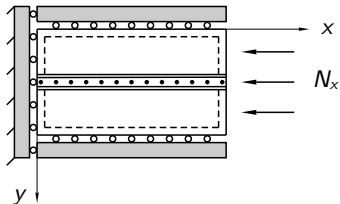
- Estudio analítico: Ritz, Von Karman

Panel articulado



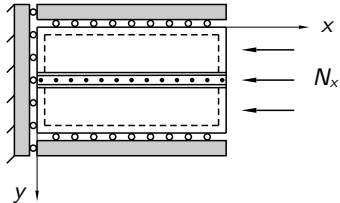
- Estudio analítico: Ritz, Von Karman
- Estudio MEF: Riks, imperfecciones, CC precisan resultados analíticos

Panel articulado



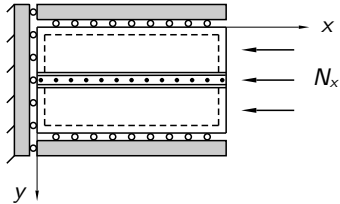
- Estudio analítico: Ritz, Von Karman
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- Larguerillo: 23000 elementos *shell*

Panel articulado

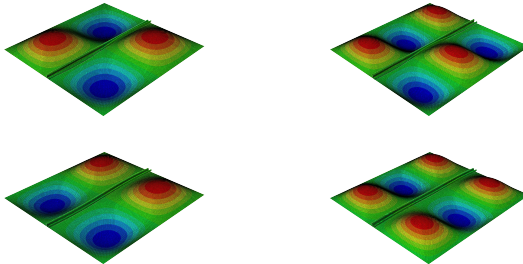


- Estudio analítico: Ritz, Von Karman
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- Larguerillo: 23000 elementos *shell*
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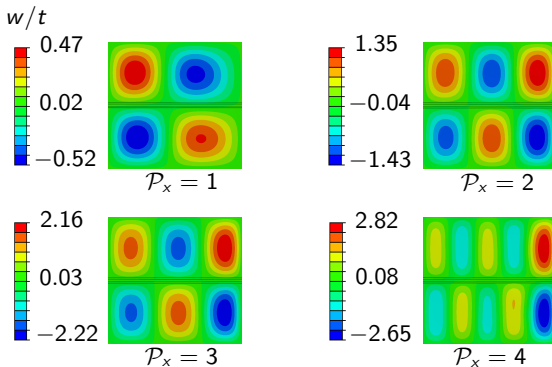
Panel articulado



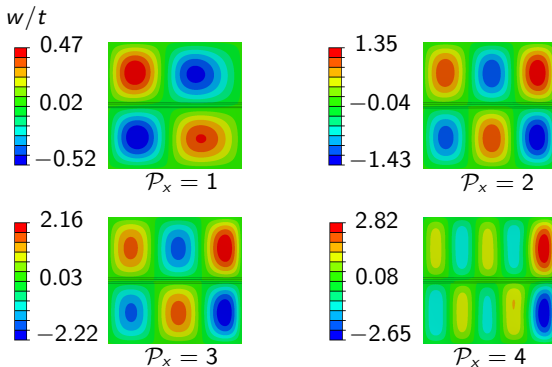
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- Tensión de pandeo local: $\sigma_{cr} \approx 4$ MPa
- Modos para w^* :



Deformada del panel - *Abaqus*[®]

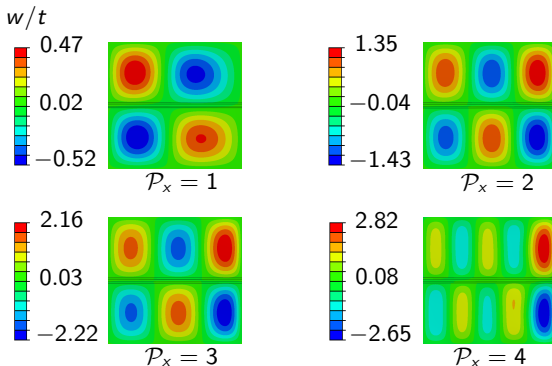


Deformada del panel - *Abaqus*[®]



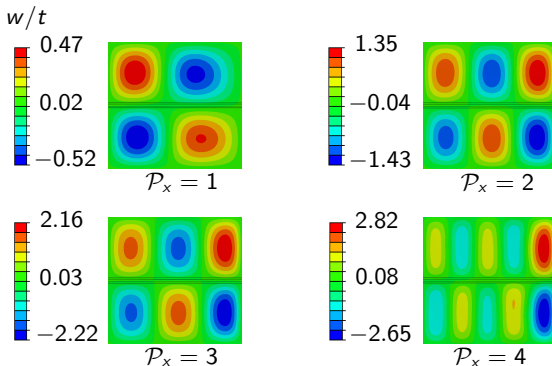
- Imperfecciones: modos no simétricos

Deformada del panel - *Abaqus*[®]



- Imperfecciones: modos no simétricos
- Pandeo en piel: diseño eficiente

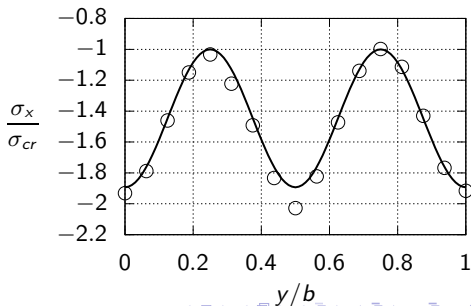
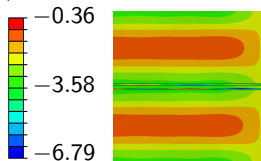
Deformada del panel - *Abaqus*[®]



- Imperfecciones: modos no simétricos
- Pandeo en piel: diseño eficiente
- Larguerillo inmóvil en $z = 0$, pero torsiona

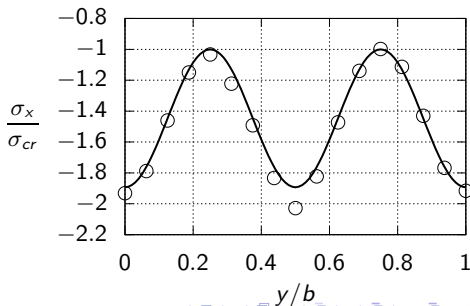
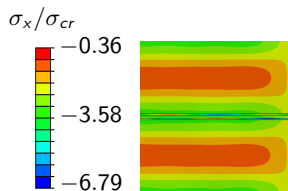
Tensión placa

$$\sigma_x / \sigma_{cr}$$



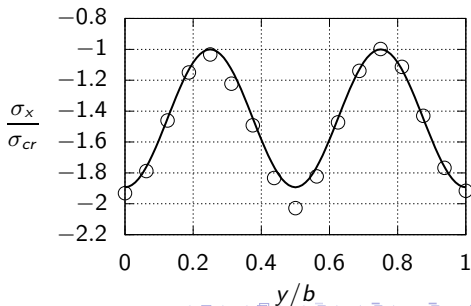
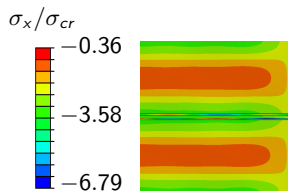
Tensión placa

- Redistribución de σ_x :



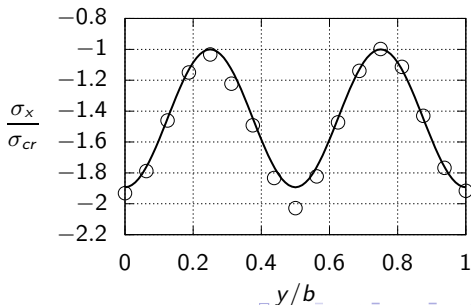
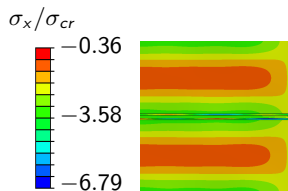
Tensión placa

- Redistribución de σ_x :
 - Bordes descargados



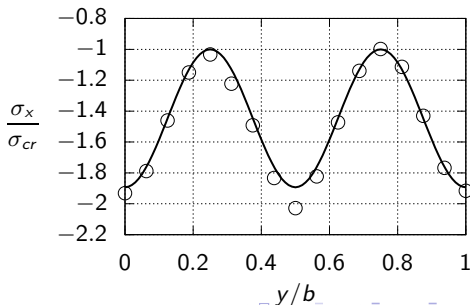
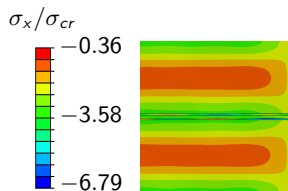
Tensión placa

- Redistribución de σ_x :
 - Bordes descargados
 - Larguerillo



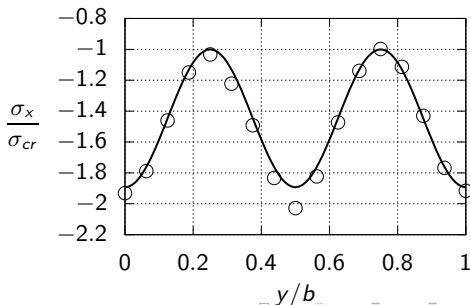
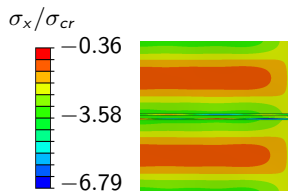
Tensión placa

- Redistribución de σ_x :
 - Bordes descargados
 - Larguerillo
- Tensión baja en placa

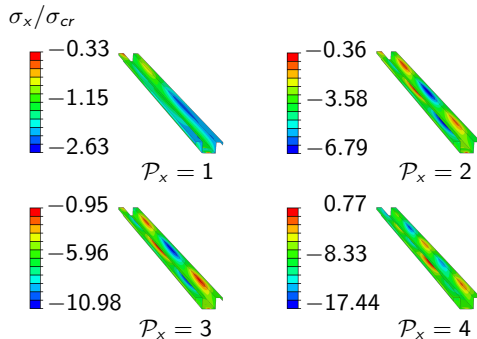


Tensión placa

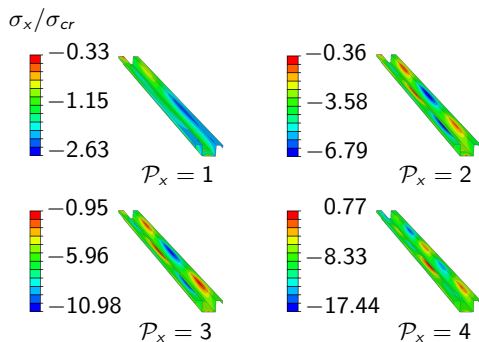
- Redistribución de σ_x :
 - Bordes descargados
 - Larguerillo
- Tensión baja en placa
- Elevada en larguerillo: compresión y torsión.



Larguerillo - Tensiones

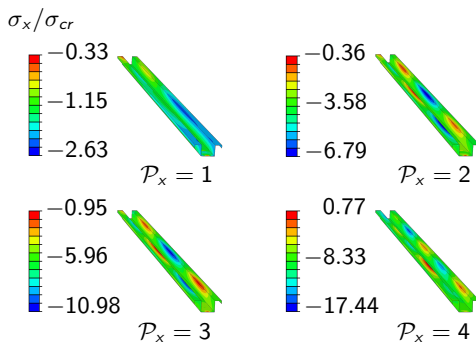


Larguerillo - Tensiones



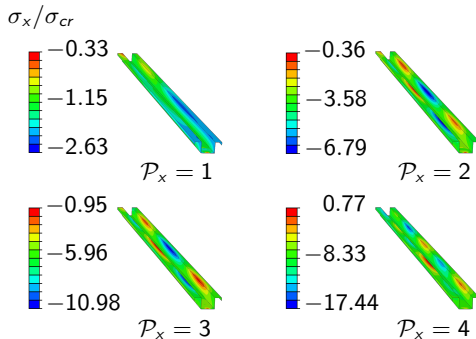
- Doble de σ_{max} que en placa

Larguerillo - Tensiones



- Doble de σ_{max} que en placa
- Tensión $\neq cte$ en sección: alabeo $\omega(s)$

Larguerillo - Tensiones



- Doble de σ_{max} que en placa
- Tensión $\neq cte$ en sección: alabeo $\omega(s)$
- Fallo por *crippling* o remaches

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5 Conclusiones

Conclusiones

- Combinación analítico-MEF:

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Conclusiones

- Combinación analítico-MEF:
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- Priorización pandeo piel: $r = a/b$, factor de diseño
- Tensión elevada en larguerillos
- Estudio de placas insuficiente para dimensionado de panel

Gracias por su atención

¿Preguntas?