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Using system simulation to search for the optimal multi-ordering policy for perishable goods

Huang, Y.C.[a1], Chang, X.Y.[a2], and Ding, Y.A.[a3]

[a]Department of Industrial Management, National Pingtung University of Science and Technology. No.1, Xuefu Rd., Neipu Township, Pingtung County 912, Taiwan (R.O.C.)

[a1] *ychuang@mail.npust.edu.tw*, [a2] *pink4141114@gmail.com*, [a3] *yian0213@gmail.com*

Abstract: This paper explores the possibility that perishable goods can be ordered several times in a single period after considering the cost of Marginal contribution, Marginal loss, Shortage, and Purchasing under stochastic demand. In order to determine the optimal ordering quantity to improve the traditional newsvendor and maximize the total expected profits, and then sensitivity analysis is taken to realize the influence of the parameters on total expected profits and decision variables respectively. In addition, this paper designed a multi-order computerized system with Monte Carlo method to solve the optimal solution under stochastic demand. Based on numerical examples, this paper verified the feasibility and efficiency of the proposed model. Finally, several specific conclusions are drawn for practical applications and future studies.

Key words: Perishable goods, Single-period, Multi-ordering, Newsvendor model, Monte Carlo method.

1. Introduction

There are many goods which are shorter period than the durable commodities in reality. As time goes by, the value of the goods will rapidly decline. This type of goods is very common in our life such as newspapers, magazines, fresh food, and milk, and so on. Before the start of the sales cycle, decision maker often needs to determine how many the goods to be ordered for the entire cycle, and no more ordering before the expiry date. This type of goods will be discussed in newsvendor model. In addition, these products are called as perishable goods or seasonal goods according to their characteristics.

There were many kinds of research on newsvendor problems in academic community; they discussed the inventory method, demand situations, single or twice orders and so on. In the past literature, several scholars have discussed the second order in a single period. If the first ordering quantity is sold out, there has time to the end of the period, then determine the second order should be taken or not, and proved that in some cases the expected profit of order twice is higher than order once. However, past literature did not discuss the single-period and multi-order situations, although the expiry period of perishable goods is very short, but if only ordered once before the sales cycle, and do not consider the situation that all the perishable goods was sold out before the

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expiry period, then it may be not an optimal ordering strategy, eventually. This paper is to improve the traditional newsvendor model, and to explore whether the perishable goods should be ordered more than one time in a single period, to achieve the goal of maximizing the total expected profit.

The aim of this paper is to determine whether a perishable commodity should be ordered more than once in order to maximize the total expected profit. The purposes of the paper are as follows:

1. To establish a stochastic model under single-period and multi-ordering.
2. Proposed an optimal ordering strategy for single-period and multi-ordering.
3. Proved the total expected profit of multi-ordering is better than the single order under stochastic demand.

2. Literature review

Perishable goods were ordered in the case of uncertain demand to meet the needs of the sales cycle. Therefore, the order should be carefully determined when ordering. There were many kinds of research on newsvendor problems which discussed the inventory method, demand situations, single, and twice orders. This paper will discuss the optimal ordering strategy for perishable commodities under single period.

2.1. Order once in a single period

Dian (1990) derived an algorithm to determine a sequence of supply quantities which minimizes total costs of over- and undersupply in the most adverse demand conditions. Fujiwara *et al.* (1997) considered the problem of ordering and issuing policies arising in controlling finite-life-time fresh-meat-carcass inventories in supermarkets. They developed a mathematical model describing actual operations and then simplify the sub-product run out period so that optimal ordering and issuing policies were easily established.

The newsvendor problem is also called Single-Period Problem (SPP). Khouja (1999) built taxonomy of the SPP literature and delineated the contribution of the different SPP extensions. Khouja (2000) extended the SPP to the case in which demand was price-dependent and multiple discounts with prices under the control of the newsvendor were used to sell excess inventory. They developed two algorithms for determining the optimal number of discounts under fixed discounting cost for a given ordering quantity and realization of demand.

Chun (2003) assumed that the customer's demand was represented as a negative binomial distribution, and determined the optimal product price based on the demand rate, buyers' preferences, and length of the sales period. For the case where the seller can divide the sales period into several short periods, finally proposed a multi-period pricing model. Dye and Ouyang (2005) extended Padmanabhan and Vrat's model (1995) by proposing a time-proportional backlogging rate to make the theory more applicable in practice. Alfares and Elmorra (2005) extended the analysis of the distribution-free newsvendor problem to the case when shortage cost was taken into consideration. A model was presented for determining both an optimal ordering quantity and a lower bound on the profit under the worst possible distribution of the demand.

Chen and Chen (2009) presented a newsvendor model with a simple reservation arrangement by introducing the willingness rate, represented as the function of the discount rate, into the models. And mathematical models were developed, and the solution procedure was derived for determining the optimal discount rate and the optimal ordering quantity.

In addition, some scholars put forward that the idea of demand forecast updated, which focus only on the trade-off between exact requirements and additional costs, and often assuming that the supplier's capabilities were unrestricted, but in real life is not the case. Zheng *et al.* (2016) investigated an extension of the newsvendor model with demand forecast updating under supply constraints. In studying the manufacturer-related effects, two supply modes are investigated: supply mode A, which has a limited ordering time scale, and supply mode B, which has a decreasing maximum ordering quantity. A comparison of the different supply scenarios demonstrated the negative effects of increased purchasing cost and ordering time and quantity restrictions when demand forecast updating implemented.

2.2. Order twice in a single period

Gallego and Moon (1993) extended the analysis to the recourse case, where there was a second purchasing opportunity; to the fixed ordering cost case, where a fixed cost was charged for placing an order; to the case of random yields; and to the multi-item case, where multiple items compete for a scarce resource. Azoury and Miller (1984) used the concept of flexibility it was anticipated that the quantity ordered under the non-Bayesian policy would be greater than or equal to that under a Bayesian policy. This result was established for the n-period non depletive inventory model. Lau and Lau (1998) considered the very common situation in which a single-period newsvendor type product may be ordered twice during a period. They extended the basic model to consider a non-negligible set-up cost for the second order; it served as an illustration of how one might want to extend their basic two-order model to handle a large number of different combinations of additional factors such as the second-order's delivery delay time and price differential.

Chung and James (2001) extended the classic newsvendor problem by introducing reactive production. Production occurs in two stages, an anticipatory stage and a reactive stage. Their model reduces to a single-period model with piecewise-linear convex costs. They obtain an analogue of the well-known critical fractile formula of the classic newsvendor model. Pando *et al.* (2013) presented of the newsvendor problem where an emergency lot can be ordered to provide for a certain fraction of shortage. This fraction was described by a general backorder rate function which is non-increasing with respect to the unsatisfied demand. An exponential distribution for the demand during the selling season was assumed. An expression was obtained in a closed form for the optimal lot size and the maximum expected profit.

2.3. Literature review

In this paper, we explored the single-period and multi-order strategy for perishable goods. The relevant literature was summarized and shown in Table 1.

3. Construction of the mathematical model

This paper proposed the concept of single-period and multi-order strategy for perishable goods, then developed the total expected profits model to determine the optimal ordering quantity and quantity of order. Furthermore, we will prove that the multi-order is superior to the single-order for perishable goods. We will introduce the simulation method and program flow chart in Section 3.7.

3.1. The assumptions of this paper

1. The model assumes no lead time. Each ordering must pay the same ordering cost. If the goods sold out in this period, then the subsequent ordering quantity can be delivered before the start of next period.
2. The demand is a random variable. The marginal contribution, marginal loss, shortage cost, salvage value, and delivery costs are all known and fixed.
3. The sales quantity of each period can be known by the POS system, and the distribution of demand can be reasonably estimated by historical data and goodness-of-fit test.
4. Do not consider the quantity discount and restrictions of storage space.

3.2. Definitions of symbols

i: The period, $i=1,2,3\ldots n$

n: The number of time intervals in expired period

j: The j^{th} ordering

X_i: The demand quantity of i^{th} time interval (X_i is a random variable)

Y_j: The total demand from j^{th} ordering to the end of sales cycle (Y_j is a random variable).

$$Y_j = \sum_{i=K_j}^{n} X_i$$

Co_j: Ordering cost of j^{th} ordering

C_P: Purchase cost per unit of perishable goods

Price: Price per unit of perishable goods

S: Salvage value per unit of perishable goods

C_S: Shortage cost per unit of perishable goods

MP: Marginal contribution, $MP=Price-C_P$, where $Price>C_P$

ML: Marginal loss, $ML=C_P-S$, where $C_P>S$

Q_j: The ordering quantity of j^{th} ordering (Q_j is a decision variable)

Table 1. The comparison between literature and this paper.

Author \ Project	Shortage cost	Salvage value	Order cost	Total expected profit maximization	Twice orders	Multi-orders	System Simulation
Azoury and Miller (1984)	✓				✓		
Dian (1990)	✓		✓				
Gallego and Moon (1993)		✓	✓		✓		
Fujiwara *et al.* (1997)	✓		✓				
Lau, H. and H. Lau (1998)					✓		
Khouja (2000)	✓	✓		✓			
Chung and James (2001)	✓				✓		
Dye and Ouyang (2005)	✓						
Pando *et al.* (2013)	✓	✓	✓	✓	✓		
This paper	✓	✓	✓	✓	✓	✓	✓

$f_j(y_j)$: Probability density function of Y_j

$F_j(y_j)$: Cumulative distribution function of Y_j

K_j: The ordering time point of j^{th} ordering

$M\pi_j(Q_j)$: Marginal profit under ordering quantity Q_j and j^{th} ordering

$MR_j(Q_j)$: Marginal revenue under ordering quantity Q_j and j^{th} ordering

$MC_j(Q_j)$: Marginal cost under ordering quantity Q_j and j^{th} ordering

$T\pi_j(Q_j)$: Total expected profit under ordering quantity Q_j and j^{th} ordering

$T\pi_1(Q_1)$: Total expected profit under ordering quantity Q_j and 1^{st} ordering

$T\pi_M(Q_1,Q_2,\ldots,Q_J)$: Sum of total expected profit under ordering quantity $(Q_1,Q_2,\ldots,Q_J)$

After symbols definition, the concept of multiple orders in single period for perishable goods can be shown in Figure 1. The expiry period can be divided into n, and $X_1,X_2,X_3,\ldots,X_n$ respectively represents the demand quantity at period 1, 2, 3…n. Only the 1st ordering time point is sure, the other ordering time points $K_1,K_2,\ldots$ are uncertain. If the demand of the entire cycle can be satisfied by the first ordering quantity, then K_2 will not happen. If the initial ordering quantity cannot satisfy the demand of the entire cycle, and reordering has a positive profit, then 2nd ordering will be taken and the time point is K_2. The others are reasoned by analogy.

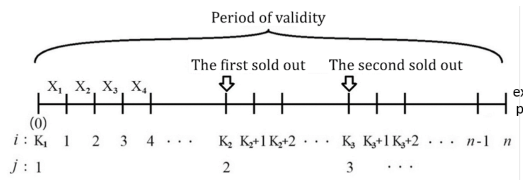


Figure 1. The schematic of single-period and multi-order structure for perishable goods.

Based on the symbol definition and Figure 1, the demand of Y_j is

$$Y_j = \sum_{i=K_j}^{n} X_i$$

3.3. Ordering strategy

This section describes the mathematical model of the ordering strategy.

3.3.1. Ordering strategy

Assuming that the demand of i^{th} period is X_i, the first ordering quantity is Q_1, and the total demand of whole period is Y_1, so

$$Y_1 = \sum_{i=1}^{n} X_i$$

The total expected profit $T\pi_1(Q_1)$ under single order strategy is shown in Equation (1):

$$T\pi_1(Q_1) = \int_0^{Q_1} [y_1 \cdot MP - (Q_1 - y_1) \cdot ML] \cdot f_1(y_1)dy_1 + \int_{Q_1}^{\infty} [Q_1 \cdot MP - (y_1 - Q_1) \cdot C_S] \cdot f_1(y_1)dy_1 - Co_1 \quad (1)$$

$T\pi_1(Q_1)$ can be taken a first order derivative with respect to Q_1 and set the result be equal to zero to obtain the optimal ordering quantity Q_1 that maximizes the total expected profit, as shown in Equation (2):

$$\frac{\partial T\pi_1(Q_1)}{\partial Q_1} = \int_0^{Q_1} ML \cdot f_1(y_1)dy_1 + \int_{Q_1}^{\infty} (MP + C_s) \cdot f_1(y_1)dy_1 = 0$$

$$\Longrightarrow F_1(Q_1) = \frac{MP + C_s}{MP + ML + C_s} \quad (2)$$

$$\therefore Q_1 = F_1^{-1}\left(\frac{MP + C_s}{MP + ML + C_s}\right) \quad (3)$$

After finding out the optimal ordering quantity and if $T\pi_1(Q_1)<0$, it means the expected profit is negative, then the decision maker will not make an order to purchase the perishable goods; Conversely, if $T\pi_1(Q_1)\geq 0$, it means the expected profit is positive, then the decision maker will make an order to purchase the perishable goods with the optimal ordering quantity (Q_1).

The second order derivative of the total expected profit $T\pi_1(Q_1)$ with respect to Q_1 to verify whether the $T\pi_1(Q_1)$ is a concave function of Q_1:

$$\frac{\partial}{\partial Q}\left(\frac{\partial T\pi_1(Q_1)}{\partial Q_1}\right) = \frac{\partial}{\partial Q}\left(\int_0^{Q_1} ML \cdot f_1(y_1)dy_1 + \int_{Q_1}^{\infty}(MP + C_s) \cdot f_1(y_1)dy_1\right)$$
$$= -ML \cdot f_1(Q_1) - (MP + C_s) \cdot f_1(Q_1)$$
$$= -(ML + MP + C_s) \cdot f_1(Q_1) < 0 \quad (4)$$

From equation (4) know that $\frac{\partial^2 T\pi_1(Q_1)}{\partial Q^2} < 0$, so the $T\pi_1(Q_1)$ is the concave function of Q_1, Therefore, $Q_1 = F_1^{-1}\left(\frac{MP + C_s}{MP + ML + C_s}\right)$ is an optimal ordering quantity, and can make $T\pi_1(Q_1)$ have a maximum value.

3.3.2. The construction of multi-order in single period problem

The multi-order means that the decision maker may deliver one or more orders during the expiry period, each ordering quantity can be denoted by $Q_1,Q_2,\ldots,Q_J$, respectively. The total expected profit is expressed by $T\pi_M(Q_1,Q_2,\ldots,Q_J)$, so we have

$$T\pi_M(Q_1,Q_2,\cdots,Q_J) = \sum_{j=1}^{J} T\pi_j(Q_j)$$

The total expected profit of j^{th} order, $T\pi_j(Q_j)$, can be expressed as shown in Equation(5):

$$T\pi_j(Q_j) = \int_0^{Q_j}\left[y_j \cdot MP - (Q_j - y_j) \cdot ML\right] \cdot f_j(y_j)dy_j + \int_{Q_j}^{\infty}\left[Q_j \cdot MP - (y_j - Q_j) \cdot C_S\right] \cdot f_j(y_j)dy_j - Co_j \quad (5)$$

where

$$Y_j \sim F_j(y_j),\ and\ Y_j = \sum\nolimits_{i=K_j}^{n} X_i$$
$$\therefore Q_j = F_j^{-1}\left(\frac{MP + C_s}{MP + ML + C_s}\right),\ j = 1, 2, \cdots, J. \quad (6)$$
$$J = Max\{\, j \mid T\pi_j(Q_j) > 0\}$$

3.3.3. Compare single-order and multi-order

Multi-order in single period will occur when the first ordering quantity was sold out and second order before the end of the sales cycle is still profitable. It can be inferred that the total expected profit of multi-order will be greater than the single-order, it means $T\pi_M(Q_1,Q_2,\ldots,Q_J)\geq T\pi_1(Q_1)$. The proof was shown in Proposition 1.

Proposition 1. $T\pi_M(Q_1,Q_2,\ldots,Q_J)\geq T\pi_1(Q_1)$

Proof: $\because T\pi_M(Q_1,Q_2,\cdots,Q_J) = \sum_{j=1}^{J} T\pi_j(Q_j)$

and $T\pi_j(Q_j) > 0,\ \forall j = 1, 2, \cdots, J$

$$T\pi_M(Q_1,Q_2,\cdots,Q_J) = T\pi_1(Q_1) + \sum_{j=2}^{J} T\pi_j(Q_j) > T\pi_1(Q_1)$$

so $T\pi_M(Q_1,Q_2,\ldots,Q_J)\geq T\pi_1(Q_1)$ Q.E.D. (7)

3.3.4. Without considering the shortage cost

When we do not consider the shortage cost, the total expected profit of perishable goods in 1st ordering $T\pi_1(Q_1)$ was shown in Equation (8):

$$T\pi_1(Q_1) = \int_0^{Q_1}\left[y_1 \cdot MP - (Q_1 - y_1) \cdot ML\right] \cdot f_1(y_1)dy_1 + \int_{Q_1}^{\infty}\left[Q_1 \cdot MP\right] \cdot f_1(y_1)dy_1 - Co_1 \quad (8)$$

Based on first order condition (so called FOC), we have Equation (9) and (10) as follows:

$$\frac{\partial T\pi_1(Q_1)}{\partial Q_1} = \int_0^{Q_1} ML \cdot f_1(y_1)dy_1 + \int_{Q_1}^{\infty} (MP + C_s) \cdot f_1(y_1)dy_1 = 0$$

$$\Longrightarrow F_1(Q_1) = \frac{MP}{MP+ML} \tag{9}$$

$$\therefore Q_1 = F_1^{-1}\left(\frac{MP}{MP+ML}\right) \tag{10}$$

When we consider the shortage cost, the optimal ordering quantity Q_1 is $F_1^{-1}\left(\frac{MP+C_s}{MP+ML+C_s}\right)$; whereas, when we do not consider the shortage cost, the optimal ordering quantity Q_1 is $F_1^{-1}\left(\frac{MP}{MP+ML}\right)$.

If $C_S=0$, then $\frac{MP+C_s}{MP+ML+C_s} = \frac{MP}{MP+ML}$, so $F_1^{-1}\left(\frac{MP+C_s}{MP+ML+C_s}\right) = F_1^{-1}\left(\frac{MP}{MP+ML}\right)$.

If $C_S>0$, then $\frac{MP+C_s}{MP+ML+C_s} > \frac{MP}{MP+ML}$, so $F_1^{-1}\left(\frac{MP+C_s}{MP+ML+C_s}\right) \geq F_1^{-1}\left(\frac{MP}{MP+ML}\right)$.

It means when the shortage cost exists, the optimal ordering quantity will increase.

3.4. Goodness-of-fit test

This paper collected sales data, and based on the historical data at different periods to take the goodness-of-fit test to estimate the demand distribution and its population parameters. The Kolmogorov-Smirnov test (K-S test) is a goodness-of-fit test. The test is a nonparametric statistical method to test the sampling data whether follows a specific theoretical distribution, such as uniform distribution, normal distribution, exponential distribution and so on. The testing steps are as follows:

Step 1: building a hypothesis

Suppose that the actual distribution function of random variable X is $F(x)$, and the specific theoretical distribution function is given as $F_0(x)$. The hypothesis of this test is:

1. Null hypothesis H_0: $X \sim F_0(x)$
2. Alternative hypothesis H_1: $\sim X_0$ (H_1 is the supplementary set of H_0)

Step 2: calculating the testing statistic

Let $x_1, x_2, \ldots, x_n$ be a set of random sample taken from the population distribution $F_0(x)$, and let $F(x)$ be the actual distribution function, the testing statistic $D = Max|F(x) - F_0(x)|$, $\forall x$, the testing statistic D is the maximum absolute difference between the actual distribution function $F(x)$ and the specific theoretical distribution function $F_0(x)$.

Step 3: rejection region

If $D>d_\alpha$, then reject H_0, where d_α is a critical value of D.

After goodness-of-fit test to estimate the demand distribution of and then construct the mathematical model to search for the optimal ordering strategy.

3.5. The additive property of distributions

Assuming that the demand distribution for each period can be estimated from past sales data through by goodness-of-fit test, and then we need to discuss whether the distribution has the property of additive.

Let, $Y_j = \sum_{i=K_j}^{n} X_i$ where $X_i \sim F_i(x_i)$, and $X_i \perp\!\!\!\perp X_j$, $\forall\ i \neq j$

And
$$E(Y_j) = E\left(\sum_{i=K_j}^{n} X_i\right) = \sum_{i=K_j}^{n} E(X_i) = \sum_{i=K_j}^{n} \mu_i \tag{11}$$

$$V(Y_j) = V\left(\sum_{i=K_j}^{n} X_i\right) = \sum_{i=K_j}^{n} V(X_i) = \sum_{i=K_j}^{n} \sigma_i^2 \tag{12}$$

If X_i follows normal distribution, it can be denoted as $X_i \sim N(\mu_i, \sigma_i^2)$, and $Y_j = \sum_{i=K_j}^{n} X_i$, then

$$Y_j \sim N\left(\sum_{i=K_j}^{n} \mu_i, \sum_{i=K_j}^{n} \sigma_i^2\right) \tag{13}$$

The common distributions are summarized in Table 2 to justify their additive property.

3.5.1. The discussion on ordering quantity

Under the premise of additive property or $\sum_{i=K_j}^{n} X_i$ follows the central limit theorem, and if MP, ML and C_s are known, then $Q_j \geq Q_{j+1}$, it means $Q_1 \geq Q_2 \geq \ldots \geq Q_J$. The proof is shown in Proposition 2.

Proposition 2. : If *MP*, *ML* and C_s are known and fixed, then $Q_1 \geq Q_2 \geq \ldots \geq Q_J$.

Proof:

Given $Y_j = \sum_{i=K_j}^{n} X_i$ and $X_i \geq 0$

so $Y_1 \geq Y_2 \geq \cdots \geq Y_J \Longrightarrow \mu_{Y_1} \geq \mu_{Y_2} \geq \cdots \geq \mu_{Y_J}$ and

$V(Y_1) \geq V(Y_2) \geq \cdots \geq V(Y_J)$ it has , and, as shown in Figure 2:

Therefore $Q_j \geq Q_{j+1}$. By the same way, we can prove that $Q_1 \geq Q_2 \geq \ldots \geq Q_J$. Q.E.D.

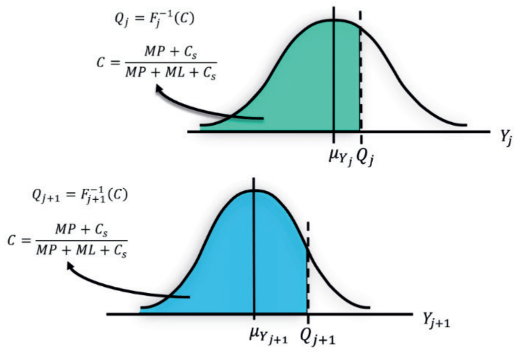


Figure 2. Schematic of $F_j^{-1}(C) \geq F_{j+1}^{-1}(C)$.

3.5.2. The discussion on total expected profit in each period

If X_i is additive, and *MP*, *ML* and C_s are known, and $Co_1 = Co_2 = \ldots = Co_J$, then $T\pi_1(Q_1) \geq T\pi_2(Q_2) \geq \ldots \geq T\pi_J(Q_J) \geq 0$. The proof is shown in Proposition 3.

Proposition 3 : If , *MP*, *ML* and C_s are known and $Co_1 = Co_2 = \ldots = Co_J$, then $T\pi_1(Q_1) \geq T\pi_2(Q_2) \geq \ldots \geq T\pi_J(Q_J) \geq 0$.

Proof:

$\because F_j(y_j)$ is an increasing function of y_j, and if $y_1 > y_2$, then $F_j(y_1) \geq F_j(y_2)$.

If $T\pi_j(Q_j) \geq 0$, then $M\pi_j(Q_j) \geq 0$, where $MR_j(Q_j) - MC_j(Q_j)$

so $MR_j(Q_j) \geq MC_j(Q_j)$, and

$MR_j(Q_j) = MP \cdot P(Y_j \geq Q_j)$; $MC_j(Q_j) = ML \cdot P(Y_j < Q_j)$

$\therefore MP \cdot P(Y_j \geq Q_j) \geq ML \cdot P(Y_j < Q_j)$

likewise $MP \cdot P(Y_{j+1} \geq Q_{j+1}) \geq ML \cdot P(Y_{j+1} < Q_{j+1})$

and because of $Y_j \geq Y_{j+1}$ and $Q_j \geq Q_{j+1}$, so

$$T\pi_j(Q_j) = \sum_{h=1}^{Q_j} \left[MP \cdot P(Y_j \geq h) - ML \cdot P(Y_j < h) \right]$$

Table 2. Additive justification of common distributions.

Distribution	$f(x)$	$\sum_{i=1} X_i$	Additive
Normal	$f(x) = \frac{1}{\sqrt{2}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$, $X_i \sim N(\mu_i, \sigma_i^2)$	$\sum_{i=1}^{n} X_i \sim N\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$	✓
Exponential	$f(x) = \lambda e^{-\lambda x}$, $X_i \sim Exp(\lambda)$, $X_i \sim iid$ (A special case of gamma distribution)	$\sum_{i=1}^{n} X_i \sim Gamma\left(\alpha = n, \beta = \frac{1}{\lambda}\right)$	✓
Uniform	$f(x) = \frac{1}{b-a}$, $X_i \sim U(0,1)$, $X_i \sim iid$	$\sum_{i=1}^{n} X_i \neq U(0,n)$	×
Gamma	$f(x) = \frac{x^{\alpha-1}}{\Gamma(\alpha) \cdot \beta^{\alpha}} \cdot e^{\frac{-x}{\beta}}$, $X_i \sim Gamma(\alpha_i, \beta)$	$\sum_{i=1}^{n} X_i \sim Gamma\left(\sum_{i=1}^{n} \alpha_i, \beta\right)$	✓
Poisson	$f(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}$	$\sum_{i=1}^{n} X_i \sim Poisson\left(\sum_{i=1}^{n} \lambda_i\right)$	✓
Bernoulli	$f(x) = p^x \cdot q^{1-x}$ (A special case of binomial distribution)	$\sum_{i=1}^{n} X_i \sim B(n,p)$	✓
Binomial	$f(x) = \binom{n}{x} p^x \cdot q^{n-x}$	$\sum_{i=1}^{n} X_i \sim B\left(\sum_{i=1}^{n} n_i, p\right)$	✓
Geometric	$f(x) = q^{x-1} \cdot p$ (A special case of negative binomial distribution)	$\sum_{i=1}^{r} X_i \sim NB(r,p)$	✓

$T\pi_{j+1}(Q_{j+1}) =$
$\sum_{h=1}^{Q_{j+1}}[MP \cdot P(Y_{j+1} \geq h) - ML \cdot P(Y_{j+1} < h)]$
also $Q_j \geq Q_{J+1}$, and $F_j(y_j^\circ) < F_{j+1}(y_j^\circ)$, so $[(1 - F_j(y_j^\circ)) > (1 - F_{j+1}(y_j^\circ))]$.

In other words, $P(Y_j > y_j^\circ) > P(Y_{j+1} > y_j^\circ)$. So $T\pi_j(Q_j) \geq T\pi_{j+1}(Q_{j+1})$. Q.E.D.

3.6. Sensitivity analysis

The sensitivity analysis is taken to realize the influences of the system parameters on total expected profit are shown as follows.

1. The influence of marginal contribution (*MP*) on total expected profit ($T\pi_j(Q_j)$) has a same changing direction.

$$\frac{\partial T\pi_j(Q_j)}{\partial MP} = \int_0^{Q_j} y_j \cdot f_j(y_j)dy_j + \int_{Q_j}^{\infty} Q_j \cdot f_j(y_j)dy_j > 0 \quad (14)$$

2 The influence of marginal loss (*ML*) on total expected profit ($T\pi_j(Q_j)$) has an opposite changing direction.

$$\frac{\partial T\pi_j(Q_j)}{\partial ML} = \int_0^{Q_j} -(Q_j - y_j) \cdot f_j(y_j)dy_j < 0 \quad (15)$$

3 The influence of shortage cost (C_s) on total expected profit ($T\pi_j(Q_j)$) has an opposite changing direction.

$$\frac{\partial T\pi_j(Q_j)}{\partial C_s} = \int_{Q_j}^{\infty} -(y_j - Q_j) \cdot f_j(y_j)dy_j < 0 \quad (16)$$

4 The influence of delivery cost (Co_j) on total expected profit ($T\pi_j(Q_j)$) has an opposite changing direction.

$$\frac{\partial T\pi_j(Q_j)}{\partial Co_j} = -1 < 0 \quad (17)$$

3.7. System simulation

The Monte Carlo simulation will be applied and introduced as follows.

3.7.1. Monte Carlo simulation

Monte Carlo simulation is a simulation; it can generate random numbers that follow a specific probability distribution. Based on the random numbers and given mathematical model to find out the optimal solution that maximizes the total expected profit or minimizes the total expected cost.

In this study, the Monte Carlo method was applied to simulate the demand of each period. After collecting the past sales data and building the demand distribution of each period by goodness-of-fit test, using a random number generator to create a random number between 0 and 1, and let it denote $F_X(X)$. Applying the inverse function of $F_X(X)$ to find out the value of random variable that follows a specific distribution. The steps of Monte Carlo simulation are as follows:

Step 1: Collecting historical sales data.

Step 2: Using goodness-of-fit test to estimate the population's parameters and demand distribution of each period.

Step 3: Using random number generator to create a random number (U) between 0 and 1, and $U \sim Uniform(0,1)$.

Step 4: Finding the cumulative distribution function of the demand distribution ($F(x)$).

Step 5: Let $U= F(X)$

Step 6: $X= F^{-1}(U)$.

Step 7: Repeat step 4 to 6 until the required random numbers are satisfied.

3.7.2. The relationship between system simulation and uniform distribution

A random variable U is generated, and $U \sim U(0,1)$, then let $F_X(X)=U$, therefore $X= F_X^{-1}(U)$. If $X \sim F_X$, where F_X is a cumulative distribution function (*c.d.f*) of X. In other words, a random number U can be obtained from the random number generator, where U has a uniform distribution between 0 and 1, and then given $X \sim F_X$ and let $F_X(X)=U$ can be used to obtain a mapping value of random variable (X). The proof is shown in Property4.

Property 4. : Given $X \sim F_X$ and $X \in C.R.V$ (Continuous Random Variable), let $F_X(X)=U$, where $U \sim U(0,1)$, then $X= F_X^{-1}(U)$.

Proof:

Let $U \sim U(0,1)$, then $F_U(u) = P(U \leq u) = \int_0^u 1 \cdot dt = u$, $u \in [0,1]$

If $F_X(X)=U$, then

$$\begin{aligned} F_U(u) &= P(U \leq u) \\ &= P[F_X(X) \leq u] \\ &= P\left[F_X^{-1} \cdot F_X(X) \leq F_X^{-1}(u)\right] \\ &= P\left[X \leq F_X^{-1}(u)\right] \\ &= F_X\left[F_X^{-1}(u)\right] \\ &= u \end{aligned}$$

And

$$\begin{aligned} P(X \leq x) &= P\left[F_X^{-1}(U) \leq x\right] \\ &= P\left[F_X\left[F_X^{-1}(U)\right] \leq F_X(x)\right] \\ &= P[U \leq F_X(x)] \\ &= F_U[F_X(x)] \\ &= F_X(x). \text{ Q.E.D.} \end{aligned}$$

Since F_X is a non-decreasing function of X, it means if $a>b$, then $F_X(a) \geq F_X(b)$, and if $X \in C.R.V$, then $F_X(a)>F_X(b)$.

3.8. The flow chart of the proposed system simulation

This paper uses the Visual Basic software to develop a multi-ordering computerized system; the system flow chart is shown in Figure 3.

4. Example analysis

This chapter will base on the statistical analysis described as above to search for the optimal ordering strategy under single-period and multi-ordering situations. At first, describes the problem and then put the data into simulation system to find out the optimal ordering quantity and total expected profit, then analysis and discuss the simulation results. Finally, sensitivity analysis is carried out to verify the feasibility and correctness of the proposed model.

4.1. Example description

Suppose there is a convenience store sells monthly magazine, and its price is \$ 120 at cost \$ 60. If it is not sold after the end of the sales cycle, it will only be worth \$ 1 sold to the recycling dealer. Considering the shortage cost is equal to the marginal contribution of the magazine, and each ordering and delivery cost is 50, and assuming that no lead time, when the expected profit of each ordering is 0 will also carry out an order to satisfy the customer's need. The sales period of the magazine is 30 days and divided into 3 periods, so each period is 10 days. We collected sales data over the past years and took the goodness-of-fit test to estimate the demand distribution of each period. We found that the demand distribution of each period is a normal distribution, which is $X_i \sim N(\mu_i, \sigma_i^2)$.

The influences of the mean and variance of three periods on the number of orders, the optimal ordering quantity and the total expected profit is discussed. Therefore, the mean and variance are classified as large, medium and small, respectively. The large, medium and small of mean were 10, 20 and 30; the large, medium and small of standard deviation were $\frac{1}{3}\mu_i$, $\frac{1}{6}\mu_i$, and $\frac{1}{9}\mu_i$. Therefore, there are 729 ($(3\times3)^3$) combinations. The random demand (X_i) for period i is calculated by the Monte Carlo simulation method. Each experiment is repeated 1000 times.

The model proposed in this paper does not limit the ordering quantity, as long as $T\pi_j(Q_j) \geq 0$, the order will be delivered. In this example, there are three possible ordering time points: the first time point is at the beginning of the sales period to meet the demand of entire period; The second time point is at the beginning of period 2 when the magazine was sold out in period 1, and reorder to meet the needs of period 2 and 3; The third time point is at the beginning of period 3 when the magazine was sold out in period 2, and then reorder to meet the need of time3. The purpose of this paper is to decide the optimal multi-ordering policy under stochastic demand to maximize the total expected profit.

4.2. General situation

Putting the values of MP, ML and C_s into Equation (6) to find out the optimum ordering quantity Q_j, and calculate the total expected profit $T\pi_j(Q_j)$ by Equation (5). If $T\pi_j(Q_j) \geq 0$, then takes an order and the ordering quantity is Q_j; If $T\pi_j(Q_j)<0$, then do not take an order.

4.2.1. Analysis of single data

We now randomly select the combination No. 8 which ordering twice in a sales period (it has three periods) to explain. The mean demand of the period 1 is 30 and its standard deviation is 10, the mean demand of period 2 is 30 and its standard deviation is 10 and the mean demand of time interval 3 is 10 and its standard deviation is 1.67. The data is shown in Table 3.

According to Table 3, it can be found that there happened 145 times of twice ordering in 1000 experiments. If order occurs twice, the second ordering quantity will be less than the first ordering quantity (11<76). Therefore, the Proposition 2 was verified. When the second order occurs in combination No. 8, the final total expected profit will be greater than which is only order once (4493.97>3116.16), so the Proposition 1 is verified.

4.2.2. Analysis of order twice data

According to the simulation results where each combination was performed 1000 times. We show partial results of order twice in Table 4.

According to Table 4, it can be found that if order occurs twice, the second ordering quantity will be less than the first ordering quantity ($Q_2<Q_1$). Therefore, the Proposition 2 is verified. In addition the final total expected profit of order twice will be greater than which is only order once ($T\pi_M(Q_1,Q_2)>T\pi_1(Q_1)$), so the Proposition 1 is verified.

In general, If Q_1 is less than or close to ($\mu_1+3\sigma_1$), then it has the opportunity to order twice and the time of second order is at the end of period 1; When Q_1 is less than or close to $\mu_1+(\mu_2+3\sigma_2)$ or close to $(\mu_1+3\sigma_1)+\mu_2$, it has the opportunity to order twice and the time of second order is at the end of the period 2. It was known from the examples that ordering twice is likely to occur in ($\mu_1\geq\mu_2\geq\mu_3$) or ($\mu_2\geq\mu_1$ and $\mu_2\geq\mu_3$) condition.

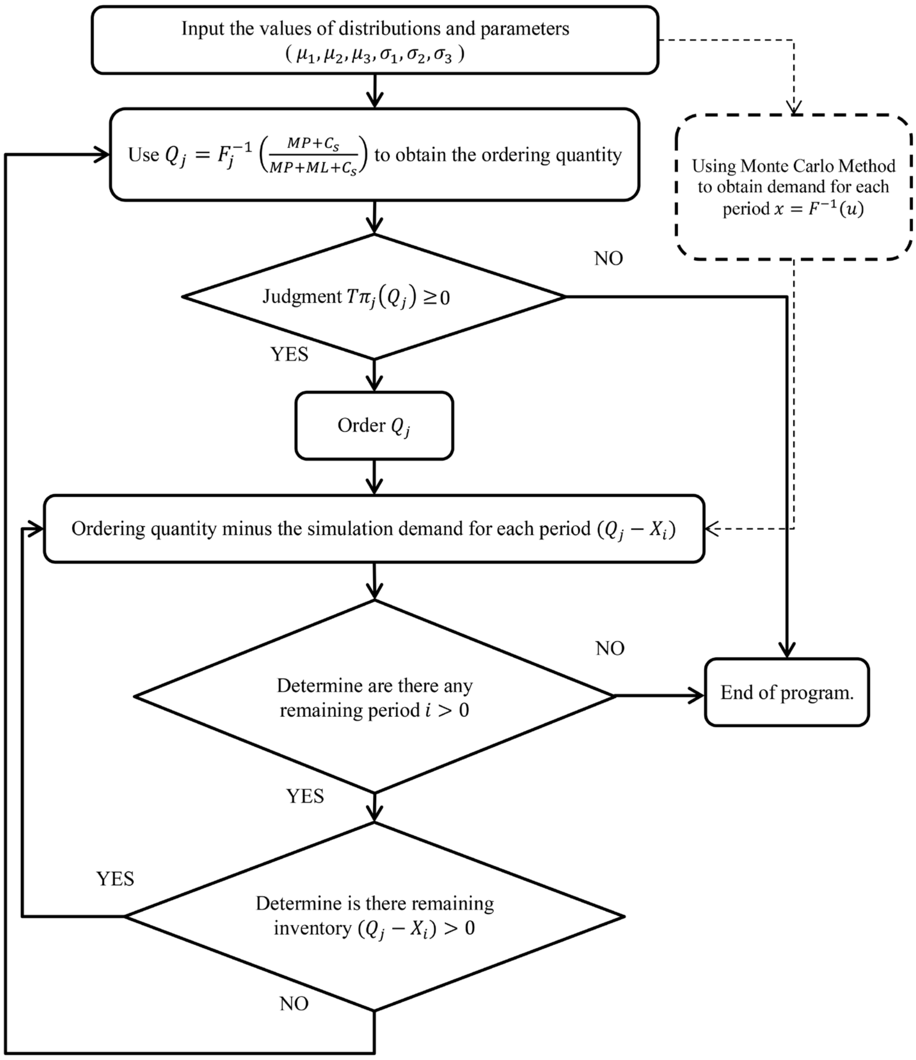


Figure 3. System flow chart.

Table 3. Total expected profit and ordering quantity of combination No. 8.

	Time 1		Time 2		Time 3		J=1			J=2		
Number	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	total times of ordering	$T\pi_1(Q_1)$ total average	Q_1	total times of ordering	$T\pi_M(Q_1,Q_2)$ total average	Q_1+Q_2
8	30	10	30	10	10	1.7	855	3,116.16	76	145	4,493.97	87

4.3. Shortage cost

Under the other parameters are fixed, we will discuss the magnitude of shortage cost that influences the optimal ordering strategy and total expected profit, simultaneously. There are three kinds of situations need to consider: (1) Thinking of the shortage cost is as the opportunity cost, it means that C_s=MP; (2) Thinking of the shortage cost is as the opportunity cost plus customer run off cost, it means that C_s>MP; (3) Thinking of the shortage costs is as fictitious loss, it means that C_s=0. Applying the simulation system developed in this paper, the results are obtained and shown in Figure 4.

According to Figure 4, it can be found that the ordering quantity will be increased when shortage cost rises. Those results just verify the inference in section 3.3.4.

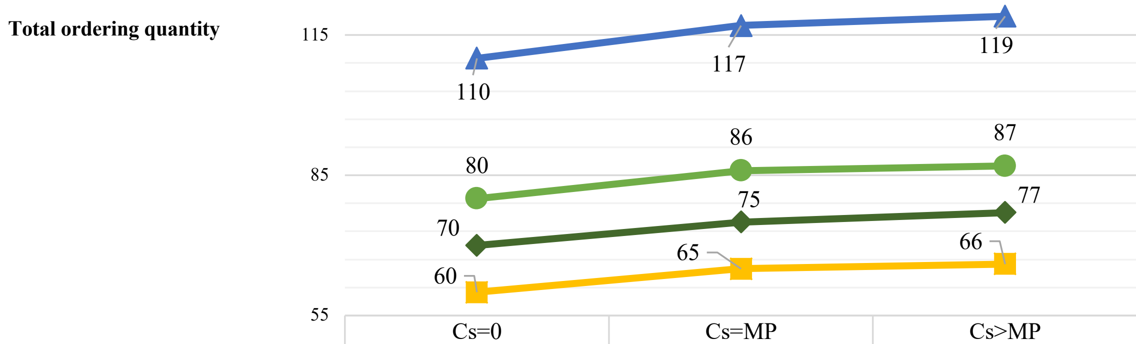
4.4. Order three times' conditions

Based on Section 3.3.4, we knew that the ordering quantity of considering the shortage cost is greater than the one of do not consider. Under do not consider the shortage cost, it can be found that ordering more than one time would occur in some particular combinations, and those results also proved that multi-ordering policy for perishable goods in expiry period (which can be divided into several periods) is worthy. The combinations of order three times are shown in Table 4.

According to we found that the situation of order three times is likely to occur only once in 1000 Table 4 times random simulations under some specific combinations. Usually it occurs at the mean and variation of period 1 are large, and the mean of period 3 is small. From Table 4 we can find the ordering quantity is decreasing each time, it means Q_1>Q_2>Q_3. Therefore, the Proposition 2 was verified.

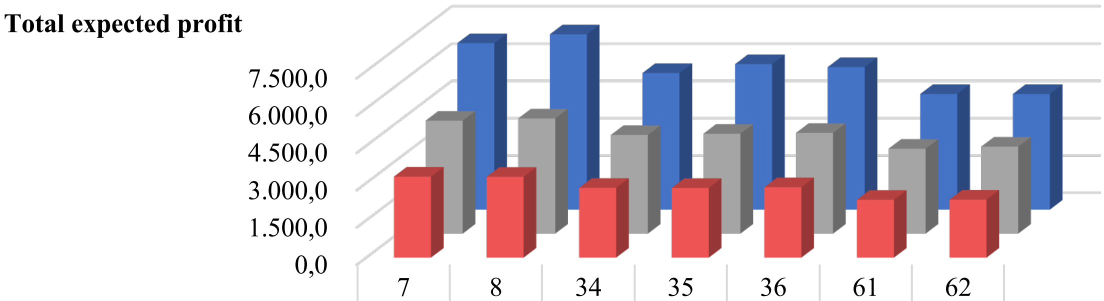
In addition, the total expected profit is shown in Figure 5. Reorder conditions are based on $T\pi_j(Q_j)\geq 0$. Therefore, that can be known the total expected profits will increase when the order number is rising, so the Proposition 1 was verified.

When we do not consider shortage cost (it means C_s=0) and then execute 1000 times simulations for each combination. It can be found that when



	Cs=0	Cs=MP	Cs>MP
NO.246 Order once Total order quantity average	80	86	87
NO.246 Order twice Total order quantity average	110	117	119
NO.333 Order once Total order quantity average	60	65	66
NO.333 Order twice Total order quantity average	70	75	77

Figure 4. The effects of various shortage costs on ordering quantity.



	7	8	34	35	36	61	62
■ Order once in thousand times Total expected profit averages	3.257,5	3.250,2	2.806,1	2.804	2.830,7	2.326,4	2.332,1
■ Order twice in thousand times Total expected profit averages	4.541,9	4.631,7	3.965,9	4.016,4	4.050,3	3.413,8	3.495,7
■ Order thrice in thousand times Total expected profit averages	6.693	7.050	5.493	5.850	5.731	4.650	4.650

Figure 5. Total expected profit under three times ordering.

$(\mu_1 \geq \mu_2 \geq \mu_3)$ and $(\mu_1, \sigma_1, \sigma_2)$ are very large, furthermore, Q_1 is less than or close to $(\mu_1+3\sigma_1)$ and Q_2 is less than or close to $(\mu_2+3\sigma_2)$, order three times situations will be happened.

4.5. Sensitivity analysis

The influences of the system parameters on total expected profit are shown as follows. According to Table 5, it can be observed that $T\pi_j(Q_j)$ will increase when MP is rising. It showed that MP and $T\pi_j(Q_j)$ has a positive correlation. Therefore, Equation (14) was verified. It can be observed that $T\pi_j(Q_j)$ will decrease when ML is rising. It showed that ML and $T\pi_j(Q_j)$ has a negative correlation. Therefore, Equation (15) was verified. It can be observed that $T\pi_j(Q_j)$ will decrease when C_s is rising. It showed that C_s and $T\pi_j(Q_j)$ has a negative correlation. Therefore, Equation (16) was verified. It can be observed that $T\pi_j(Q_j)$ will decrease when Co_j is rising. It showed that Co_j and $T\pi_j(Q_j)$ has a negative correlation. Therefore, Equation (17) was verified.

Table 4. Total expected profit and ordering quantity for order twice and three times.

	i=1		*i*=2		*i*=3		*J*=1			*J*=2			*J*=3		
No.	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	total times of ordering	$T\pi_1(Q_1)$ total average	Q_1 average	total times of ordering	$T\pi_M(Q_1,Q_2)$ total average	Q_1+Q_2 average	total times of ordering	$T\pi_M(Q_1,Q_2,Q_3)$ total average	$Q_1+Q_2+Q_3$ average
7	30	10	30	10	10	3.3	870	3,092.94	76	130	4,466.06	76+11.3	-	-	-
49	30	10	20	2.2	20	6.7	981	3,375.84	75	19	4,891.47	75+23	-	-	-
89	30	5	30	10	10	1.7	899	3,396.94	75	101	4,588.55	75+11	-	-	-
99	30	5	30	5	10	1.1	961	3,692.56	73	39	4,683.49	73+10	-	-	-
116	30	5	20	6.7	10	1.7	940	2,999.72	64	60	4,008.30	64+11	-	-	-
134	30	5	20	2.2	10	1.7	993	3,194.83	63	7	3,896.00	63+11	-	-	-
269	20	6.7	30	3.3	10	1.7	951	3,061.81	63	49	4,046.29	63+11	-	-	-
656	10	1.1	30	10	10	1.7	919	2,304.68	55	81	3,481.16	55+11	-	-	-
7	30	10	30	10	10	3.3	734	3,257.5	70	265	4,541.9	70+10.1	1	6,693	70+40+10
8	30	10	30	10	10	1.7	743	3,250.2	70	256	4,631.7	70+10	1	7,050	70+40+10
34	30	10	20	6.7	10	3.3	796	2,806.1	60	203	3,965.9	60+10.3	1	5,493	60+30+10
35	30	10	20	6.7	10	1.7	797	2,804	60	202	4,016.4	60+10.1	1	5,850	60+30+10
36	30	10	20	6.7	10	1.1	791	2,830.7	60	208	4,050.3	60+10.1	1	5,731	60+30+10
61	30	10	10	3.3	10	3.3	806	2,326.4	50	193	3,413.8	50+11.3	1	4,650	50+20+10
62	30	10	10	3.3	10	1.7	811	2,332.1	50	188	3,495.7	50+11.4	1	4,650	50+20+10

Table 5. The influence of MP, ML, C_s, Co_j on Q_1 and $T\pi_1(Q_1)$.

Number		183 (μ_1=30, σ_1=3.33, μ_2=30, σ_2=3.33, μ_3=30, σ_3=3.33)		547 (μ_1=10, σ_1=3.33, μ_2=10, σ_2=3.33, μ_3=10, σ_3=3.33)	
		Q_1	$T\pi_1(Q_1)$	Q_1	$T\pi_1(Q_1)$
MP	60	93	4,978.4	33	1,363.4
	70	93	5,868.8	33	1,655.4
	80	93	6763.8	33	1943.4
ML	49	93	5,002.9	33	1,417.1
	59	93	4,978.4	33	1,363.4
	69	92	4,942.1	32	1,339.2
C_s	0	90	5,076.1	30	1,474.5
	60	93	4,978.4	33	1,363.4
	80	93	4,953.7	33	1,351
Co_j	0	93	5,016.9	33	1,423
	50	93	4,978.4	33	1,363.4
	100	93	4915.5	33	1332.1

5. Conclusions

This paper establishes a single-period and multi-ordering mathematical model to revise the traditional newsvendor model and based on numerical examples to verify its feasibility and profitability. The purpose of this paper is to modify the traditional newsvendor model from single-order to multi-order to maximize the total expected profit. With consideration of marginal contribution, marginal loss, and shortage cost, the total expected profit for multiple orders will be better than for single order, and the amount of each order placed under multiple orders and its corresponding expected profit will gradually decrease. Based on numerical examples, the perishable goods will be ordered three times only in few cases. The most order times is once and twice, and as long as order times is more than once, the total expected profit will increase when the times of ordering is increasing. In this paper, Monte Carlo method is used to simulate stochastic demand in each period, and we also designed a computerized system to search for the optimal multi-ordering strategy to maximize the total expected profit. Finally, numerical examples are proposed to demonstrate the effectiveness and feasibility of the proposed model.

Finally, this paper only studies a single perishable goods, it can be studied for multi-perishable goods in the future. The model can be added in different limiting factors such as space or budget.

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