Dipartimento di Ingegneria
Corso di Laurea in Ingegneria delle tecnologie della comunicazione e dell'informazione

Modelling and Performance evaluation of Duty-Cycled Wireless Sensors Networks with Energy Harvesting

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Resum

Aquesta tesi de màster té com a objectiu desenvolupar un nou model matemàtic per a una xarxa de sensors sense fils que operen amb cicle de treball i estan equipats amb maquinari de recol·lecció d'energia. El protocol emprat és S-MAC i la xarxa és modelada amb una Cadena de Markov de temps discret 3-D (DTMC) l'estat de la qual està definit pel vector (i,k,b). Els elements de l'aquest vector són: el nombre de paquets en la cua d'un dispositiu, el nombre de nodes actius en un cicle i el nivell discret de càrrega de la bateria. Els elements de la matriu de transició s'obtenen combinant les probabilitats que definixen el resultat de l'accés al canal i les probabilitats que definixen el consum i l'arribada d'energia. Els paràmetres de rendiment provenen directament de la distribució estacionària i s'analitzaran en diferents escenaris. A més, s'ha calculat la distribució del temps que passa el sensor amb energia en la bateria quan no es pot collir-la..

Resumen

Esta tesis de máster tiene como objetivo desarrollar un nuevo modelo matemático para una red de sensores inalámbricos que operan con ciclo de trabajo y están equipados con hardware de recolección de energía. El protocolo empleado es S-MAC y la red es modelada con una Cadena de Markov de tiempo discreto 3-D (DTMC) cuyo estado está definido por el vector (i,k,b). Los elementos del dicho vector son: el número de paquetes en la cola de un dispositivo, el número de nodos activos en un ciclo y el nivel discreto de carga de la batería. Los elementos de la matriz de transición se obtienen combinando las probabilidades que definen el resultado del acceso al canal y las probabilidades que definen el consumo y la llegada de energía. Los parámetros de rendimiento provienen directamente de la distribución estacionaria y se analizarán en diferentes escenarios. Además, se ha calculado la distribución del tiempo que pasa el sensor con energía en la batería cuando no se puede cosecharla.

Abstract

This master's thesis aims to develop a new mathematical model for a network of wireless sensors that operate with duty cycle and are equipped with energy harvesting hardware. The protocol used is S-MAC and the network is modeled with a discrete-time 3-D Markov Chain (DTMC) whose state is defined by the vector (i, k, b). The elements of the said vector are: the number of packets in the queue of a device, the number of active nodes in a cycle and the discrete level of charge of the battery. The elements of the transition matrix of the DTMC are obtained by combining the probabilities that define the ways to access to the channel and the probabilities that define the energy consumption and arrival. The performance parameters come directly from the stationary distribution and will be analyzed in different scenarios. In addition, we have calculated the distribution of time that the sensor passes with power in the battery when no energy can be harvested.

Riassunto

Oggigiorno molte applicazioni richiedono l'implementazione di sensori che raccolgono informazioni su una determinata area geografica per poi successivamente trasmetterle a un gateway centrale. Questi dispositivi possono condividere il problema di una limitata energia a causa della batteria e pertanto il loro consumo deve essere ridotto al minimo. Quando la sostituzione o la ricarica delle batterie sul campo non è fattibile, un meccanismo di raccolta dell'energia potrebbe essere la scelta migliore per prolungare la durata della rete. Questa tesi di laurea mira a sviluppare un nuovo modello matematico con il quale saremo in grado di valutare le prestazioni di reti di sensori wireless Duty-Cycled dotati di hardware per la raccolta di energia. Il modello considererà un singolo sistema di nodi in grado di inviare pacchetti di informazione solo a un nodo centrale (Sink Node), ma si potrebbero considerare la possibilità di integrare più sistemi per formare una rete più grande. Tutti i nodi funzionano in regime di Duty-Cycle (DC) in modo che, all'interno di un ciclo, si alternino periodi attivi e inattivi al fine di risparmiare energia. Per essere considerato attivo, un sensore dovrebbe avere abbastanza energia e una quantità minima di pacchetti all'interno della coda. Si supporrà che l'arrivo di pacchetti a un dispositivo segua un processo di Poisson. Inoltre, quando un dispositivo vince l'accesso al canale, esso trasmette un frame contenente più pacchetti la cui lunghezza è limitata da un valore massimo o dal numero di pacchetti nella coda. Il protocollo utilizzato sarà il S-MAC con un handshake RTS/CTS/DATA/ACK basato su un CSMA/CA e in cui i timer di back-off vengono resettati ad ogni ciclo iterativo. Difatti, la probabilità di vincere l'accesso al canale è strettamente correlata alla selezione del valore di back-off.

La rete sarà modellata con una catena di Markov tempo discreta tridimensionale (DTMC) il cui stato è definito dal vettore (i,k,b). Gli elementi vettoriali sono: i) il numero di pacchetti nella coda di un dispositivo; ii) il numero di nodi attivi in un ciclo; iii) il livello effettivo di carica della batteria. Di fatto, per limitare la cardinalità dello spazio degli stati DTMC, l'energia raccolta dai nodi e il loro consumo di energia saranno anche discretizzati in modo adeguato. Pertanto, uno dei contributi dello studio sarà quello di definire l'energia consumata dai nodi per ciascuno dei diversi modi di accesso al canale, come le trasmissioni avvenute con successo o le collisioni. Gli

elementi della matrice di transizione sono ottenuti combinando le probabilità che definiscono il modo di accedere al canale e le probabilità relative al consumo o arrivo di energia. Inoltre, il tutto sarà formulato con una matrice di transizione associata a un processo di Quasi-Nascita-Morte, in modo tale che un algoritmo di riduzione dello stato a blocchi verrà utilizzato per ottenere la distribuzione stazionaria. I parametri di prestazione, fra cui il ritardo medio, il throughput e il consumo di energia, provengono direttamente dalla distribuzione stazionaria e saranno analizzati in diversi scenari. Ad esempio, si cambierà la dimensione della rete, la dimensione della coda, la probabilità di arrivo dell'energia e così via. In particolare, impostando quest'ultima uguale a zero, sarà possibile determinare la distribuzione del tempo trascorso dal dispositivo fino alla scarica della batteria. Può essere modellato come una distribuzione di fase, cioè una distribuzione del tempo fino all'assorbimento da parte di uno stato DTMC. Tale analisi potrà consentire di comprendere al meglio il consumo energetico della rete.

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Chapter 1

Introduction

1.1 Prologue

Nowadays, wireless sensor networks have assumed fundamental importance not only in the field of scientific research but also in the everyday life of any citizen. In fact, thanks to the *Internet of Things* (IoT), both our homes and our cities are becoming increasingly ”Smart” in order to make every type of service more efficient. Examples of such applications will be the lighting control in cities, the arrangement of sensors for environmental and territorial control and the network of security cameras positioned in the various streets of the city.

An omnipresent problem of such networks is energy consumption. In fact, many sensors are located in places that are difficult to connect to the mains electricity and this involves the use of a battery power supply. To prolong the network lifetime, there are two ways we can operate, that are both to guarantee the lowest possible consumption of energy and implement a technology of energy harvesting (EH) . On the first point, many standards such as Bluetooth Low Energy (BLE), ZigBee, ANT, and other Low Power Wide Area Networks (LPWAN) have been created. In the course of my thesis, however, I will mainly focus on the second point, that is on the situations that provide for the collection of energy from external factors such as solar rays or electromagnetic energy.

1.2 Previous work

Throughout my research period, I have developed a mathematical model that represents the operating way of a sensor network thanks to a three-dimensional Discrete Time Markov Chain (DTMC). A 2-D model has already been developed in [3], such that I have continued their work by considering the actual battery charge level. The remaining two dimensions are the number of packets in the queue of a device and the number of active nodes in a cycle. Respectively, their maximum values are Q and K.

The protocol employed is the S-MAC with CSMA/CA-based RTS/CTS/DATA/ACK handshake, where back-off timers are reset at each cycle iteration. In fact, the probability to win access to the channel is strictly related to the selection of the back-off value. The model considers a single cluster of sensor nodes that are able to send information packets only to a central Sink Node (SN), but it might contemplate multiple clusters to form a larger network. All of the sensors contemporaneously active at the beginning of the same cycle will contend the access to the channel in order to send information to the SN.

All the nodes work in a Duty-Cycle (DC) regime so that, within a cycle, they periodically alternate an active and inactive period with the goal of saving energy.

1.3 Targets, methodology and contribution

To limit the cardinality of the DTMC state space, one of the contributions of the study has been to discretize the energy harvested and the energy consumed by the nodes for each of the different channel access outcomes, such as successful transmissions or collisions.

It has been assumed that, inside a cycle, the energy harvesting hardware does not send energy as long as its value overcomes a tunable threshold, then it passes such discrete amount of energy to the corresponding sensor battery with a probability Pr_{EP}. The threshold is C times the maximum energy E_{tx}^{max} that a successful transmission can consume and it is equal, as well, to the energy stored inside one notch of the battery out of the possible B notches. In other words, after an energy acquisition from the EH hardware, the battery level increases

of one, whereas after C full frame successful transmission, it decreases of one. Subsequently, the consumption of every other node operation has been normalized to this threshold such that the respective values assume the probabilistic view of a fraction of cycles in which the RN consumes energy. For example, if $E_{tx}^{max} = 10$ mJ and $C = 20$, then 1 cycle every 20 of maximum transmission produces a consumption of one energy notch and Pr_{EP} is the fraction of cycles in which at least $20 * 10$ mJ are harvested.
Another contribution of my work on [3] has been the definition of a threshold bm that specifies the minimum number of packets in the queue under which is not energetically convenient to transmit.
The transition matrix elements of the DTMC are obtained by combining the probabilities that define the channel access outcome and the probabilities that define the energy consumption and the arrival of packets and harvested energy. Moreover, it will be formulated as a transition matrix associated with a Quasi-Birth-Death process and hence a block state reduction algorithm will be used to reduce the computational cost required to obtain the stationary distribution.

The performance parameters, including the average delay, the throughput and the energy consumption, directly come from the stationary distribution and they will be analyzed in different scenarios. For example, by changing the size of the network, the size of the queue, the energy arrival probability, and so on. In particular, by setting the latter greater than the energy consumed, it will be possible to determine the distribution of the time the device spends with power until the battery runs out. It can be modelled as a phase type distribution, that is a distribution of the time until absorption in an absorbing DTMC. Its analysis will allow us to better understand the energy consumption of the network.

1.4 Contents

The remainder of this document is structured as follows: chapter two provides a brief overview of the basic concept of DTMC; chapter three defines the reference network scenario and the way to discretize the energy is further explained; chapter four focuses on how the probabilities have been combined to define the transition matrix elements and how to calculate both the stationary distributions and the main performance parameters; chapter five includes the conclusions and the future works.

Chapter 2

Introduction to Markov Chains

Since the subject of the Markov Chains was completely new to my knowledge, under the guidance of Professor Martinez I have learned the main basic notions behind this theory, such as the Markov Chains and some probabilistic distributions. Following, I have summarized some of these notions that will be useful to follow the next chapters.

2.1 Discrete Time Markov Chains

Since my work is mainly based on Markov chains in a permanent regime, I have first studied their general properties. These properties apply to my study because both the trend of the number of packets inside a waiting buffer of a sensor node and the amount of energy of the sensor can be represented by a stochastic processes $N(t)$. Then, if the time axis is discretized in cycles of length T and the stochastic process is memoryless, then $N(t_i)$ becomes a DTMC with $t_i = iT$.

Each m state that the stochastic process can take for that particular cycle is characterized by a probability $p_m(t_i)$. Thanks to the Markovian property of the absence of memory, the probability of transition from a state m to a n, ie $p_{mn}(t_i, t_{i+1})$, does not depend on the past instants but only on the current and future ones.

From the previous definitions we can easily find the Chapman-Kolmogorov relation (2.1) which defines the transition probability between two states, i.e. m and b, placed in two non-adjacent time instants, for example t_0 and t_2, with $t_0 < t_1 < t_2$. [2].

$$p_{mb}(t_0, t_2) = \sum_n p_{mn}(t_0, t_1) p_{nb}(t_1, t_2) \tag{2.1}$$

In the course of my study I have only dealt with homogeneous time chains, so the probabilities of transition between states do not depend on the individual moments but only on their difference (2.2).

$$p_{mn}(t_0, t_0 + k) = p_{mn}(k) \quad \forall\, t_0, k \tag{2.2}$$

Moreover, the states are aperiodic and recurrent and the chain is irreducible [2].
A periodic state means that one can return to the same state after a fixed number, i.e. a period, of transitions.
A state is called recurrent if the probability of returning to that state is equal to 1. In particular, the state is positively recurring if the numbers of transitions to return to a state is a finite number.
Finally, a chain is irreducible if all states communicate with each other.
A more complete classification of states is visible in Fig. 2.1.

Under these assumptions, after an infinite number c of cycles T (or jumps) from an initial i state, we obtain the so-called stationary probabilities π_j of the j state [1], i.e.:

$$\lim_{c \to \infty} p^c_{i \to j} = \pi_j \tag{2.3}$$

These probabilities are independent of the initial state (ergodic chain) and interpretable in two main ways:

- π_j represents the probability of finding the chain in the j state at a random time slot.

- π_j it is the fraction of cycles that the system spends in the j state in the stationary regime

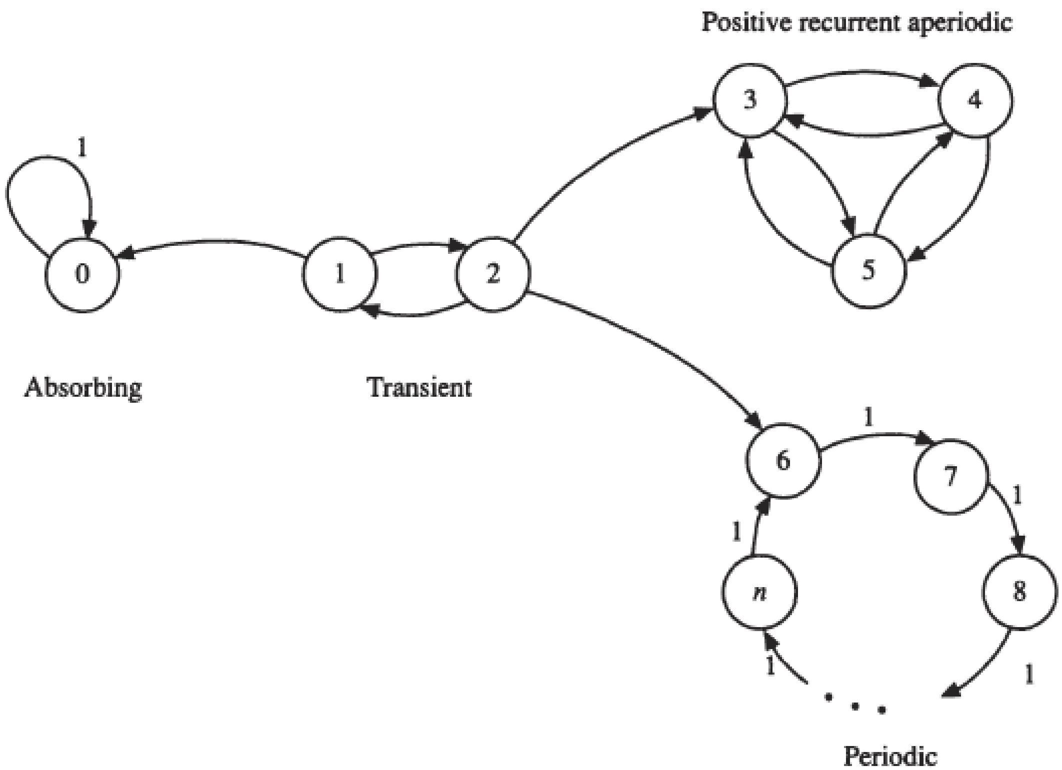


Figure 2.1: Classification of the states [1]

The vector $\underline{\pi} = [\pi_1 ... \pi_j ...]$ includes all the possible states and can be obtained by the Global Balance Equations (2.4). It can be defined thanks to the transition matrix $\mathbf{P}$ which consists of all $p_{i \to j} \forall\ i, j$. This system of equations, usually solved in matrix form, must be flanked by the condition that the sum of the elements on the same row of $\mathbf{P}$ must be equal to 1 [1].

$$\begin{cases} \pi_j = \sum_{k=0}^{\infty} \pi_k p_{k,j} \\ \sum_{j=0}^{\infty} \pi_j = 1 \end{cases} \iff \begin{cases} \underline{\pi} = \underline{\pi} \cdot \mathbf{P} \\ \|\underline{\pi}\| = 1 \end{cases} \tag{2.4}$$

2.2 Probabilistic elements

Many elements related to the study of probabilities have been used, thus in the following lines I will describe what are the most common distributions that the stochastic processes present in the problems related to the queuing theory.

First of all, we must divide the discrete-time case from the continuous-time one. Furthermore, for each of the two cases, the distributions of the processes related to the following three factors have been analysed:

- the number of arrivals in a given time
- the waiting time for the first arrival
- the waiting time for the first K arrivals

2.2.1 Discrete-time case

The arrival or absence of a packet within a time slot, or cycle, is a Bernoulli random variable with the probability of success (arrival) p and probability of failure (non-arrival) $1 - p = q$. Multiple consecutive slots form a Bernoulli process without memory. Let k be the number of arriving and n the number of consecutive slots to be analysed, the probability density function that describes the number of packets arriving in n slots of time is the binomial distribution in (2.5).

$$f_K(k, n, p) = \binom{n}{k} p^k (1-p)^{n-k} \tag{2.5}$$

while its expected value and variance are expressed in (2.6).

$$\begin{aligned} E[K] &= \sum_k k f_K(k) = np \\ Var[K] &= E[K^2] - E[K]^2 = p(1-p) \end{aligned} \tag{2.6}$$

If we were interested in the number of time slots n that we have to wait before the first success, the distribution of this phenomenon is no longer binomial but geometric (2.7).

$$\begin{aligned} f_N(n,p) &= (1-p)^{n-1}p \\ E[N] &= 1/p \\ Var[N] &= (1-p)/p^2 \end{aligned} \tag{2.7}$$

Finally, if what we are looking for is the number of necessary cycles to receive K arrivals, the distribution to be used is the Pascal one (2.8). This distribution is easily obtained from the (2.5) since it corresponds to the probability of having $k-1$ arriving in $n-1$ slots multiplied by the probability of having an arrival in the n-th slot.

$$\begin{aligned} f_N(n,k,p) &= \binom{n-1}{k-1} p^{k-1}(1-p)^{t-k}p \\ E[N] &= k/p \\ Var[N] &= k(1-p)/p^2 \end{aligned} \tag{2.8}$$

2.2.2 Continuous-time case

The continuous-time case is based on the discrete case when the time duration τ of the time slots is infinitesimal ($\tau \approx \delta$). By defining the rate describing the average number of arrivals per second with λ, the probability of K arriving during δ is:

$$f_K(k,\delta,\lambda) = \begin{cases} 1-\lambda\delta, & \text{se } k=0 \\ \lambda\delta, & \text{se } k=1 \\ 0, & \text{se } k>1 \end{cases} \tag{2.9}$$

By combining the (2.9) and the (2.5) we get the probability density function relative to the number of arrivals within a period of t:

$$\begin{aligned} f_K(k,t,\lambda) &= \frac{(\lambda t)^k e^{-\lambda t}}{k!} \\ E[K] &= \lambda t \\ Var[K] &= \lambda t \end{aligned} \tag{2.10}$$

The (2.10) pdf is known as Poisson distribution and is widely used to model totally random and independent entries in a time interval of length t.

The time between consecutive arrivals is modelled with an exponential pdf (2.11) and it is interesting to note how the distribution of Poisson in the (2.10) becomes the exponential one when $k = 1$.

$$\begin{aligned} f_T(t, \lambda) &= \lambda e^{-\lambda t} \\ E[T] &= 1/\lambda \\ Var[T] &= 1/\lambda^2 \end{aligned} \tag{2.11}$$

Finally, the time-related distribution t to detect k arrivals equals the probability that $k-1$ arrive in time $[0, t-\delta]$, that is the (2.10) written with $k-1$ times the probability of receiving a packet at the next δ instant (2.9). The result is the distribution of Erlang (2.12).

$$\begin{aligned} f_T(t, \lambda, k) &= \frac{(\lambda t)^{k-1}}{(k-1)!} e^{-\lambda t} \lambda \\ E[T] &= k/\lambda \\ Var[T] &= k/\lambda^2 \end{aligned} \tag{2.12}$$

The table 2.1 summarizes the names of the distributions in the time-discrete and time-continuous case for the various parameters that we want to analyse [4].

Table 2.1: Distributions with discrete and continuous time

Parameter	**Discrete-time distribution**	**Continuous-time distribution**
Arrivals number in a given time	Binomial	Poisson
Waiting time for the first arrival	Geometric	Exponential
Waiting time for K arrivals	Pascal	Erlang

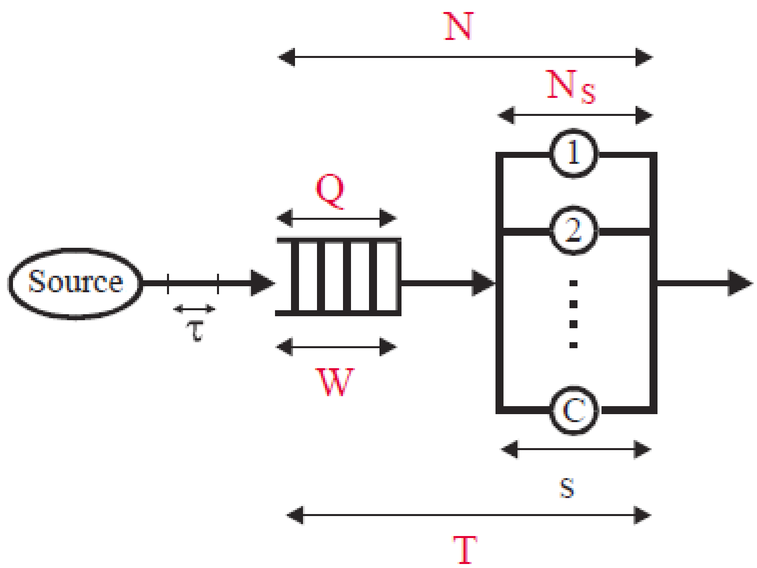


Figure 2.2: Parameters in a waiting system [2]

2.3 Queuing theory

In queueing theory, to represent a waiting system, one must first get familiar with the notation about some common parameters. All the parameters in Fig. 2.2 are random variables with different distributions. Their meaning is described in table 2.2.

These parameters can be related to each other thanks to Little's Law. This law is easy to understand, but it took several years to prove it. If with λ we call the rate of customers entering and leaving the system and with $\overline{T_s}$ the average service time value s, Little's law says that the average values of the random variables in figure 2.2 depend on each other thanks to the formulas in (2.13).

$$\begin{aligned} \overline{N} &= \lambda\overline{T} \\ \overline{Q} &= \lambda\overline{W} \\ \overline{N_S} &= \lambda\overline{T_S} \end{aligned} \tag{2.13}$$

Table 2.2: Meaning of the parameters in Fig.2.2 [2]

Parameter	**Function**
τ	Customer inter-arrival time
s	Customer service time
W	Waiting time in the queue
T	Residence time in the system
N_S	Number of customers served simultaneously
Q	Number of customers waiting in the queue
N	Number of customers in the system

Chapter 3

Scenario

After explaining the basic notions of Markov Chains, I am now going to illustrate the scenario under study. First, I will describe the topology of the network, how the RN can be modelled according to the DTMC and the Medium Access Protocol that has been deployed. Then, I will explain a way to treat the energy as a discrete variable such that the cardinality of the state space of the DTMC will be limited. Finally, I will talk about the activation and inactivation of a node due to the absence or presence of energy in the battery or packets in the queue.

3.1 Sensors network

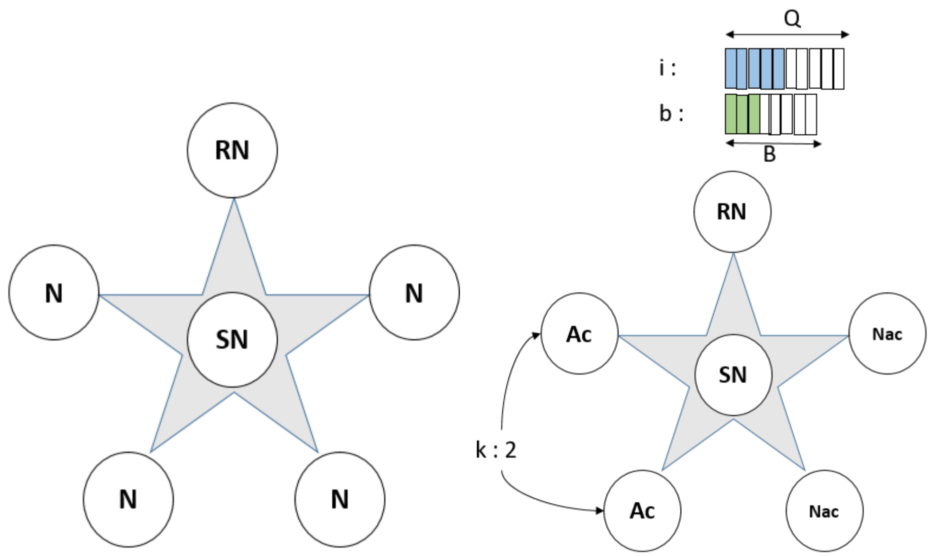
The sensors network under study respects a star topology, i.e. each node sends data to a central server called sink node (SN) (Fig. 3.1 (a)). Each node has an antenna for data transmission and hardware to harvest energy from the environment. The DTMC is designed for all the network but I am going to refer to the behaviour of a generic reference node (RN) by assuming homogeneity in nodes behaviour.

At a random cycle (remember that time is discretized in cycles of duration T), the RN may be in different states. We represent the network states by a state vector of three dimensions $(i\ k\ b)$, that are, respectively, the number of packets in the buffer of the RN waiting to be sent ($0 \leq i \leq Q$), the number of active nodes for that cycle in addition to the RN ($0 \leq k \leq K$) and the actual battery charge level ($0 \leq b \leq B$).

A clarifying example is given in Fig. 3.1 (b) where the RN presents $i = 5$ packages ready to be sent, $k = 2$ other nodes that want to access the channel and $b = 3$ battery notches; hence, the state of the chain at that cycle is $(5, 2, 3)$.

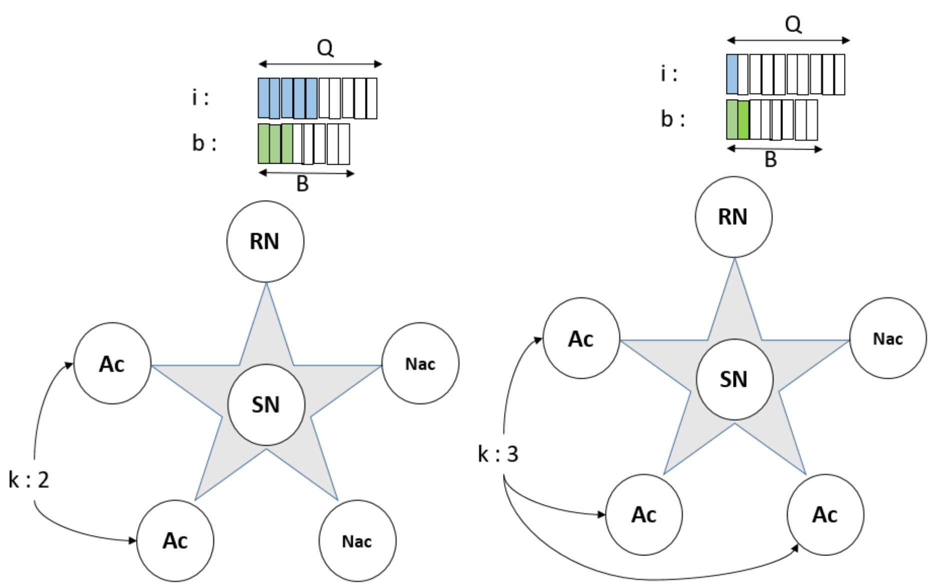
At this point, the main purpose is to understand how and with what probability the state transitions occur from one cycle to another. In other words, if at cycle n the state of the RN is (i, k, b), what is the probability that at the next cycle $n + 1$ the RN is in the state (j, l, d)? Taking up the Markov chain theory described above, this corresponds to defining the elements of the transition matrix **P**.

For example, assuming that neither data packets nor energy arrive at the RN at the end of cycle n, the element $\mathbf{P}_{(i,k,b)(j,l,d)}$ with $j < i$, $l > k$ and $d < b$ is equivalent to the probability that there has been a correct sending of packets (therefore i decreases and becomes j) times the probability that the number of active nodes in the system increases (therefore k rises and becomes l) and the whole is multiplied by the probability that the battery level decreases by one (therefore b decreases and becomes d). An example is visible in Fig. 3.2.



(a) Star topology (b) RN in the state (5,2,3)

Figure 3.1: (a) shows the network topology; (b) represents the situation when the RN is in a state (i, k, b)=(5,2,3).



(a) RN in the state (5,2,3) (b) RN in the state (1,3,2)

Figure 3.2: (a) shows the state at the beginning of the cycle t_0; (b) illustrates the system after the transition, i.e. at the beginning of the next cycle t_1.

3.2 S-MAC

The operation of the Medium Access Protocol (MAC) definitely affects the energy consumption since the transmission and overhearing events depends on the protocol used.

S-MAC is one of the most studied protocols in the literature thanks to its effectiveness and simplicity. Its main qualities are substantial energy savings, good scalability and the ability to avoid collisions in packet transmission. The main purpose of the S-MAC is to save energy on four factors: collisions, receiving packets that are intended for other nodes (overhearing), sending and receiving control packets, and finally listening to the channel when no package has been sent [5].

Two extensions have been added to the classic operation of S-MAC. The first refers to the definition of a parameter F that indicates how many packets are jointly sent into a frame throughout a cycle [3]. The second, instead, consists of specifying the minimum number bm of packets in the queue that are necessary to activate the node. This threshold might be tunable and allow the node to save energy by reducing the number of consecutive transmissions. Moreover, it has been assumed that no data packets can be lost due to a collision because they will be retransmitted an infinite number of times.

We can divide the S-MAC cycle into three parts: synchronization, data and sleep period (Fig. 3.3).

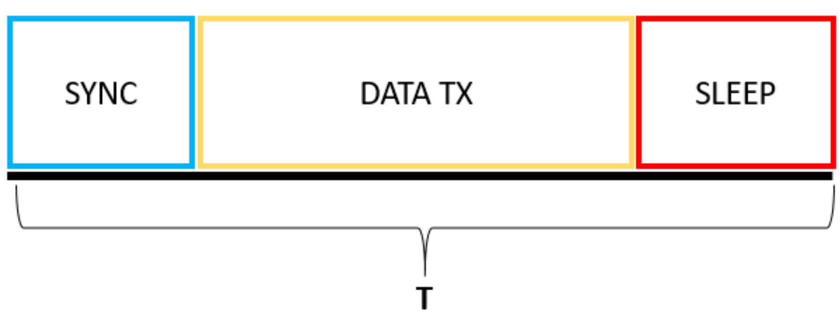


Figure 3.3: Periods in a S-MAC cycle

In a synchronization period, every node decides its sleep-awake schedule and communicate it to the other nodes thanks to a SYNC packet.

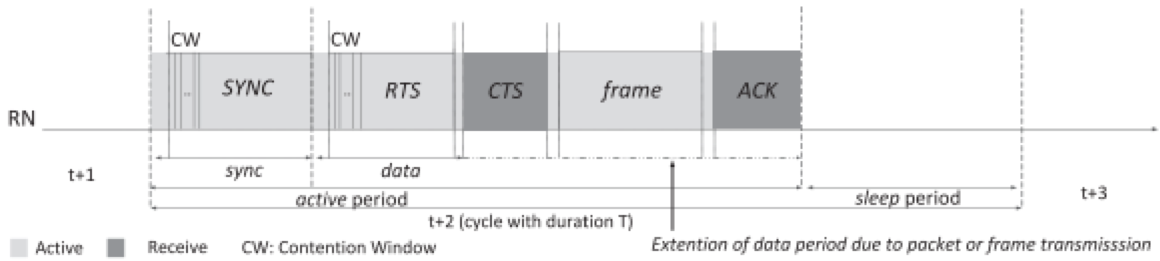


Figure 3.4: Operational way of an active node with S-MAC [3]

At that point, if a node has less than bm packets, then it suddenly goes to the sleeping mode; otherwise, it randomly selects a value that indicates a slot inside a contention window (CW) of length W. Depending on the back-off time values of the other nodes, the node under study goes to sleep after:

- losing the contention
- colliding with other nodes during the RTS packet
- receiving the last packet of a RTS/CTS/DATA/ACK handshake that says the data have been received correctly by the Sink Node.

As example, Fig. 3.4 shows the operation of S-MAC in the most significant case of a node that manages to send a frame of packets.

3.2.1 S-MAC parameters

The access to the channel is ruled by probabilistic laws and hence I have illustrated in Table 3.1 the parameters for each channel outcome of a node that uses S-MAC.

Table 3.1: Probabilistic parameter that define the access to the channel [3]

Symbol	Formula	Description
$P_{s,k}$	$\sum_{w=0}^{W-1} \frac{1}{W}(\frac{W-1-w}{W})^k$	Probability that RN transmits successfully in a cycle with other k active nodes, thanks to selecting a back-off value smaller than the ones chosen by other k active nodes
$P_{sf,k}$	$\sum_{w=0}^{W-1} \frac{1}{W}(\frac{W-w}{W})^k$	Probability that RN transmits (successfully or causing a collision) with other k nodes active
$P_{f,k}$	$P_{sf,k} - P_{s,k} = \frac{1}{W}$	Probability that RN transmits and a collision happens with other k nodes active
$\hat{T}_k$	$1 - (k+1)P_{s,k} - P_{f,k}$	Probability that the RN does not transmit when contending with other k active nodes and two or more of the other nodes collide
S_k	$kP_{s,k-1}$	Probability that an active node transmits successfully in a cycle where k active nodes contend, that is the active node plus $k-1$ contenders and the RN
$BT_{s,k}$	$\frac{1}{P_{s,k}} \sum_{w=0}^{W-1} w\frac{1}{W}(\frac{W-1-w}{W})^k$	Average number of slots of back-off time conditioned on a successful packet transmission by the RN when contending with other k active nodes
$BT_{f,k}$	$\sum_{w=0}^{W-1} w[(\frac{W-w}{W})^k - (\frac{W-1-w}{W})^k]$	Average number of slots of back-off time conditioned on an unsuccessful packet transmission by the RN when contending with other k active nodes

3.3 Energy consumption in S-MAC

Due to the fact that the synchronization period is a constant for each state and the sleep period always provides for a very small amount of power, the most relevant results regarding the energy consumption come from the data period.

The analysis will be flanked by numerical examples obtained in Matlab with the following data:

Table 3.2: Numerical data to be used in the examples [3]

Parameter	Data	Description
T	60 ms	Duration of a cycle
t_{SYNC}	0.18 ms	Duration of a synchronization packet
t_{RTS}	0.18 ms	Duration of a request to send packet
t_{CTS}	0.18 ms	Duration of a clear to send packet
t_{ACK}	0.18 ms	Duration of an acknowledgment packet
D_p	0.001 ms	Propagation delay
t_{DATA}	1.716 ms	Duration of a data packet
t_{slot}	0.001 ms	Duration of one slot in the contention window
W	128	Number of slots in the contention window
P_{tx}	52 mW	Transmission power
P_{rx}	59 mW	Reception power
P_{sl}	3 μW	Sleep power
N_{sc}	10	Number of cycles between one sync packet and another
Q	10	Maximum number of packets in the queue
N	13	Number of nodes inside the network
bm	1	Threshold of packets in the queue to make the node active or not
F	5	Maximum frame size
dc	0.5	Duty cycle: fraction of cycle in which the system is active

3.3.1 Synchronization

The synchronization period is omnipresent in all cycles and the node transmits a SYNC packets once every N_{sc} cycles. The energy consumed is:

$$E_{sync} = \frac{1}{N_{sc}} \cdot [t_{SYNC} \cdot P_{tx} + (T_{sync} - t_{SYNC}) \cdot Prx] + \frac{N_{sc}-1}{N_{sc}} \cdot (t_{SYNC} \cdot Prx) \tag{3.1}$$

where T_{sync} is the total synchronization period time that includes the maximum back-off time $(W-1)$, the sending time of the package t_{SYNC} and the propagation time D_p.
With the numeric data of the table 3.2 we get $E_{sync} = 0.085$ mJ.

3.3.2 Data period

According to the channel outcome, the events that can occur in the data period are shown in Fig. 3.5. At each of them correspond a different usage of transmission and reception power and hence the energy consumption of the RN is different in every case.

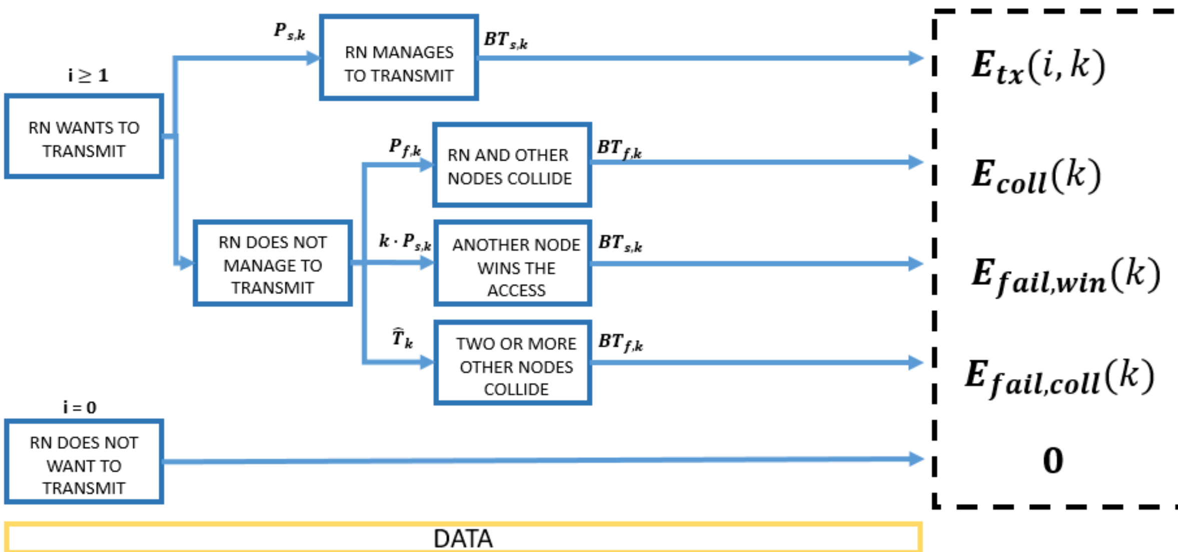


Figure 3.5: Channel outcomes and the respective energy consumed

Based on the description of S-MAC in Fig. 3.6 we can derive the formula related to the energy that is normally used within a cycle in which the RN manages to win the contention of the channel (3.2).

$$E_{tx}(i,k) = t_{RTS}P_{tx} + [t_{CTS} + t_{ACK} + 4D_p]P_{rx} + \alpha t_{DATA}P_{tx} + BT_{s,k}t_{slot}P_{rx} \quad (3.2)$$

where α ranges from 1 to the maximum number of packets per frame ($\alpha = min(i, F)$).

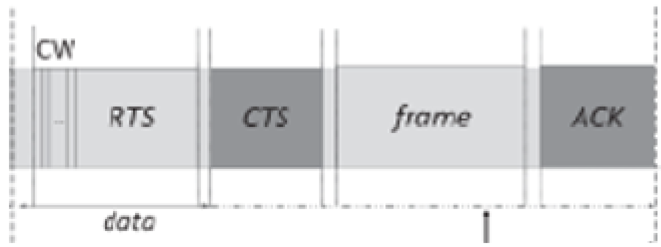


Figure 3.6: S-MAC cycle in a case of a successful transmission

In the same way, we can calculate the energy used by the node during the data period of the other channel outcomes. (3.3), (3.4) and (3.5) respectively define the energy consumed by the RN whether it collides with another node $E_{coll}(k)$, another node wins the access $E_{ovh,tx}(k)$ or two or more other nodes collide $E_{ovh,coll}(k)$.

$$E_{coll}(k) = BT_{s,k}t_{slot}P_{rx} + t_{RTS}P_{tx} + 2D_pP_{rx} \tag{3.3}$$

$$E_{ovh,tx}(k) = (BT_{s,k}t_{slot} + D_p)P_{rx} \tag{3.4}$$

$$E_{ovh,coll}(k) = (BT_{f,k}t_{slot} + D_p)P_{rx} \tag{3.5}$$

The numerical results obtained with the data in table 3.2 are in Fig. 3.7 and Fig. 3.8. The x-axis represents the number of active node in the network in addition to the RN, the y-axis the number of packets in the queue of the RN.
We notice that the energy in Fig. 3.7 does not increase once the size of the tx frame reaches F. Moreover, in Fig. 3.8 we see that the more nodes are active, the smaller is the value of $BT_{s,k}$ or $BT_{f,k}$ that must be considered for the transmission or collision and hence the smaller the consumed power is.

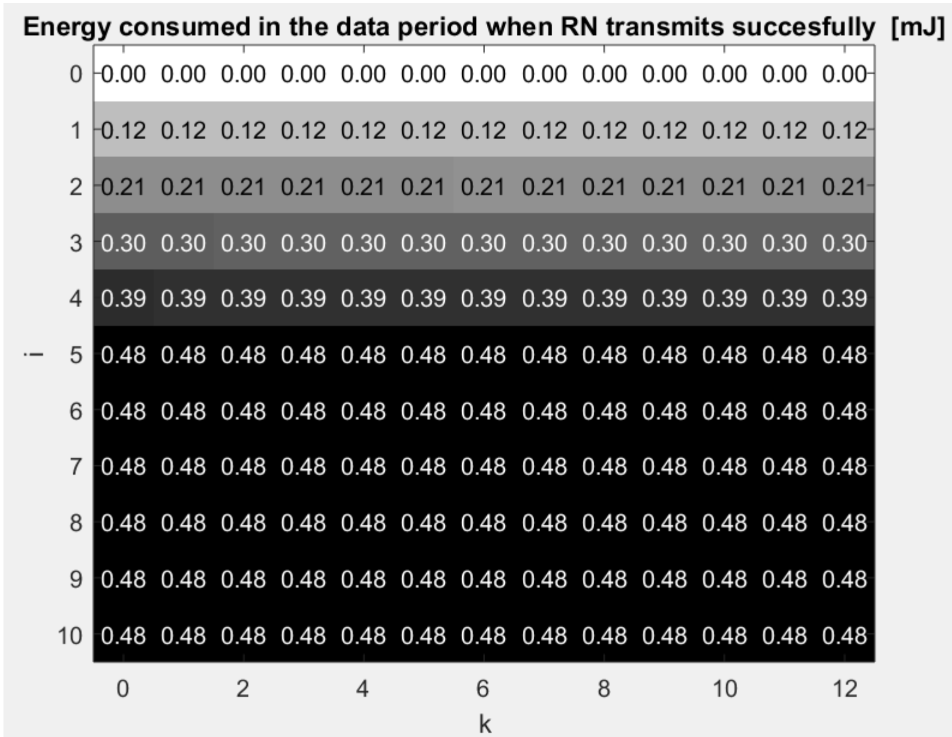


Figure 3.7: Energy consumption for cycles where the RN manages to transmit [mJ]

3.3.3 Sleep

With the same arguments used to determine the energy consumption during the data period, we could calculate the energy consumed in the sleeping period for every different event. However, the values are so small that we consider them negligible, thus I am not going to represent the results as I did in the previous section.

3.4 Energy Discretization

In order to start the process of energy discretization, let us assume that one notch of the battery is C times the maximum energy that can be consumed by a node in a random cycle. Next, let us normalize every element of the table in Fig. 3.7 according to that maximum and we further divide everything by C.

If we take as example the new value $Pr_{E,tx}(i \geq F, k = 0) = E_{tx}/(C * max)$, it means that, in

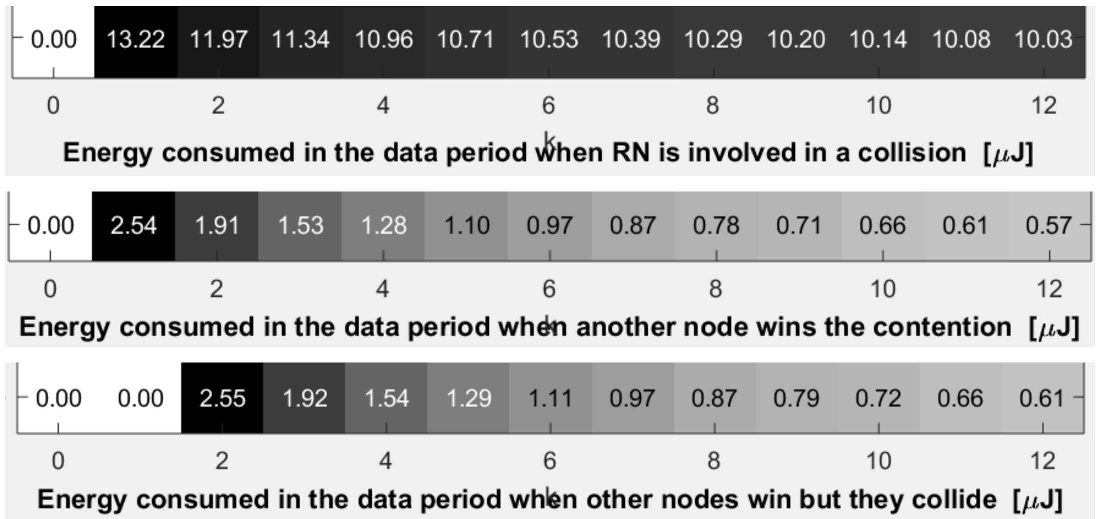


Figure 3.8: Energy consumption for cycles where the RN does not manage to transmit [μJ]

this state and in the current cycle, we consume the $1/C$ of an energy notch. In other words, every C occurrences of this event there is a consumption of 1 notch of energy. Consequently, $Pr_{E,tx}$ might be considered as the probability to consume one energy notch in the battery of the RN when the event occurs.

We can apply the same arguments to other channel outcomes by paying attention to use the same normalization factor. By doing that and by referring to Fig. 3.5, in addition to $Pr_{E,tx}$ we get:

- $Pr_{E,coll}$
- $Pr_{E,ovh,tx}$
- $Pr_{E,ovh,coll}$

3.4.1 Energy harvesting

Initially, we assume that the environmental conditions are error-free but it has been thought of specifying such conditions in the future to model the energy harvesting process by a Markov Chain in a more elaborated and realistic way.

We have also assumed that the energy harvesting hardware does not send energy as long as its value overcomes a tunable threshold, then it passes such discrete amount of energy to the RN battery. This occurs with probability Pr_{EP} in a random cycle. The threshold corresponds to C times the maximum energy E_{tx}^{max} that a successful transmission can consume and it is equal, as well, to the energy stored inside one notch of the battery out of the possible B notches (Fig. 3.9). Therefore, we interpret Pr_{EP} as the fraction of cycles in which a packet of energy reaches the RN and subsequently provokes an increase of one notch on it. We could have modelled the energy arrival according to some type of distribution, as it has been done in [6] and [7], but the result would always be a fraction between 0 and 1 which indicate, respectively, that no energy at all will arrive in any cycle or that it arrives in every cycle. Consequently, in the model the energy arrival is controlled only by varying the value of Pr_{EP}.

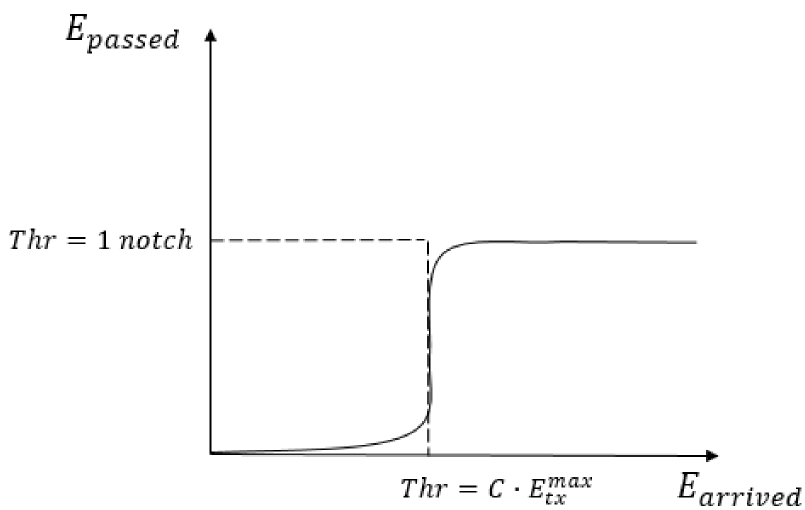


Figure 3.9: How the harvesting hardware passes energy to the node battery

3.4.2 Energy state transition

By combining the concepts of energy consumption and collection, it is easy to understand that the energy state transition can only be of three types: the RN remains at the same energy level, it gains an extra energy notch or it consumes a notch (Fig. 3.10).

In a certain state (i, k) the RN gains a notch when the energy consumed is not sufficient

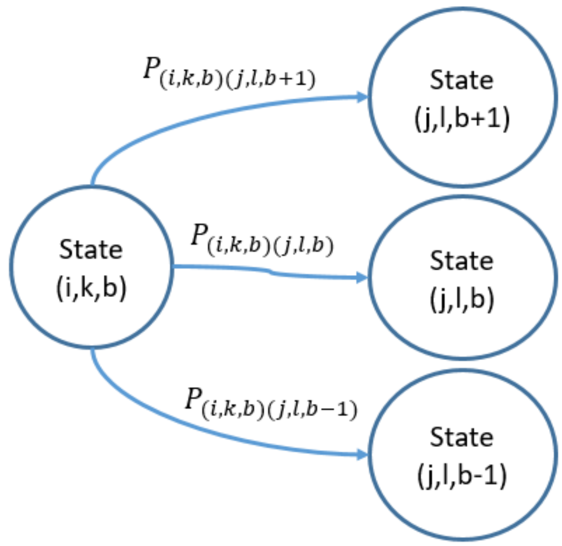


Figure 3.10: The three possible transitions

to decrease the level and, at the same time, we witness the arrival of a energy package (3.6).

$$Pr^{+}_{E,tx}(i,k) = Pr_{EP} \cdot (1 - Pr_{E,tx}(i,k)) \tag{3.6}$$

The second transition can occur in two ways: either the energy has arrived and, at the same time, a notch has been consumed, or an energy package has not arrived but the energy consumed has not been sufficient to lower the number of notches (3.7).

$$Pr^{=}_{E,tx}(i,k) = Pr_{EP} \cdot Pr_{E,tx}(i,k) + (1 - Pr_{EP}) \cdot (1 - Pr_{E,tx}(i,k)) \tag{3.7}$$

Finally, the probability that the RN loses a notch is equal to the probability that the energy consumed is sufficient to decrease the level times the probability that no energy package reaches the RN in that cycle (3.8).

$$Pr^{-}_{E,tx}(i,k) = (1 - Pr_{EP}) \cdot Pr_{E,tx}(i,k) \tag{3.8}$$

We might fall in two particular cases, namely when $b = 0$ or $i < bm$, that is the RN has no energy or no sufficient packets to transmit, or when $b = B$, that is the RN battery is totally full.

In the former case, we suppose that the RN is not capable to send anything, thus it just waits as long as some energy or more packets arrive. Consequently, the formulas (3.6) and (3.7) are simpler and become, respectively, Pr_{EP} and $(1 - Pr_{EP})$.

In the latter, we have the reversed problem, namely the RN cannot acquire energy any more but only consume it. Consequently, $Pr_{EP} = 0$ and the (3.7) and (3.8) change according to this simplification.

It is important to notice that, no matter in which case we are, by summing up the energy formulas relative to three possible transitions the result is always 1. This simple property leads to the fact that the row of the transition matrix, which I am going to build in the next section, will be equal to one so that the transition matrix is still stochastic.

3.5 Packets in the queue

Once the energy factor has been discussed, we should define how the number of packets in the queue of the RN evolves with time. We have first considered the fraction of cycles in which i packets arrive (A_i) by using a Poisson process with arrival rate λ (3.9). In addition to that, we have also defined the probability that i or more packets arrive in the same cycle $A_{\geq i}$. At the beginning of the new cycle, the number of packets j in the RN queue can vary from $i - \alpha$ and Q, with $\alpha = min(i, F)$.

$$
\begin{aligned}
&A_i = e^{-\lambda t}(\lambda T)^i / i! \\
&A_{\geq i} = 1 - \sum A_i \\
&\text{with } i - \alpha \leq j \leq Q
\end{aligned}
\tag{3.9}
$$

3.6 Activation or Inactivation of other nodes

About the dimension k, that is the number of active nodes that contend the channel with the RN, we are interested in two phenomena:

- the probability that n nodes, out of $K-k$ that are inactive, become active in the next cycle (Bk_n)
- the probability that s nodes, out of k that are active, remain active in the next cycle (S_s)

If we indicate with $\underline{Bk}$ the vector such that Bk_n occupies the n-th position and with $\underline{S}$ the one referring to S_s, then each element of convolution vector $\underline{Pl}$ in (3.10) indicates the combinations of the previous probabilities such that their joint values produces l active nodes at the end of the cycle.

$$
\begin{aligned}
&\underline{Pl}(l) = \underline{S} * \underline{Bk} = [Pl_0....Pl_l....Pl_K] \\
&\text{with} \\
&\underline{Bk}(k) = [Bk_0...Bk_n...Bk_{K-k}] \\
&\underline{S}(k) = [S_0...S_s...S_k] \\
&\text{Ex: if } k=3, K-k=6 \text{ then } Pl_4 = B_1S_3 + B_2S_2 + B_3S_1 + B_4S_0
\end{aligned}
\tag{3.10}
$$

All the next formulas depend on the stationary distribution $\pi_{i,k,b}$ whose ikb-th element tells us the fraction of cycles the RN spends in the state (i,k,b). Initially, we have given random values to this vector but the algorithm is iterated with a fixed point method as long as the error between the current $\pi_{i,k,b}$ and the old $\pi_{i,k,b}$ is smaller than a threshold, for example 10^{-10}.

3.6.1 Bk_n

Bk_n strictly depends on how we define $P_{act/inac}$, that is the probability of the RN turning to active conditioned on the fact that in the previous cycle it was inactive. Thus, we first define G_{inact} by summing the fraction of cycles in which a node either does not have enough packet or its battery is empty (3.11).

$$G_{inact}(k) = \sum_{i=0}^{bm-1}\sum_{b=1}^{B}\pi_{i,k,b} + \sum_{i=0}^{Q}\pi_{i,k,0} \tag{3.11}$$

Then, we must sum up:

- the fraction of cycles in which the node receive more than bm packets while it already has enough energy;
- the fraction of cycles in which it receives both energy and packets;
- the fraction of cycles in which it receives only some energy because it already has enough packets in his buffer;

$P_{act/inact}$ has been obtained by dividing the sum of the previous elements by G_{inact} (3.12).

$$\begin{aligned} P_{act/inact}(k) =& \frac{1}{G_{inact}} \cdot \sum_{i=0}^{bm-1} \sum_{b=1}^{B} \pi_{i,k,b} \cdot A_{\geq bm-i} \\ &+ \frac{1}{G_{inact}} \cdot \sum_{i=0}^{bm-1} \pi_{i,k,0} \cdot Pr_{EP} \cdot A_{\geq bm-i} \\ &+ \frac{1}{G_{inact}} \cdot \sum_{i=bm}^{Q} \pi_{i,k,0} \cdot Pr_{EP} \\ P_{inac/inac}(k) =& 1 - P_{act/inact}(k) \end{aligned} \tag{3.12}$$

Finally, Bk_n can be obtained by considering all the n combination out of $K - k$ (3.13).

$$Bk_n(k) = \binom{K-k}{n} P^{n}_{act/inac} P^{(K-k)-n}_{inac/inac} \tag{3.13}$$

3.6.2 S_s

In order to define the probability that s nodes, out of k that are active, remain active in the next cycle, we first describe the probabilities that a single node turns inactive according to its channel outcomes:

- $P_{inact/succ}(k)$: probability to turn inactive conditioned on the fact that the node successfully transmits;
- $P_{inact/coll}(k)$ probability to turn inactive conditioned on the fact that the node is involved in a collision;

- $P_{inact/ovh/tx}(k)$ probability to turn inactive conditioned on the fact of being overhearing a successful transmission;
- $P_{inact/ovh/col}(k)$ probability to turn inactive conditioned on the fact of being overhearing a collision;

In order to determine $P_{inact/succ}$, we must first consider the fraction of cycles G_{succ} in which a successful transmission takes place (3.14). Then, we take into account for $b > 2$ the fraction of cycles in which:

- the successful transmission leads to an empty buffer and less then bm packets arrive;
- the successful transmission brings to have $1 \leq i < bm$ packets in the queue and less then $bm - 1 - i$ packets arrive;

whereas for $b = 1$:

- the successful transmission produces a consumption of one notch of energy ($Pr^-_{E,tx}(i,k)$) while b was equal to 1;
- the successful transmission empties the buffer, does not consume energy ($1-Pr^-_{E,tx}(i,k)$), but less then bm packets arrive;
- the successful transmission does not empty the buffer or consume energy, but less then $bm - 1 - i$ packets arrive;

If we sum up the previous probabilities and divide everything by G_{succ}, we finally obtain $P_{inact/succ}$ (3.15) or (3.16).

$$G_{succ}(k) = \sum_{i=bm}^{Q} \sum_{b=1}^{B} \pi_{i,k,b} \cdot P_{s,k} \tag{3.14}$$

$$\begin{aligned}P_{inact/succ}(k) =& \frac{1}{G_{succ}} \cdot \sum_{i=bm}^{F} \sum_{b=2}^{B} \pi_{i,k,b} \cdot P_{s,k} \cdot \sum_{i=0}^{bm-1} A_i \\ &+ \frac{1}{G_{succ}} \cdot \sum_{i=F+1}^{\beta} \sum_{b=2}^{B} (\pi_{i,k,b} \cdot P_{s,k} \cdot \sum_{j=0}^{(F+bm-1)-i} A_j) \\ &+ \frac{1}{G_{succ}} \cdot \sum_{i=bm}^{Q} \pi_{i,k,1} \cdot P_{s,k} \cdot Pr^{-}_{E,tx}(i,k) \\ &+ \frac{1}{G_{succ}} \cdot \sum_{i=bm}^{F} \pi_{i,k,1} \cdot P_{s,k} \cdot (1 - Pr^{-}_{E,tx}(i,k)) \cdot \sum_{i=0}^{bm-1} A_i \\ &+ \frac{1}{G_{succ}} \cdot \sum_{i=F+1}^{\beta} (\pi_{i,k,1} \cdot P_{s,k} \cdot (1 - Pr^{-}_{E,tx}(i,k)) \cdot \sum_{j=0}^{(F+bm-1)-i} A_j) \end{aligned} \tag{3.15}$$

with

$\beta = min(Q, F + bm - 1)$ and $bm \leq F$

On the other hand, if $bm > F$:

$$\begin{aligned}P_{inact/succ}(k) =& \frac{1}{G_{succ}} \cdot \sum_{i=bm}^{\beta} \sum_{b=2}^{B} (\pi_{i,k,b} \cdot P_{s,k} \cdot \sum_{j=0}^{(F+bm-1)-i} A_j) \\ &+ \frac{1}{G_{succ}} \cdot \sum_{i=bm}^{Q} \pi_{i,k,1} \cdot P_{s,k} \cdot Pr^{-}_{E,tx}(i,k) \\ &+ \frac{1}{G_{succ}} \cdot \sum_{i=bm}^{\beta} (\pi_{i,k,1} \cdot P_{s,k} \cdot (1 - Pr^{-}_{E,tx}(i,k)) \cdot \sum_{j=0}^{(F+bm-1)-i} A_j) \end{aligned} \tag{3.16}$$

with

$\beta = min(Q, F + bm - 1)$ and $bm > F$

$P_{inact/coll}$, $P_{inact/ovh/tx}$ and $P_{inact/ovh/col}$ cannot consider the events that lead to inactivity due to packets transmission so that their formula are, respectively, (3.17), (3.18) and (3.19).

$$
\begin{aligned}
&P_{inact/coll}(k) = \frac{1}{G_{coll}} \cdot \sum_{i=bm}^{Q} \pi_{i,k,1} \cdot P_{f,k} \cdot Pr^{-}_{E,coll}(k) \\
&\text{with} \\
&G_{coll}(k) = \sum_{i=bm}^{Q} \sum_{b=1}^{B} \pi_{i,k,b} \cdot P_{f,k} \\
&k \geq 1
\end{aligned}
\tag{3.17}
$$

$$
\begin{aligned}
&P_{inact/ovh/tx}(k) = \frac{1}{G_{ovh/tx}} \cdot \sum_{i=bm}^{Q} \pi_{i,k,1} \cdot kP_{s,k} \cdot Pr^{-}_{E,ovh,tx}(k) \\
&\text{with} \\
&G_{ovh/tx}(k) = \sum_{i=bm}^{Q} \sum_{b=1}^{B} \pi_{i,k,b} \cdot kP_{s,k} \\
&k \geq 1
\end{aligned}
\tag{3.18}
$$

$$
\begin{aligned}
&P_{inact/ovh/col}(k) = \frac{1}{G_{ovh/col}} \cdot \sum_{i=bm}^{Q} \pi_{i,k,1} \cdot \hat{T}_k \cdot Pr^{-}_{E,ovh,coll}(k) \\
&\text{with} \\
&G_{ovh/col}(k) = \sum_{i=bm}^{Q} \sum_{b=1}^{B} \pi_{i,k,b} \cdot \hat{T}_k \\
&k \geq 2
\end{aligned}
\tag{3.19}
$$

By taking the conjugate of each of the previous formulas, we define the probabilities that the RN remains active according to the different channel outcomes (3.20). Moreover, the probability $P_{s,k}$, $P_{f,k}$, $kP_{s,k}$ and $\hat{T}_k$ can go out from the summary and be simplified with the same values of the denominator.

$$\begin{aligned} &P_{act/succ}(k) = 1 - P_{inact/succ} \\ &P_{act/coll}(k) = 1 - P_{inact/coll} \\ &P_{act/ovh/tx}(k) = 1 - P_{inact/ovh/tx} \\ &P_{act/ovh/col}(k) = 1 - P_{inact/ovh/col} \end{aligned} \tag{3.20}$$

Once we get the (3.20), we can continue with the calculation of S_s that changes according to the channel outcome of the RN.

RN successfully transmits $P_{s,k}$

If the RN successfully transmits, then the other k nodes are overhearing the transmission. It means that s out of k nodes could remain active due to $P_{act/ovh/tx}$, while $k - s$ turn to active thanks to the (3.18). Therefore:

$$S_{s,succ}(k) = \binom{k}{s} P^{s}_{act/ovh/tx} \cdot P^{k-s}_{inact/ovh/tx} \tag{3.21}$$

RN overhears a successful transmission $kP_{s,k}$

If the RN overhears a transmission, then one active node is successfully transmitting while the others $k - 1$ are overhearing the transmission. Thus, we are interested to the probability that the transmitting node remains active ($a = 1$), it turns to inactive, c out of $k - 1$ overhearing nodes remain active and $k - 1 - c$ turn to inactive (3.22).

$$\begin{aligned} &P(a, 1-a, c, k-1-c) = \\ &\binom{k-1}{c} \cdot P^{a}_{act/succ} \cdot P^{1-a}_{inac/succ} \cdot P^{c}_{act/ovhtx} \cdot P^{k-1-c}_{inac/ovh/tx} \end{aligned} \tag{3.22}$$

Finally, the probability $S_{s,ovhtx}$ that s nodes remain active is in (3.23).

$$S_{s,ovh/tx}(k) = \sum_{a=\alpha}^{\beta} P(a, 1-a, s-a, k-1-s+a)$$
$$\text{with}$$
$$\alpha = max(0, s-(k-1));\ \beta = min(1, s) \tag{3.23}$$
$$\sum_{s=0}^{k} S_{s,ovh/tx}(k,s) = 1$$

If the RN is not active and a successful transmission among other nodes happens ($kP_{s,k-1}$), then the probability $S_{s,ANtx} = S_{s,ovh/tx}$ (3.24) thanks to the fact that we can simplify the $kP_{s,k-1}$ to the denominator and numerator.

$$S_{s,ANtx}(k) = \sum_{a=\alpha}^{\beta} P(a, 1-a, s-a, k-1-s+a) \tag{3.24}$$

RN collides $P_{f,k}$

The RN can collide with $n = 1, 2$ or k active nodes and, consequently, $k-1$, $k-2$...or 0 active nodes are overhearing the channel. Thus, we are interested to the probability that a colliding remain actives, $n-a$ colliding turn to inactive, c out of $k-n$ overhearing nodes remain active and $k-n-c$ overhearing turn to inactive (3.25).

$$P(a, n-a, c, k-n-c) = \binom{n}{a}\binom{k-n}{c} \cdot P^{a}_{act/coll} \cdot P^{n-a}_{inac/coll} \cdot P^{c}_{act/ovh/coll} \cdot P^{k-n-c}_{inac/ovh/coll} \tag{3.25}$$

The probability that the RN collides with n out of k active nodes is given by:

$$P_{f,k,n} = \binom{k}{n} \sum_{w=1}^{W} \frac{1}{W} \cdot \left(\frac{1}{W}\right)^{n} \cdot \left(\frac{W-w}{W}\right)^{k-n}$$
$$\text{such that } \sum_{n=1}^{k} P_{f,k,n} = P_{f,k} \tag{3.26}$$

Finally, the probability $S_{s,coll}$ that s nodes remain active is the sum between all the possible combination of n and a (3.27).

$$\begin{aligned} & S_{s,coll}(k) = \sum_{n=1}^{k}\sum_{a=\alpha}^{\beta} P(a, n-a, s-a, k-n-s+a) \cdot \frac{P_{f,k,n}}{P_{f,k}} \\ & \text{with} \\ & \alpha = max(0, s-(k-n));\ \beta = min(n,s) \\ & \sum_{s=0}^{k} S_{s,coll}(k) = 1 \end{aligned} \tag{3.27}$$

RN overhears a collision $\hat{T}_k$

If the RN overhears a collision between $n = 2, 3$ or k nodes, then $k-2$, $k-3$ or 0 nodes are overhearing that collision. We can define the probability of overhearing a collision between n nodes as we did in (3.26) but without considering the influence of the RN.

$$\begin{aligned} & \hat{T}_{k,n} = \binom{k}{n}\sum_{w=1}^{W}\left(\frac{1}{W}\right)^{n} \cdot \left(\frac{W-w}{W}\right)^{k-n+1} \\ & \text{such that } \sum_{n=2}^{k}\hat{T}_{k,n} = \hat{T}_k \end{aligned} \tag{3.28}$$

The probability $S_{s,ovhcoll}$ (3.29) that s nodes remain active is the sum between all the possible combination between (3.25) and (3.28).

$$\begin{aligned} & S_{s,ovh/coll}(s,k) = \sum_{n=2}^{k}\sum_{a=\alpha}^{\beta} P(a, n-a, s-a, k-n-s+a) \cdot \frac{\hat{T}_{k,n}}{\hat{T}_k} \\ & \text{with} \\ & \alpha = max(0, s-(k-n));\ \beta = min(n,s) \\ & \sum_{s=0}^{k} S_{s,ovh/coll}(k,s) = 1 \end{aligned} \tag{3.29}$$

If the RN is not active and the other nodes provoke a collision ($\hat{S}_k = 1 - kP_{s,k-1}$), then (3.28) becomes the (3.30) and (3.29) becomes the (3.31).

$$\hat{S}_{k,n} = \binom{k}{n} \sum_{w=1}^{W} (\frac{1}{W})^n \cdot (\frac{W-w}{W})^{k-n}$$
$$\text{such that } \sum_{n=2}^{k} \hat{S}_{k,n} = \hat{S}_k \tag{3.30}$$

$$S_{s,ANcoll}(s,k) = \sum_{n=2}^{k} \sum_{a=\alpha}^{\beta} P(a, n-a, s-a, k-n-s+a) \cdot \frac{\hat{S}_{k,n}}{\hat{S}_k}$$
$$\text{with}$$
$$\alpha = max(0, s-(k-n));\ \beta = min(n,s) \tag{3.31}$$
$$\sum_{s=0}^{k} S_{s,ANcoll}(k) = 1$$

3.6.3 Pl_l

As I previously said, we are interested in the vector that indicates the combinations of the probabilities S_s and Bk_n such that their joint value produces l active nodes at the end of the cycle. Because of the difference between the channel outcomes, also (3.10) has been broken down in different probabilities, just like it is indicated in the following summary table 3.3.

Table 3.3: Probabilistic parameters that define the activity of a node

Symbol	Formula	Description
$P_{act/inac}$	(3.12)	Probability to make a node active conditioned on the fact that in the previous cycle was inactive
Bk_n	(3.13)	Probability that n nodes, out of $K-k$ that have their queues or battery empty or both, become active in the cycle
$S_{s,event}$	(3.21) (3.23) (3.24) (3.27) (3.29) (3.31)	Probability that s out of k active nodes remain active in the cycle when the RN presents a predetermined channel outcome
$\underline{Pl}_{succ}$	$\underline{S}_{succ} * \underline{Bk}$	Probability that l out of K nodes are active in the next cycle when the RN successfully transmits
$\underline{Pl}_{ovhtx}$	$\underline{S}_{ovhtx} * \underline{Bk}$	Probability that l out of K nodes are active in the next cycle when the RN overhears a successful transmission
$\underline{Pl}_{coll}$	$\underline{S}_{coll} * \underline{Bk}$	Probability that l out of K nodes are active in the next cycle when the RN is involved in a collision
$\underline{Pl}_{ovhcoll}$	$\underline{S}_{ovhcoll} * \underline{Bk}$	Probability that l out of K nodes are active in the next cycle when the RN overhears a collision
$\underline{Pl}_{ANsucc}$	$\underline{S}_{ANsucc} * \underline{Bk}$	Probability that l out of K nodes are active in the next cycle when the RN is not active and a node successfully transmits
$\underline{Pl}_{ANcoll}$	$\underline{S}_{ANcoll} * \underline{Bk}$	Probability that l out of K nodes are active in the next cycle when the RN is not active and a collision happens

Chapter 4

3-D Transition matrix and Results

In this chapter, I will first discuss how to define the state transition probabilities by combining the probabilistic parameter described previously; then, I will show how the performance can be calculated and how they behave by varying the scenario.

4.1 Creation of the 3-D transition matrix

The RN can behave in five ways depending on whether or not it has energy and packets to send or whether it is the only active node or not. According to these behaviours, in the following sections it is described how the elements of the transition matrix can be defined by combining all the previous concepts.

4.1.1 No node is active

In order to be considered *no active*, the RN must have less than *bm* packets or no energy at all or both. The energy consumed is close to 0 mJ because the RN suddenly goes to the sleeping mode at the beginning of the cycle. Due to the absence of consumption, the modified transition probabilities, which are in (4.1), may only refer, in addition to the probabilities of packet arrivals and activation of other nodes, either to the gain of a notch or to keep the same energy level.

$$
\begin{aligned}
&P_{(i,0,b)(j,l,b+1)} = Bk_l \cdot A_{j-i} \cdot Pr_{EP} \\
&P_{(i,0,b)(j,l,b)} = Bk_l \cdot A_{j-i} \cdot (1 - Pr_{EP}) \\
&\qquad \text{with} \\
&0 \le i < bm \quad \text{or} \quad bm \le i \le Q \\
&k = 0 \\
&b = 0 \quad \text{or} \quad 0 \le b < B \text{ or} \quad b = B \\
&i \le j < Q \; or \; j = Q \\
&0 \le l \le K
\end{aligned}
\tag{4.1}
$$

The (4.1) cannot be used in the case in which the battery is totally full, that is $b = B$, because the energy level clearly cannot further grows. In this state, the RN is programmed to maintain the same level of energy.
Moreover, if $j = Q$, it means that $Q - i$ packets or more have arrived, thus we cannot use A_{j-i} any more but $A_{\ge Q-i}$.

4.1.2 The RN is not active

This section refers to the case when $b = 0$ or $i < bm$ or both, i.e. the RN is not active, while there is at least another node ready to transmit. In the most general case, the formula that must be used are in (4.2) and its meaning is described as follows. The first line of the sum represents those cycles in which one of the other active node manages to win the contention for the channel, while the second describe the event in which the active node does not manage to transmit because it collides with other active nodes.

$$
\begin{aligned}
P_{(i,k,b)(j,l,b+1)} =& S_k \cdot Pl_{l,ANtx} \cdot A_{j-i} \cdot Pr_{EP} \\
& + \hat{S}_k \cdot Pl_{l,ANcoll} \cdot A_{j-i} \cdot Pr_{EP} \\
P_{(i,k,b)(j,l,b)} =& S_k \cdot Pl_{l,ANtx} \cdot A_{j-i} \cdot (1 - Pr_{EP}) \\
& + \hat{S}_k \cdot Pl_{l,ANcoll} \cdot A_{j-i} \cdot (1 - Pr_{EP}) \\
& \text{with} \\
& 0 \le i < bm \quad \text{or} \quad bm \le i \le Q \\
& 1 \le k \le K \\
& b = 0 \quad \text{or} \quad 0 < b < B \text{ or } \quad b = B \\
& i \le j < Q \; or \; j = Q \\
& 0 \le l \le K
\end{aligned}
\tag{4.2}
$$

Just like I said in the previous section, the element A_{j-i} changes to $A_{\ge Q-i}$ when $j = Q$ while the RN is programmed to maintain the same level when $b = B$.

4.1.3 Only the RN is active

If RN is the only active node, then the RN surely transmits $\alpha = min(i, F)$ packets and the energy that is consumed in this cycle is $Pr_{E,data,tx}(i, k = 0)$ (4.3).

$$
\begin{aligned}
P_{(i,0,b)(j,l,b+1)} &= P_{s,0} \cdot Bk_l \cdot A_{j-i+\alpha} \cdot Pr^{+}_{E,tx}(i,0) \\
P_{(i,0,b)(j,l,b)} &= P_{s,0} \cdot Bk_l \cdot A_{j-i+\alpha} \cdot Pr^{=}_{E,tx}(i,0) \\
P_{(i,0,b)(j,l,b-1)} &= P_{s,0} \cdot Bk_l \cdot A_{j-i+\alpha} \cdot Pr^{-}_{E,tx}(i,0)
\end{aligned}
\quad \text{with} \quad
\begin{aligned}
& bm \le i \le Q \\
& k = 0 \\
& 0 < b < B \; or \; b = B \\
& i - \alpha \le j < Q \; or \; j = Q \\
& 0 \le l \le K
\end{aligned}
\tag{4.3}
$$

Also for this section we might fall into the case in which $j = Q$ or $b = B$, wherein the RN is programmed to maintain the same level of energy as long as it does not consume something.

4.1.4 Contention: RN and other nodes are active

The fourth section is the most interesting and general one because it describes the contention of the channel between $k+1$ active nodes, including the RN. The next formulas follows the following pattern:

$$\begin{aligned} P_{(i,k,b)(j,l,d)} =& Pr_1\{ \text{ RN manages to transmit}\} \\ &+ Pr_2\{ \text{ another node wins the access } \} \\ &+ Pr_3\{ \text{ RN collides } \} \\ &+ Pr_4\{ \text{ two or more other nodes collide } \} \end{aligned} \tag{4.4}$$

If we break down each line of (4.4), it becomes the (4.5) that is an example of a transition to an higher level of energy.

$$\begin{aligned} &P_{(i,k,b)(j,l,b+1)} = \\ &P_{s,k} \cdot Pl_{l,succ} \cdot A_{j-i+\alpha} \cdot Pr^{+}_{E,tx}(i,k) \\ &+kP_{s,k} \cdot Pl_{l,ovh/tx} \cdot A_{j-i} \cdot Pr^{+}_{E,ovh,tx}(k) \\ &+P_{f,k} \cdot Pl_{l,coll} \cdot A_{j-i} \cdot Pr^{+}_{E,coll}(k) \\ &+\hat{T}_k \cdot Pl_{l,ovh/coll} \cdot A_{j-i} \cdot Pr^{+}_{E,ovh,coll}(k) \end{aligned} \quad \text{with} \quad \begin{aligned} &bm \le i < Q \ or \\ &bm \le i \le Q \Rightarrow A_{\ge Q-i} \\ &1 \le k \le K \\ &0 < b < B \ or \\ &b = B \Rightarrow Pr_{EP} = 0 \\ &i \le j < Q \ or \\ &j = Q \Rightarrow A_{\ge Q-i} \\ &0 \le l \le K \end{aligned} \tag{4.5}$$

The first line of (4.5) says that the RN wins the access to the channel ($P_{s,k}$) and it receives $j-i+\alpha$ packets in the same cycle ($A_{j-i+\alpha}$). In addition to that, the RN gains one energy notch ($Pr^{+}_{E,tx}(i,k)$) and l other nodes will be active once the cycle ends ($Pl_{l,succ}$).
The second line describes the probability that another node of the network wins the contention ($kP_{s,k}$) and that the RN receives $j-i$ packets. Moreover, everything is multiplied by the probability of winning an energy notch thanks ($Pr^{+}_{E,ovh,tx}(k)$).
The third takes into account the probabilities relative to a collision that includes the RN and

hence the probability of gaining a notch is $Pr^{+}_{E,coll}(k)$.

Finally, the last line provides for the case when a collision between other nodes but the RN occurs ($\hat{T}_k$). All the lines are multiplied for their respective probability to find l active nodes at the beginning of the next cycle.

The last special case that must be included is when $i - \alpha \leq j \leq i - 1$. We can obtain these values of j only if the RN has won the channel and thus the only line which remains is the first while the others will be set to 0.

4.1.5 Impossible transitions

All the other cases not included by the previous lists of indices must be considered impossible transitions and their values are equal to 0.

4.1.6 Resuming table and visual interpretation

A summary of all the different cases is illustrated in table 4.1. Moreover, a visual interpretation of the transitions that might occur is given in Fig. 4.1. It has been obtained by using:

- $Q = 3$: maximum number of packets in the queue
- $K = 2$: number of nodes in the system, except the RN
- $F = 2$: maximum frame size
- $B = 3$: maximum state of the battery
- $C = 10$: fraction of notch consumed due to a fully transmission
- $\lambda = 2$: arrival rate of the packets [units of arrival every second]
- $bm = 2$: threshold of packets in the queue to make the node active or not
- $Prep = 0.1$: fraction of cycles in which one energy notch arrives

Fig. 4.1 must be read in the following way. The y-axis shows the possible states at the cycle t_0, while the x-axis at t_1 after a transition. The probability of such transition is denoted by the colour, i.e. the darker the grey is, the more likely the transition can occur. I have divided the matrix into square cells, each of dimension $(Q + 1)(K + 1) \times (Q + 1)(K + 1)$, that indicate the battery transition. For instance, the cell at the second line and second column includes all the transitions of Q and K that provide for being in the same battery state $b = 1 \hookrightarrow b = 1$. The cell on its right side, instead, includes all the transitions of the other dimensions providing that an increase of the energy happens $b = 1 \hookrightarrow b = 2$. On the contrary, the cell on the left represents the transitions referring to consumption of energy $b = 1 \hookrightarrow b = 0$.

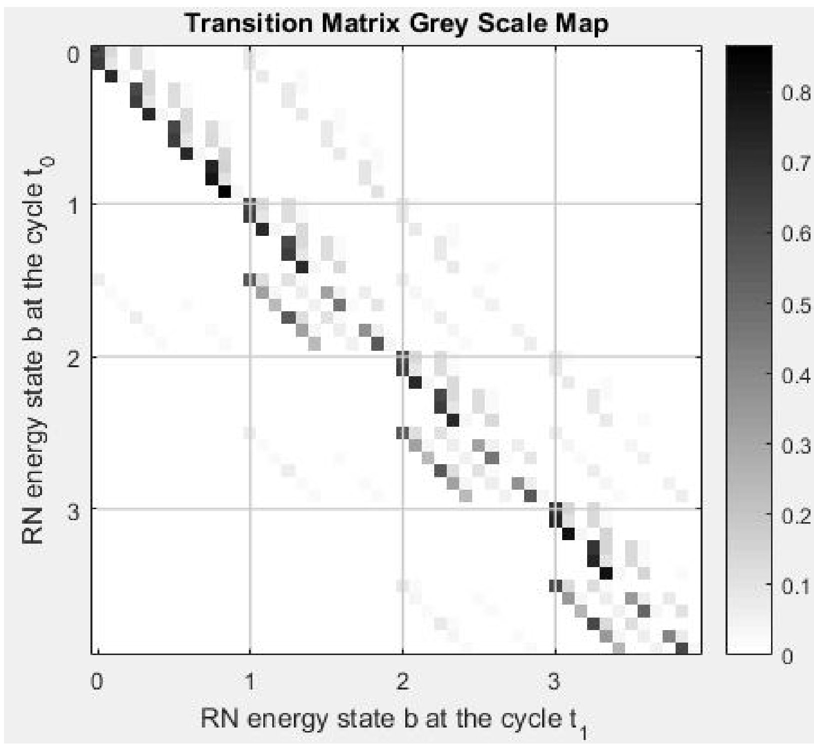


Figure 4.1: Visual representation of a transition matrix of the RN states into a small network with a high probability of energy arrival and high consumption every time a packet is sent

Table 4.1: Elements of the 3-D transition matrix

No node is active

$P_{(i,0,b)(j,l,b+1)} = Bk_l \cdot A_{j-i} \cdot Pr_{EP}$

$P_{(i,0,b)(j,l,b)} = Bk_l \cdot A_{j-i} \cdot (1 - Pr_{EP})$

$0 \leq i < bm;\ k = 0;\ 0 \leq b < B\ or\ b = B;\ i \leq j < Q\ or\ j = Q;\ 0 \leq l \leq K$

or

$bm \leq i \leq Q;\ b = 0$

The RN is not active

$$
\begin{aligned}
P_{(i,k,b)(j,l,b+1)} =& S_k \cdot Pl_{l,ANtx} \cdot A_{j-i} \cdot Pr_{EP} \\
&+ \hat{S}_k \cdot Pl_{l,ANcoll} \cdot A_{j-i} \cdot Pr_{EP} \\
P_{(i,k,b)(j,l,b)} =& S_k \cdot Pl_{l,ANtx} \cdot A_{j-i} \cdot (1 - Pr_{EP}) \\
&+ \hat{S}_k \cdot Pl_{l,ANcoll} \cdot A_{j-i} \cdot (1 - Pr_{EP})
\end{aligned}
$$

with

$0 \leq i < bm$ or $0 \leq i \leq Q$

$1 \leq k \leq K$

$b = 0$ or $0 \leq b < B$ or $b = B$

$i \leq j < Q\ or\ j = Q$

$0 \leq l \leq K$

Only the RN is active

$P_{(i,0,b)(j,l,b+1)} = P_{s,0} \cdot Bk_l \cdot A_{j-i+\alpha} \cdot Pr^{+}_{E,tx}(i,0)$

$P_{(i,0,b)(j,l,b)} = P_{s,0} \cdot Bk_l \cdot A_{j-i+\alpha} \cdot Pr^{=}_{E,tx}(i,0)$

$P_{(i,0,b)(j,l,b-1)} = P_{s,0} \cdot Bk_l \cdot A_{j-i+\alpha} \cdot Pr^{-}_{E,tx}(i,0)$

$bm \leq i \leq Q;\ k = 0;\ 0 < b < B\ or\ b = B;\ i - \alpha \leq j < Q\ or\ j = Q;\ 0 \leq l \leq K$

Contention: RN and other nodes are active

$$
\begin{aligned}
P_{(i,k,b)(j,l,b+1)} =& P_{s,k} \cdot Pl_{l,succ} \cdot A_{j-i+\alpha} \cdot Pr^{+}_{E,tx}(i,k) \\
&+ kP_{s,k} \cdot Pl_{l,ovhtx} \cdot A_{j-i} \cdot Pr^{+}_{E,ovh,tx}(k) \\
&+ P_{f,k} \cdot Pl_{l,coll} \cdot A_{j-i} \cdot Pr^{+}_{E,coll}(k) \\
&+ \hat{T}_k \cdot Pl_{l,ovhcoll} \cdot A_{j-i} \cdot Pr^{+}_{E,ovh,coll}(k) \\
P_{(i,k,b)(j,l,b)} =& P_{s,k} \cdot Pl_{l,succ} \cdot A_{j-i+\alpha} \cdot Pr^{=}_{E,tx}(i,k) \\
&+ kP_{s,k} \cdot Pl_{l,ovhtx} \cdot A_{j-i} \cdot Pr^{=}_{E,ovh,tx}(k) \\
&+ P_{f,k} \cdot Pl_{l,coll} \cdot A_{j-i} \cdot Pr^{=}_{E,coll}(k) \\
&+ \hat{T}_k \cdot Pl_{l,ovhcoll} \cdot A_{j-i} \cdot Pr^{=}_{E,ovh,coll}(k) \\
P_{(i,k,b)(j,l,b-1)} =& P_{s,k} \cdot Pl_{l,succ} \cdot A_{j-i+\alpha} \cdot Pr^{-}_{E,tx}(i,k) \\
&+ kP_{s,k} \cdot Pl_{l,ovhtx} \cdot A_{j-i} \cdot Pr^{-}_{E,ovh,tx}(k) \\
&+ P_{f,k} \cdot Pl_{l,coll} \cdot A_{j-i} \cdot Pr^{-}_{E,coll}(k) \\
&+ \hat{T}_k \cdot Pl_{l,ovhcoll} \cdot A_{j-i} \cdot Pr^{-}_{E,ovh,coll}(k)
\end{aligned}
$$

$bm \leq i \leq Q;\ 1 \leq k \leq K;$

$0 < b < B\ or\ b = B;$

$i \leq j < Q\ or\ j = Q;\ 0 \leq l \leq K$

$P_{(i,k,b)(j,l,b+1)} = P_{s,k} \cdot Pl_{l,succ} \cdot A_{j-i+\alpha} \cdot Pr^{+}_{E,tx}(i,k)$

$P_{(i,k,b)(j,l,b)} = P_{s,k} \cdot Pl_{l,succ} \cdot A_{j-i+\alpha} \cdot Pr^{=}_{E,tx}(i,k)$

$P_{(i,k,b)(j,l,b-1)} = P_{s,k} \cdot Pl_{l,succ} \cdot A_{j-i+\alpha} \cdot Pr^{-}_{E,tx}(i,k)$

$bm \leq i \leq Q\ \ 1 \leq k \leq K;$

$0 < b < B\ or\ b = B;$

$\mathbf{i} - \alpha \leq \mathbf{j} < \mathbf{i};\ 0 \leq l < K$

4.2 Results

All the next results have been calculated with the following parameters: $Q = 7$, $K = 9$, $B = 6$, $C = 10$.

4.2.1 Steady-state Probabilities

Once we know all the elements of the transition matrix **P**, we can proceed to the calculation of the steady-state probabilities and the performance parameters.

We might use many approaches to calculate $\pi_{i,k,b}$, but only two of them have been considered. The first (4.6) is the simplest and derives directly from the theory of Markov Chains, as explained in [2].

$$\begin{aligned} \underline{\pi} \cdot \mathbf{P} &= \underline{\pi} \\ \underline{\pi} \cdot \underline{1} &= 1 \end{aligned} \quad \Rightarrow \quad \underline{\pi} = \underline{1}^T \cdot (\mathbf{P} + \mathbf{E} - \mathbf{I})^{-1} \tag{4.6}$$

On the other hand, because the transition matrices are of dimension $(Q+1)(K+1)(B+1) \times (Q+1)(K+1)(B+1)$, the second approach consists in an iteration of inversions of smaller matrices. The method's name is *block state reduction* and it is described in [8] for Quasi-Birth-Death processes. These processes are characterized by the same diagonality of the transition matrix that we can observe in Fig. 4.1. The method starts with a division of the transition matrix $(Q+1) \times (K+1) \times (B+1)$ in smaller ones of dimension $(Q+1) \times (K+1)$ (4.7).

$$\mathbf{P}_{(B+1)\times(B+1)} \quad \Rightarrow \quad \begin{bmatrix} \mathbf{A_{00}} & \mathbf{U_{00}} & \mathbf{0} & \dots & \dots & \mathbf{0} \\ \mathbf{D_{21}} & \mathbf{A_{11}} & \mathbf{U_{23}} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{D_{32}} & \mathbf{A_{11}} & \mathbf{U_{23}} & \dots & \mathbf{0} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \mathbf{0} & \dots & \dots & \mathbf{D_{32}} & \mathbf{A_{11}} & \mathbf{U_{23}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{D_{B,B+1}} & \mathbf{E_B} \end{bmatrix} \tag{4.7}$$

Then, we can use the algorithms described below in order to solve the linear system in (4.8).

$$\begin{cases} \underline{\pi}_0 = \underline{\pi}_0 E_0 \\ \underline{\pi}_{B-b} = \underline{\pi}_{B-b-1} R_{B-b} \\ \sum_{b=0}^{B} \underline{\pi}_b \underline{1} = 1 \end{cases} \quad with \quad \begin{aligned} & R_{B-b} = U_{23}(I - E_{B-b})^{-1} \\ & R_0 = U_{00}(I - E_{B-b})^{-1} \\ & \quad and \\ & E_{B-1} = A_{11} + U_{23}(I - E_{B-b+1})^{-1} D_{B,B+1} \\ & E_{B-b} = A_{11} + U_{23}(I - E_{B-b+1})^{-1} D_{32} \\ & E_0 = A_{00} + U_{00}(I - E_1)^{-1} D_{21} \end{aligned} \tag{4.8}$$

Data: A_{00} U_{00} D_{21} A_{11} U_{23} D_{32} E_B $D_{B,B+1}$ B

Result: Steady-State probabilities $\underline{\pi}_{i,k,b}$

$R_B = U_{23}(I - E_B)$

for $1 \leq b \leq B - 1$ **do**

  if *b=B-1* **then**

    $E_{B-b} = A_{11} + U_{23}(I - E_{B-b+1})^{-1} D_{B,B+1}$

  end

  else

    $E_{B-b} = A_{11} + U_{23}(I - E_{B-b+1})^{-1} D_{32}$

  end

  $R_{B-b} = U_{23}(I - E_{B-b})$

end

$E_0 = A_{00} + D_{21}(I - E_1)U_{00}$

`/* Solution of the linear system */`

$$\underline{\pi}_{ik,b=0} = \begin{bmatrix} (I - E_0) \\ 1..1..1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$

for $1 \leq b \leq B$ **do**

  $\underline{\pi}_{ik,b} = \underline{\pi}_{ik,b-1} R_b$

end

`/* Normalization */`

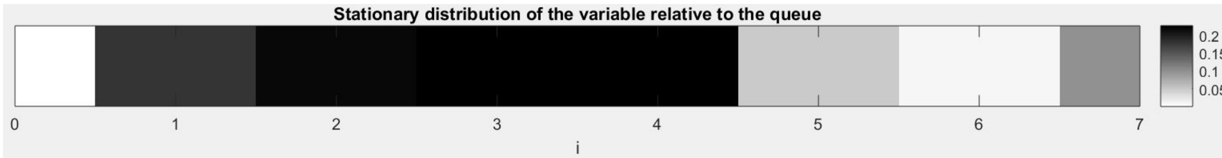
$$sum = \sum_{b=0}^{B} \sum_{ik=0}^{(Q+1)\times(K+1)} \underline{\pi}_{ik,b}$$

for $0 \leq b \leq B$ **do**

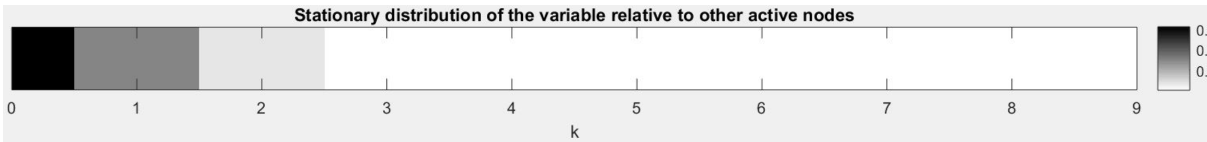
  $\underline{\pi}_{ik,b} = \underline{\pi}_{ik,b} \, / \, sum$

end

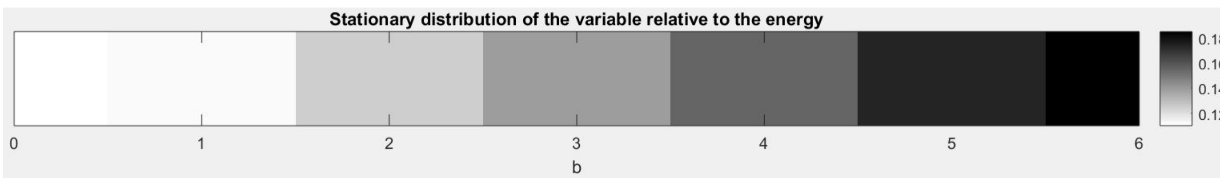
Algorithm 1: Block state reduction algorithm



(a) $\pi_i = \sum_{k=0}^{K} \sum_{b=0}^{B} \pi_{i,k,b}$



(b) $\pi_k = \sum_{i=0}^{Q} \sum_{b=0}^{B} \pi_{i,k,b}$



(c) $\pi_b = \sum_{i=0}^{Q} \sum_{k=0}^{K} \pi_{i,k,b}$

Figure 4.2: Single dimension stationary distributions obtained with the same inputs of Fig. 4.1

Once we have found out which are $\underline{\pi}_b$ and, consequently, $\underline{\pi}_{i,k,b}$, we can obtain the marginal probability $\underline{\pi}_i$ (Fig. 4.2 (a)) by simply summing over all the values for k and b and vice versa for $\underline{\pi}_k$ (Fig. 4.2 (b)) and $\underline{\pi}_b$ (Fig. 4.2 (c)). Because at the beginning of the process, that is before building the transition matrix, we needed some initial values of $\underline{\pi}_{i,k,b}$, all the previous instructions are repeated as long as the difference between the current $\underline{\pi}_{i,k,b}$ and the one calculated in the previous iterations is less than 10^{-10}.

4.2.2 Performance parameters

Thanks to the stationary distributions previously found, we can now calculate different performance parameters.

Throughput of a node

It is the average number of packets successfully delivered per cycle by a node. Indeed, we obtain it by summing the fraction of states in which the RN node is active multiplied by the number of packets that are sent and the probability that the transmission succeeds (4.9).

$$\eta = \sum_{i=bm}^{Q} \sum_{k=0}^{K} \sum_{b=1}^{B} \alpha \pi_{i,k,b} P_{s,k} \quad [\frac{\text{packets}}{\text{cycles}}] \tag{4.9}$$

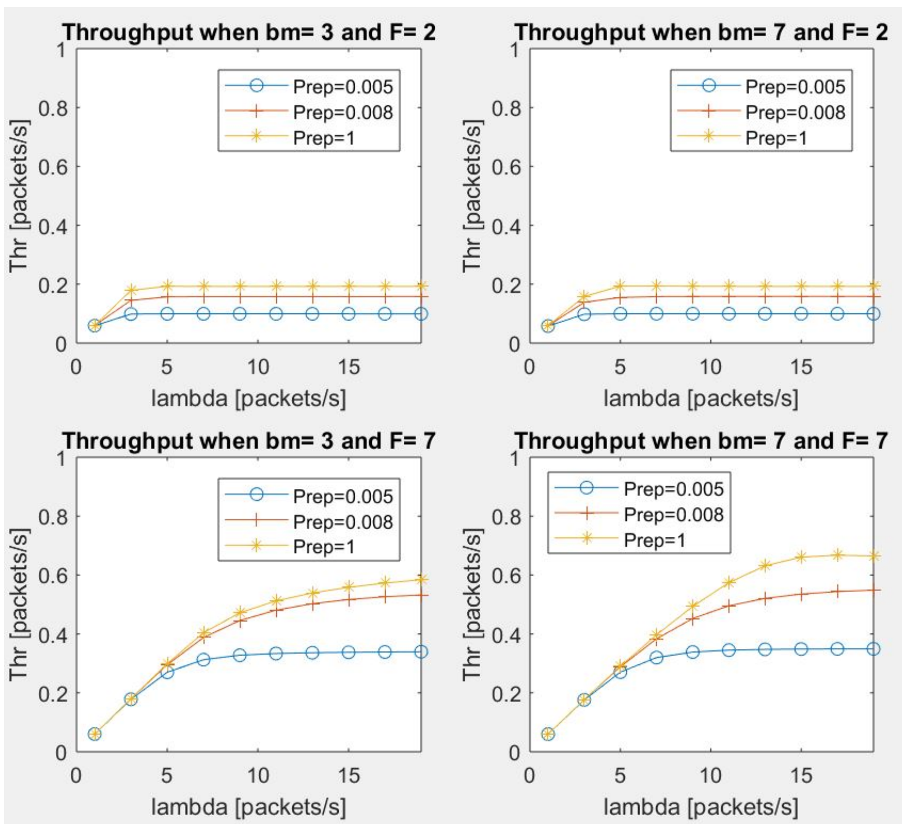


Figure 4.3: Throughput of a single node as a function of λ and with different F, bm and Pr_{EP} values

We suddenly notice from Fig. 4.3 that the throughput gets worse by reducing the probability of harvesting energy. This is due to the fact that the node is more likely to empty the battery and hence stop transmitting.

Moreover, the throughput grows proportionally with F and such behaviour states again the importance of the aggregated packet transmission to improve the performances of the system [3].

Finally, with low packets and energy arrival rates, we do not see changes between throughputs with different bm but, with high arrival rate and a constant energy supply, the higher is bm, the more likely transmission with many packets will be.

Successful Transmission in a Random Cycle

If we are just interested in the probability that an active node manages to transmit conditioned on the fact of being active, then the (4.9) becomes the (4.10).

$$P_s = \frac{1}{\sum_{i=bm}^{Q}\sum_{k=0}^{K}\sum_{b=1}^{B}\pi_{i,k,b}} \sum_{i=bm}^{Q}\sum_{k=0}^{K}\sum_{b=1}^{B}\pi_{i,k,b}P_{s,k} \tag{4.10}$$

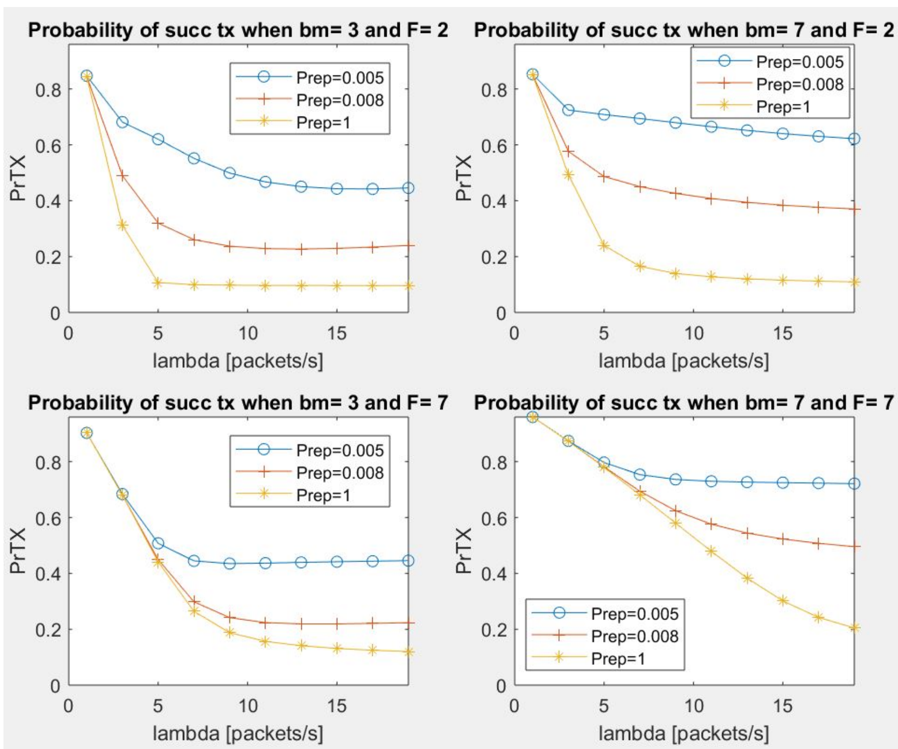


Figure 4.4: Probability of successful transmission as a function of λ and with different F, bm and Pr_{EP} values

The interesting result coming from Fig. 4.4 is that the probability of successful transmission is higher when the energy is harvested less frequently. One reason of such a behaviour could be that the other nodes are less likely to be active, then $P_{s,k}$ is higher. Moreover, everything is conditioned on the fact of being already active, thus the event of being with no energy is not taken into account for the RN.

Packets in the queue

The average number of packets in the queue is given by summing the amount of packets in the queue in a state times the probability of being in that state (4.11).

$$\overline{Q} = \sum_{i=1}^{Q} \pi_i \cdot i \text{ [packets] } with \ \pi_i \ = \sum_{k=0}^{K} \sum_{b=0}^{B} \pi_{i,k,b} \tag{4.11}$$

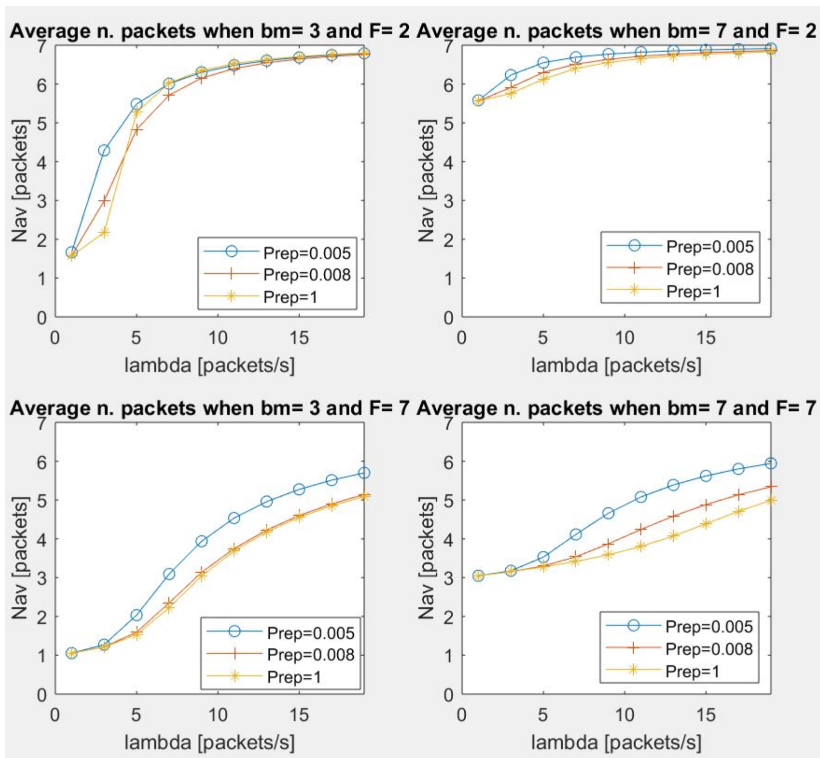


Figure 4.5: Average number of packets in the queue as a function of λ and with different F, bm and Pr_{EP} values

Fig. 4.5 shows that the average number of packets in the queue grows proportionally with bm and inversely proportional with Pr_{EP}. As a matter of fact, both the events produce a reduction of transmissions and hence a rise in the queue of the node.

Average Delay

Once we know η and $\overline{Q}$, thanks to the Little's Law that has been explained in the first chapter, we sort out the average delay in the queue for each packet (4.12).

$$\overline{W} = \frac{\overline{Q}}{\eta} \quad \text{[portion of cycles]} \tag{4.12}$$

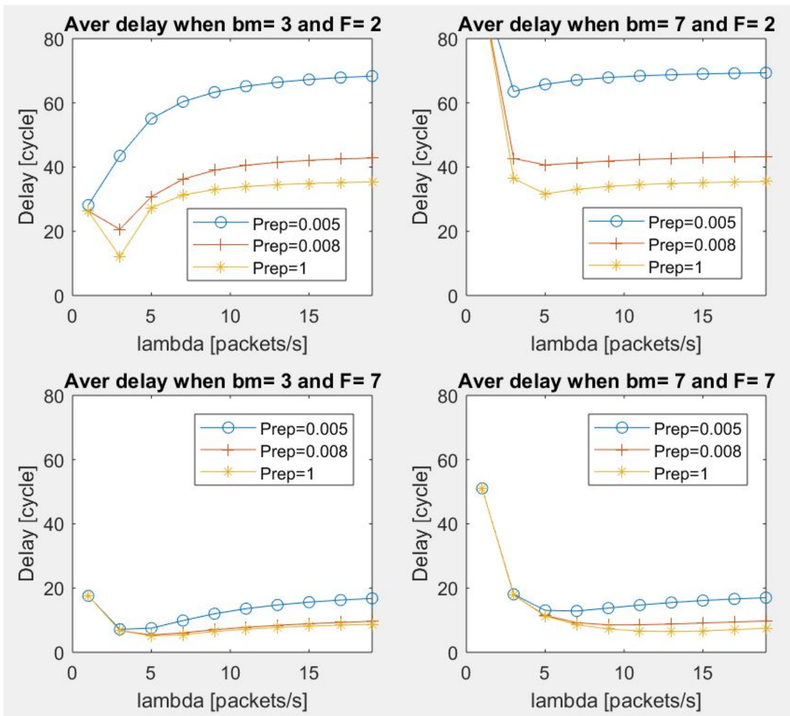


Figure 4.6: Average delay for a packet as a function of λ and with different F, bm and Pr_{EP} values

Fig. 4.6 illustrates that enlarging bm causes an abrupt increase of the average delay for low packets arrival rates. The curves first rapidly decrease until a minimum and then rises gently until becoming stable for high rates. From this and the previous figures it is clear that, the lower is Pr_{EP}, the worse will be the global performances.

Average energy consumed

The average energy consumed depends on the channel outcome we are referring to:

$$\overline{E}_{av,tx} = \sum_{i=bm}^{Q}\sum_{k=0}^{K}\sum_{b=1}^{B}\pi_{i,k,b}\cdot P_{s,k}\cdot E_{tx}(i,k) \qquad \overline{E}_{av,ovh,coll} = \sum_{i=bm}^{Q}\sum_{k=2}^{K}\sum_{b=1}^{B}\pi_{i,k,b}\cdot \hat{T}_k\cdot E_{ovh,coll}(k)$$
$$\overline{E}_{av,coll} = \sum_{i=bm}^{Q}\sum_{k=1}^{K}\sum_{b=1}^{B}\pi_{i,k,b}\cdot P_{f,k}\cdot E_{coll}(k) \qquad \overline{E}_{av,ovh,tx} = \sum_{i=bm}^{Q}\sum_{k=1}^{K}\sum_{b=1}^{B}\pi_{i,k,b}\cdot kP_{s,k}\cdot E_{ovh,tx}(k) \tag{4.13}$$

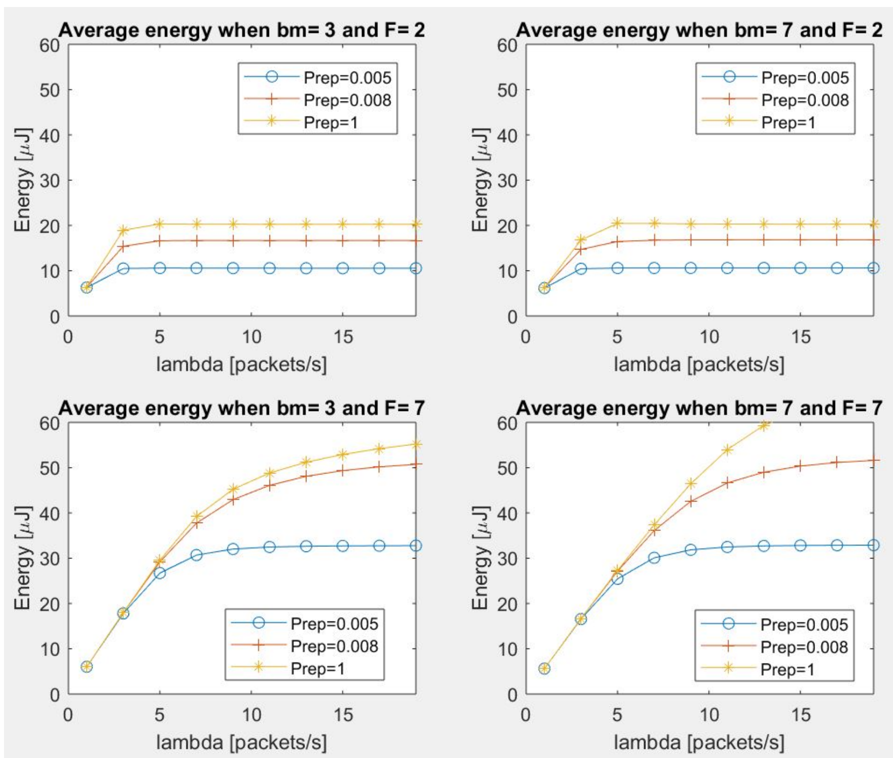


Figure 4.7: Average energy transmitted as a function of λ and with different F, bm and Pr_{EP} values

If the energy arrival is more unlikely, the average number of transmission doubtless falls off and hence the average energy consumption is lower.

Phase type distribution

When we give as input a probability of energy arrival equal to 0, the RN works thanks to its stored energy but, after a while, it will surely fall into the absorbing state of no energy at all, that is $b = 0$. In this case, basing on the 2-D process in [8], we have derived the phase type distribution for a 3-D DTMC, defined as the distribution of the time until absorption in an absorbing DTMC. To obtain it, the first step is to rearrange the P matrix as you see in Fig. 4.8 and decide in a vector $\underline{\alpha}$ which are the initial probabilities that the system starts from a transient state b. I have supposed that the RN starts to work with a full battery so that $\alpha_{00,B} = 1$ while the other elements of $\underline{\alpha}$ are 0.

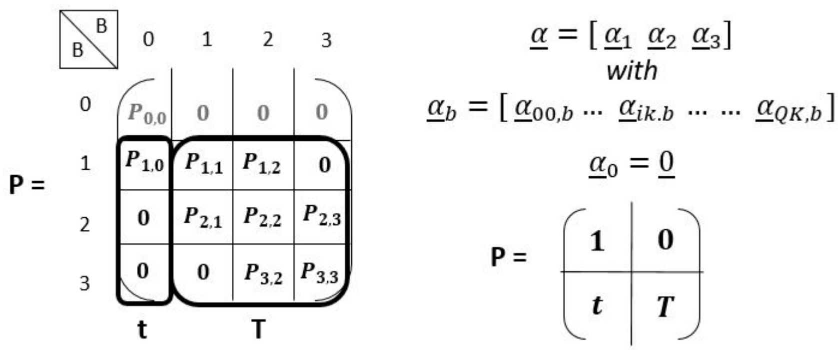
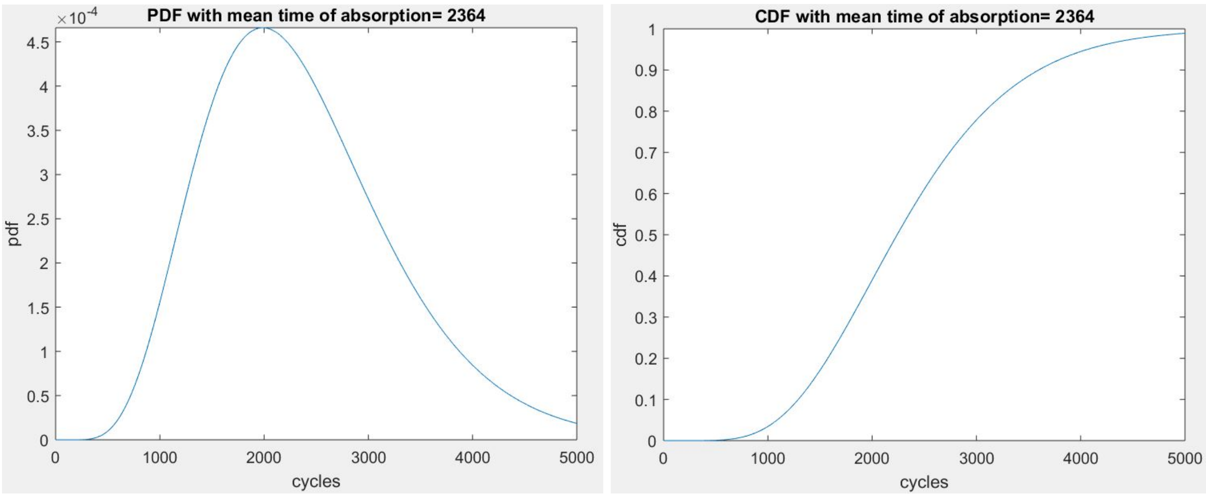


Figure 4.8: Rearranging of the transition matrix and declaration of the α vector

After deriving $\boldsymbol{t}$, we have calculated the probability p_c that the RN falls into the absorption state $b = 0$ at the cycle c (4.14). An example of such a probability density function and the associated cumulative distribution function is in Fig. 4.9.
Finally, we can express the mean value by using the (4.15).

$$
\begin{aligned}
p_0 &= \underline{\alpha}_0 \underline{1}' = 0 \\
p_c &= (\underline{\alpha} \mathbf{T}^{c-1} \boldsymbol{t}) \underline{1}'
\end{aligned}
\tag{4.14}
$$

$$
\mu = \underline{\alpha} (\mathbf{I} - \mathbf{T})^{-1} \underline{1}' \tag{4.15}
$$



(a) Probability density function (b) Cumulative distribution function

Figure 4.9: Example of pdf and cdf relative to a random variable related with the probability of falling in the state $b = 0$ during a specific cycle ($F = 7, \lambda = 3, bm = 7$)

Validation

In Fig. 4.10 you can see a comparison between the performance obtained by using the Matlab code of the model and the same parameters coming from a network simulator. The relative error $\frac{(mat-sim)}{sim}$ is represented in the last column. We notice that, above all for $\lambda = 3$, the relative error is very small and hence we could say my model come up with quite accurate results.

lambda=2	Matlab	Simulation	ERROR
D_c	28,7051217	2,53E+01	1,35E-01
Thr	0,09711503	1,00E-01	-3,12E-02
Edata	1,03E-05	1,05E-05	-1,63E-02
Nav	2,78769882	2,53E+00	1,00E-01

lambda=3	Matlab	Simulation	ERROR
D_c	54,4143128	5,44E+01	7,23E-04
Thr	0,09961967	1,00E-01	-7,77E-03
Edata	1,06E-05	1,05E-05	6,96E-03
Nav	5,42073605	5,46E+00	-7,03E-03

Figure 4.10: Comparison between the model and the simulation results with the following input parameters: $Q = 10, N = 10, B = 10, C = 10, F = 2, bm = 2$

Chapter 5

Conclusion

In the thesis, an analytical model based on a discrete-time Markov chain (DTMC) has been elaborated to evaluate the performances of a wireless network. The nodes communicate with a common sink node with the classic S-MAC protocol to which two important extensions have been added. The first refers to the number of packets that are jointly sent together in the same cycle, whereas the second is the number minimum of packets in the queue under which a node cannot be considered active.

During each cycle, every channel outcome causes its specific energy consumption and one energy notch is harvested with probability Pr_{EP}. We have first calculated each element of the transition matrix and realized that its form belongs to the family of Quasi Birth and Death processes. This allows to exploit specific algorithms to solve the stationary distribution that greatly reduce the computational complexity, particularly when the cardinality of the state space is large. From such a distribution, we have derived the performance parameters expressions, such as throughput, average number of packets in the queue of a node, average packet delay in a node buffer, i.e., since packet arrival until it is successfully transmitted, and energy consumption. Finally, we have analysed the behaviour of a single node when it starts with full battery but no energy can be harvested.

As previously said, the thesis describes a novel methodology to treat the energy as a dimension of a DTMC. The main limitation is due to the huge dimensionality of the transition matrix. Indeed, a high number of notches would better represent the behaviour of the energy and hence increase the model precision. However, we must mediate between such a number and the computational complexity. If more precision is required, we can use a block state reduction algorithm to solve the stationary distribution.

With some combination of input parameters, i.e. a very low Pr_{EP}, the main algorithm, that uses a fixed point iteration, does not converge properly. Thus, it would be useful to carry out a deeper study on such an issue in the future.

The energy harvested has been represented for simplicity with a Bernoulli process. Further analysis of the energy arrival distribution according to the different environments might give results useful for a real-life application.

Finally, because a channel error-free has been assumed, adapting the model to a channel error-prone would be interesting by introducing it as a further dimension.

Acknowledgements

This thesis is the final step of a long stair I have begun to climb more than five years ago. Throughout my academic career, I have crossed three countries and faced many obstacle that I have been able to get over also thanks to my friends, my girlfriend and my family.

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Finally, I want to thank my parents for the education that they have given to me. In particular my mum, who alone has always been able to direct me on the straight road that has led me here. I hope one day I will be able to be a strong father as much as she has been a strong mother for me.

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