Computing the minimum construction cost of a building's external wall taking into account its energy efficiency

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Abstract

The construction of a building's external wall is subject to many restrictions such as budget, workforce, availability of materials, thickness, maintenance cost, time limit and specially, energy efficiency legislation intended at mitigating the negative effects of the energy consumption and obtaining a more sustainable and healthier indoor environment. The choice of the appropriate material and thickness composing each layer of an external wall can significantly reduce the energy consumption of the building without adversely affecting the cost of the wall.

By using Integer Linear Programming (ILP), the aim of this paper is to obtain this best choice of materials and thicknesses to minimize the construction cost of an external wall while complying with the abovementioned restrictions. A case study is presented with more than 5.5 million combinations of different selected materials and their thicknesses for the different layers of the wall. The ILP problem has been solved for 165 scenarios that take into account different maximal allowed thermal transmittances and a range of the most usual thicknesses and material options of an external wall.

Keywords

Integer Linear Programming; external wall; building process; budget; thermal transmittance.

1. Introduction

Linear Programming (LP) [1-3] has proven its efficiency to mathematically model many real-world problems aiming at the maximization or minimization of a certain function (objective function) that is linearly dependent on a set of variables related to each other through a set of linear constraints. It is well-known that a LP problem has polynomial complexity when all variables are real and continuous. However, if all variables must be integer (ILP) or it is a mixed case (MILP) where there are both continuous and integer variables, the optimization problem has exponential complexity. In the last two cases, several iterative procedures have been developed to obtain the optimal solution, although, of course, they cannot guarantee that the optimal solution will be found in all the instances within a reasonable time. Sherali and Driscoll [4] provide an interesting discussion of the evolution of the technique and philosophy leading to the current state-of-the-art for modeling and solving ILP problems.

As stated before, many optimization problems in all fields of real life can be modeled as LP problems and the number of applications of LP to real-word problems is

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continuously increasing. We mention four recently published applications as example. Ilaya-Ayza et al. [5] use ILP to solve an optimization problem with the aim of improving the technical management of intermittent water supply systems. Skrzipek [6] uses LP to recover signals from given samples. With an adequate number of samples, the signal will be reconstructed. Otherwise, the frequency, amplitude and bandwidth will be approximated by the proposed procedure. Omidi et al. [7] use LP in an economic problem where the investor wants either to maximize the expected value of the total return subject to some chance constraints, or to minimize the investment risk of the total return with some chance constraints. Finally, Soler et al. [8] use ILP with binary variables to model the problem of minimizing the thermal transmittance of an external wall under certain restrictions.

Particularly interesting are the applications of LP in the field of energy related to buildings, in which this work fits. In addition to the above cited paper [8], among others, Privitera el at. [9] use LP to minimize the cost of renewable energy technologies to meet the reduction of carbon emissions. Ashouri et al. [10] use MILP to obtain the best selection of heating and cooling systems, thermal and electrical storages, and renewable energy sources. Lindberg et al. [11] investigate solutions for Zero Energy Buildings with a financial perspective. MILP is used to optimize both the investments in technology and the operation of the energy technologies. Finally, Ogunjuyigbe et al. [12] use MILP to allocate electrical power to home appliances in residential building with intermittent photovoltaic source, with the aim of maximizing the sub-load points available at each period of the day. They use binary variables to model whether an electrical appliance must be on or off.

A key concept is present in the last five cited articles: *energy efficiency*. According to the definition given by the International Energy Agency [13], energy efficiency includes the managing and restraining of growth in energy consumption. A building is considered more energy efficient if it delivers more services for the same energy input, or if it ensures the same services for less energy input.

Since the approval of the EU Directive 2010/31 [14], European legislation encourages energy saving. Buildings are a key factor as they account for 40% of the total EU's energy consumption [15]. The EU also foresees three energy targets for 2020: a) 20% reduction of the produced greenhouse gases, b) at least 20% coverage of energy consumption by renewable energies and c) 20% reduction of the primary energy needs by improving energy efficiency [16].

In buildings, energy efficiency is strongly related to the building envelope, which is the physical separator between the interior and exterior of the building and plays a crucial role in controlling the thermal energy transfer and avoiding excessive noise inside the enclosures. Externals walls, roofs, floors, doors and windows are the typical components of a building envelope. With respect to external walls, historically they were resolved previously as load-bearing walls, usually made of stone and then made of bricks. In the first half of the 20th century concrete structures became much more usual. The conventional façade was freed from its structural function and could therefore become lighter and thinner, reaching just a 120 mm thickness [17]. Subsequently, a second interior panel was added creating an intermediate air cavity to improve its thermal performance, with the option to introduce a thermal insulation in order to mitigate the energy consumption of the building [18]. Fig. 1 shows this evolution between 1972 and 1990 [19].

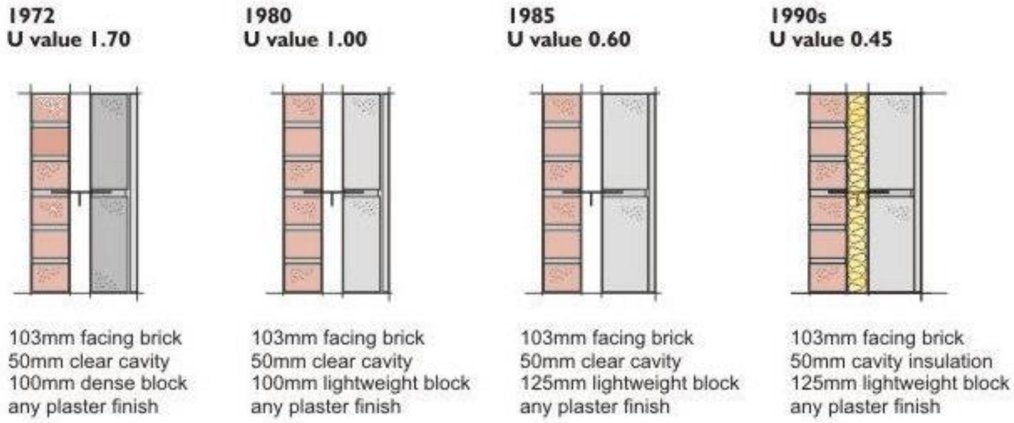

Fig. 1. Evolution of the external wall between 1972 and 1990 [19].

The *U*-value that appears in Fig. 1 is the thermal transmittance, a key magnitude in building efficiency that describes the insulation capacity of a building structure. Transmittance is a property of materials and depends on temperature. Its units are Watts per meter squared Kelvin ($Wm^{-2}K^{-1}$). If a wall material has a *U*-value of 1 $Wm^{-2}K^{-1}$, for every degree of temperature difference between the inside and the outside façade, 1 Watt of heat energy will flow through each square meter of its surface. Thus, the lower the *U*-value, the better the insulation of the building. The thermal transmittance of a wall consisting of *n* layers is given by Eq. (1) as described by McMullan [20]:

$$U = \frac{1}{\frac{1}{h_{int}} + \sum_{i=1}^{n} \frac{e_i}{\lambda_i} + \frac{1}{h_{ext}}} \quad (1)$$

Where λ_i ($Wm^{-1}K^{-1}$) and e_i (m) represent the thermal conductivity and the thickness respectively of layer *i*, and $1/h_{ext}$ and $1/h_{int}$ (m^2KW^{-1}) represent the standard external and internal conductivity respectively for the air layers connected with the envelope.

Legislation in Spain [21] divides the territory into five climate zones according to winter climate severity, from A (less severe) to E (most severe). This legislation allows a maximal thermal transmittance for the different parts of the building envelope depending on the zone where the building is located. Specifically, for the external wall, which is the object of study in this paper, the maximum allowed thermal transmittances are:

- Zone A: 1.25 $Wm^{-2}K^{-1}$.
- Zone B: 1.00 $Wm^{-2}K^{-1}$.
- Zone C: 0.75 $Wm^{-2}K^{-1}$.
- Zone D: 0.60 $Wm^{-2}K^{-1}$.
- Zone E: 0.55 $Wm^{-2}K^{-1}$.

As far as we know, the scientific literature has not developed procedures to minimize the construction cost of a building's external wall, while complying with the inherent economic constraints (budget, workforce, dimensions, availability of materials, maintenance cost, time limit, etc.), and the current energy efficiency legislation in relation to thermal transmittance. An external wall consists of several layers and each layer can admit different materials with their own thermal transmittance and different thicknesses. Therefore, thousands or even millions of possible combinations of materials and their thicknesses must be evaluated to obtain the combination that minimizes the construction cost of the wall without violating any restriction imposed to the constructor.

The aim of this paper is to use ILP to model and to solve the above described optimization problem.

The rest of the paper is organized as follows. Section 2 introduces all the variables and parameters involved in the problem and presents its ILP formulation. Section 3 shows the computational results obtained on a case study consisting of a 6-layer external wall with more than 5.5 million combinations of the different selected materials and their thicknesses for the different layers of the wall, and where the ILP formulation is applied for 165 scenarios consisting of combinations of thickness bounds and maximal allowed thermal transmittances. Finally, Section 4 presents conclusions and some suggestions for future research.

2. Formulation of the ILP problem

The problem of minimizing the construction cost of the external wall of a building under the restrictions cited above is formulated in this section as an ILP problem. To this aim, we first need to present the used variables and parameters, and to give some notations and suppositions for a better understanding of the formulation.

1. Given the total surface in m^2 of the external wall, this surface will be taken into account on preliminary simple calculations, such as determining the availability of certain materials for the entire wall or if the total value of that material exceeds the budget we are willing to pay for that material or its corresponding layer. However, the prices and other parameters of the materials are usually given by m^2 in the databases. Therefore, the goal will be to minimize the cost per m^2 of the external wall.
2. Let *n* be the number of layers of the wall, which will be enumerated from inside to outside. Each layer $i \in \{1, \dots, n\}$ is made of one of the m_i different materials available for this layer, and given a layer $i \in \{1, \dots, n\}$, the material $j \in \{1, \dots, m_i\}$ is available in r_{j_i} different thicknesses.
3. For each $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m_i\}$ and $k \in \{1, \dots, r_{j_i}\}$, the following parameters are defined:
   - $e_{i,j,k}$ thickness corresponding to material *j* with type of thickness *k* available for layer *i* (note that *k* indicates the type of thickness, not the thickness).
   - $c_{i,j,k}$ cost of placing in layer i 1m^2 of material *j* with type of thickness *k* available for layer *i*.
   - $t_{i,j,k}$ time of placing in layer i 1m^2 of material *j* with type of thickness *k* available for layer *i*.
   - $mc_{i,j,k}$ maintenance cost for a given period of time for 1 m^2 of material *j* with type of thickness *k* and located in layer *i*.
4. Given two consecutive layers, there may exist incompatibilities between some materials and thicknesses corresponding to these layers. A detailed example of incompatibility between two materials corresponding to two consecutive layers will be given in the last paragraph of this section.
5. The total thickness of the external wall is comprised between bounds e_{min} and e_{max}.
6. Let U_{max} be the maximum thermal transmittance allowed for the external wall.
7. Let t_{max} be the maximum time allowed to construct 1 m^2 of the external wall.
8. Let mc_{max} be the maximum maintenance cost for 1 m^2 of the external wall for a given period of time.

9. The variables of the ILP problem are the binary variables $x_{i,j,k}$ whose value are 1 if layer *i* is made with material *j* and type of thickness *k*, and 0 otherwise, $i \in \{1, \dots, n\}$, $j \in \{1, \dots, m_i\}$ and $k \in \{1, \dots, r_{j_i}\}$.
10. Given a material *j*, with $j \in \{1, \dots, m_i\}$ for some $i \in \{1, \dots, n\}$, and let λ_j be its thermal conductivity, according to Eq. (1) the constraint to comply with the thermal transmittance upper bound is:

$$\frac{1}{\frac{1}{h_{int}} + \sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} \frac{e_{i,j,k}}{\lambda_j} x_{i,j,k} + \frac{1}{h_{ext}}} \leq U_{max} \qquad (2)$$

Note that Eq. (2) is not a linear constraint of variables x_{ijk}, but U_{max}, h_{int} and h_{ext} are constant, as well as e_{ijk} and λ_j for all the involved subscripts. It is easy to see that Eq. (2) is equivalent to Eq. (3), which is a linear constraint in the variables x_{ijk}:

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} \frac{e_{i,j,k}}{\lambda_j} x_{i,j,k} \geq \frac{1}{U_{max}} - \frac{1}{h_{int}} - \frac{1}{h_{ext}} \qquad (3)$$

Taking into account all the concepts, restrictions and suppositions given above, the problem of minimizing the construction cost of an external wall can be formulated mathematically as the following ILP problem, defined through Eqs. 4 to 11:

$$Minimize \sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} c_{i,j,k} x_{i,j,k} \qquad (4)$$

s.t.:

$$\sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} x_{i,j,k} = 1 \quad \forall i \in \{1, \dots, n\} \qquad (5)$$

$$e_{min} \leq \sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} e_{i,j,k} x_{i,j,k} \leq e_{max} \qquad (6)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} \frac{e_{i,j,k}}{\lambda_j} x_{i,j,k} \geq \frac{1}{U_{max}} - \frac{1}{h_{int}} - \frac{1}{h_{ext}} \qquad (7)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} t_{i,j,k} x_{i,j,k} \leq t_{max} \qquad (8)$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{r_{j_i}} mc_{i,j,k} x_{i,j,k} \leq mc_{max} \qquad (9)$$

$$x_{i,j,k} + x_{(i+1),j',k'} \leq 1 \quad \forall\, (i,j,k - (i+1),j',k') - \ incompatible \qquad (10)$$

$$x_{i,j,k} \in \{0,1\} \quad \forall\, i \in \{1,\dots,n\}, j \in \{1,\dots,m_i\}, k \in \left\{1,\dots,r_{j_i}\right\} \qquad (11)$$

Where:

Eq. (4) is the objective function, that is, the construction cost of 1m^2 of the wall.

Eq. (5) guarantees that each layer is made exactly of one material with a specific thickness.

Eq. (6) restricts the total thickness of the external wall within the established bounds.

Eq. (7) is the key restriction with respect to energy efficiency. It ensures that the wall does not exceed the maximal allowed thermal transmittance.

Eq. (8) guarantees that the established time limit to construct 1 m^2 of the wall will not be exceeded. Note that deadlines are very important in building construction, because a project consists of a set of tasks such that some of them must be finished before others can start. In fact, the aim of the well-known Critical Path Method [22] is to find the best sequence of tasks with the least amount of slack and to have a tool to control and adapt deadlines.

Eq. (9) ensures that the maintenance cost of 1 m^2 of the external wall will not exceed a certain amount. Note that the approximate maintenance costs for certain period of time of the different materials are usually available on the databases. To ensure a low maintenance cost can be considered a good option on a sales policy.

Eq. (10) forbids to place a material *j'* with thickness *k'* in the next layer to the one (layer *i*) containing the material *j* with thickness *k* (we denote this fact (*i*,*j*,*k*-(*i*+1),*j'*,*k'*)-incompatibility). At most one of the two materials will appear in the corresponding layer.

Finally, Eq. (11) defines the variables of the problem as binary ones.

Note that the above formulation contains the usual constrains given in the construction of an external wall, but it could include other types of linear constraints in order to fit as much as possible the real restrictions involved in each specific situation. In the same way, some constrains given above could be removed or modified conveniently, for instance, if maintenance cost is not taken into account.

Note also that, although for mathematical reasons the number of layers is considered fixed, in the real problem some layers of the external wall could be optional. This is not a handicap for the presented ILP formulation because, if a layer is optional, an imaginary material for that layer with zero construction cost, zero construction time, zero maintenance cost, zero thickness and any conductivity different from zero can be considered. If the optimal solution to the ILP problem assigns this material to the corresponding layer, this means that in the real solution this layer does not exist. This is the case, for instance, of an internal air chamber, that in many cases is optional.

But the idea of an imaginary material can also be useful to address the case of incompatibility between materials of adjacent layers. For instance, it is usual for the exterior panel of an external wall to be made of solid brick, concrete block or facing brick. If the exterior panel is made by solid brick or concrete block, for aesthetic reasons, an external layer consisting of some kind of coating is added. But if the external panel is made of facing brick, as its name indicates, it makes no sense to add an external coating covering the facing bricks and therefore preventing their view. Therefore, there is an incompatibility between external coating and facing brick in the external panel, but as the number of layers must be fixed, the problem is how our model can guarantee that if the external panel is made of facing brick, the external coating will not exist. To face this issue, we consider an imaginary material for the external coating layer as described above (zero cost, zero thickness, etc.). In this case, Eq. (10) must be used to declare incompatibility between facing brick in the penultimate layer and all options of external coating in the last layer except for the imaginary one, and to declare incompatibility between the imaginary external coating in the last layer and the solid brick or the concrete block in the penultimate layer. Both the optionality of the air chamber and the incompatibility between external layers occur in the case study given in the next section.

3. Case study

Our case study consists of a 6-layer façade which includes an internal coating (IC) and panel (IP), a layer with thermal insulation (TI), an optional air chamber (AC), an external panel (EP) and finally an external coating (EC) (see Fig. 2). This façade is a common and representative constructive solution for an external wall and is included in the Catalog of Constructive Elements of the Spanish Technical Act [23].

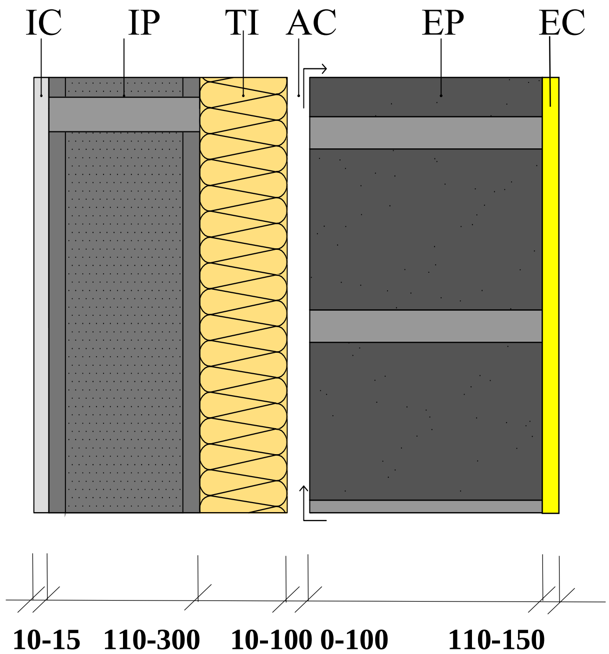

Fig.2. Case study.

In order to make this constructive solution more flexible and global, we have chosen different materials for each layer. The internal panel (IP) shows six materials (air brick, perforated brick, solid concrete block, light concrete block, terracotta brick and perforated concrete brick). The eight chosen thermal insulation materials (TI) present an increasing conductivity from 0.013 up to 0.09 W/mK^{-1}. In order to avoid a poor behavior of the thermal insulation its ideal position is inside the chamber and attached to the internal panel. Projected polyurethane, extruded and expanded polystyrene, mineral wool, sandwich panel can be classified as conventional materials and cork and wood chips as green and alternative materials, as described by Schiavoni et al. [24]. We also want to highlight the nanoporous aerogel as a quite new and very performing insulation material with the lowest conductivity. A ventilated façade aims to improve the behavior of a conventional envelope in terms of static stability, thermal performance, weather tightness and flexibility. Our air cavity (AC) presents 3 options: light ventilated, not ventilated or absence. The external panel (EP) can consist of solid brick, concrete block, perforated brick, facing brick or pressed facing brick with similar thicknesses but quite variable thermal conductivity. Finally, the external coating (EC) shows options for continuous or discontinuous envelope, as described in the introduction. If a facing brick is used as external panel, obviously external coating is not necessary, which implies that there is an incompatibility between both materials. All materials can show furthermore different thickness as shown in Table 1, which summarizes all options.

The cost of the different materials with their corresponding thiknesses have been consulted in the cost generator website of CYPE Ingenieros [25] during November 2017. Materials, staff and site facilities are included in the cost. Maintenance costs for 10 years are also available. Table A1 in Appendix shows all the necessary data correspondig to the different materials and thicknesses for the 6 layers: conductivity, cost per m^2, 10-year maintenance cost per m^2 and variable name in the ILP problem. Note that from Table A1 we obtain a total number of 134 binary variables in the ILP problem. Moreover, the Spanish Technical Act (CTE), Basic Document of Energy Saving (DB_HE) [21] recommends a specific thermal resistance for the air layers close to the external and internal surfaces, which values for vertical air flow are: $1/h_{ext} = 0.04\ m^2KW^{-1}$ and $1/h_{int} = 0.13\ m^2KW^{-1}$.

Table 1
Composition of the layers of the case study.

Layer	Function	Material	Thickness [mm]
Layer 1	Internal coating	Plaster	10, 12, 14, 15
Layer 2	Internal panel	Air brick	110, 115
		Perforated brick	115
		Solid concrete block	150, 200
		Light concrete block	200, 250, 300
		Terracotta bricks	140, 190, 240
		Perforated concrete brick	120
Layer 3	Thermal insulation	Nanoporous aerogel	10, 20
		Projected Polyurethane	20 up to 80
		Extruded polystyrene	30 up to 60
		Mineral wool	30 up to 100
		Expanded polystyrene	30 up to 80
		Cork	25 up to 60
		Sandwich panel	25, 35, 50
		Wood chips	15, 25, 35, 50
Layer 4	Air cavity	Light ventilated	30, 50, 80, 100
		not ventilated	30, 50, 80, 100
		absence	0
Layer 5	External panel	Solid brick	115
		Concrete block	150
		Perforated brick	110
		Facing brick	115
		Pressed facing brick	120
Layer 6	External coating	Regular plaster	10 up to 20
		Thermal plaster	10 up to 20
		Metallic plate	15, 20
		Stone plate	30, 40
		Composite plate	40
		Ceramic plate	10
		Absence	0

In this case study the number of possible combinations of the different selected materials and thicknesses for the different layers is 5,598,720. We can check that 2,169,504 of them do not violate Eq. (10) of the ILP formulation, which corresponds to the incompatibilities between different materials in layers 5 and 6. That is, a total amount of 2,169,504 possible constructive solutions remain for this external wall. The builder or the designer should evaluate which one of these constructive solutions has the lowest construction cost, according to the rest of restrictions that must be met regarding the thickness of the wall, the maintenance cost, the maximum allowed thermal transmittance, the time limit to build the wall, etc. In order to obtain trends and conclusions, particularly on how the maximum allowed thermal transmittance affects the minimum construction cost, we have decided to evaluate 165 scenarios among hundreds of different possibilities, depending on different values given to the parameters mentioned above and taking into account the following circumstances:

- The most usual thicknesses of an external wall are between 25 and 40 cm. Therefore, we have considered 15 different intervals of thickness, each one with a width of one cm: [25+i,25+i+1[for each $i \in \{0,1,\ldots,14\}$.
- The maximum allowed *U*-Value for the most severe winter climate zone in Spain is 0.55 $Wm^{-2}K^{-1}$. But as stated in Section 1, the lower the *U*-value, the better the insulation of the structure. Furthermore, the Perfil de Calidad (Quality Profile) of the Instituto Valenciano de la Edificación [26] is a special label with point system, that considers improvements in energy savings, environmental protection, acoustic comfort, accessibility or spatial quality. Extra points are given if the thermal transmittance *U* is reduced by 40% or 60%. Therefore, we have decided to consider 11 maximum allowed *U*-values $\{0.25+0.05i\}_{i=0}^{10}$, which allows studying the 3 most critical climatic zones (E,D,C) and go further. Note that with these *U*-values, Spanish legislation about thermal transmittance will always be fulfilled for climate zones A and B.
- We do not consider any time limit to build the wall, i.e. Eq. (8) is not considered.
- As 10-year maintenance costs are known for all the involved materials, and in order to make use of Eq. (9) with a logical argument, for each one of the 165 combinations of thickness and maximum *U*-value, we have solved and ILP problem to obtain the minimum 10-year maintenance cost for 1m^2 of the wall. To do this, in the ILP formulation given in Section 2 we have replaced the triple summation of Eq. (4) by the triple summation of Eq. (9) and deleted Eq. (9). Among all the obtained minimum maintenance costs, we have used the maximum of these values, which is 12.82€, as an upper bound in Eq. (9) for all 165 ILP problems to obtain the minimum construction cost. In this way, we guarantee that the wall will have a low maintenance cost (which makes the property more attractive to hypothetical buyers), but at the same time, by selecting the maximum of those minimums, we try to guarantee the existence of a feasible solution in as many scenarios as possible.

To solve both the 165 ILP problems to obtain minimum 10-year maintenance costs and the 165 ILP problems to obtain minimum construction costs, we have run *Mathematica* 11.0 [27] on a PC Intel® Core™ I7-6700 with 4 processors, 3.46GHz and 8GB RAM. Note that *Mathematica* is a widely used tool to solve mathematical, physical and engineering problems. It contains several functions to solve ILP problems and its own programming language that allows solving the 165 ILP problems with a single execution. Table 2 shows the minimum construction costs obtained for the 165 scenarios.

Note that in 25 of these scenarios, which correspond to the lowest maximum *U*-values and the smallest wall thicknesses, the ILP problem was unfeasible. This can be concluded since *Mathematica* guarantees that it obtains the optimal solution if the ILP problem has a feasible solution, but in these 25 scenarios, *Mathematica* concluded that it was not able to find any feasible solution.

Table 2
Minimum construction cost in euros of $1m^2$ of the external wall given an interval of thickness (row) and a maximal allowed thermal transmittance (column). A blank means a lack of solution.

	0.25	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**
[0.25,0.26[						232.7	232.7	232.7	138.3	134.09	134.09
[0.26,0.27[					195.8	195.8	195.8	96.84	81.37	80.4	80.4
[0.27,0.28[					199.93	83.68	82.35	82.35	80.44	80.44	80.44
[0.28,0.29[				234.84	85.57	84.55	80.82	80.82	80.82	80.82	80.82
[0.29,0.30[				86.64	83.48	81.37	80.4	80.4	80.4	80.4	80.4
[0.30,0.31[			88.28	84.53	82.11	81.4	80.44	80.44	80.44	80.44	80.44
[0.31,0.32[		90.13	89.88	82.75	80.82	80.4	80.4	80.4	80.4	80.4	80.4
[0.32,0.33[		91.53	83.41	81.47	81.47	80.44	80.44	80.44	80.44	80.44	80.44
[0.33,0.34[		88.52	84.53	82.11	80.82	80.82	80.82	80.82	80.82	80.82	80.82
[0.34,0.35[		89.88	82.75	81.37	80.4	80.4	80.4	80.4	80.4	80.4	80.4
[0.35,0.36[	94.91	83.41	82.11	81.4	80.44	80.44	80.44	80.44	80.44	80.44	80.44
[0.36,0.37[	94.41	87.54	82.75	80.4	80.4	80.4	80.4	80.4	80.4	80.4	80.4
[0.37,0.38[	91.53	83.41	81.47	80.44	80.44	80.44	80.44	80.44	80.44	80.44	80.44
[0.38,0.39[	95.21	84.53	80.82	80.82	80.82	80.82	80.82	80.82	80.82	80.82	80.82
[0.39,0.40[	89.88	82.75	81.47	81.47	81.47	81.47	81.47	81.47	81.47	81.47	81.47

Also notice, as a logical trend, that given a thickness interval, the lower the maximum *U*-value, the higher the minimum construction cost (we need more expensive materials to reduce the thermal transmittance). Finally, as a general rule, given a maximum *U*-value, the lower the thickness of the wall, the greater the minimum cost. However, the existence of different thicknesses for the air layer, all with zero cost (but with different thermal transmittance), causes that some minimum construction costs are the same when increasing the thickness in some cm, because the thickest wall has the same composition except for the air layer, which has a greater thickness. In fact, among the 140 minimum costs obtained, the lowest one (80.40€), which from now on will be called the global minimum cost, appears in several scenarios corresponding to different thicknesses and different maximum *U*-values due to the use of different thicknesses for the air layer.

Figure 3 shows a radar diagram of costs vs transmittance for six of the most representative thickness intervals according to the results given in Table 2. Radar diagrams can be very effective to display jointly three or more quantitative variables in a two-dimensional chart. In our case we have considered as variables the cost associated to a specific *U*-value depending on thickness. The concentric circles represent increasing values for costs from 0 €/m^2 (in the center) up to 250 €/m^2 (in the external circle). Values for thermal transmittance *U* are displayed in the external part of the chart and start with 0.25 $Wm^{-2}K^{-1}$ reaching clockwise the maximum value of 0.75 $Wm^{-2}K^{-1}$. We have considered six thicknesses (normal and dotted lines) of the 15 intervals. In these diagrams, the form of the obtained polygon for a specific thickness (complete, incomplete, regular, irregular) show the variation of the given thicknesses.

For example, we can recognize how there is a little difference between the intervals of thickness [0.35,0.36[and [0.39,0.40[for the highest thermal transmission. All related polygons are quite near the center of the circle meaning lower reached final construction cost. The absence of part of the polygon means presence of unfeasible problem for that *U* in that sector, e.g. in intervals [0.25,0.26[, [0.26,0.27[, [0.28,0.29[and [0.31,0.32[. The first thickness interval reaches the maximum construction cost of 232.70€ for *U* between

0.5 $Wm^{-2}K^{-1}$ and 0.6 $Wm^{-2}K^{-1}$ while for interval [0.28,0.29[a similar maximum cost of 234.84€ corresponds to a U value of 0.4 $Wm^{-2}K^{-1}$.

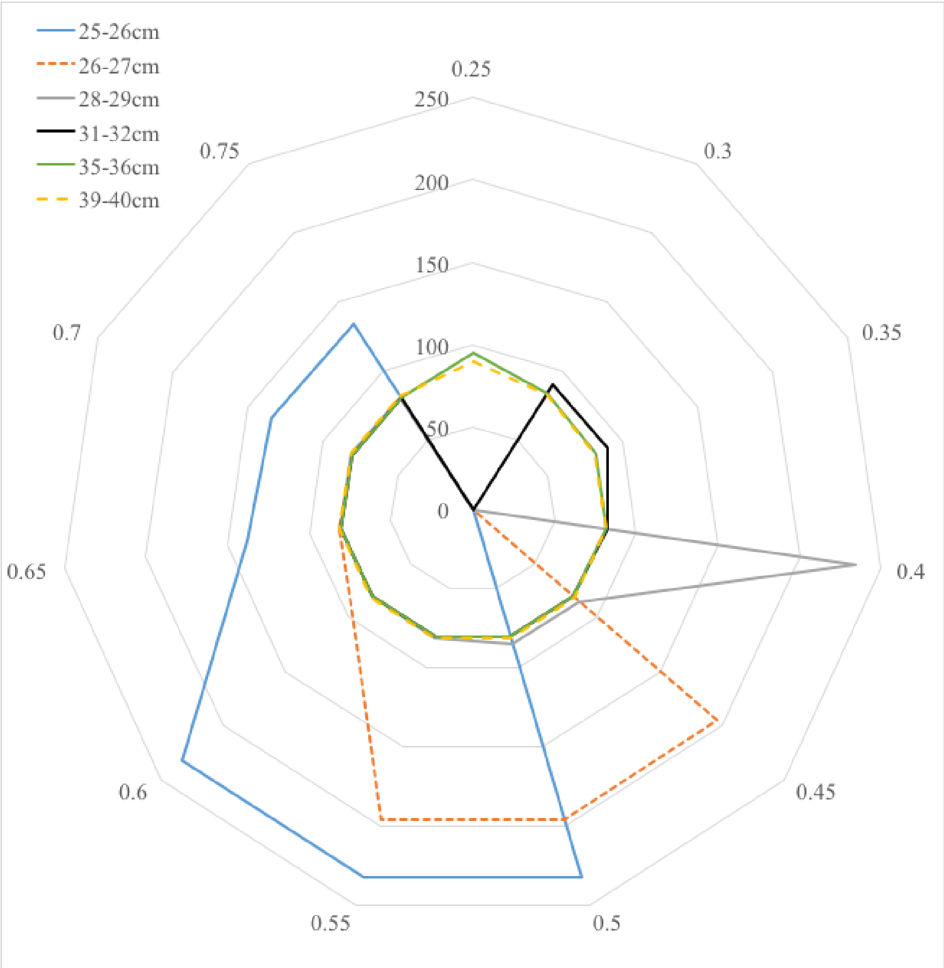

Fig.3. Radar diagram of cost (values in upper vertical radius, in €/m^2) vs transmittance (values in the direction of clockwise, in $Wm^{-2}K^{-1}$) for six representative thickness intervals.

For interval [0.26,0.27[the maximum cost drops up to 195.80€ for U between 0.45 $Wm^{-2}K^{-1}$ and 0.55 $Wm^{-2}K^{-1}$.

Table 3 shows the CPU time in seconds to obtain each one of the optimal solutions whose costs are given in Table 2. To obtain each optimal solution *Mathematica* took only a few hundredths of a second. These times are insignificant, so we do not consider necessary to add additional information about them, as average or maximum or minimum values. Note that in some cases *Mathematica* reported 0 CPU time. According to *Mathematica*'s assumption, this means that the calculation took no measurable CPU time. Note also that, regarding the set of 25 ILP problems without feasible solution, *Mathematica* took between 18 and 22 seconds to determine that each problem was unfeasible. We consider this time reasonable due to the fact that the algorithm has to check up to 5,598,720 combinations of layers, all of them unfeasible for the given conditions.

Table 3
CPU time in seconds to obtain the minimal construction cost solution for each interval of thickness (row) and maximal allowed thermal transmittance (column).

	0.25	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**	**0.55**	**0.6**	**0.65**	**0.7**	**0.75**
[0.25,0.26[	18.719	18.813	18.984	18.672	18.781	0.016	0	0.016	0.047	0.016	0.016
[0.26,0.27[	8.813	18.641	18.813	18.688	0.016	0.047	0.078	0.078	0.016	0	0.016
[0.27,0.28[	18.750	18.609	18.703	18.609	0.063	0.031	0.016	0.094	0.016	0.016	0.016
[0.28,0.29[	18.672	18.875	18.563	0.016	0.063	0.047	0.016	0.016	0.016	0.016	0
[0.29,0.30[	18.734	18.625	18.813	0.031	0.063	0.047	0.016	0.016	0	0.016	0.016
[0.30,0.31[	18.625	18.531	0.031	0.031	0.047	0.031	0.016	0.016	0.031	0	0.016
[0.31,0.32[	18.594	0	0.063	0.031	0.016	0.016	0.016	0.016	0.016	0	0.016
[0.32,0.33[	18.922	0.078	0.016	0.047	0.047	0.016	0.016	0.016	0	0.016	0.016
[0.33,0.34[	18.641	0.063	0.063	0.031	0.016	0.016	0.016	0.031	0.016	0.016	0.016
[0.34,0.35[	22.047	0.047	0	0.016	0	0.031	0	0.016	0	0.016	0
[0.35,0.36[	0.031	0.016	0.047	0.031	0.016	0.016	0.016	0.016	0	0.016	0.016
[0.36,0.37[	0.016	0.063	0.016	0.016	0.016	0	0	0.016	0	0.016	0
[0.37,0.38[	0.094	0.047	0.031	0	0.016	0.016	0	0.016	0.016	0	0.016
[0.38,0.39[	0.125	0.094	0.016	0.016	0.016	0.016	0	0.016	0	0.016	0
[0.39,0.40[	0.047	0.031	0.031	0.016	0.016	0.016	0.016	0.016	0	0.016	0.016

Note that for each one of the minimum construction cost solutions, in addition to this cost, *Mathematica* provides the values of the variables. Therefore, it is easy to know which are the chosen materials and their thicknesses for each one of the six layers in the optimal solution. For obvious space reasons, we omit the selected materials for each one of the optimal solutions obtained. Instead, Table 4 shows all the data corresponding to the obtained solution for six representative scenarios, most of them extreme:

- Solution with global minimum cost and minimum *U*-value.
- Solution with global minimum cost and minimum thickness.
- Least cost solution for the minimum U_{max} (0.25 $Wm^{-2}K^{-1}$).
- Least cost solution using nanoporous aerogel, which is a quite new insulation material with the lowest conductivity and the lowest thickness of those available for this wall.
- Least cost solution using projected polyurethane, which is one of the most used insulating materials.
- Least cost solution using a facing brick for layer 5, which means absence of layer 6 (external coating).

From Table 4 we observe that only in the scenario with minimum U_{max} the thickness of the plaster in layer one is different, while layer 2 is the same for the six scenarios studied (air brick 33x16x11).

The insulation layer 3 presents 2 insulation materials, projected polyurethane and nanoporous aerogel, with 2 different fixation options: dots and mechanical fixing. It is very interesting to highlight that, although its price is high, the use of nanoporous aerogel allows obtaining a feasible solution for the minimum considered thickness of the wall (0.25m). In fact, the combination of nanoporous aerogel and facing brick allows obtaining an external wall with just four layers, with a thickness of only 0.255m, and a thermal transmittance (0.47 $Wm^{-2}K^{-1}$) compatible with all 5 climate zones in Spain. Nevertheless, the price of this constructive solution is very high (232.7€) with respect to the global minimum cost (80.4€).

For layer 4 two options appear: light ventilated air gap (with two thicknesses) and no air gap. Note that the composition of the wall for both scenarios with global minimum cost is the same, except for layer 4 that, as expected, it has no air gap for minimum thickness and it has the maximum allowed thickness of light ventilated air gap for minimum thermal transmittance. This fact was commented above, regarding that the air gap has zero cost independently of its thickness, but the greater its thickness, the lower its thermal transmittance.

For layer 5 and layer 6 we have five equal solutions with perforated brick 33x30x11 and regular plaster with thickness 0.01m. The only different scenario is the one with the facing brick commented above.

Finally, we observe that in this case study, the cost of the least cost solution using projected polyurethane is the global minimum cost. This fact endorses that projected polyurethane is one of the most used insulating materials, but the demand of final clients with increasing environmental or esthetical awareness could also bring to different and maybe more expensive constructive solutions which includes the use of green insulating materials or even the use of lime stone plates for the external coating. This is just a business issue from the constructor point of view and not part of our paper, but our method is flexible enough in order to attend any input or constraints in this sense.

Table 4
Best solution for six scenarios of interest.

	Global minimum cost with minimum U_{max}	Global minimum cost with minimum thickness	Minimum U_{max}	Use of nanoporous aerogel	Use of projected polyurethane	Use of facing brick
Cost	80.4	80.4	94.41	134.09	80.4	232.7
Thickness interval	[0.36,0.37[	[0.26,0.27[	[0.36,0.37[	[0.25,0.26[	[0.26,0.27[	[0.25,0.26[
Exact thickness	0.36	0.26	0.369	0.25	0.26	0.255
U_{max}	0.4	0.7	0.25	0.7	0.7	0.5
Exact U	0.3871	0.6792	0.2495	0.6548	0.6792	0.4711
Layer 1	Plaster 0.01	Plaster 0.01	Plaster 0.014	Plaster 0.01	Plaster 0.01	Plaster 0.01
Layer 2	Air brick 33*x*16*x*11	Air brick 33*x*16*x*11	Air brick 33*x*16*x*11	Air brick 33*x*16*x*11	Air brick 33*x*16*x*11	Air brick 33*x*16*x*11
Layer 3	Project. polyu. dots 0.02	Project. polyu. dots 0.02	Project. polyu. dots 0.075	Npour aerogel mec. fix. 0.01	Project. polyu. dots 0.02	Npour aerogel mec. fix. 0.02
Layer 4	Light vent. air gap 0.1	No air gap	Light vent. air gap 0.05	No air gap	No air gap	No air gap
Layer 5	Perforat. brick 33*x*30*x*11	Perforat. brick 33*x*30*x*11	Perforat. brick 33*x*30*x*11	Perforat. brick 33*x*30*x*11	Perforat. brick 33*x*30*x*11	Facing brick 24*x*11.5*x*5 waterproof
Layer 6	Regular plaster 0.01	Regular plaster 0.01	Regular plaster 0.01	Regular plaster 0.01	Regular plaster 0.01	Absence of plaster

4. Conclusion

ILP has proven its effectiveness to solve many real-world optimizations problems, particularly in the field of energy efficiency related to building. This paper presents a new application of ILP in this field. The tackled problem consists on minimizing the construction cost of a building's external wall, fulfilling on the one hand with the economic constraints inherent in any constructive process, and on the other, with the current thermal transmittance legislation, to guarantee the building's energy efficiency. Among thousands or even millions of possible combinations of materials and their thicknesses for the different layers of the wall, our ILP procedure selects the minimum construction cost combination complying with all restrictions imposed to the builder/designer.

A case study consisting of a 6-layer external wall has been presented, with more than 5.5 million combinations of the different selected materials and their thicknesses for the different layers of the wall. The ILP problem has been solved for 165 scenarios corresponding to different thicknesses of the wall and maximum allowed thermal transmittances. *Mathematica* 11.0 has been used as ILP solver and it has been able to obtain each optimal solution in just a few hundredths of a second.

We are convinced that this tool can allow the builder to decide the best option for the construction of an external wall by analyzing different scenarios. An adequate selection of the materials and a small variation of the wall's thickness can considerable reduce its construction cost without decreasing the energy efficiency of the building.

In future works, we plan to apply a similar ILP modelization to other components of the building envelope or even to the whole building envelope, including the opaque and transparent part and the roof. All envelope elements (walls, floors, roofs, fenestrations and doors) show a "similar" structure to that of the external wall: several layers with different options for each layer, with different prices, thicknesses, *U*-values and other technical or aesthetic characteristics. Therefore, both a similar objective function and a restriction set can be stated for each part of the building envelope, depending on the budget for that part, the technical conditions imposed, the preferences of potential buyers, etc.

Moreover, an ILP problem can also take into account restrictions involving different parts of the building envelope. For instance, the ILP problem could decide the ratio between opaque and transparent part for a specific façade, within stipulated limits. In this case, the ILP formulation will be more complex because the variables will not be binary, the number of them will increase because they must be different for the different façades of the building, and the objective function would not be the cost per m2, but the total cost of the studied parts of the building envelope. If possible, a final ILP model will minimize the construction cost of the whole building envelope taking into account its energy efficiency.

Likewise, we think that the ideas presented in this paper could be applied to thermal refurbishment of building envelopes.

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Appendix

Table A.1
Layers and chosen materials with their characteristics.

Layer	Material	Thickness [m]	Conductivity [W/mK^{-1}]	Cost [€/m^2]	10 years maint Cost [€/m^2]	Variable name
1. Internal coating						
	Plaster	0.010	0.26	22.89	3.20	$x_{1,1,1}$
		0.012	0.26	25.03	*3.50*	$x_{1,1,2}$
		0.014	0.26	27.17	*3.80*	$x_{1,1,3}$
		0.015	0.26	28.25	3.96	$x_{1,1,4}$
2. Internal panel						
	Air brick *24x11.5x11.5*	0.115	0.49	23.93	0.72	$x_{2,1,1}$
	Air brick *33x16x11*	0.11	0.49	17.74	0.53	$x_{2,2,1}$
	Perforated brick *24x11.5x9*	0.115	0.76	23.29	0.70	$x_{2,3,1}$
	Solid Concrete block *40x20x15*	0.15	0.46	22.46	0.67	$x_{2,4,1}$
	40x20x20	0.20	0.46	25.73	0.77	$x_{2,4,2}$
	Expanded clay light	0.20	0.3	42.21	1.27	$x_{2,5,1}$
	concrete block	0.25	0.3	53.94	1.62	$x_{2,5,2}$
		0.30	0.3	63.23	1.90	$x_{2,5,3}$
	Terracotta brick	0.14	0.28	21.42	0.64	$x_{2,6,1}$
	without additive	0.19	0.28	25.36	0.76	$x_{2,6,2}$
		0.24	0.24	29.10	0.87	$x_{2,6,3}$
	Perforated concrete brick *25x12x9.5*	0.12	0.543	26.06	0.78	$x_{2,7,1}$
3. Thermal insulation						
	Nanoporous aerogel	0.01	0.013	59.71	1.19	$x_{3,1,1}$
		0.02	0.013	121.42	2.43	$x_{3,1,2}$
	Projected Polyurethane *density 40kg/m^3*	0.02	0.028	6.02	0.12	$x_{3,2,1}$
		0.025	0.028	6.99	0.14	$x_{3,2,2}$
		0.03	0.028	7.97	0.16	$x_{3,2,3}$
		0.035	0.028	9.30	0.19	$x_{3,2,4}$
		0.04	0.028	10.17	0.20	$x_{3,2,5}$
		0.045	0.028	11.19	0.22	$x_{3,2,6}$
		0.050	0.028	12.26	0.25	$x_{3,2,7}$
		0.055	0.028	12.51	0.25	$x_{3,2,8}$
		0.06	0.028	13.90	0.28	$x_{3,2,9}$
		0.065	0.028	14.14	0.28	$x_{3,2,10}$
		0.07	0.028	15.50	0.31	$x_{3,2,11}$
		0.075	0.028	15.75	0,32	$x_{3,2,12}$
		0.08	0.028	17.15	0.34	$x_{3,2,13}$
	Extruded polystyrene *Dots*	0.03	0.034	7.02	0.14	$x_{3,3,1}$
	Adhesive mortar	0.03	0.034	9.00	0.18	$x_{3,4,1}$
	Mechanical fixing	0.03	0.034	8.52	0.17	$x_{3,5,1}$

Dots	0.04	0.034	8.07	0.16	$x_{3,3,2}$
Adhesive mortar	0.04	0.034	10.05	0.20	$x_{3,4,2}$
Mechanical fixing	0.04	0.034	9.57	0.19	$x_{3,5,2}$
Dots	0.05	0.034	9.10	0.18	$x_{3,3,3}$
Adhesive mortar	0.05	0.034	11.08	0.22	$x_{3,4,3}$
Mechanical fixing	0.05	0.034	10.60	0.21	$x_{3,5,3}$
Dots	0.06	0.034	10.15	0.20	$x_{3,3,4}$
Adhesive mortar	0.06	0.034	12.15	0.24	$x_{3,4,4}$
Mechanical fixing	0.06	0.034	11.65	0.23	$x_{3,5,4}$
Mineral wool					
Dots	0.03	0.035	8.73	0.17	$x_{3,6,1}$
Adhesive mortar	0.03	0.035	10.40	0.23	$x_{3,7,1}$
Mechanical fixing	0.03	0.035	9.52	0.19	$x_{3,8,1}$
Dots	0.04	0.035	10.35	0.21	$x_{3,6,2}$
Adhesive mortar	0.04	0.035	13.03	0.26	$x_{3,7,2}$
Mechanical fixing	0.04	0.035	11.14	0.22	$x_{3,8,2}$
Dots	0.05	0.035	11.91	0.24	$x_{3,6,3}$
Adhesive mortar	0.05	0.035	14.50	0.29	$x_{3,7,3}$
Mechanical fixing	0.05	0.035	12.70	0.25	$x_{3,8,3}$
Dots	0.075	0.035	16.64	0.33	$x_{3,6,4}$
Adhesive mortar	0.075	0.035	19.31	0.39	$x_{3,7,4}$
Mechanical fixing	0.075	0.035	17.42	0.35	$x_{3,8,4}$
Dots	0.10	0.035	19.86	0.40	$x_{3,6,5}$
Adhesive mortar	0.10	0.035	22.53	0.45	$x_{3,7,5}$
Mechanical fixing	0.10	0.035	20.64	0.41	$x_{3,8,5}$
Expanded polystyrene					
Dots	0.03	0.036	6.06	0.12	$x_{3,9,1}$
Adhesive mortar	0.03	0.036	8.04	0.16	$x_{3,10,1}$
Mechanical fixing	0.03	0.036	7.43	0.15	$x_{3,11,1}$
Dots	0.04	0.036	6.44	0.13	$x_{3,9,2}$
Adhesive mortar	0.04	0.036	8.42	0.17	$x_{3,10,2}$
Mechanical fixing	0.04	0.036	7.80	0.16	$x_{4,11,2}$
Dots	0.05	0.036	7.09	0.14	$x_{3,9,3}$
Adhesive mortar	0.05	0.036	9.07	0.18	$x_{3,10,3}$
Mechanical fixing	0.05	0.036	8.46	0.17	$x_{3,11,3}$
Dots	0.06	0.036	7.73	0.15	$x_{3,9,4}$
Adhesive mortar	0.06	0.036	9.71	0.19	$x_{3,10,4}$
Mechanical fixing	0.06	0.036	9.10	0.18	$x_{3,11,4}$
Dots	0.07	0.036	8.37	0.17	$x_{3,9,5}$
Adhesive mortar	0.07	0.036	10.35	0.21	$x_{3,10,5}$
Mechanical fixing	0.07	0.036	9.74	0.19	$x_{3,11,5}$
Dots	0.08	0.036	9.03	0.18	$x_{3,9,6}$
Adhesive mortar	0.08	0.036	11.01	0.22	$x_{3,10,6}$
Mechanical fixing	0.08	0.036	10.39	0.21	$x_{3,11,6}$
Agglomerate of expanded cork					
Dots	0.025	0.036	12.64	0.25	$x_{3,12,1}$
Mechanical fixing	0.025	0.036	13.36	0.27	$x_{3,13,1}$
Dots	0.03	0.036	14.44	0.29	$x_{3,12,2}$
Mechanical fixing	0.03	0.036	15.17	0.30	$x_{3,13,2}$
Dots	0.04	0.036	18.05	0.36	$x_{3,12,3}$
Mechanical fixing	0.04	0.036	18.78	0.38	$x_{3,13,3}$
Dots	0.05	0.036	21.68	0.43	$x_{3,12,4}$
Mechanical fixing	0.05	0.036	22.40	0.45	$x_{3,13,4}$
Dots	0.06	0.036	25.29	0.51	$x_{3,12,5}$
Mechanical fixing	0.06	0.036	26.01	0.52	$x_{3,13,5}$
Dots	0.07	0.036	25.69	0.51	$x_{3,12,6}$
Mechanical fixing	0.07	0.036	26.42	0.53	$x_{3,13,6}$
Dots	0.08	0.036	26.09	0.52	$x_{3,12,7}$
Mechanical fixing	0.08	0.036	26.82	0.54	$x_{3,13,7}$
Sandwich panel					
Mechanical fixing	0.025	0.056	17.25	0.25	$x_{3,14,1}$
Mechanical fixing	0.035	0.056	18.76	0.38	$x_{3,14,2}$
Mechanical fixing	0.05	0.056	22.27	0.45	$x_{3,14,3}$
Wood chips					
Mechanical fixing	0.015	0.09	14.96	0.30	$x_{3,15,1}$
Mechanical fixing	0.025	0.09	16.83	0.34	$x_{3,15,2}$

	Mechanical fixing	0.035	0.09	18.26	0.37	$x_{3,15,3}$
	Mechanical fixing	0.05	0.09	21.00	0.42	$x_{3,15,4}$
4. Air gap						
	Light ventilated air gap	0.03	0.08	0	0	$x_{4,1,1}$
		0.05	0.09	0	0	$x_{4,1,2}$
		0.08	0.09	0	0	$x_{4,1,3}$
		0.10	0.09	0	0	$x_{4,1,4}$
	Not ventilated air gap	0.03	0.17	0	0	$x_{4,2,1}$
		0.05	0.18	0	0	$x_{4,2,2}$
		0.08	0.18	0	0	$x_{4,2,3}$
		0.10	0.18	0	0	$x_{4,2,4}$
	No air	0	-	0	0	$x_{4,3,1}$
5. External panel						
5a	**Solid brick**	0.115	0.85	25.46	1.28	$x_{5,1,1}$
	Concrete block	0.15	0.46	22.63	1.14	$x_{5,2,1}$
	Perforated brick *33x30x11*	0.11	0.35	20.53	1.03	$x_{5,3,1}$
5b	**Face brick**					
	24x11.5x5 waterproof	0.115	0.76	70.65	6.36	$x_{5,4,1}$
	24x11.3x5.2 clinker	0.115	0.76	79.44	7.15	$x_{5,5,1}$
	29x11.5x5 waterproof	0.115	0.76	72.79	6.55	$x_{5,6,1}$
	Pressed face brick 24x12x4	0.12	0.76	109.95	9.90	$x_{5,7,1}$
	Pressed face brick 24x12x5	0.12	0.76	103.81	9.34	$x_{5,8,1}$
6. External coating						
6a. External coating	**Regular plaster**	0.010	0.93	13.22	0.66	$x_{6,1,1}$
		0.013	0.93	14.46	0.72	$x_{6,1,2}$
		0.015	0.93	15.29	0.76	$x_{6,1,3}$
		0.018	0.93	16.52	0.83	$x_{6,1,4}$
		0.020	0.93	17.35	0.87	$x_{6,1,5}$
	Thermal plaster	0.010	0.67	24.41	1.22	$x_{6,2,1}$
		0.013	0.67	25.81	1.28	$x_{6,2,2}$
		0.015	0.67	27.22	1.35	$x_{6,2,3}$
		0.018	0.67	28.62	1.42	$x_{6,2,4}$
		0.020	0.67	30.02	1.50	$x_{6,2,5}$
	Metallic plate corten	0.015	0.58	121.65	6.27	$x_{6,3,1}$
	S355J0WP	0.02	0.58	130.16	6.71	$x_{6,3,2}$
	Limestone plate *(Spain)*	0.04	3.5	93.07	8.72	$x_{6,4,1}$
	Limestone plate *(Brasil)*	0.04	3.5	53.71	5.03	$x_{6,5,1}$
	Marble plate *(Spain)*	0.03	2.09	55.51€	5.20	$x_{6,6,1}$
	Marble plate *(Italy)*	0.03	2.09	102.33	9.59	$x_{6,7,1}$
	Composite plate	0.04	3.38	113.19	19.24	$x_{6,8,1}$
	Ceramic plate					
	exposed fixing 30x60	0.10	1.3	124.10	28.54	$x_{6,9,1}$
	hidden fixing 30x60	0.10	1.3	170.35	39.18	$x_{6,10,1}$
6b. Without external coating with 5b		0	-	0	0	$x_{6,11,1}$