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Sanchez Saez, F.; Sánchez Galdón, AI.; Villanueva López, JF.; Carlos Alberola, S.; Martorell Alsina, SS. (2018). Uncertainty analysis of a large break loss of coolant accident in a pressurized water reactor using non-parametric methods. Reliability Engineering & System Safety. 174:19-28. https://doi.org/10.1016/j.ress.2018.02.005



The final publication is available at

https://doi.org/10.1016/j.ress.2018.02.005

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Additional Information

Uncertainty analysis of a Large Break Loss of Coolant Accident in a Pressurized Water Reactor using non-parametric methods

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ABSTRACT

The safety analysis of nuclear power plant is moving towards a realistic approach in which the simulations performed using best estimate computer codes must be accompanied by an uncertainty analysis, known as the Best Estimate Plus Uncertainties approach. The most popular statistical method used in these analyses is the Wilks' method, which is based on the principle of order statistics for determining a certain coverage of the Figures-of-Merit with an appropriate degree of confidence. However, there exist other statistical techniques that could provide similar or even better results. This paper explores the performance of alternative non-parametric methods as compared to the Wilks' method of obtaining such Figure-of-Merits tolerance intervals. Three methods are investigated, i.e. Hutson and Beran-Hall methods and a bootstrap method. All the techniques have been used to perform the uncertainty analysis of a Large-Break Loss of Coolant Accident. The Figure-of-Merit of interest in this application is the maximum value reached by the Peaking Clad Temperature. In order to analyze the results obtained by the different methods, four performance metrics are proposed to measure the coverage, dispersion, conservativeness, and robustness of the tolerance intervals.

Keyword: Uncertainty; Best-Estimate plus uncertainty; deterministic safety analysis; Large Break Loss of Coolant Accident; Wilks' method; order statistics, non-parametric methods

NOTATION

 α Acceleration parameter in BC α method

B(n,p) Binomial distribution of parameters n and p

 C_j Coverage of the sample j

CC Conservativeness

 $F(\cdot)$ Cumulative distribution function (cdf)

 \hat{F}^{-1} Inverse of the empirical cdf

 $f(\cdot)$ Probability density function (pdf)

I[c] Indicator function. This function is equal to 1 if c is true and 0 if c is false

 $I_x(a,b)$ Incomplete beta function of parameters x,a and b

Z Reference distribution sample size
 n Sample size for estimating TL
 N Number of samples (repetitions)
 p Coverage of the tolerance interval

 $U_{n'p:n}$ Dirichlet process

 $X_{i:n}$ i^{th} order statistics from a sample of size n of independent and identically distributed

random variables

 z_0 Bias correction parameter in BC_{\alpha}[\gamma] method z_{γ} z score from the standard normal distribution Confidence level of the tolerance interval

 ξ_p p percentile

 $\Phi(\cdot)$ Standard normal cdf

ACRONYMS

 $BC_{\alpha}[\gamma]$ Bias Corrected Accelerated bootstrap method

BE Best Estimate

BEPU Best Estimate Plus Uncertainty
CD Coverage standard deviation

CHF Critical Heat Flux CM Coverage Mean

CV Coverage Coefficient of Variation
DSA Deterministic Safety Analysis
ECCS Emergency core cooling systems

FOM Figure of Merit FOS First Order Statistic

GAMA Working Group on Accident Management and Analysis

IAEA International Atomic Energy Agency
LBLOCA Large-Break Loss of Coolant Accident

LPIS Low-Pressure Injection System

NPP Nuclear Power Plant OS Order Statistic

PCT Peaking Clad Temperature

PIRT Process Identification and Ranking Tables

 $\begin{array}{lll} PWR & Pressurized \ Water \ Reactor \\ SV_{ref} & Reference \ Safety \ Value \\ TH & Thermal \ Hydraulic \\ TL & Tolerance \ limit \\ UA & Uncertainty \ analysis \end{array}$

TRACE TRAC-RELAP Advanced Computational Engine

SNAP Symbolic Nuclear Analysis Package

1. INTRODUCTION

The International Atomic Energy Agency's guidance (IAEA) on the use of deterministic safety analysis (DSA) for the design and licensing of nuclear power plants (NPPs) "Deterministic Safety Analysis for Nuclear Power Plants Specific Safety Guide, Specific Safety Guide No. SSG-2" [1] (hereinafter referred to as SSG-2) addresses four options for DSA applications. Due to the importance of taking the current understanding of physical phenomena into account, and thanks to the availability of reliable tools for more realistic safety analyses without compromising plant safety, many countries have chosen Option 3.

Option 3 involves the use of best-estimate codes and data together with an evaluation of the uncertainties, the so-called Best Estimated Plus Uncertainty (BEPU) methodologies. Table 1 shows the different options addressed in the SSG-2 guide.

Option	Computer code	Availability of systems	Initial and boundary conditions
1. Conservative	Conservative	Conservative assumptions	Conservative input data
2. Combined	Best Estimate	Conservative assumptions	Conservative input data
3. Best estimate	Best Estimate	Conservative assumptions	Realistic input data plus uncertainty; partly most unfavourable conditions ^a
4. Risk-informed	Best Estimate	Derived from probabilistic safety analysis	Best Realistic input data with uncertainties ^a

Table 1: SSG-2 DSA options

The IAEA Safety Report Series N°.23 "Accident Analysis for Nuclear Power Plants" [2] recommends a sensitivity and uncertainty analysis if Best Estimate (BE) codes are used in the licensing analysis. A comprehensive overview of uncertainty methods can be found in the IAEA Safety Report Series N°.52 "Best Estimate Safety Analysis for Nuclear Power Plants: Uncertainty Evaluation", issued in 2008 [3]. References [4,5,6,7] deal with the evolution of BEPU analysis and describe some of the most frequently used techniques. Some of these techniques have been developed by International programs which have discussed the BEPU approaches in order to address the issue of the capabilities of best-estimate computational tools and uncertainty analysis.

This is the case, for example, of the BEMUSE, promoted by the Working Group on Accident Management and Analysis (GAMA) of the OECD. These discussions have led to the development of BEPU approaches insofar as they have been accepted for performing deterministic safety analysis by the regulatory authorities. The scope of BEMUSE Phase V, in which fourteen participants from twelve organizations and ten countries participated, is the uncertainty analysis of a Large Break Loss-Of-Coolant-Accident (LBLOCA) in a Pressurized Water Reactor. The results and the main lessons learned from this BEMUSE program are presented in reference [8].

In a BEPU design-basis accident it is normally assumed that the uncertainty in the safety outputs [i.e., the figures of merit (FOMs) involved in the acceptance criteria of the analysis] derives from the uncertainties in the input parameters (initial and boundary conditions) and those arising from the computational model [4].

^a Realistic input data are used only if the uncertainties or their probabilistic distributions are known. For those parameters whose uncertainties are not quantifiable with a high level of confidence, conservative values should be used.

These FOMs are usually extreme values (minima, maxima) of safety variables during the transient, such as Peak Clad Temperature (PCT), Critical Heat Flux (CHF), etc. Current BEPU methodologies mainly rely on a probabilistic description of the uncertainty and on the use of statistical techniques to estimate it [9, 10, 11, 12, 13]. In this framework, the uncertainty of a FOM can be identified with its probability distribution.

Most BEPU approaches accepted by the regulatory authorities are based on the propagation of input uncertainties and make use of methods based on Wilks' formula, which is based on the principle of order statistics for determining a certain coverage of the Figures-Of-Merit (FOM) with a certain degree of confidence. The German Technical Safety Organisation (TSO) Gesellschaft für Anlagen- und Reaktorsicherheit (GRS) was the first to introduce Wilks' tolerance limits in uncertainty analyses with TH codes, so that this type of analysis is renowned as the GRS method [9]. This method determines the number of code runs needed to obtain a sample of outputs, i.e. FOMs, which are required to verify compliance with acceptance criteria. In accordance with current regulatory practice a 95% coverage with a 95% confidence level is required. So, if a one-side FOM tolerance interval is applied based on the use of the First Order Statistics (FOS) with a 95/95 coverage/confidence level a sample size of n=59 runs is required.

This paper focuses on the deterministic safety analysis of a Large-Break Loss of Coolant Accident (LBLOCA) scenario in a PWR NPP based on a BEPU approach and the use of order statistics according to the current practice for the formulation, propagation, and analysis of uncertainties. In addition, the paper introduces alternative non-parametric methods to the traditional first order statistics based on Wilks' formulae. The results of the alternative methods are compared with those of the traditional method based on appropriate performance metrics also proposed in this paper. The study specifically focuses on the analysis of the uncertainty associated with the maximum of the PCT (Peak Cladding Temperature) as the FOM.

2. OVERVIEW OF THE BEPU APPROACH

Figure 1 outlines a typical procedure used in BEPU approaches [14], which consist of the following twelve steps:

- 1. Selection of the accident scenario. Reactor system and transient selection.
- 2. Selection of the safety criteria linked to the accident scenario under study and the FOM involved in the acceptance criteria.
- 3. Identification and ranking of relevant physical phenomena based on the safety criteria.
- 4. Selection of the appropriate uncertain TH (Thermal Hydraulic) parameters to represent those phenomena.
- 5. Identification of relevant safety-related functions and systems involved in the accident scenario
- 6. Identify relevant trains and components of the safety-related functions and systems developing their possible redundancies.
- 7. Development of the TH computer model of the accident scenario, e.g. developing an input for the TRACE integrated into the SNAP platform [15, 16, 17].
- 8. Allocation of PDF (Probability Density Functions) for each selected uncertain TH parameter.
- 9. Establishing conservative assumptions on the availability of trains/components of safety systems.
- 10. Random sampling of the selected uncertain TH parameters according to PDF. Sample size (n) will depend on the particular statistical method and the acceptance criteria

- adopted to verify compliance with safety criteria. Perform n computer runs to obtain the value of the FOM for each run.
- 11. Processing the results of the multiple computer runs (n) to estimate either the probability distribution of the FOM, or rather some descriptor of this distribution, such as for example a percentile of the FOM, or a tolerance limit of FOM using OS, etc.
- 12. Verify compliance of acceptance criteria for each FOM depending on the particular statistical method and acceptance criteria adopted.

Despite SSG 2 recommends the development of uncertainty analysis based on the use of OS (normally first order to produce FOM with 95/95 tolerance limit (TL)), several alternatives can be explored integrating not only uncertainty but also sensitivity analyses to produce more realistic and accurate results while maintaining an acceptable computational cost. Some of these alternatives are based on the use of non-parametric methods.

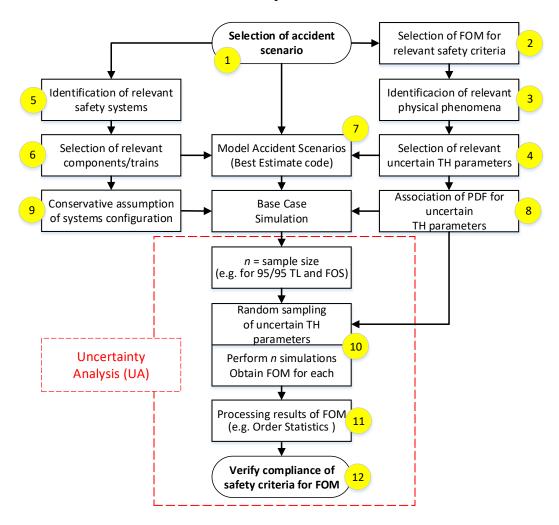


Figure 1. Overview of a typical BEPU approach.

3. UNCERTAINTY ANALYSIS BASED ON TOLERANCE INTERVALS

Since BEPU analyses have been performed by the Nuclear Regulatory Commission (NRC) [18], several uncertainty approaches have been proposed and applied in analyses approved by the nuclear industry [19]. The Wilks' method [20, 21] is the most popular non-parametric statistical method used in thermo-hydraulic codes uncertainty analysis [9]. It is based on the idea of order statistics and requires a minimum number of thermal-hydraulic code simulations

in order to infer a certain tolerance limit of a given output from the code simulations. The use of the Wilks' method and first order statistics mostly leads to conservative results. Another alternative to solve this problem consists of using higher order statistics, which can avoid overconservative results because of outliers.

Alternative non-parametric approaches have also been proposed in the literature with the aim of reducing the level of conservatism. These approaches are aimed at meeting a one-sided confidence limit for a percentile, which is also a one-side tolerance limit. Thus, Beran & Hall [22] and Hutson [23] proposed two approaches to estimate a non-parametric confidence interval for percentiles. The bootstrap method is also a data-based method that can be used to estimate a confidence interval for a parameter of interest, e.g. percentiles. These methods are outlined in the following subsections.

3.1. Wilks' method

The main advantage of using First Order Statistics based on Wilks' formulae to derive the 95/95 TL is that it provides a conservative result with only a few computer code runs. This way, the computational cost is kept reasonable, since the simulation of the evolution of the plant transient for each sample of inputs using complex TH (Thermal Hydraulic) NPP models is expensive in terms of computational cost. However, FOS often provides excessively conservative results. Several authors have explored the advantage of using order statistics of higher levels, which reduce the conservatism at the expense of a higher computational cost [24, 25]. Others propose the use of sensitivity and uncertainty analysis in an integrated manner within the framework of the BEPU approach [2, 8]. Non-parametric methods have been proposed with a similar purpose, such as the Bootstrap method introduced in reference [26].

Wilks' method uses the Order Statistics (OS) as tolerance limits. Let X be a (continuous) random variable with a cumulative distribution function (cdf), and $X_1, X_2, ..., X_n$ be a simple random sample of size n of X. Let $X_{1:n} \le X_{2:n} \le ... \le X_{r:n} \le ... \le X_{n:n}$ be the order statistics from the sample. According to Wilks' method [14, 15] the smallest number n of code runs to be performed to obtain the r-th order one-sided tolerance interval, or tolerance limit, of a given FOM is given by the following inequality:

$$1 - \sum_{k=n-r+1}^{n} {n \choose k} p^k (1-p)^{n-k} \ge \gamma \tag{1}$$

where n is the sample size, p is the coverage probability and γ is the confidence level of the one-sided tolerance interval. It is common practice in BEPU applications, for example to LBLOCA scenarios, to use the 95/95 tolerance limit (p/γ) according to current regulatory practice. When first order statistics (FOS) (i.e. r=1) is employed, it imposes the use of a sample size of 59 sets of TH input parameters randomly selected from their pdf and performing 59 code runs, one for each set.

However, in most cases using FOS leads to conservative results. Applying the Wilks' formula to higher order statistics usually produces more accurate FOM tolerance limits but increases the computational burden, as more runs are needed. For example, if r is 3 and p/γ are both 0.95/0.95, the minimum number of code runs needed to satisfy such criteria is n=124. Table 2 shows the minimum number of code runs from the 1st to 4th order statistics necessary to determine a 95/95 TL.

Table 2: Minimum number of code runs

Order Wilks' formula	Minimum number of code runs (n)
1	59
2	93
3	124
4	153

3.2. Beran and Hall's simple linear interpolation

Beran and Hall's simple linear interpolation provides an interesting approach to obtaining non-parametric confidence intervals for population percentiles based on a random sample of size n. They used a convex combination of sample percentiles to develop linearly interpolated confidence intervals with interpolation weights based on binomial distribution.

Let $n(p, \gamma)$ be the minimum sample size required for the calculation of a p/γ one-sided tolerance limit obtained from Eq. (1) for r=1, i.e. FOS. For a sample of size $n' \ge n(p, \gamma)$ let $X_{s:n}$ be the one-sided upper tolerance limit obtained using the Wilks' method with a confidence level $\gamma_l \ge \gamma$. Additionally, the confidence level of $X_{s-1:n}$ is $\gamma_2 < \gamma$. An estimation of $X(s^*)$ can be obtained by linear interpolation between $(\gamma_2, X_{s-1:n})$ and $(\gamma_l, X_{s:n})$. Thus, for a confidence level γ :

$$\gamma_1 = \Pr(\mathbf{B}(n, p) \le s - 1) = \Pr(\xi_p \in [-\infty, X_{s:n}]) \ge \gamma$$
(2)

$$\gamma_2 = \Pr(\mathbf{B}(n, p) \le s - 2) = \Pr(\xi_p \in [-\infty, X_{s-1:n}]) < \gamma$$
(3)

Where B is the binomial distribution of parameters n and p and ξ_p is the p percentile. Therefore, p/γ upper tolerance limit for any distribution is a γ upper confidence limit for the p percentile of the distribution.

Beran and Hall showed that using linear interpolation the value of the one-sided upper tolerance limit corresponding to a coverage p and nominal confidence γ is given by:

$$\lambda_2 \cdot X_{s:n} + (1 - \lambda_2) \cdot X_{s-1:n} \tag{4}$$

so that

$$\lambda_2 = \frac{\gamma - \Pr(B(n, p) \le s - 2)}{\Pr(B(n, p) = s - 1)} \tag{5}$$

3.3. Hutson fractional statistics

Hutson [23] proposes an approach to constructing non-parametric confidence intervals for percentiles based on fractional statistics. Thus, if $U_{n'p:n}$ is a Dirichlet process with $p \in [0,1]$ indexed by n'p where n'=n+1 then $U_{n'p:n}$ is the fractional uniform order statistic and $X_{n'p:n}$ is the fractional order statistic for the distribution $F_x(.)$. Stigler [27] outlines the properties of $U_{n'p:n}$:

- a) $U_{n'p:n}$ has a beta distribution with parameters n'p and n'(l-p),
- b) $E[U_{n'p:n}]=p$ and $Var[U_{n'p:n}]=p(1-p)/(n+2)$.
- c) The random variable $U_{n'p:n}$ has the same distribution as the uniform order statistic $U_{i:n}$ at n'p=i, i=1,2,...,n.
- d) The random variable $F^{-1}(U_{n'p:n})$ has the same distribution as the order statistic $X_{i:n}$ at n'p=i, i=1,2,...,n being F^{-1} the inverse distribution function.

The estimator of the *p*-percentile based on the fractional order statistic is defined as:

$$\hat{\xi}_p = X_{n'p:n} \tag{6}$$

In general, $X_{n'p:n}$ cannot be calculated from the sample since n'p need not be an integer. In reference [28] is shown that the distribution of the fractional order statistic is well approximated by the distribution of linear combinations of order statistics, under moderate regularity conditions. So, the distribution of the fractional order statistic $\xi_p = X_{n'p:n}$ can be approximated by the distribution of the linear interpolation estimator of the percentile function as

$$\hat{\xi}_{p} = (1 - \varepsilon) \cdot X_{\lfloor n'p \rfloor n} + \varepsilon \cdot X_{\lfloor n'p \rfloor + 1:n} \tag{7}$$

where $\varepsilon = n'p - \lfloor n'p \rfloor$, being $\lfloor \ \rfloor$ the floor function.

Hutson obtains the γ nonparametric confidence interval for the p percentile as $[-\infty, \xi_{p1}]$ where p_I is determined numerically by solving the equation

$$I_{p}[n' p_{1}, n'(1-p_{1})] = 1 - \gamma \tag{8}$$

where I_p is the incomplete beta function and ξ_{p1} is given by Eq. (7).

3.4. Bootstrap method

The bootstrap method is a data-based method in which statistical inference is obtained by data sampling. The bootstrap is a computationally intensive, nonparametric technique for assessing the accuracy of a parameter estimator which requires very few assumptions or analysis [27, 28, 29, 30]. The goal of bootstrap confidence interval theory is to calculate confidence limits for a parameter of interest θ from the bootstrap distribution of $\hat{\theta}$.

It is possible to obtain confidence bounds of percentiles from the bootstrap sampling distribution. Different methods have been proposed in the literature to obtain confidence bounds from the bootstrap, for example, percentile method, bias-corrected accelerated method, approximated bootstrap confidence interval, bootstrap-t method or a combination of Ensemble-Based Sensitivity Analysis and the Bootstrap method [26].

In this paper, the bias-corrected accelerated (BC $_{\alpha}[\gamma]$) method is used for calculating one-side upper 95/95 TL. This method has a number of advantages: 1) it corrects for bias and skewness in the distribution of bootstrap estimates, 2) no estimates of the standard deviation are required and 3) minimum coverage error.

The BC $_{\alpha}[\gamma]$ procedure [24] is a method of setting approximate confidence intervals for θ from the percentiles of the bootstrap histogram. Suppose θ is a parameter of interest, in this case, the p-percentile (ξ_p) , $\hat{\theta}(x)$ is an estimate of θ based on the observed data x and $\hat{\theta}^* = \hat{\theta}(x^*)$ is a bootstrap replication of $\hat{\theta}$ obtained by resampling x^* . Let $\hat{F}(c)$ be the empirical cumulative distribution function of B bootstrap replications $\hat{\theta}^*(b)$ with b=1,2,...,B, thus is:

$$\widehat{F}(c) = \#\{\widehat{\theta}^*(b) < c\}/B \tag{9}$$

The upper confidence interval limit $\hat{\theta}_{BC\alpha}[\gamma]$ of a one-side level- γ BC $_{\alpha}$ interval is defined as a function of \hat{F} , empirical cdf, and two parameters: The bias correction z_0 , which is related to the proportion of bootstrap estimates that are less than the observed statistic, and the acceleration α , proportional to the skewness of the bootstrap distribution. By definition, $\hat{\theta}_{BC\alpha}[\gamma]$ is the upper confidence interval limit, which is given by:

$$\hat{\theta}_{BC_{\alpha}[\gamma]} = \hat{F}^{-1}\Phi\left(z_0 + \frac{z_0 + z_{\gamma}}{1 - \alpha(z_0 + z_{\gamma})}\right)$$

$$\tag{10}$$

Where \hat{F}^{-1} is the inverse empirical cdf of the bootstrap sampling distribution of $\hat{\theta}(b)$, z_{γ} is the critical value of the normal standard distribution for a given γ thus is $z_{\gamma} = \Phi^{-1}(\gamma)$, and Φ is the standard normal cdf.

The bias-correction z_0 is estimated by

$$z_0 = \Phi^{-1}(\hat{F}(\hat{\theta})) \tag{11}$$

And so can be computed directly from the bootstrap distribution. The acceleration factor α can be approximated [28] by:

$$\alpha = \frac{SKEW \, \hat{l}_{\theta}(\hat{\theta})}{6} \tag{12}$$

SKEW $\hat{l}_{\theta}(\hat{\theta})$ being the skewness at parameter value $\theta = \hat{\theta}$ of the score statistic $\hat{l}_{\theta}(\hat{\theta})$. $\hat{l}_{\theta}(\hat{\theta})$ is the first derivate of the log-likelihood l for θ , thus $\hat{l}_{\theta}(\hat{\theta}) = \frac{\partial}{\partial \theta} \log \left\{ f_{\theta}(\hat{\theta}) \right\}$, where $f_{\theta}(\hat{\theta})$ is the probability density function, $\hat{\theta}$ is the sample estimate of θ and the likelihood is taken under bootstrap sampling.

3.5. Performance metrics

In nuclear deterministic safety analysis using a BEPU approach, the results of the uncertainty analysis must meet the regulatory requirements. The Wilks' method is the widest uncertainty technique used in this field, although has been proved to be over-conservative in some cases. It therefore seems necessary to find metrics able to evaluate the goodness of the results found by the alternative methods and compare them to the Wilks' method in terms of conservatism.

In this work, the metrics proposed in reference [31] have been adapted as follows. First, a reference sample of size Z, e.g. Z=1000, is generated using the cdf of the input parameters, i.e. Z sets of input values for the BE code are obtained. The BE code is then run for each set of the input values, so that Z simulations of the code output are obtained, which permits us to obtain Z FOM values, i.e. one for each simulation result. Let y_i be the i-th FOM value (i=1,2,...,Z), which is the result of the i-th BE code simulation.

Next, a size n is adopted from the set $n=\{59, 93, 124, 153\}$, depending on the OS of interest. For example, n=59 is considered to obtain the FOS. The process explained below can be repeated for each n value to cover all the OS of interest.

Then, from the complete set of Z FOM values, N samples of a given size n are extracted, which are analyzed with the different non-parametric methods described in Sections 3.1 to 3.4. The results of these analyses are estimations of 95/95 TL of the FOM according to each method and for each of the N samples of size n. From these FOM 95/95 TL results the following performance metrics can be obtained of interest for the comparison of the several non-parametric methods.

On one hand, the coverage reached under each method for each sample j=1,...,N of size n is calculated, which represents the percentage of the reference distribution covered by the j 95/95 tolerance limit (TL_j) estimated for each j=1,...,N. It can be evaluated using the following expression:

$$C_{j} = \frac{1}{Z} \left(\sum_{i=1}^{Z} I[(y_{i}) \le TL_{j})] \right)$$

$$\tag{13}$$

where y_i is the *i*-th FOM value, TL_j is the 95/95 tolerance limit estimated with some of the uncertainty methods for each *j* case, and I[x] is the indicator function, which equals 1 if *x* is true and 0 if *x* is false. With the C_j , the different descriptive measures are evaluated:

- 1) Coverage Mean (CM) of the individual coverages as a measure of coverage performance of the method.
- 2) Coverage Standard Deviation (CD), as a measure of variability of the results.
- 3) Coefficient of Variation (CV). It is a measure of the robustness of the result, which is evaluated as follows:

$$CV = \frac{CD}{CM} \cdot 100 \tag{14}$$

On the other hand, it is also possible to quantify the degree of conservativeness (CC) of each method, which represents the number of results out of the total of N samples that are conservative for each method. This can be evaluated as follows:

$$CC = \frac{1}{N} \left(\sum_{j=1}^{N} I[(TL_j) \ge SV_{ref}) \right) \cdot 100$$
(15)

where SV_{ref} is the reference safety value, which, for the purpose of this study, is the 95%-percentile of the distribution of the FOM, and can be estimated using the reference sample of size Z=1000 >> n.

4. CASE STUDY

The methods introduced in the previous sections are applied to the uncertainty analysis of an LBLOCA in the cold leg of a typical 4-loops PWR-Westinghouse reactor design, whose plant of reference is Zion NPP. When the break occurs the primary system rapidly depressurizes and activates the SCRAM signal due to low pressure. This leads to the safety injection through accumulators followed by actuation of the Low-Pressure Injection System (LPIS) to prevent core uncovery.

4.1. Safety variables and acceptance criteria

Following the methodology described in Fig. 1, it is necessary to determine the safety variables and their acceptance criteria to be considered in the study. Several safety criteria were established by USNRC for LOCA accidents in 1974 and last reviewed in 2015 [32]. These criteria can be mainly summarized into three requirements that must be in compliance: the peak cladding temperature, the maximum cladding oxidation of the core, and the maximum of the total amount of hydrogen produced during the transient. The safety variable usually followed in thermo-hydraulic safety analysis is the Peak Cladding Temperature (PCT), with a limiting maximum value of 1477 K. As this criterion was also used in this study, the PCT represents the safety variable of interest and the maximum value of the PCT, i.e. PCTmax is the FOM to be considered in the BEPU analysis.

4.2. Relevant physical phenomena, thermal hydraulics parameters, and PDF

Once the accident scenario (LBLOCA), the safety variable of interest (PCT), the corresponding FOM (PCTmax) and the acceptance criterion (PCTmax below 1477 K) have been selected, it is necessary to identify and rank the important phenomena and the most important TH parameters affecting the transient evolution and consequently also the value of the PCTmax reached.

A Process Identification and Ranking Tables (PIRT) process is often used to select the most important phenomena and parameters. Reference [6] contains a comparison of three PIRTs for LBLOCA in 4 loops PWR NPP, corresponding to three studies performed by CSAU [33], AREVA [19] and Westinghouse/EPRI [34]. In this comparison, common phenomena in the three studies are observed, such as heat transfer in single and two-phase flow, rod behavior, etc. However, significant differences were also found in the three studies. The latest studies on BEMUSE [4] and PREMIUM [35] have brought new findings, for example, on the re-flooding phenomena.

In the present study, the relevant phenomena and parameters identified in these PIRT studies and other parameters that have been incorporated into the new modeling capabilities introduced in the BE TRACE code V5.0 Patch 4 using the SNAP suite have been considered. Table 3 shows the 68 TH parameters selected, together with their nominal value and their uncertainty taken from the above references, the latter being introduced through a multiplicative or additive factor that acts as either multiplier or additive factor for the nominal values of the parameters.

The probability density functions that characterize each of these parameters are normal, uniform and lognormal distributions, as shown in Table 3, where Normal distributions are

truncated in ± 2 standard deviations. The two lognormal distributions have μ =0, σ =0.3536, with a shift to 0.5 and truncated to 2.

Table 3. Description of uncertain TH input parameters

Normal Distributed Parameters Nominal Value Factor Mín M				
Normai Distributeu Farameters	(units)	type	181111	Máx
Initial thermal power	3250 (MW)	Mult.	0.98	1.02
Peaking factor	1.2468 (-)	Mult.	0.95	1.05
Hot gap size average	5.4e-5 (m)	Mult.	0.8	1.03
Hot gap size hot rod	5.4e-5 (m)	Mult.	0.8	1.2
Residual power multiplier	1 (-)	Mult.	0.92	1.08
UO ₂ thermal conductivity	1 (-)	Mult.	0.9	1.1
UO ₂ specific heat	1 (-)	Mult.	0.98	1.02
Intact loops pump speed multiplier	1 (-)	Mult.	0.98	1.02
Break loop pump speed multiplier	1 (-)	Mult.	0.9	1.1
Accumulator pressure setpoint	4.14 (MPa)	Add.	-0.2	0.2
Accumulator liquid temperature	334 (K)	Add.	-10	10
LPI mass flow rate	88 (kg/s)	Mult.	0.95	1.05
Initial pressurizer pressure	15.5 (MPa)	Add.	-0.1	0.1
Initial pressurizer level	8.8 (m)	Add.	-0.1	0.1
Initial total primary mass flow rate	17357 (kg/s)	Mult.	0.96	1.04
Initial cold legs average temperature	565 (K)	Add.	-2	2
Lognormal Distributed Parameters	303 (11)	Tiud.		
Accumulator loss coeff.	1 (-)	Mult.	0.5	2
Surge line coss coeff.	1 (-)	Mult.	0.5	2
Uniform Distributed Parameters	1 ()	TVIGIT.	0.5	
Core loss coeff.	0.51 (-)	Mult.	0.95	1.05
Bypass loss coeff.	13.575 (-)	Mult.	0.95	1.05
Downcomer loss coeff.	0.54 (-)	Mult.	0.95	1.05
Low core plate loss coeff.	0.2 (-)	Mult.	0.95	1.05
Break cold leg loss Coeff.	0.68 (-)	Mult.	0.95	1.05
Vessel liquid wall friction multiplier	1 (-)	Mult.	0.95	1.05
Wall friction multiplier	1 (-)	Mult.	0.95	1.05
Liquid choke flow coeff.	1 (-)	Mult.	0.95	1.05
Vapour/two-phase choke flow coeff.	1 (-)	Mult.	0.95	1.05
Gas-gap heat transfer Coeff.	6300 (W/m ² K)	Mult.	0.9	1.1
gap conductance coeff.	1 (-)	Mult.	0.9	1.1
Clad thermal conductivity	1 (-)	Mult.	0.9	1.1
Clad density	6551.4 (kg/m ³)	Mult.	0.95	1.05
Maximum containment pressure	3.5e5 (Pa)	Mult.	0.85	1.15
Single phase liquid to wall heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Nucleate boiling heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Single phase vapor to wall heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Dispersed flow film boiling heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Film to transition boiling Tmin criterion	1 (-)	Mult.	0.9	1.1
Transition boiling heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Critical Heat Flux (CHF) multiplier	1 (-)	Mult.	0.8	1.2
Form loss coeff.	1 (-)	Mult.	0.9	1.1
Interfacial drag (bubbly) coeff.	1 (-)	Mult.	0.9	1.1
Interfacial drag (churn) coeff.	1 (-)	Mult.	0.9	1.1
Interfacial drag (annular) coeff.	1 (-)	Mult.	0.9	1.1
Interfacial drag (droplet) coeff.	+ '	Mult.	+	

Interfacial drag (dispersed flow film boiling) coeff.	1 (-)	Mult.	0.9	1.1
Interfacial drag (inverted slug flow) coeff.	1 (-)	Mult.	0.9	1.1
Interfacial drag (inverted annular flow) coeff.	1 (-)	Mult.	0.9	1.1
Wallis c for counter current flow limitation	0.8625 (-)	Mult.	0.9	1.1
Accumulator liquid volume	10.564 (m ³)	Add.	-0.5	0.5
Accumulator gas volume	13.583 (m ³)	Add.	-0.8	0.8
Vapour to wall inverted annular heat transfer	1 (-)	Mult.	0.9	1.1
coeff.				
Liquid to wall inverted annular heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Initial upper head temperature	570 (K)	Add.	0	10
Liquid to interface bubbly-slug heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Liquid to interface annular-mist heat transfer	1 (-)	Mult.	0.9	1.1
coeff.				
Liquid to interface transition heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Liquid to interface stratified heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Vapour to interface bubbly-slug heat transfer coeff.	1 (-)	Mult.	0.9	1.1
Vapour to interface annular-mist heat transfer coeff.	1(-)	Mult.	0.9	1.1
Vapour to interface transition heat transfer coeff.	1(-)	Mult.	0.9	1.1
Vapour to interface stratified heat transfer coeff.	1(-)	Mult.	0.9	1.1
Sub-cooled boiling heat transfer coeff.	1(-)	Mult.	0.9	1.1
Departure from nucleate boiling/CHF	1(-)	Mult.	0.9	1.1
Fuel thermal conductivity before burst coeff.	1(-)	Mult.	0.9	1.1
Cladding metal-water reaction rate coeff.	1(-)	Mult.	0.9	1.1
Rod internal pressure coeff.	1(-)	Mult.	0.9	1.1
Burst temperature coeff.	1(-)	Mult.	0.9	1.1
Burst strain coeff.	1(-)	Mult.	0.9	1.1

4.3. Identification of relevant safety-related systems, trains, and components. Assumptions of systems configuration

The safety-related systems involved in the accidental transient are the reactor protection system, accumulators, and low-pressure injection systems. There are four accumulators and four LPIS injections. Thus, when the break occurs reactor scram is produced and the reactor coolant pumps stop. The pressure inside the reactor coolant system rapidly falls and, when it reaches 4.14 MPa, the injection begins through the accumulators. The pressure cannot be recovered and when it reaches 1.42 MPa LPIS injection starts.

The conditions imposed for the thermal hydraulic transient simulation related to safety systems were extracted from the BEMUSE project [4] and are summarized as follows:

- No actuation of the high-pressure injection system (HPIS).
- Injection from accumulators at 4.14 MPa.
- Low-pressure injection system (LPIS) initiate at 1.42 MPa.
- Containment pressure imposed as a function of time after the break.
- Reactor coolant pumps velocity imposed as a function of time after the break.
- Power after scram imposed by means of a multiplier as a function of time after the break.

4.4. TRACE model of the 4-loops PWR-Westinghouse

Figure 2 outlines the primary system modeled for TRACE V5.0 Patch 4 using the SNAP suite, which includes a three-dimensional component type VESSEL, which represents the reactor pressure vessel including the core.

The reactor pressure vessel (RPV) consists of 31 axial, 5 radial and 8 azimuthal nodes. The tridimensional vessel permits represent peak clad temperature in a more realistic way. The core is composed of the cells comprised to 6 to 23 axial nodes, 1 to 3 radial nodes, and for all 8 azimuthal nodes.

It also includes the four cooling loops (PIPES, 4 SGs, PRZ and 4 PUMPS). The safety systems involved in the accidental sequence are also modeled. The accumulators are modeled using PIPE and VALVE components in each loop, e.g. PIPE-190 and VALVE 194 in loop one. The LPIS injections are simulated as boundary conditions using a FILL component in each loop, e.g. FIL-192 in loop1. No safety injections were modeled in the broken loop, as it is considered that the coolant injected is completely lost through the break. Finally, the break is simulated as a double guillotine using three VALVE (VALVE 505, VALVE 507 and VALVE 213) and two BREAK (BREAK 500 and BREAK 501) components in the cold leg of Loop 2.

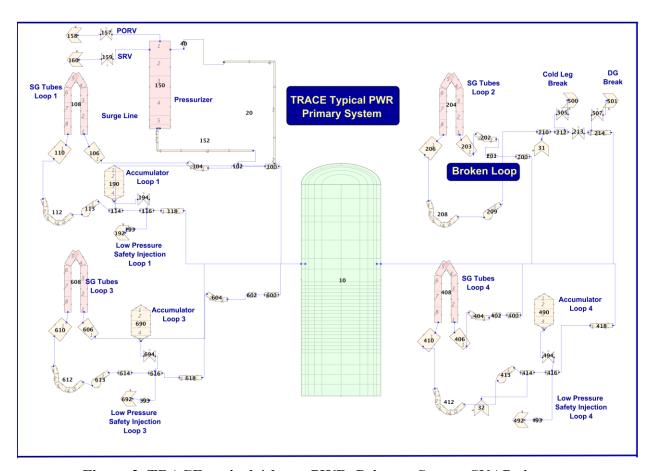


Figure 2. TRACE typical 4-loops PWR. Primary System SNAP view.

5. RESULTS OF THE UNCERTAINTY ANALYSIS

This section summarizes the results of the uncertainty analysis of the BEPU approach highlighted in Figure 1 (Section 2). The uncertainty analysis was carried out using the methods introduced in Section 3 and the model and data provided in Section 4. The results of the alternative methods are compared by means of the performance metrics proposed in Section 2.

5.1. PCT evolution for the base case

The LBLOCA transient was simulated using the SNAP-TRACE code model with the input parameters at their nominal value, which results in the PCT evolution shown in Figure 3. This simulation of the Base Case (see Figure 1) reached a PCTmax of 1221.1 K.

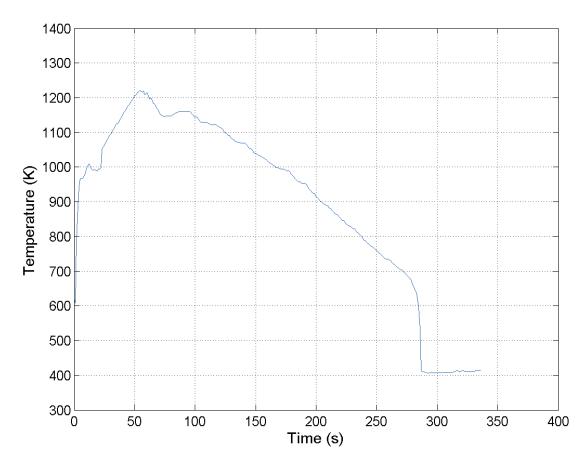


Figure 3. TRACE result for the PCT evolution with nominal input parameters

5.2. 95-percentile of the PCTmax as a reference value

To obtain the 95-percentile of the PCTmax, Z = 1000 simulations of the transient were performed using the SNAP-TRACE code, varying the nominal values of the parameters given in Table 3 according to their probability distributions and ranges of variation. After all the simulations, the set of PCT evolutions depicted in Figure 4 were obtained. It can be seen that the PCT evolutions have similar shapes and different widths and lengths.

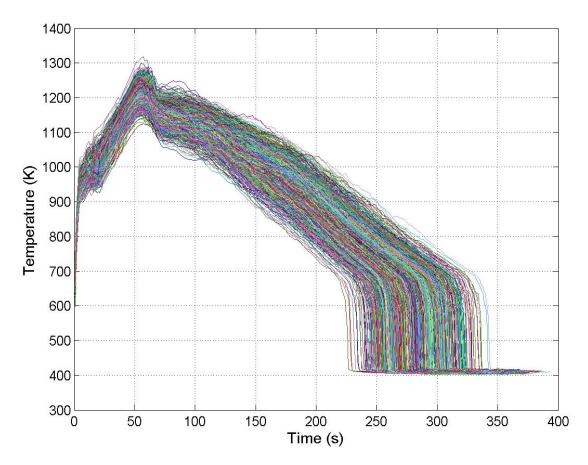


Figure 4. TRACE results for the PCT evolution of one thousand simulations

These simulations are intended to be used for the purpose of estimating the density function of the PCTmax empirically. Thus, Figure 5 shows the empirical probability density function (pdf) and the histogram obtained for PCTmax from the 1000 simulations. In order to compute the performance metrics in the comparison of the different uncertainty analysis methods, the 95-percentile of the pdf is considered to be the SV_{ref} in Eq. (15), which has a value of SV_{ref}=1259.5 K.

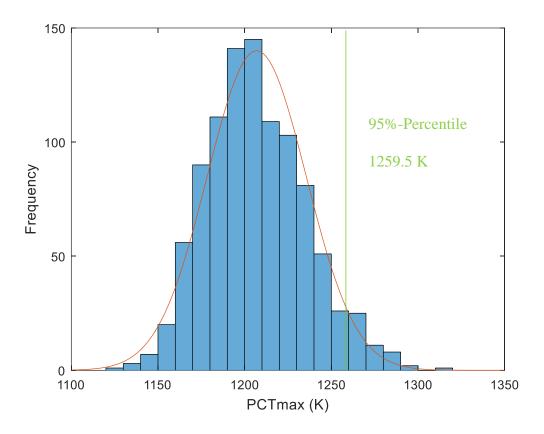


Figure 5. Histogram and probability density function (pdf) obtained from 1000 simulations of TRACE code for PCTmax.

5.3. Uncertainty analysis: Wilks versus other non-parametric methods

This section gives the results of the estimation of the PCTmax 95/95 TL by the different methods described in Section 3, i.e. Wilks (W), Beran and Hall (BH), Hutson (HU) and biascorrected accelerated bootstrap (BO) methods. The estimation was performed adopting the first and higher order statistics (2, 3 and 4 order), which correspond with sample sizes $n = \{59, 93, 124, 153\}$, respectively.

In obtaining each particular PCTmax 95/95 TL (identified as TL_j in Section 3.5) a subset of n samples of PCTmax values are randomly extracted from the Z = 1000 simulations carried out and the TL_j is determined for each non-parametric method from the fundamentals introduced in Sections 3.1 to 3.4. This process is repeated N = 999 times, so that a sample of TL_j values is obtained, j=1,N, for each method, which permits the distribution of the PCTmax TL_j to be estimated according to each method. This distribution will provide the performance measures for each method (see Section 5.4). This process is repeated for the different sizes of $n = \{59, 93, 124, 153\}$.

Table 4 shows the mean, standard deviation, minimum and maximum value of the PCTmax TL estimation obtained using the Wilks, Beran-Hall, Huston and Bootstrapped methods for different samples sizes, n. Thus, the FOS (n=59) with the Wilks' method results in a mean PCTmax TL value of 1280.97 K. Also given is the dispersion of the PCTmax TL values among the different repetitions, which can be confirmed by the standard deviation of 14.62 K. The alternative methods based on fractional order statistics provide results similar to Wilks for the

same sample size (*n*=59), since the mean values for Beran-Hall and Hutson are 1280.98 K and 1280.62 K, respectively, and their dispersions are practically the same as Wilks', since Beran-Hall gives a standard deviation of 14.5 K and Hutson 14.38 K. The Bootstrap method gives a slightly lower mean value than the others (1277.38 K) and also lower dispersion, with a standard deviation of 13.29 K.

Figure 6 represents the Box-Whisker plot of the PCTmax TL distribution for several n values. The figure shows a positive asymmetry in the PCTmax TL distribution obtained using the different non-parametric methods with small sample sizes (n=59). This asymmetry decreases as the sample size increases. It can be seen that the most accurate estimates are obtained by increasing the sample size. It also confirms the very conservative estimation of the Wilks method for a sample size of 59.

For sample size n=93 the results were similar to those for n=59, with the lowest mean and standard deviation values for the Bootstrap method, with values of 1272.78 K and 8.7 K. respectively. The other methods provide similar results, but slightly higher than the Bootstrap values.

For higher sample sizes, (n=124 and 153) the results tend to become similar for all the uncertainty analysis methods. The Hutson and Bootstrap results have lower dispersion than Wilks and Beran-Hall. For example, for n=153, Bootstrap has a lower standard deviation value (6.19 K) than Hutson (6.53 K), Beran-Hall (6.67 K) and Wilks (7.28 K). The mean values of all four methods are practically identical.

Table 4. PCTmax 95/95 TL estimation results (in K).

Method	Mean	Std Dev	Minimum	Maximum
Wilks n=59	1280.97	14.62	1241.6	1318.9
Wilks n=93	1274.06	9.13	1243.6	1300.0
Wilks $n=124$	1271.78	7.71	1249.2	1294.7
Wilks n=153	1271.33	7.28	1248.1	1294.7
Beran-Hall n=59	1280.98	14.50	1242.5	1318.7
Beran-Hall n=93	1273.88	9.10	1249.6	1300
Beran-Hall n=124	1271.28	7.27	1249.4	1294.7
Beran-Hall n=153	1270.30	6.67	1250.0	1287.6
Hutson n=59	1280.62	14.38	1241.1	1318.6
Hutson n=93	1274.19	9.19	1242.6	1299.6
Hutson n=124	1271.24	7.23	1249.4	1294.6
Hutson n=153	1270.36	6.53	1255.4	1288.2
Bootstrap n=59	1277.38	13.29	1241.9	1318.9
Bootstrap n=93	1272.78	8.70	1246.2	1299.2
Bootstrap n=124	1271.04	7.26	1248.1	1291.8
Bootstrap n=153	1269.42	6.19	1248.5	1288.0

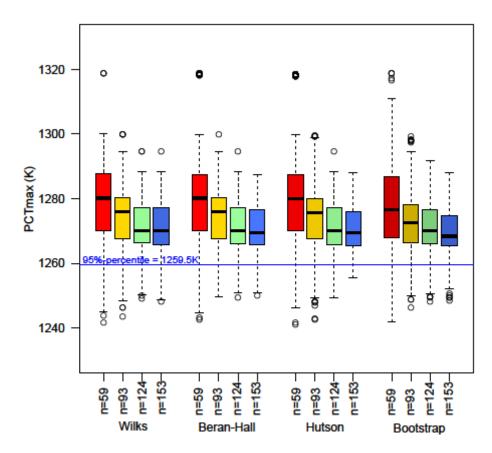


Figure 6. Box-Whisker plot for PCTmax (in K) 95/95 TL using different uncertainty analysis methods

5.4. Performance metrics

The PCTmax 95/95 TL_j distributions of each non-parametric method given above are used here to obtain the performance measures for each method following the procedure introduced in Section 3.5. This procedure is carried out for several sizes of $n = \{59, 93, 124, 153\}$ to permit the comparison of the results provided by the four uncertainty methods according to the order statistic adopted.

Table 5 shows the coverage mean (CM), standard deviation coverage (CD), the coefficient of variation (CV) and conservativeness (CC) estimated for the Wilks, Beran-Hall, Hutson and bootstrap methods for different sample sizes. The results in this table show that the four methods provide good CM values of around 97-98%. In terms of CD, which measures dispersion, the values obtained by all methods are also quite similar, but for higher n sizes (n=124 and 153) the Bootstrap and Hutson methods gave better results than Beran-Hall and Wilks. However, for the lowest n sample size (n=59) the Bootstrap method had the worst result in terms of dispersion, i.e. CD value. The Wilks' method obtained the worst results in terms of dispersion for n=124 and n=153. However, Beran-Hall and Wilks are more robust than Bootstrap and Hutson for lower n sizes (59, 93), based on the CV metric, while the robustness of the Bootstrap and Hutson methods is better than the Beran-Hall and Wilks methods for higher n sizes (124, 153), based on the same CV metric. Regarding the CC metric, Hutson and Bootstrap provide values no higher than 95% for n=59 and n=93, while Wilks and Beran-Hall provide CC values higher than 95% for all the n sample sizes. In general, both the Beran and Hall and the Wilks' methods are conservative for any sample size n, the former being less conservative than Wilks.

Table 5. Performance metrics for the PCTmax 95/95 TL estimation

Method	Coverage	Coverage	Coeff. of	Conservative-
	Mean (CM)	Std Dev (CD)	Var. (CV)	ness (CC)
Wilks n=59	98.47	1.55	1.57	96.6
Wilks n=93	97.94	1.42	1.45	96.4
Wilks n=124	97.67	1.32	1.35	96.2
Wilks n=153	97.62	1.30	1.33	96.5
Beran-Hall n=59	98.43	1.55	1.57	95.7
Beran-Hall n=93	97.91	1.43	1.46	95.3
Beran-Hall n=124	97.61	1.26	1.29	96.3
Beran-Hall n=153	97.47	1.23	1.27	95.3
Hutson n=59	98.37	1.59	1.62	95.0
Hutson n=93	97.89	1.50	1.53	94.6
Hutson n=124	97.57	1.25	1.29	96.2
Hutson n=153	97.47	1.16	1.19	96.5
Bootstrap n=59	98.10	1.70	1.73	93.0
Bootstrap n=93	97.73	1.41	1.45	94.6
Bootstrap n=124	97.58	1.30	1.34	95.2
Bootstrap n=153	97.32	1.17	1.20	95.6

6. CONCLUDING REMARKS

The main advantage of using FOS based on Wilks' formulae is that it usually provides a conservative result with a small number of computer code runs. Although the Wilks' method is mainly used in this kind of analysis, other non-parametric methods could also be used to estimate the 95/95 TL. The study presented in this paper compares the use and performance of alternative non-parametric methods in obtaining FOM tolerance intervals within a BEPU framework, based on 1st to 4th order statistics. Three methods are investigated, i.e. Hutson and Beran-Hall methods in the context of fractional statistics and a Bootstrap method. These methods are applied to estimate the PCTmax 95/95 TL within the uncertainty analysis of a Large-Break Loss of Coolant Accident in the cold leg of a Pressurized Water Reactor, using the thermal-hydraulic code TRACE to perform the simulations. As expected, in all the methods the higher (2nd to 4th) order statistics usually produced less conservative and more accurate results at the expense of additional code runs.

Four performance metrics were proposed to evaluate the goodness of the results. From the case study it was found that for lower n sizes (n=59 and 93) the Bootstrap method provided the best results in terms of coverage mean (CM). However, it had the largest coverage standard deviation (CD) values, which gave the worst scores and thus affected robustness (CV). Bootstrap also provided conservativeness (CC) values lower than 95%. According to the performance metrics obtained for these sizes, the best results were thus found by the Beran-Hall and Wilks methods, which provided similar results for robustness and both obtained very good CC values above 95%. However, for higher n sizes (n=124 and 153), the Bootstrap method provided the best results. The Beran-Hall method gave very similar results to those obtained by the Bootstrap method for n=124. Thus, in general, the performance of the methods improves with sample size as all four methods performed in a very similar way for the largest n sizes. The main differences were found for the lowest n sizes, with which the Beran-Hall method performed best. Even so, with the exception of the Bootstrap method, one could conclude that there are no significant differences between the methods.

This kind of analysis should be extended to other transients considered important in nuclear safety, in which the figures of merit may change from the one proposed in this paper. Moreover, it could be interesting to analyze if the consideration of one figure of merit is sufficient to quantify the transient uncertainty or more variables should be considered in the uncertainty analysis.

ACKNOWLEDGEMENTS

This work was carried out partially with the support of the *Programa de Apoyo a la Investigación y Desarrollo of the Universitat Politècnica de València* (PAID UPV).

The authors are also grateful to the Spanish *Consejo de Seguridad Nuclear* for the financial support they provided for Research Projects SIN/4078/2013/640 (MASA Project) and STN/4524/2015/640 (CAMP Project).

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