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Tables for cumulative probability of the ex-Gaussian distribution

Tablas de probabilidad acumulada para la distribución ex-Gaussiana

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C. Moret-Tatay[1*], D. Gamermann[2] and P. Fernández de Córdoba[3]

[1] Facultad de Psicología, Magisterio y Ciencias de la Educación. Universidad Católica de Valencia San Vicente Mártir.
[2] Departamento de Física. Universidade Federal do Rio Grande do Sul.
[3] Departamento de Matemática Aplicada. Universitat Politécnica de València.
[*] Correspondencia: Universidad Católica de Valencia San Vicente Mártir. Facultad de Psicología, Magisterio y Ciencias de la Educación. Calle Guillem de Castro, 175. 46008 Valencia. España. *E-mail*: mariacarmen.moret@ucv.es

ABSTRACT

The ex-Gaussian distribution provides a good fit to multiple empirical positive skewed distributions, such as reaction time. This fit is a useful tool when estimating confidence intervals and determining hypothesis testing for skewed variables. In order to create tables for significance testing, the ex-Gaussian distribution was parametrize in terms of its average, standard deviation and a new skewness parameter $\lambda = \sqrt[3]{(t/2)}$. To this end, x was standardized in order to normalize the ex-Gaussian distribution for average of 0 and standard deviation of 1. In this way, the distribution has only one free parameter ($0 \leq \lambda < 1$).

KEYWORDS: *ex-Gaussian distribution*, *significance testing*, *statistical tables*.

RESUMEN

La distribución ex-Gaussiana proporciona un buen ajuste a múltiples distribuciones asimétricas positivas, como el tiempo de reacción. Este ajuste es una herramienta útil para estimar los intervalos de confianza y determinar las pruebas de hipótesis para las variables asimétricas. A los efectos de desarrollar tablas para la prueba de significación, la distribución ex-Gaussiana fue parametrizada en términos promedio, desviación estándar y un nuevo parámetro de asimetría $\lambda = \sqrt[3]{(t/2)}$. Para este fin, x se estandarizó, tratando de normalizar la distribución ex-Gaussiana para un promedio 0 y desviación estándar 1. De este modo, la distribución presenta un solo un parámetro libre ($0 \leq \lambda < 1$).

PALABRAS CLAVE: *distribución ex-Gaussiana*, *prueba de significación*, *tablas estadísticas*.

The ex-Gaussian fit has turned into one of the preferable options when dealing with positive skewed distributions in behavioral sciences (Gu, Gau, Tzang y Hsu, 2013; Luce, 1986; Moreno-Cid et al., 2015; Moret-Tatay et al., 2017). The characterization of response latencies to an ex-Gaussian fit, generally is employed to examine the positively skewed shapes of most Reaction time distributions and it can be quantified using three parameters: μ (the mean of the normal component), σ (the standard deviation of the normal component), and τ (both the mean and standard deviation of the exponential component). This technique provides a good fit to multiple empirical data, such as reaction times. In this work, we present statistical tables for cumulative probability of the ex-Gaussian distribution.

In order to create tables for the ex-Gaussian distribution, a parametrization was carried out in terms of average M, standard deviation S and a new skewness parameter $\lambda = \sqrt[3]{(t/2)}$. Finally, the point x was standardized ($z = (x - M) / S$) in order to normalize the ex-gaussian distribution for average 0 and standard deviation 1 (in terms only of its asymmetry):

$$f_\lambda(z) = \frac{1}{2\lambda} e^{\frac{1}{2\lambda^2}(-2z\lambda-3\lambda^2+1)} \operatorname{erfc}\left(\frac{-z+\frac{1}{\lambda}-2\lambda}{\sqrt{2}\sqrt{1-\lambda^2}}\right).$$

METHOD

The calculations were performed in a computer running under linux operational system with a processor icore7 (four kernels) and 6Gb of RAM memory. The integrations were performed by a python script programmed by the authors. The script used a slightly modified gaussian quadrature method: the whole integration interval was divided in 1000 smaller intervals and the integral over each one of these intervals was performed via the 20 point gaussian quadrature method. In the numerical integral the minus infinity was substituted by the value –8 (8 standard deviations from the average). At this point (8), for any value of λ the value of the ex-Gaussian function is smaller than the floating point precision of the computer (meaning that -8 is as good as minus infinity for practical numerical integration).

RESULTS

How to read the tables

The tables present the area of the left tail for a given z point for an ex-Gaussian with asymmetry given by the λ parameter. In order to use the tables, one should first know the three parameters deffining a particular ex-Gaussian probability density. There are two different sets of three parameters that one must know: the ex-Gaussian components parameters (μ, σ and τ) and the statistical parameters (M, S and t or λ). The parameter λ (assymmetry) indicates which table one should consult and knowing the point x one is interested in, one calculates the table point z he must consult via $z = (x - M) / S$.

The point z should be looked for in the tables first rows and columns, in the following WAY: the point z is the result of summing up the first number in a column with the first number in a row. For example, if one is interested in the point −1.78, the first row should indicate the number −1.8 and the first column 0.02. For example, considering the following extract from the table:

z	0.00	⋯	0.04	⋯ 0.09	
-2.0	0.0000	⋯	0.0000	⋯	0.0000
⋮	⋮	⋱	⋮	⋱	⋮
-1.1	0.0308	⋯	0.0419	⋯	0.0593
⋮	⋮	⋱	⋮	⋱	⋮
0.6	0.7964	⋯	0.8045	⋯	0.8143
⋮	⋮	⋱	⋮	⋱	⋮

It is possible to read that for z = −1.06 one has F(z) = 0.0419 while for z = 0.64, F(z) = 0.8045. This means that for the particular value of λ in this table, 4.19% of the distribution is below the point z = −1.06 and 80.45 % is below the point z = 0.64. Tables are attached in the following Appendix: http://meblaboratory.blogspot.it/p/response-latency-components.html.

CONCLUSIONS

The ex-Gaussian fit has turned into one of the preferable options when dealing with positive skewed distributions. This technique provides a good fit to multiple empirical data, such as reaction times (a popular variable in Psychology due to its sensibility to underlying cognitive processes). Thus, in this work we present statistical tables for cumulative probability of the ex-Gaussian distribution This work provides tables for significance testing with an ex-Gaussian distribution, which were currently not available in the literature. Their potential advantage are related to significance testing in terms of determining the Type I error %.

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